Journal of Experimental Child Psychology 122 (2014) 104–121

Contents lists available at ScienceDirect

Journal of Experimental Child Psychology

journal homepage: www.elsevier.com/locate/jecp

Working memory and language: Skill-specific or domain-general relations to mathematics?

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ARTICLE INFO

Article history:
Received 1 July 2013
Revised 21 December 2013
Available online 15 February 2014

Keywords:
Mathematics
Numeracy
Working memory
Language
Vocabulary
Preschool
Kindergarten

ABSTRACT

Children's early mathematics skills develop in a cumulative fashion; foundational skills form a basis for the acquisition of later skills. However, non-mathematical factors such as working memory and language skills have also been linked to mathematical development at a broad level. Unfortunately, little research has been conducted to evaluate the specific relations of these two non-mathematical factors to individual aspects of early mathematics. Thus, the focus of this study was to determine whether working memory and language were related to only individual aspects of early mathematics or related to many components of early mathematics skills. A total of 199 4- to 6-year-old preschool and kindergarten children were assessed on a battery of early mathematics tasks as well as measures of working memory and language. Results indicated that working memory has a specific relation to only a few—but critically important—early mathematics skills and language has a broad relation to nearly all early mathematics skills.

Introduction

The successful acquisition and development of mathematics skills and concepts is a critical aspect of children's early academic growth (Baroody, Lai, & Mix, 2006; Jordan, Hanich, & Uberti, 2003). Early

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http://dx.doi.org/10.1016/j.jecp.2013.12.009

mathematical performance is one of the strongest predictors of later academic and career outcomes (Duncan et al., 2007; Lubinski & Benbow, 2006). Unfortunately, many children fail to achieve early success in mathematics, and these early difficulties tend to persist and become more pronounced over time (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004; Baroody & Ginsburg, 1990). The cumulative nature of early mathematical development—later competencies building on earlier ones—underscores the need for early prevention and intervention with children at risk for developing mathematics difficulties. To effectively intervene in early skills, it is particularly important to understand how these individual skills develop and what factors influence that development. It is evident that a range of both mathematical and non-mathematical factors (e.g., working memory, language) affect children's early mathematical development (Fuchs et al., 2005, 2008, 2010; Gathercole, Pickering, Knight, & Stegmann, 2004; Jarvis & Gathercole, 2003; Purpura, Hume, Sims, & Lonigan, 2011; Raghubar, Barnes, & Hecht, 2010); however, the specificity of the relation between these non-mathematical domains and early mathematics skills is not well understood. The central goal of this study was to identify how these important non-mathematical factors differentially were related to specific early mathematics skills.

Development of early mathematics skills

There is clear evidence that the strongest predictor of *later* mathematics success is *early* mathematical performance (Claessens, Duncan, & Engel, 2009; Duncan et al., 2007; Fuchs et al., 2010). This is because mathematics skills develop as a progression of interrelated facts and concepts (Baroody, 2003; Gersten & Chard, 1999; National Mathematics Advisory Panel [NMAP], 2008; Purpura, Baroody, & Lonigan, 2013) called a learning trajectory (Gravemeijer, 2002; Sarama & Clements, 2009; Simon & Tzur, 2004). Advanced mathematical knowledge is dependent on the acquisition and retention of more basic prerequisites; therefore, missing (or having an underdeveloped ability in) one or more prerequisites limits an individual's ability to acquire the more advanced skills. For example, at the early elementary school level, for a child to successfully (and reliably) acquire fluency in basic arithmetic, the child not only should know the process of adding or subtracting but also must (a) associate specific number word names with the appropriate Arabic numerals (e.g., know that "two" is equal to "2"), (b) associate specific quantities with the appropriate number words and the appropriate Arabic symbols (e.g., know that "•••" is equal to "three" and "3"), and (c) understand the meaning behind operational symbols (e.g., know that "+" means to add). Without developing a strong foundation of these early mathematics skills, children are likely to experience difficulties in acquiring later mathematics skills and be at a higher risk for developing mathematics difficulties than children who do develop a strong foundation of early mathematical knowledge (Baroody & Ginsburg, 1990).

When discussing the early mathematics skills children learn in schools (e.g., basic arithmetic computations), the term *formal* mathematics is typically used. Formal mathematics encompasses those skills and concepts that are taught in school and require the use of abstract written numerical notation such as written arithmetic algorithms using numerals, place-value tasks, knowledge of the base-ten mathematics system, and decimal knowledge (Ginsburg, 1977). However, there are a range of skills called *informal* (or early) mathematics skills that form the basis for the acquisition of formal mathematical knowledge (Bryant, Bryant, Kim, & Gersten, 2006; Chard et al., 2005; Committee on Early Childhood Mathematics, Center for Education, Division of Behavioral & Social Sciences & Education, & National Research Council, 2009; Geary, 1994; Ginsburg, Klein, & Starkey, 1998; Griffin & Case, 1997; Jordan, Kaplan, Ramineni, & Locuniak, 2009; NMAP, 2008;). Informal mathematical knowledge consists of those competencies often learned before or outside of school that typically do not require knowledge of the formal Arabic numeral system (Ginsburg, 1977).

Children's informal mathematics skills are composed of several distinct, but highly related, components (Jordan, Kaplan, Locuniak, & Ramineni, 2007; National Research Council, 2009; Purpura & Lonigan, 2013) that vary in their complexity and difficulty to acquire. These components of informal knowledge are believed to develop in three overlapping phases (Krajewski, 2008; Krajewski & Schneider, 2009). In the first phase, children separately learn to distinguish between small quantities (comparing sets) and learn the verbal counting sequence (number word sequence). In the second phase, they apply the counting sequence to fixed sets (one-to-one counting) and make links between all of the number words and their respective quantities (e.g., they learn to subitize and to apply cardinal number knowledge). This second phase entails understanding that each of the number words (e.g., "three") represents a distinct quantity (e.g., "•••"). The third phase of informal mathematical development involves being able to combine number words and quantities into new number words and quantities without using physical objects (e.g., story problems). These early skills, coupled with written symbolic-based skills (e.g., numeral naming, comparing numerals), lay a foundation for the acquisition of formal mathematical knowledge.

Non-mathematical factors related to mathematical development

Although early mathematical knowledge is the critical basis for developing more advanced mathematical knowledge (De Smedt, Verschaffel, & Ghesquiere, 2009; Jordan et al., 2007; Purpura et al., 2013), other *non*-mathematical factors are believed to affect the development of mathematical knowledge as well. Two specific factors that have been related to general academic achievement—and specifically to mathematical development—are working memory (Alloway & Passolunghi, 2011; Holmes & Adams, 2006; Raghubar et al., 2010) and language (Fuchs et al., 2005, 2008, 2010; Purpura et al., 2011). In fact, deficits in both of these areas can often co-occur and result in broader impairment in mathematical functioning (Reimann, Gut, Frischknecht, & Grob, 2013).

Working memory

Working memory is an individual's ability to hold information in memory while simultaneously processing other information (Baddeley, 1992; Engle, Tuholski, Laughlin, & Conway, 1999; Just & Carpenter, 1992). It is generally described as having three primary components: the central executive, the phonological loop, and the visuospatial sketchpad (Baddeley, 2000; Baddeley & Hitch, 1974). Although these components have been found to be distinct, the central executive is the overarching component and is highly correlated to the other two components in preschool children (rs > .70; De Smedt, Janssen, et al., 2009). Working memory has been shown to be a significant predictor of children's academic achievement—particularly in mathematics (Berg, 2008; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Jarvis & Gathercole, 2003). Although some evidence suggests that individual components of working memory are differentially related to mathematics (Simmons, Willis, & Adams, 2012; Wilson & Swanson, 2001), other evidence indicates that working memory as a whole (rather than a specific aspect of working memory) is related to mathematical development (Swanson, 2012; Zheng, Swanson, & Marcoulides, 2011).

In elementary school, children with mathematics difficulties tend to perform lower on working memory tasks than their typically achieving peers (Geary, Hoard, Nugent, & Bailey, 2012; Passolunghi & Siegel, 2004; Raghubar et al., 2010), and working memory has been found to be a significant predictor of later mathematics success through middle school (Nunes, Bryant, Barros, & Sylva, 2012). Working memory is likely important for mathematical development in elementary school because fluently solving complex mathematical problems has multiple cognitive demands, and children with more developed working memory resources are generally better able to retrieve arithmetic facts or connect new knowledge with previously learned information than their peers with less developed working memory skills (Geary, Hoard, & Nugent, 2012). However, working memory has not consistently been found to be related to all aspects of mathematics; rather, it has been found to be related to specific components of mathematics. For example, Nyroos and Wiklund-Hornqvist (2012) found that, at third grade, working memory accounted for significant variance in basic problem solving but not in procedural algorithm usage. Similarly, Fuchs et al. (2005) found that in a sample of first-grade students, working memory was a significant predictor of curriculum-based measures of computation, mathematical concepts, and applications, but not of computation and addition fluency. These findings generally suggest that working memory is related to complex but applied components of mathematics rather than more simple procedural tasks where there are few steps involved in completing the tasks.

Although the majority of research connecting working memory and mathematics achievement has been conducted with elementary school students and focused on formal aspects of mathematics, some evidence has suggested that working memory and informal mathematics skills are related in preschool and kindergarten (e.g., Bull, Espy, & Wiebe, 2008; Chiappe, Hasher, & Siegel, 2000). Specifically, Bull et al. (2008) found that working memory at the start of preschool predicted broad mathematics achievement, but not reading achievement, through first grade. Similarly, Östergren and Träff (2013) used a latent variable model to assess the relation of working memory to informal and formal mathematics skills. They found that verbal working memory was a strong predictor of both informal and formal skills. Yet, unlike these studies with older children and more advanced mathematical concepts, there is little research that evaluates the relation of working memory to specific aspects of informal mathematics.

It is evident that working memory plays a critical role in the development of early mathematics skills. Yet, with informal mathematics being the synthesis of an array of early mathematical competencies—and some skills being more complex than others—it is likely that the role working memory plays in early mathematical development is specific to individual aspects of early mathematics. The more complex skills (e.g., those that require multiple steps) are likely to have stronger relations to working memory than more basic skills (e.g., those that have only one step or are more procedural in nature). These more complex skills include cardinal number knowledge, subitizing, exact set comparison, number order, and story problems. For example, cardinal number knowledge tasks require children to count out a specified set size from a larger set. In such tasks, children must maintain the target number while keeping track of the objects being counted. In subitizing, children must rapidly enumerate a set without counting and maintain the quantity in memory while identifying its verbal name. When children compare exact sets, they must identify the set sizes of multiple sets, maintain each set size in memory while identifying the size of other sets, and then identify the largest (or smallest) of the sets. To identify the missing number in a number sequence, children must be able to manipulate the counting sequence and use it at a deeper level than simple recitation. Finally, several studies have shown that working memory is related to story problems with elementary school children (Fuchs et al., 2005; Rasmussen & Bisanz, 2005), and it is likely to be similar with younger children who have fewer cognitive resources.

In contrast to the more complex components of early mathematics, there are several equally critical, but less complex, aspects of mathematics for which working memory is not likely to be a predictor. These components include verbal counting, one-to-one counting, comparing numeral magnitudes, naming numerals, and connecting sets to numerals. Verbal counting and one-to-one counting are skills that simply require children to use the counting sequence from the start without maintaining additional information at the same time (procedural). Numeral comparison, unlike set comparison, is unlikely to require working memory because the set sizes are already identified and mapped onto the symbols. Naming numerals likely does not require working memory because it is simply applying names to symbols. Similarly, connecting quantities to numerals simply requires children to match numerals and quantities and is also unlikely to require working memory. Overall, there is a critical need to investigate the relation of working memory to *specific* components of mathematics across all developmental phases (Raghubar et al., 2010), particularly early mathematical development.

Language skills

General language skills have been found to be related to, and predictive of, broad mathematical performance across a range of ages (Hooper et al., 2010; Purpura et al., 2011; Romano et al., 2010). In elementary school, general language skills have been found to be significantly related to story problems but not to calculation problems (Fuchs et al., 2005, 2008, 2010). The differential relation of language skills to these two types of problems is typically associated with the linguistic demands of story problems. In story problems, children not only need to be able to complete the mathematical computations but also need to understand that a range of mathematical words can mean the same thing and can be used interchangeably (e.g., "plus," "and," "add," "together"). Similarly, other research also has found differential relational patterns of language across various other mathematical domains in elementary school children (LeFevre et al., 2013; Vukovic & Lesaux, 2013a). Vukovic and Lesaux (2013a) found that, with elementary school children, broad language skills were related to tasks that involved understanding conceptual meaning but not to procedural calculations. Prior research at the preschool level (Purpura et al., 2011) has found similar results where language was predictive of broad measures of informal mathematics skills but not of a broad measure of formal calculation skills. The role language plays in informal mathematical development appears to be relatively broad. LeFevre et al. (2010) found that young children's early linguistic skills were related to both early numeracy and geometry; however, they did not investigate individual components within the broader mathematical domains.

As children develop their informal mathematics skills, they are primarily connecting quantitative knowledge to words and symbols—or making meaning of early mathematical concepts (Krajewski & Schneider, 2009; LeFevre et al., 2010). Children also need to understand linguistic concepts (or terminology) such as "more" and "less" in addition to being able to perceptually discriminate between sets. Thus, it is likely that language skills play a role in the development of mathematics skills at all three phases of informal development. Yet, similar to the relation between working memory and early mathematics skills, little research has been conducted to evaluate this relation directly.

The current study

It is evident that early mathematics skills form the foundation for the development of later mathematics skills (Fuchs et al., 2010), and these early skills build on one another in overlapping, but distinct, phases (Krajewski & Schneider, 2009). There also has been consistent evidence suggesting that both working memory and language broadly are related to the development of early mathematics skills (Bull et al., 2008; LeFevre et al., 2010). However, there is a dearth of research investigating the links of working memory and language to *specific* aspects of *informal* mathematics skills at the preschool and kindergarten age levels. Thus, the purpose of this study was to systematically evaluate the unique relations of working memory and language to a range of specific early mathematics skills in a sample of preschool- and kindergarten-aged children. Identifying whether these two domains have specific or general relations to early mathematics skills and concepts will provide key information in developing an integrated mathematical learning trajectory that combines both mathematical and non-mathematical factors. It was hypothesized that language skills have a general relation to all of the early mathematics skills and concepts because children at this age are in the process of linking number words, quantities, and symbols. Furthermore, it was hypothesized that the relation of working memory and early mathematics would be found only in those skills or concepts that require multiple steps or the integration of multiple earlier skills and concepts, specifically in cardinality, subitizing, set comparison, number order, and story problems.

Method

Participants

Data were collected in 45 public and private preschools and kindergartens serving children from families of low to middle socioeconomic statuses (SES). The 199 children who completed all assessments were approximately evenly split by sex (51.8% female and 48.2% male) and approximately representative of the demographics of the area (59.8% Caucasian, 28.6% African American, and 11.6% other race/ethnicity). Approximately half of the children ($n = 106$) were in kindergarten, and the others ($n = 93$) were in their second year of preschool. Children ranged in age from 4.05 to 6.83 years ($M = 5.54$ years, $SD = 0.75$), were primarily English speaking, and had no known developmental disorders. This work was approved by the institutional review board, and parental consent was obtained for each participating child.

Measures

Early mathematics tasks

Ten tasks served to measure different aspects of early mathematical knowledge. The individual early mathematics tasks were developed as part of a broader measure of early mathematics skills (Purpura, 2010). The specific tasks selected assess skills from the different phases of informal knowledge noted earlier and/or other aspects of early mathematics that have been found to be strong predictors of later mathematics. Items on each of the tasks were derived by a process using item response theory, which ensured that each item was related to its intended construct, had adequate discrimination (*a* parameter), and did not duplicate the difficulty level (*b* parameter) of other items on the same task. In the measure development sample (Purpura, 2010), all tasks exhibited strong item response theory standard error scores (which are sample-independent measures of reliability) across a range of ability levels. Cronbach's alphas from the development sample are presented for each measure below.

Verbal counting. Children were asked to count as high as possible. When children made a mistake, or correctly counted to 100 without making a mistake, the task was stopped. Spontaneous self-corrections were not scored as incorrect, and children were allowed to continue counting. The highest number counted to was converted to a score based on a seven-point scale. Children were awarded one point each for correctly counting to 5, 10, 15, 20, 25, 40, and 100.

One-to-one counting. Children were presented with a set of 3, 6, 11, 14, or 16 dots on a page and asked to count the set. Children were awarded one point for each item if they correctly counted each dot only once. This task had an internal consistency (Cronbach's alpha) of .79.

Cardinality (counting a subset). In the first part of this task, children were presented with a specific quantity of objects (e.g., 15) and were asked to count out ("give me *n*") a smaller set of objects (e.g., 4) from the larger set. Set sizes to be counted out were 3, 4, 8, and 16. In the second part of this task, children were presented with pictures of both dogs and cars. Children were instructed to count all of one type of picture (e.g., "count all the dogs"). Set sizes to be counted were 3, 8, 16, and 20. This task had an internal consistency of .82.

Subitizing. Children were briefly presented (2 s) with a set of pictures (set sizes from 1 to 7 presented in a linear fashion, e.g., ••••) and instructed to say how many dots or pictures were presented. There were a total of seven items on this task. For each correct response, children were awarded one point. This task had an internal consistency of .69.

Number comparison. Children were asked to identify which of four numbers was the largest or smallest. Half of the items were presented visually with Arabic numerals, and half of the items were presented verbally. There were a total of six items on this task. This task had an internal consistency of .74.

Set comparison. For each of the six items, children were presented with four sets of dots on a page representing different quantities (e.g., | ••• | •• | •••• | • |). They were then asked which set had the most dots (three items) or fewest dots (three items). Children received one point for pointing to the correct set. This task had an internal consistency of .77.

Number order. Children were shown a sequence of numbers with one number missing in between two numbers (1 2 3 _ 5 6). They were asked what number comes before or after another number (e.g., "What number comes before 5?"). The six items include identifying the numbers before 2, 5, and 15 and the numbers after 2, 9, and 15. This task had an internal consistency of .87.

Numeral identification. Children were presented with flashcards of nine numbers (1, 2, 3, 7, 8, 10, 12, 14, and 15). They were shown the flashcards one at a time and were asked, "What number is this?" For each correct response, children were awarded one point. This task had an internal consistency of .90.

Set to numerals. On the first three items in this task, children were presented with a numeral at the top of the page (e.g., 3) and five sets of dots below (e.g., | •••• | • | ••• | •• | ••••• |). They were instructed to identify which of the sets meant the same thing as the number at the top of the page. On the last two items of this task, children were presented with a set of dots at the top of the page (e.g., ••••) and five numerals at the bottom (e.g., 4, 2, 3, 1, 5). They were instructed to identify which of the numerals meant the same thing as the set of dots at the top of the page. Children were awarded one point for each correct response. This task had an internal consistency of .80.

Story problems. Children were presented verbally with story problems that did not contain distracters (e.g., irrelevant information). These story problems were simple addition problems (three items) or subtraction problems (four items) that were appealing to children. For example, one question was, "Johnny had one cookie and his mother gave him one more cookie. How many cookies does he have now?" Children were awarded one point for each correct response. This task had an internal consistency of .71.

Primary predictor skills

Both working memory and language tasks were included as primary predictor tasks.

Language. The Expressive One-Word Picture Vocabulary Test–Third Edition (EOWPVT; Brownell, 2000) was used to measure children's expressive vocabulary ability. In this task, children were shown a colored picture of an object(s) and were asked, "What is this?", "What is this for?", or "What are these?" The EOWPVT has excellent reliability (αs = .95–.97 for 4- to 7-year-old children in the validation sample; Brownell, 2000) and validity (*r*s = .67–.90 with 12 other measures of vocabulary and *r*s = .71–.85 with five other measures of broader language skills; Brownell, 2000). Given such high correlations with other language skills, the EOWPVT was used as a proxy for general language skills. Children received one point for each correct response. Basal (eight correct in a row) and ceiling (six incorrect in a row) rules were administered per the instruction manual.

Working memory. To assess working memory, we used a word recall task similar to the Automated Working Memory Assessment listening recall task (AWMA; Alloway, 2007). This task is primarily a verbal working memory task (typically used to measure the central executive and phonological loop aspects of working memory). A verbal working memory task was selected for this study, rather than a visual–spatial working memory task, for two reasons. First, prior research with children (Wilson & Swanson, 2001) found that the relation between working memory and mathematics is better predicted by verbal working memory than by visual working memory. Second, the complexity of information for these early mathematics tasks is generally in the verbal components and not in the visual components of the tasks. The difference between the task used in this study and the AWMA is primarily in the method of administration. The AWMA is computerized, and the verbal working memory task in this study was not computerized.

The task was divided into four blocks. The first block consisted of trials that included only one question, the second block had trials with two questions, and so on up to four questions. Each block had three trials. In each trial, participants heard a series of questions and decided whether the answer was yes or no. After they heard one to four questions in a row that they answered with yes or no, they were asked to recall the last word of each question. For example, in a trial with two questions, participants heard "Do dolls read?", said yes or no, then heard "Do boys run?", said yes or no, and then were asked to say the last word of each of the questions ("read, run"). Participants were introduced to Block 1 with three practice items, to Block 2 with two practice items, and to Block 3 with one practice item. No practice items were used to introduce Block 4. Practice items were similar to the items given in each set except that they were explained using picture cards (e.g., a picture of a doll holding a book was shown for "Do dolls read?"). Children were explicitly informed that they would need to remember the last word(s). Feedback was provided during the practice trials to ensure that children understood the rules of the task. No pictures or feedback were used for the test trials. Children completed trials within each block until they did not recall any words from the questions in each trial for all three trials in the same block. Thus, the working memory task measured the ability to remember words from several questions while also answering yes or no to the questions. The outcome measure was the recall score—the number of times participants accurately recalled the last word in a question from each trial. Children were awarded one point for each correct last word they identified regardless of whether or not it was in the correct order. The test–retest reliability for the AWMA measure is .81 (Alloway, Gathercole, & Pickering, 2006).

Covariates

Children's chronological age, sex (scored 1 for male and 0 for female), grade in school (scored 1 if children were in kindergarten and 0 if they were in preschool), and general calculation skills were included as covariates.

General calculation. The Woodcock–Johnson III Calculation subtest (WJ-III Calculation) was used to assess general calculation ability. WJ-III Calculation is a paper-and-pencil arithmetic test where children are asked to solve addition and subtraction problems. This test has been shown to have a reliability of between .96 and .97 for 5- and 6-year-old children (Woodcock, McGrew, & Mather, 2001). Children were awarded one point for each correct answer. This task was used to control for children's general computational knowledge.

Procedure

Assessment procedure

Children were assessed on all tasks during the spring of the academic year. Individuals who either had completed or were working toward completion of a bachelor's degree in psychology or education conducted the assessments. Assessments took place in the local preschools or kindergarten classrooms during non-instructional time in a quiet room designated by the individual school directors or teachers.

Analytic procedure

To evaluate the relations of language and working memory with specific early mathematics skills, a series of separate mixed-effects regression analyses (Raudenbush & Bryk, 2002) were conducted. In each of the analyses, school was included as a random effect to account for variance across schools. There were a total of 45 schools with an average of 4.4 children per school. Age, grade (kindergarten or preschool), sex, and WJ-III Calculation were included as fixed-effect covariates. Working memory and language tasks were included in the model as fixed-effect predictors. Benjamini–Hochberg corrections were applied within each regression analysis to correct for multiple comparisons. Based on our first hypothesis—that language would be related to all aspects of informal mathematics—we expected the language task to be a significant predictor of *each* early mathematics skill. Based on our second hypothesis—that working memory would be related to only specific aspects of informal mathematics skills—we expected the working memory task to be a significant predictor of only the five specific early mathematics tasks identified earlier (cardinality, subitizing, set comparison, number order, and story problems).

Results

Preliminary analyses

Means, standard deviations, skew, and kurtosis are presented in Table 1. All tasks were normally distributed, and no tasks exhibited significant skew or kurtosis. Correlations among tasks are presented in Table 2. All tasks were significantly correlated with one another.

Primary analyses

Ten separate mixed-effects regression analyses were conducted. Results of each analysis are presented in Table 3. In Table 4, a summary of the significance values for predicting each early mathematics skills is presented. All analyses were conducted using raw scores. When analyses were conducted with age-regressed standardized scores, the results were comparable to those presented here. Further-

Table 1
Means, standard deviations, ranges, skew, and kurtosis of the sum scores of the mathematics, language, and working memory tasks.

Task	Mean	*SD*	Range[a]	Skew	Kurtosis
Mathematics					
Verbal counting	5.18	1.88	0–7	−0.83	−0.60
One-to-one counting	4.32	1.07	0–5	−1.76	2.76
Cardinality	6.17	1.89	0–8	−1.12	0.88
Subitizing	4.80	1.31	0–7	−0.20	0.00
Number comparison	3.78	2.03	0–6	−0.43	−1.17
Set comparison	4.99	1.51	0–6	−1.51	1.34
Number order	4.46	1.96	0–6	−1.14	0.16
Number identification	7.48	2.13	0–9	−1.39	1.21
Set to numerals	4.00	1.24	0–5	−1.29	1.12
Story problems	4.67	2.01	0–7	−0.57	−0.71
WJ-III calculation	3.51	3.70	0–16	0.95	0.36
Non-mathematics					
EOWPVT	49.21	17.06	0–91	−0.20	−0.59
Working memory	15.24	7.56	0–28	−0.66	−0.70

Note. $N = 199$.
[a] The range indicates both the possible and actual range of scores for all tasks except the WJ-III Calculation task, the One-Word Picture Vocabulary task (EOWPVT), and the working memory task. For these three tasks, only the actual range is presented because the tasks are designed for wide age ranges of individuals.

more, when analyses were conducted without controlling for broad calculation ability, the results were also consistent with the presented results.

Working memory

Working memory was a significant predictor of only three of the five hypothesized early mathematics skills and concepts: cardinality (counting a subset), set comparison, and number order. Working memory was also a marginally significant predictor of one of the other hypothesized early mathematics skills (subitizing). However, it was not a significant or marginally significant predictor of story problems. As predicted, working memory was not a significant predictor of the other five early mathematics skills (verbal counting, one-to-one counting, number comparison, numeral identification, and set to numerals).

Language

As expected, language was a significant predictor of nearly all mathematics skills and concepts with the exception of verbal counting, one-to-one counting, and subitizing. However, it was a marginally significant predictor of both verbal counting and one-to-one counting.

Table 2
Correlations among the sum scores of all tasks.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1. Verbal counting	–												
2. One-to-one counting	.62	–											
3. Cardinality	.70	.63	–										
4. Subitizing	.38	.44	.45	–									
5. Number comparison	.63	.64	.62	.49	–								
6. Set comparison	.46	.53	.55	.42	.62	–							
7. Number order	.64	.63	.76	.50	.66	.64	–						
8. Number identification	.75	.59	.72	.41	.63	.51	.76	–					
9. Set to numerals	.52	.54	.66	.42	.57	.57	.89	.61	–				
10. Story problems	.59	.59	.62	.49	.62	.63	.65	.58	.55	–			
11. Working memory	.37	.32	.50	.37	.40	.45	.55	.42	.40	.42	–		
12. Language	.52	.62	.56	.44	.63	.52	.62	.57	.52	.57	.52	–	
13. Calculation	.55	.57	.52	.44	.60	.48	.60	.51	.42	.59	.41	.58	–

Note. $N = 199$. All correlations were significant at $p < .001$.

Table 3
Mixed-effects regression predicting the early numeracy measures.

	Estimate	*SE*	*F*	*p*
Verbal counting				
Age	.30	.30	0.99	.322
Grade	.81	.45	3.23	.075
Sex	−.37	.20	3.19	.076
Calculation	.11	.04	8.62	.004
Working memory	.01	.02	0.35	.552
Language	.02	.01	5.70	.018^{+}
One-to-one counting				
Age	.38	.20	3.69	.056
Grade	−.09	.29	0.10	.757
Sex	−.29	.14	4.64	.033^{+}
Calculation	.04	.02	2.73	.100
Working memory	.01	.01	0.35	.553
Language	.01	.01	3.99	.047^{+}
Cardinality				
Age	.79	.29	7.35	.007
Grade	−.01	.45	0.00	.983
Sex	.40	.20	4.03	.046^{+}
Calculation	.08	.04	4.71	.031
Working memory	.04	.02	7.36	.007
Language	.02	.01	7.56	.007
Subitizing				
Age	.45	.24	3.72	.055
Grade	−.25	.34	0.52	.473
Sex	−.24	.16	2.12	.147
Calculation	.08	.03	6.69	.010^{+}
Working memory	.02	.01	2.85	.093
Language	.01	.01	2.11	.148
Number comparison				
Age	.95	.30	10.11	.002
Grade	−.29	.42	0.46	.497
Sex	−.10	.21	0.21	.647
Calculation	.13	.04	11.29	.001
Working memory	.00	.02	0.06	.804
Language	.04	.01	19.48	<.001
Set comparison				
Age	.45	.25	3.17	.077
Grade	.09	.38	0.06	.807
Sex	−.05	.17	0.083	.774
Calculation	.06	.03	3.38	.068
Working memory	.04	.01	7.21	.008
Language	.02	.01	6.97	.009
Number order				
Age	1.26	.27	21.98	<.001
Grade	−.63	.38	2.77	.099
Sex	−.20	.19	1.16	.284
Calculation	.10	.03	8.74	.003
Working memory	.06	.01	15.46	<.001
Language	.02	.01	8.27	.005
Numeral identification				
Age	.80	.33	5.72	.018
Grade	.38	.50	0.58	.448
Sex	−.36	.23	2.52	.114
Calculation	.06	.04	1.79	.182
Working memory	.02	.02	1.19	.276
Language	.03	.01	12.91	<.001

(*continued on next page*)

Table 3 (*continued*)

	Estimate	*SE*	*F*	*p*
Set to numerals				
Age	.54	.21	3.35	.013
Grade	−.04	.31	0.01	.909
Sex	.01	.15	0.01	.923
Calculation	.01	.03	0.27	.604
Working memory	.02	.01	2.20	.140
Language	.02	.01	9.18	.003
Story problems				
Age	.70	.31	4.92	.028^{+}
Grade	−.34	.45	0.58	.449
Sex	.43	.22	3.92	.049^{+}
Calculation	.17	.04	17.63	<.001
Working memory	.02	.02	2.03	.156
Language	.03	.01	9.98	.002

Note. N = 199. A superscript plus sign ($^{+}$) indicates marginally significant after the Benjamini–Hochberg correction was applied.

Table 4
Significance of predictor variables and covariates in the mixed-effect regressions for each early mathematics skill.

Task	Age	Grade	Sex	Calculation	Working memory	Language
Verbal counting		+	+	*		+
One-to-one counting	+		+			+
Cardinal knowledge	*		+	*	*	*
Subitizing	+			+	+	
Number comparison	*			*		*
Set comparison	+			+	*	*
Number order	*	+		*	*	*
Numeral identification	+					*
Set-to-numerals	*					*
Story problems	*		+	*		*

Note. N = 199. All p values not included in the table were greater than p = .100. An asterisk (*) indicates a significant predictor after applying the Benjamini–Hochberg correction. A plus sign (+) indicates a marginally significant predictor after applying the Benjamini–Hochberg correction.

Covariates

Age was a significant, or marginally significant, predictor of nearly all early mathematics skills, indicating that older children performed better than younger children. Grade was not a significant predictor of any early mathematics skills, indicating that grade and level of instructional focus at school did not significantly alter the results. After the application of the Benjamini–Hochberg correction for multiple comparisons, sex was a marginally significant predictor only of verbal counting and one-to-one counting in the direction favoring girls and of cardinal knowledge and story problems in the direction favoring boys. Finally, the calculation task was significantly, or marginally significantly, related to all early mathematics skills with the exception of numeral identification, set to numerals, and one-to-one counting.

Discussion

The results of this study suggest that both language skills and working memory are related to a range of early mathematics skills. Although both domains are correlated with all early mathematics skills that were assessed, the nature of these relations differs when accounting for the other domains and controlling for background variables. As hypothesized, language skills appear to have a *general* relation to early mathematics skills because, even after controlling for the background variables (including a general measure of calculation skills) and working memory, it was significantly related to nearly all early mathematics skills that were assessed. In contrast to language, but also as expected, our hypothesis that working memory has a *specific* relation to particular aspects of early mathematics was supported because it was significantly related to only a few individual mathematics skills—specifically cardinality, subitizing, set comparison, and number order. These differential relations are important to consider in the broader construction of models of early mathematical development as well as in the design of curricula and instructional techniques. Furthermore, these findings underscore the importance of evaluating relations between individual mathematics skills and non-mathematical domains at a relatively fine-grained level, and this is the first study to do so at the preschool and kindergarten ages with such a broad range of early mathematics skills.

Language

Findings

In building a learning trajectory of mathematical development, it is important to understand how different non-mathematical factors integrate into the model. The general relation of children's language skills with nearly all of the early mathematics skills was not unexpected but is critical to understand. The finding was not unexpected because, for each early mathematics skill, children must either (a) know number names (word knowledge that is inherently a vocabulary task), (b) connect the number names with specific quantities (connect word knowledge with words' "definitions" or "meanings"), (c) connect the number names with numerals, (d) both of the previous two items (b and c), or (e) understand the meanings of comparative terms. This relation between language and mathematics is critical to understand because language skills could play a key role in the acquisition of new knowledge and the integration of that knowledge with prior knowledge. Because language is a strong predictor of general early mathematics development (LeFevre et al., 2010; Purpura et al., 2011), and language appears to underlie each of the aspects of early mathematics, when language is underdeveloped, it is likely to be an impediment to the successful acquisition of early mathematical knowledge. This may be particularly true for children from low SES families or English language learners. Both groups have been found to exhibit significantly lower mathematical performance on achievement tests than their middle to higher SES peers or native English speakers; however, when the linguistic demands of the tests are lessened, the gap in performance also decreases (but does not completely disappear; Abedi & Lord, 2001). Thus, language may underlie the development of symbolic mathematics skills, such as those assessed in this study, as well as the expression and application of this knowledge. As such, it may be necessary to account for children's language skills when developing and individualizing interventions, general instruction, and assessments.

Interestingly, the one task for which language was not a significant predictor was subitizing—rapidly enumerating small sets without counting. This finding aligns with prior research that has found that subitizing may be part of the preverbal (or nonsymbolic) number system (Gelman & Butterworth, 2005)—even though it is the application of a cardinal number to a quantity. It is suggested that subitizing, in tandem with the ability to approximately represent large quantities (the approximate number system; Halberda, Mazzocco, & Feigenson, 2008), underlies symbolic (or verbal) mathematics. Prior research has shown that the nonsymbolic system is correlated with the symbolic system even after controlling for language and intelligence (Libertus, Feigenson, & Halberda, 2011); however, it is not clearly defined how these skills interact in their development and whether language acts as a bridge that connects the two mathematical systems. By expanding this evaluation and more fully understanding how non-mathematical domains fit into a broader mathematical learning trajectory, it may be possible to develop more accurate and reliable methods to identify the children who are likely to experience later mathematics difficulties before these difficulties manifest themselves and, thereby, provide appropriate and targeted interventions.

Implications

Some empirical evidence has suggested that a direct intervention in language skills might not have a direct and immediate impact on mathematics skills (Jordan, Glutting, Dyson, Hassinger-Das, & Irwin, 2012). However, other research has found that an intensive classroom-based *mathematics* curriculum can result in positive outcomes for *oral language* (Sarama, Lange, Clements, & Wolfe, 2012). Such findings may suggest three explanations for the relations identified in this study. First, mathematics skills actually may be the causal variable in the relation between language and mathematics. Second, general language skills may actually have an indirect relation to mathematics skills in such a way that a language intervention might not result in immediate effects on mathematical outcomes but may enable children to better benefit from subsequent mathematical interventions. Third, general language skills (in this study and others) may have served as a proxy for specific mathematical language skills whereby the aspects of language most related to mathematics may be what could change mathematical outcomes or be changed by mathematical interventions and not more general language interventions. If a language intervention (specific to mathematics or general) has a direct or indirect impact on individual aspects of children's early mathematical knowledge, then it would be important to use such an intervention as an integral part of general mathematical instruction—particularly in efforts for preventing and/or remediating mathematics difficulties. Further research from a causal framework is needed to determine the directionality of the causal relationship (or whether it is bidirectional) and whether a form of language intervention may be a necessary precursor to students benefiting from instruction in early mathematics.

Working memory

Findings

The relation of working memory to early mathematics skills appears to be more specific than the relation between language and early mathematics skills. Similar to prior findings with children in elementary school (Mannamaa, Kikas, Peets, & Palu, 2012; Nyroos & Wiklund-Hornqvist, 2012; Simmons et al., 2012), working memory skills were related to only a few early mathematics skills. Interestingly, the specific early mathematics skills that were related to working memory are typically viewed as some of the strongest predictors of later mathematics success (Kroesbergen, Van Luit, Van Lieshout, Van Loosbroek, & Van de Rijt, 2009; Palmer & Baroody, 2011; Sarnecka & Carey, 2008). Each of these tasks also required children to complete multiple steps in order to answer correctly. For example, to complete the cardinality task, children not only needed to hold the requested set size (or the type of items to be counted in the second part of that task) in their memory but also needed to perform the act of counting out a set until they reach the specified (or cardinal) number. Similarly, in the set comparison task, children needed to be able to enumerate each of the sets, hold the total set sizes in their memory, and then identify which set was the largest. Even though the set comparison and number comparison tasks are similar in overall task structure (e.g., identifying "most" or "fewest"), working memory did not significantly predict the number comparison task. This difference is likely because the working memory load was lessened in the number comparison task given that children did not need to enumerate the quantity of each number because it was already provided. This finding differs from Simmons et al.'s (2012) study, which found that working memory was significantly related to numeral magnitude comparison. However, most of the items in their task were multidigit, and the authors noted that prior studies (e.g., LeFevre et al., 2010) had not found a significant relation between working memory and single-digit comparison tasks. The task in the current study, like the task used in LeFevre et al.'s (2010) study, involved mostly single-digit numbers—likely lessening the memory demands. Finally, although some prior research has found that story problems can be related to working memory (Rasmussen & Bisanz, 2005), those findings were not supported in this study. This is likely because the story problems in the current study did not include extraneous information, they were very simple problems, and—similar to Fuchs et al. (2006)—language was accounted for in the model (cf. Rasmussen & Bisanz, 2005). Overall, the findings from this study support the predicted hypothesis that only specific—more complex—tasks would be predicted by working memory.

Implications

The relation of working memory to these specific early mathematics skills may indicate that the instructional methodology used for these skills may need to be distinct from the instructional methodology used for the other skills that were not related to working memory. Given the significant working memory demands for specific skills, it may be important to account for children's working memory skills in the instruction of these specific domains by either (a) modifying classroom curricula or individualized instructional activities (by reducing task demands, including instructional aids, or scaffolding) to minimize the impact of working memory deficits during learning or (b) intervening in children's working memory skills separate from mathematical instruction.

A number of studies have provided experimental evidence that cognitive interventions can lead to improvements in working memory (Klingberg, Forssberg, & Westerberg, 2002; Thorell, Lindqvist, Nutley, Bohlin, & Klingberg, 2009) and that there are potentially subsequent gains in general mathematical performance (Holmes, Gathercole, & Dunning, 2009). However, in Holmes et al.'s (2009) study, transfer effects on mathematics were not causal in nature because it was a pre–post comparison and there was no comparison group. Furthermore, the effects on mathematics were not found immediately after the intervention; rather, they were found at a 6-month follow-up. Further work should be conducted to evaluate the causal connections between working memory and specific aspects of mathematics skills, particularly at the preschool and kindergarten levels where the specific relations were identified in the current study.

Covariates

It is also important to note specific patterns that were found regarding the covariates. To begin, grade was not a significant covariate for any of the measures. Second, the general mathematics measure was a significant or marginally significant predictor of all variables except for one-to-one counting, numeral identification, and set to numerals. Other than age, language skills was the only variable to significantly predict the numeral identification and set to numerals tasks—suggesting that these skills may actually be better defined as distinct language-based skills rather than as "mathematics" skills. Prior research has indicated that these two domains—together termed numeral knowledge—mediate the relation between informal and formal mathematics skills (Purpura et al., 2013). Such a distinction may provide one specific connection between mathematics and reading development—particularly in identifying potential pathways linking mathematics and reading difficulties.

Limitations and future directions

The distinct relations found in this study provide important foundational information that can be used to understand the multidimensional development of early mathematics skills. However, it must be noted that these findings are not causal in nature. Additional research in a causal framework will enable researchers and educators to more clearly understand how both language and working memory might affect the development of specific mathematics skills. It is critical, from both research and practical educational perspectives, to determine whether or not intervening in one of these two domains has either a direct or indirect impact on early mathematical development.

It is also important to note that in this study we used only singular tasks of language (expressive vocabulary) and working memory (verbal). Although consistent with work by others (Fuchs et al., 2005; Wilson & Swanson, 2001), alternative aspects of language and working memory may be differentially related to various aspects of mathematical development. Notably, the working memory task used in this study was a verbal working memory task (comprising both the central executive and the phonological loop). Although recent evidence found the central executive to be highly related to the other aspects of working memory (De Smedt, Janssen, et al., 2009), other work has suggested that working memory as a whole (rather than a specific aspect of working memory) is related to mathematical development in school-age children (Swanson, 2012; Zheng et al., 2011; cf. Wilson & Swanson, 2001, for research suggesting that verbal working memory is the key working memory component related to mathematics skills). Such findings have yet to be evaluated with younger children and with the broad range of early mathematics skills. Future research should be conducted to understand whether different aspects of working memory may produce distinct patterns of results across the spectrum of early mathematics skills. Furthermore, it is also possible that IQ, which was not measured in this study, could account for some of the variance attributed to either language or working memory. This is unlikely given that we included general mathematical achievement as a covariate; however, the role of IQ warrants further investigation.

Relatedly, working memory is an aspect of the broader construct executive functioning (Lehto, Juujärvi, Kooistra, & Pukkinen, 2003; Miyake et al., 2000), and Espy et al. (2004) found that other aspects of executive functioning such as response inhibition may be more important than working memory in predicting mathematical performance. Yet, the results of such evaluations are mixed, with other studies finding that working memory predicts mathematical performance even when accounting for response inhibition (Georgiou, Tziraki, Manolitsis, & Fella, 2013; Halberda et al., 2008; Swanson & Beebe-Frankenberger, 2004). These differences are likely because, at the preschool and early elementary school levels, it has been found in some research that executive functioning is a unitary factor (Wiebe, Espy, & Charak, 2008)—meaning that the different components such as working memory, response inhibition, and attention shifting are indistinguishable at this age. Although other research does suggest that executive functioning is multidimensional at this age (Miller, Giesbrecht, Müller, McInerney, & Kerns, 2012; Schoemaker et al., 2012). Further investigation into how the structure and measurement of executive functioning affects the development of early mathematics skills is warranted.

Another key consideration is that this study focused primarily on aspects of the symbolic (or verbal) number system and did not evaluate the link of language and working memory to the "approximate" (nonsymbolic/nonverbal) number system (Dehaene, 1992). Extending this evaluation to the nonsymbolic/nonverbal mathematics system could reveal important pathways that link the nonsymbolic and symbolic mathematics system. This is particularly critical given recent evidence suggesting that executive functioning skills may underlie symbolic mathematics skills rather than nonsymbolic skills. Two recent studies (Fuhs & McNeil, 2013; Gilmore et al., 2013) found that when including measures of executive functioning (specifically inhibition) in models predicting symbolic mathematics, the relation between nonsymbolic mathematics and symbolic mathematics became nonsignificant. However, it is also possible that aspects of executive functioning are the means by which children map their nonsymbolic knowledge to the symbolic domain.

Unlike working memory, there has been little work identifying how different aspects of language relate to mathematical development with young children. In the current study, we used a measure of expressive language as a proxy for general language skills. However, language skills—like mathematics and working memory—are multifaceted. Although some research suggests that general language skills are related to mathematics (Fuchs et al., 2005; LeFevre et al., 2010; Vukovic & Lesaux, 2013b), individual components of language may be differentially related to specific aspects of early mathematics at early ages, but this has yet to be evaluated. Relatedly, certain aspects of language (e.g., receptive, expressive) could be important for the learning of new mathematical concepts, and others could be important for the expression of known concepts. In the cases of both working memory and language, further systematic research is needed to evaluate these relations at a more fine-grained level.

Conclusion

Overall, the findings from this study provide a unique framework for the evaluation of early numeracy skills that can be used to enhance both research and teaching of early mathematical development. By understanding the connections between mathematical and non-mathematical constructs, a model learning trajectory for mathematical development can be delineated—and such a model can be used to guide instruction through the identification of which mathematics and non-mathematics skills have causal relations in their development. The links of working memory and language to specific mathematics skills list forth a targeted set of future research that can help to address important developmental and instructional issues—particularly those related to instructional methodology and sequencing in preschool and kindergarten.

Acknowledgments

This work was supported by grants from the Institute of Education Sciences, U.S. Department of Education (R305B04074 and R305B100017). Views expressed in this article are solely those of the authors and were not reviewed or cleared by the grantors.

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