Does the usage of an online EFL workbook conform to Benford’s law?

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Abstract. The aim of this paper is to explore if English as a Foreign Language (EFL) learners’ usage of an online workbook follows Benford’s law, which predicts the frequency of leading digits in numbers describing natural phenomena. According to Benford (1938), one can predict the frequency distribution of leading digits in numbers describing natural datasets, e.g. river lengths. In such numbers, the digit 1 occurs most frequently, while the digit 9 occurs least-frequently. This counter-intuitive phenomenon attracted the attention of researchers seeking inconsistencies in data, e.g. false tax claims (Miller, 2015). We show that the practical application of Benford’s law could extend to detecting abnormal learner behaviour in online EFL products. First, we show that the distributions of leading digits of the number of online activities submitted by EFL learners on an e-learning platform and the time spent on those activities do indeed follow Benford’s law. Then, we show that some learners whose behaviour does not conform to Benford’s law show online behaviour that is abnormal relative to their peers – in particular, they submit many activities in a few days, which could suggest, for example, poor time management.

Keywords: Benford’s law, EFL, e-learning, time on task.

1. Introduction

Benford (1938) stated that it is possible to predict the frequency distribution of leading digits in numbers composed of four or more digits describing such natural
datasets as river lengths or city populations. In such numbers, the digit 1 is expected to be the most frequently occurring leading digit (about 30% of cases), while the digit 9 is expected to occur least-frequently (fewer than about 5% of cases), even though the chance of occurrence is intuitively expected to be the same for all leading digits. In recent years, Benford’s law attracted the attention of researchers because of its practical use, e.g. identifying tax or vote frauds (Miller, 2015).

In education, Benford’s law has been applied to evaluating the chance of picking the correct answer among distractors on a multiple-choice test (Slepkov, Ironside, & DiBattista, 2015). We know of no previous work exploring the application of Benford’s law to e-learning of EFL, hence the present study.

2. Method

2.1. Data

According to Nigrini (2012, pp. 21-22), numbers in a dataset are expected to conform to Benford’s law if they describe natural events or facts such as city populations (rather than, say, computer-generated bank account numbers) and if the dataset has no inherent limit (which excludes, say, exam scores).

We focused on the number of online learning activities completed by EFL learners and the time spent on those activities. The data comes from MyEnglishLab for Speakout Pre-intermediate 1st edition (henceforth ‘MyEnglishLab’), an e-learning platform with exercises accompanying a textbook. The online activities comprise twelve units, each of which contain about thirty activities. The platform is aimed at institutions, so most learners analysed here were enrolled in a course set up by their teacher or instructor. The anonymised dataset contains 3,218,624 first attempts of MyEnglishLab activities from 35,265 learners from 18 different countries (speaking 12 different languages).

2.2. Analysis

To see if the number of MyEnglishLab activities completed by learners conforms to Benford’s law, we counted the total number of activities submitted (i.e. attempted) daily by each learner. Days with no learner activity were not included. Resubmitting the same activity did not increase the count. For example: if learner A submits 11 activities on Monday and three activities on Tuesday, and Learner B submits four
activities on Tuesday and six on Wednesday, the dataset contains the observations \{11, 3, 4, 6\}. The frequency distribution of the leading digits of these measurements was plotted and compared with the expected trend according to Benford’s law.

To see if time spent on those activities conforms to Benford’s law, we listed the time (in seconds) that every learner spent on every first submission of a MyEnglishLab activity. Again, the frequency distribution of the leading digits was plotted and compared with Benford’s distribution. We ran a Pearson’s Chi-squared Goodness-of-Fit test, which is one of several tests used to evaluate if a dataset conforms to Benford’s law. Of several such tests available in the BenfordTests R package (Version 1.2.0; Joenssen, 2015), this one was the fastest. Data processing and visualisation were performed in R (Version 3.2.4; R Core Team, 2016) running in RStudio (Version 0.99.893; RStudio, 2016).

### 3. Discussion

A visual inspection of Figure 1 shows that the distribution of leading digits of the number of activities submitted daily per learner on MyEnglishLab follows the Benford’s law curve closely, with the exception of the digit 1. This means there were more cases of learners submitting either one or between 11 and 19 activities per day than predicted by Benford’s law. Despite this, it could be stated that the number of submitted activities (roughly) conforms to Benford’s law.

Figure 1. Distribution of leading digits of the number of submitted activities compared to Benford’s distribution

A visual inspection of Figure 2 shows that the distribution of leading digits of time spent on single activities submitted on MyEnglishLab also closely follows Benford’s
distribution. Although the digit 1 is an exception again, the fit is better. A similar result was observed for the first two leading digits of time (not shown in this figure).

Figure 2. Distribution of leading digits of time spent on MyEnglishLab activities compared to Benford’s distribution

![Figure 2](image)

Figure 3 shows learners whose behaviour does not conform to Benford’s law. Each thin line represents a learner. Pearson’s Chi-squared Goodness-of-Fit test showed that of 12,427 learners who submitted at least 100 MyEnglishLab activities, time on task follows Benford’s law for 74% of learners and does not follow Benford’s law for 26% of learners ($\alpha = 0.05$).

Figure 3. Distribution of leading digits of time spent on MyEnglishLab activities by learners whose behaviour does not conform to Benford’s law

![Figure 3](image)

While exploring backend logs of learner interactions with MyEnglishLab, we noticed that some of the 26% of learners whose behaviour does not conform to
Benford’s law share three characteristics. First, even if they were enrolled in a course that lasted a couple of months, they used the platform to submit exercises only for a couple of days. Second, on those few days of activity, the learners submitted an unusually high number of activities, often receiving high scores. Third, learners seemed to have worked with these activities simultaneously, i.e. they opened one activity after another in quick succession (probably in separate browser tabs although front-end interactions such as browser focus were not tracked here) and then, after some time, quickly submitted one activity after another. This could be an indication of cramming.

Figure 4 shows an example of one such learner. This learner took part in what seemed to have been an intensive two-month course, judging by the online activity of other participants in that course. While other learners in the course submitted activities relatively frequently, this learner submitted 195 activities on three different days (within a span of 10 days), scoring ~96% per activity, on average. On each such day, the learner opened a number of activities almost at once, spent more time on each following activity, and then submitted them all almost at once. This happened towards the end of the course, so completing online activities might have been a course requirement.

Figure 4. Distribution of leading digits of time spent on MyEnglishLab activities by a learner whose behaviour does not conform to Benford’s law

4. Conclusions

We showed that the distributions of leading digits of the number of online activities completed by EFL learners and the time spent on those activities closely
follow Benford’s law. The approach used in this paper shows how insights can be revealed in noisy online data, such as the time data, which the standard methods of analysis would not reveal.

Benford’s law has been applied for tax fraud detection and our results show that it may also be worth applying it for detection of abnormal learner behaviour. Whereas we do not know if the learners whose behaviour did not conform to Benford’s law in this particular study behaved so because of poor time management skills or other factors, Benford’s law could help flag such learners to teachers who would then choose the best course of intervention by talking to learners.

Still, our findings are directional and future research should focus on validating such an approach, and its usefulness to teachers. Another strand of research could focus on comparing the computational performance of different tests for evaluating conformity to Benford’s law (with operationalising large-scale detection in mind) and comparing this approach to other methods of detection and prediction of learner performance, e.g. those that rely on simpler metrics, such as login frequency.

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References

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