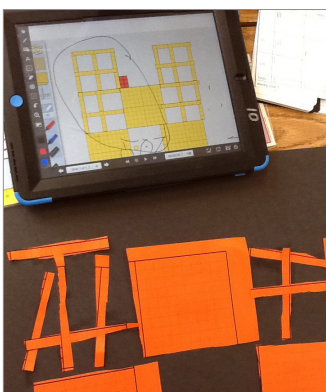




## Productive Struggle in Mathematics

Marian Pasquale



When struggle is reframed as both a natural part of the learning process and a worthwhile challenge to undertake, students are encouraged to persevere in the development of deep mathematical understanding.

### Overarching Issues

Despite the time and effort, children will plug away for hours at a puzzle until they have solved it or at least made substantial progress. They will practice a required skill repeatedly to make the team. Why?

- » What makes solving a puzzle something worth sticking with?
- » What makes practicing a jump shot worthwhile?

It seems that these tasks, unlike many mathematics problems, are worth the challenge because it feels as if you can make headway. “When you do believe you have a chance—when you know your own strength and the task looks accessible—challenge is appealing” (Goldenberg et al., 2015).

“Mathematics educators and researchers suggest that struggling to make sense of mathematics is a necessary component of learning mathematics with understanding” (Hiebert & Grouws, 2007). Yet, “student’s struggles with learning mathematics are often viewed as a problem and cast in a negative light in mathematics classrooms” (Hiebert & Wearne, 1993; Borasi, 1996).

Unfortunately, several school factors may prevent students and teachers from viewing struggle in mathematics as a positive and productive practice. In middle and high schools, school schedules provide insufficient time and opportunity for students to grapple with challenging problems. Curriculum-pacing guides with strict calendars force teachers and students to march through topics, moving from one to the next no matter whether students have grasped the concepts. Typical norms and expectations in most mathematics classrooms tend to value behaviors that do not support good problem-solving routines. Often, individual performance is prized over group problem solving, having the right answer over reasoning and understanding, and being the first to finish over explaining and communicating.

These expectations have engendered in students the idea that mathematics insight is something one is born with. “Students who lack the skills or the confidence—ones who are

(or perceive themselves as) weak—are also typically reluctant to put in the effort that might make them stronger” (Goldenberg et. al., 2015). There is often a flawed belief that you are either good at mathematics or you are not, and struggling with a mathematical task is not viewed as an opportunity to learn but rather a weakness. The good news is that struggle can be positive and can be fostered in students with good teacher support and the right classroom norms and expectations.

### What Is *Productive Struggle*?

In the Common Core Standards for Mathematical Practice, the first standard states that students should “make sense of problems and persevere in solving them” (NGA Center for Best Practices, 2010). Perseverance, or continuing forward irrespective of struggle or difficulty, is an essential element in problem solving because the first or second approach or strategy may not result in a reasonable solution. As students engage with a task, they must be mindful about the strategy they employ and assess whether it is productive. When they find they are at a dead end, they must be willing to abandon one strategy for another. When students labor and struggle but continue to try to make sense of a problem, they are engaging in *productive struggle*.

“Great works are performed not by strength, but perseverance.” — Samuel Johnson

This metacognitive ability—reflecting on one’s own problem-solving approach—is developmental and takes practice and time. Productive struggle, similar to other executive functions (i.e., cognitive skills that help the brain organize and act on information), is supported by a developmental progression in thinking and learning. This developmental progression can and should be nurtured. Students need support to learn how “to move through a progression or range of solution methods” (Fuson, Carroll, & Drucek, 2000). In addition, there are different kinds of struggles that students encounter as they work on a problem, for example, “when students encounter difficulty in figuring out how to get started or carry out their task, are unable to piece together and explain their emerging ideas, or express an error in problem solving” (Warshauer, 2014b). How students express these struggles is easy to spot. When beginning a task, students might say that they don’t understand the directions, have never done a problem like this one before, or just stare at the paper. Students may have a “plan of attack” but are unable to

implement it because they have made errors or forgotten how to proceed. Even when solving a problem correctly, explaining what they did is often their most difficult struggle.

### What Factors Influence Productive Struggle?

Research suggests that students’ out-of-school experiences influence their motivation and perseverance with difficult mathematical tasks. Tasks that have a familiar or real-life context are more meaningful. Because students are able to draw on their everyday experience to solve these kinds of tasks, they are more motivated to stick with the task (Taylor, 2015).

Productive struggle is also highly influenced by the socio-mathematical norms of the school class, particularly a student’s perception about her mathematical ability relative to other members of the class. “Students compare themselves against the norms of mathematical competence in their class[es], construct sets of ‘stories’ that define their own proclivities and handicaps, and use these stories to help them decide when and to what extent they will engage in the social activity of doing mathematics” (Middleton, Tallman, Hatfield, & Davis, 2015). They often think that perseverance is something that some students are skilled at rather than a behavior that everyone can develop.

Besides students’ mathematical self-image, their disposition to struggle with a challenging mathematical task depends largely on whether they (1) find the task interesting, (2) believe that they know enough mathematics to be able to solve it, and (3) believe that solving it is worth the effort (Star, 2015). A student’s belief that effort is more important than innate ability is key. “In order to persevere, one needs to view the struggle that may inevitably be a part of problem solving as an opportunity to learn. Motivation enables a solver to see struggle as a natural part of the learning process, and to see that confronting and working through struggle can ultimately be helpful” (Star, 2015).

### What Can Teachers Do?

The National Council of Teachers of Mathematics (NCTM) policy document, *Principles to Actions: Ensuring Mathematical Success for All*, “notes that an effective teacher provides students with appropriate challenges, encourages perseverance in solving problems, and supports productive struggle in learning mathematics” (NCTM, 2014, p.11, as cited in Star, 2015). This requires that the demand of the mathematical task is high to the

extent that it provides a cognitive stretch for the student and builds on student thinking. Student struggle must be supported so that it is a positive endeavor and not one full of difficulties and frustration (Warshauer, 2014a). The kind of questions teachers ask and the kind of support that teachers offer are critical. “The kind of guidance and structure that teachers provide may either facilitate or undermine the productive efforts of students’ struggles” (as cited in Warshauer, 2014a).

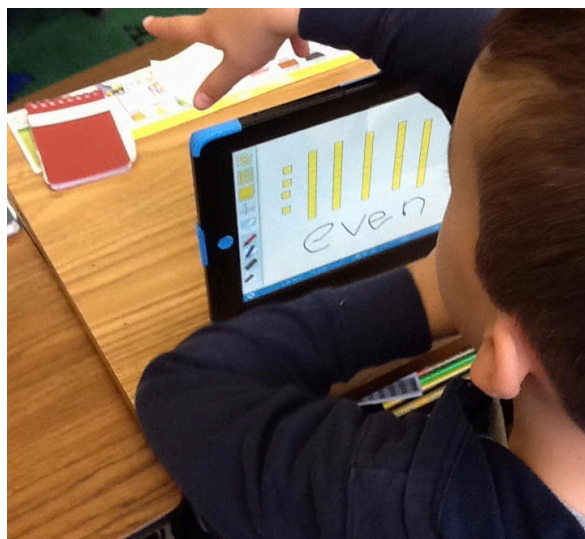
Warshauer (2015) describes the following four strategies to support students’ productive struggle:

**Strategy 1** – Teachers ask questions that help students focus on their thinking and identify the source of their struggle, then encourage students to look at other ways to approach the problem.

**Strategy 2** – Teachers encourage students to reflect on their work and support student struggle in their effort and not just in getting the correct answers.

**Strategy 3** – Teachers give time and help students manage their struggles through adversity and failure by not stepping in too soon or helping too much and thus take the intellectual work away from the students.

**Strategy 4** – Teachers acknowledge that struggle is an important part of learning and doing mathematics.



During my RISK Training, I watched children from other schools explain their thinking and assumed they were just “smarter” than the kids I work with. Now my students are those “smart” kids because they can explain and show their thinking! Thanks for teaching me as a teacher how to model thinking for open-ended questions. It feels as though teachers teach for the “right” answer versus the strategies, which far outweighs the answer for my students.

— Tanya Smith, 2nd Grade Teacher,  
Washburn School, Maine

Teachers’ responses to student struggles generally fall into four types (Warshauer, 2014a):

1. **Telling** – When using the telling strategy, teachers often suggest a new approach, correct an error, or supply information.
2. **Directed guidance** – Directed guidance involves redirecting student thinking by asking open-ended questions, breaking down the problem into smaller parts, and narrowing down what the student might try next.
3. **Probing guidance** – Probing guidance puts the struggle back into the student’s lap. Here the teacher offers ideas based on the student’s thinking, asks for an explanation that might surface an error, or asks for reasons and justifications.
4. **Affordance** – Affordance provides an opportunity for students to continue thinking with little help from the teacher other than encouragement.

All of these approaches are useful as long as the level of cognitive demand remains high, and student thinking is supported (Warshauer, 2011).

The following practices also help support student struggle and make it productive:

- » Set goals at the beginning of the lesson and keep track of student progress during the lesson.
- » Set problems in a familiar setting whenever possible, such as a sport or a familiar everyday task.
- » Support students by providing appropriate tasks, tools, and representations.

- » Group students heterogeneously, which helps struggling students.
- » Establish high mathematical expectations (i.e., doing mathematics requires effort).
- » Use good questioning techniques, such as asking students to explain how they solved a problem and why a strategy works or ask them to describe another way to solve the same problem.
- » Provide time for group reflection during problem-solving activities. This can help students recognize unproductive strategies.
- » Compare student outcomes at the end of the lesson to your original goals.



Establish class norms that support productive struggle, such as these examples:

- » Being wrong is an opportunity to learn.
- » Being correct is an impetus to help others.
- » Everyone is responsible for each other's learning.

## Conclusion

"Productive struggle is complex. A student's effort to make sense of mathematics, to figure something out that is not immediately apparent can advance the students in their thinking and play an important role in deepening students' understanding, if supported carefully toward a resolution and given appropriate time" (Hiebert & Grouws, 2007).

Teachers need to carefully select tasks that require students to struggle and provide the support that students need without diminishing the cognitive demand of the task or giving students too much help. Students need sufficient time, not only to solve difficult mathematical problems, but also to "develop genuine curiosity and stamina" (Goldenberg, et. al., 2015). Finally, teachers must create a classroom culture that demonstrates "struggle as a natural part of the learning process" (Star, 2015) and allows students to see the potential in persevering.

## References

- Borasi, R. (1996). *Reconceiving mathematics instruction: A focus on errors*. Norwood, NJ: Ablex.
- Fuson, K., Carroll, W., & Drueck, J. (2000). Achievement results for second and third graders using the standards-based curriculum *Everyday Mathematics*. *Journal for Research in Mathematics Education*, 31(3), 277–295
- Goldenberg, P. E., Mark, J., Kang, J., Fries, M., Carter, C. J., & Cordner, T. (2015). *Making sense of algebra: Developing students habits of mind*. Portsmouth, NH: Heinemann.
- Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371–404). Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.405.3591&rep=rep1&type=pdf>
- Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal*, 30(2), 393–425.
- Johnson, S. (n.d.). The history of Rasselas, Prince of Abissinia: Chapter 13. Lynch, J. (Ed.) Retrieved from <https://andromeda.rutgers.edu/~jlynch/Texts/rasselas.html#12>
- Middleton, J. A., Tallman, M. A., Hatfield, N., & Davis, O. (2015, April). Taking the severe out of perseverance: Strategies for building mathematical determination (White paper). Chicago, IL: Spencer Foundation. Retrieved from <http://hub.mspnet.org/index.cfm/28128>



National Governors Association (NGA) Center for Best Practices, Council of Chief State School Officers. (2010). Common Core State Standards: Standards for Mathematical Practice. Washington, DC: Author. Retrieved from <http://www.corestandards.org/Math/Practice/>

Star, J. R. (2015, April). When not to persevere – Nuances related to perseverance in mathematical problem solving (White paper). Chicago, IL: Spencer Foundation. Retrieved from <http://hub.mspnet.org/index.cfm/28127/>

Taylor, E. V. (2015, April). Cultural considerations in support of mathematical perseverance: The role of context activation (White paper). Chicago, IL: Spencer Foundation. Retrieved from <http://hub.mspnet.org/index.cfm/28130>

Warshauer, H. K. (2011). The role of productive struggle in teaching and learning middle school mathematics (Unpublished dissertation). The University of Texas at Austin, Austin, TX.

Warshauer, H. K. (2014a). Productive struggle in middle school mathematics classrooms. *Journal of Mathematics Teacher Education*, 17(4). Advance online publication. doi: 10.1007/s10857-014-9286-3

Warshauer, H. K. (2014b). Productive struggle in teaching and learning middle school mathematics. *Journal of Mathematics Education*, 17(4), 3–28.

Warshauer, H. K. (2015). Strategies to support productive struggle. *Mathematics Teaching in the Middle School*, 20(7), 390–393.

## Resources

Bass, H. & Ball, D. (2015, April). Beyond “you can do it!” – Developing mathematical perseverance in elementary school (White paper). Chicago, IL: Spencer Foundation. Retrieved from <http://hub.mspnet.org/index.cfm/28132>

Berry I., Robert Q., & Thunder, K. (2015, April). Black learners’ perseverance with mathematics: A qualitative metasynthesis (White paper). Chicago, IL: Spencer Foundation. Retrieved from <http://hub.mspnet.org/index.cfm/28131>

Clements, D. H., & Sarama, J. (2015, April). Learning executive function and early math (White paper). Chicago, IL: Spencer Foundation. Retrieved from <http://hub.mspnet.org/index.cfm/28129>

Hiebert, J., & Stigler, J. W. (2004). A world of difference. *National School Development Council*, 25(4), 10–15.



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