

Abstract Title Page.

Title: Who repeats algebra, and how does initial performance relate to improvement when the course is repeated?

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Abstract Body

Background / Context:

Although many high school students repeat algebra I, a course widely considered to be the “gateway” to advanced high school math and science (U.S. Department of Education, 1997), few studies have examined students’ performance when they repeat the course. Knowing how algebra I repeaters perform may help educators determine whether to promote a student to a higher math course (usually geometry) or have the student repeat algebra I when that student initially performs at a level that is less than ideal. This study seeks to help educators make informed math placement decisions.

Algebra I is often the first course in which students engage in the abstract reasoning and symbolism that make math powerful (Kieran, 1992; Vogel, 2008), marking an important step beyond arithmetic (Carraher & Schliemann, 2007; Kieran, 1989). Early success in algebra I leads students to take more advanced math courses (Atanda, 1999; Ma, 2000; Paul, 2005; Smith, 1996; Spielhagen, 2006) and achieve higher math scores (Gamoran & Hannigan, 2000; Kurlaender, Reardon, & Jackson, 2008; Smith, 1996).

However, many students struggle with algebra I. The National Mathematics Advisory Panel (2008, p. xii) concluded that “[a]lthough our students encounter difficulties with many aspects of mathematics, many observers of educational policy see Algebra as a central concern.” Student difficulties with algebra I, documented in national and international assessments (Blume & Heckman, 1997; Schmidt, McKnight, Cogan, Jakwerth, & Houang, 1999), may cause students to repeat the course. Approximately 212,000 California students in grades 8–11 in 2008 repeated the algebra I California Standards Test (CST), implying that these 212,000 students were most likely repeating the algebra I course (EdSource, 2009, fig. 4).¹ EdSource identified as test repeaters 2 percent of grade 8 test takers, 38 percent of grade 9 test takers, 52 percent of grade 10 test takers, and 52 percent of grade 11 test takers.

Repeating algebra I may be costly for both the student and the education system. Having to repeat a course may demoralize the student or turn the student off the subject, possibly resulting in the student not performing any better when he or she repeats the course. For instance, in a study of nine school districts in California, approximately half the students who were successful in algebra I in grade 8 and were placed in algebra I again in grade 9 received either the same or a lower grade in their second experience (Waterman, 2010). Repeating students also miss the opportunity to take a new and different course. At the education system level the approximately 212,000 students in grades 8–11 who repeated.

Purpose / Objective / Research Question / Focus of Study:

The information provided in this report shows how students perform when they repeat algebra I and how the level of improvement varies depending on initial course performance and the academic measure (course grades or CST scores). This information can help inform decisions and policies regarding whether and under what circumstances students should repeat the course.

The study examined four research questions:

- How many students repeat algebra I after taking it for the first time?
- How do student characteristics (such as race/ethnicity, gender, grade 7 math performance and initial algebra I performance) relate to the likelihood of repeating algebra I?
- How well do students perform when they repeat algebra I compared with the first time they took the course?
- How does that difference in performance vary based on student characteristics?

Setting:

This study was conducted in the East Side Union High School District, located in Silicon Valley, CA. The data for this study are from this district and five elementary school districts that feed into the East Side Union High School District: Alum Rock Union Elementary School District, Evergreen School District, Franklin-McKinley Elementary School District, Mt. Pleasant Elementary School District, and Oak Grove Elementary School District.

Population / Participants / Subjects:

The sample started with 5,391 first-time grade 7 students in 2006/07, comprising 1,356 students from Alum Rock Union Elementary School District, 1,400 students from Evergreen School District, 984 students from Franklin-McKinley Elementary School District, 337 students from Mt. Pleasant Elementary School District, and 1,314 students from Oak Grove Elementary School District. (Four grade 7 students in the 2006/07 school year who were repeating the entire grade level were not included in the sample.) The following students were dropped from the sample: 87 students who did not have a state student identification number; 65 students with multiple district or state identification numbers; 68 students in math courses with no math course grade for any school term or school year; 791 students without data showing that they took algebra I in any middle school or comprehensive high school; and 980 students without data showing math course enrollment in the years before or after taking algebra I. The final analytic sample included 3,400 students, with 632 students from Alum Rock Union Elementary School District, 1,049 students from Evergreen School District, 595 students from Franklin-McKinley Elementary School District, 208 students from Mt. Pleasant Elementary School District, and 916 students from Oak Grove Elementary School District

Intervention / Program / Practice:

The study focused on repeating algebra. The reasons students repeat algebra I likely vary by student, school, and district. Most districts rely on course grades and teacher recommendations to determine math course placement (Bitter & O’Day, 2010), with standardized math test scores, student and parent preferences, and counselor recommendations also factoring into the decision (Hallinan, 2003). Oakes, Muir, and Joseph (2000, p. 16) further note that “increasingly, school systems do not use fixed criteria to assign students to particular course levels. Teacher and counselor track-placement recommendations include, in addition to test scores and grades, highly subjective judgments about students’ personalities, behavior, and motivation.” The district analyzed in this study (East Side Union High School District) uses various placement criteria, including course grades, CST scores, teacher recommendations, and participation in summer intervention programs (Flamm et al., 2011). Although the exact reason each particular student in this study’s dataset did or did not repeat algebra I cannot be ascertained, the study’s results show that low student performance (measured by course grades and CST scores)

correlates with repeating

Research Design:

the study examined algebra repetition rates and success rates among students based on their characteristics and initial performance. To identify students who repeated algebra I, the study team first needed to identify instances of algebra I enrollment. A student who took a one-year algebra I course was defined as having taken algebra I. A student enrolled in the first year of a two-year algebra I sequence was not considered to have taken algebra I in that year, but a student enrolled in the second year of a two-year algebra I sequence was considered to have taken algebra I in that year. The two-year algebra I courses were confirmed with each of the school districts participating in the study. More specifically, the study team confirmed with each district whether successful students in each course with “algebra” in the title should have been promoted to geometry. Algebra I courses that did not promote students to geometry the following year even when the students performed very well were not defined as algebra I and were assumed to be two-year algebra I sequences.

To isolate the relationship between various student characteristics and the probability of repeating algebra I while holding other characteristics constant, a mixed-effects logistic regression was performed.

Data Collection and Analysis:

Data from the elementary feeder districts, which span from kindergarten through grade 8, were collected for the 2005/06–2008/09 school years, and data from the high school district, which spans from grade 9 through grade 12, were collected for the 2007/08–2011/12 school years. Student-level longitudinal data, collected from each district, include variables such as identification number, race/ethnicity, gender, math course name, final course letter grade received, math CST taken, CST scale score, and CST performance level (see appendix A for more detailed information about the dataset). Construction of the sample began with a cohort of first-time grade 7 students in 2006/07 who attended one of the five feeder districts included in the analysis. Because this study seeks to identify students who repeat algebra I, the sample included only students for whom data were available indicating the student’s math course enrollment in the years before and after taking algebra I. For instance, if a student took algebra I in 2008/09, but data were not available on the student’s math course enrollments in 2009/10, it could not be determined whether the student repeated algebra I in 2009/10. Similarly, if a student took algebra I in 2008/09 (and, for instance, pre-algebra in 2006/07 and geometry in 2009/10), but the student’s math course enrollments in 2007/08 could not be observed, it could not be determined whether the student was repeating algebra I in 2008/09 or taking it for the first time.

A logistic regression was used to identify the relationship between student characteristics and algebra repetition. Mixed effects were incorporated into the model to account for the fact that each high school may have its own way of determining which students repeat algebra I. The mixed-effects logistic regression used was of the following form:

$\Pr(\text{Repeater} = 1) = \text{logit}^{-1}(\beta_0 + \beta_1' \text{Characteristics} + \zeta_s + \varepsilon_{is})$, where *Repeater* is a dichotomous variable indicating that the student repeated algebra I, β_0 is the intercept, β_1 is a vector of parameters to be estimated from the data, *Characteristics* is a vector of student

characteristics, ζ is the school-level random effects (in this case a random intercept for each high school), and ε is the independent and identically distributed error term. Subscript i refers to the student and subscript s refers to the high school that the student attended. Odds ratios are reported for ease of interpretation.

Findings / Results:

Of the 3,400 students in the sample, 44.3 percent repeated algebra I. While grades and standardized test scores are the most common reasons to retake the course, other considerations may also factor in, such as parent preferences and teacher or counselor recommendations, depending on the school and district. The rates of repeating varied across student characteristics, with students in special education (69.6 percent), Hispanic students (61.1 percent), and English language learner students (56.7 percent) exhibiting the highest rates. Many, but not all, students who repeated algebra I had performed poorly when they first took the course. For instance, among students whose initial algebra I grades averaged between a “B” and an “A”, 8.4 percent repeated the course. And among students who scored “proficient” on the algebra I California Standards Test (CST) the first time they took algebra I, 22.2 percent repeated the course. Students’ performance improved on average by approximately half a letter grade and a little less than a third of a CST performance level when they repeated algebra I. But the data showed variation in improvement levels among higher achieving students. For instance, repeating students who initially received algebra I course grades averaging greater than a “C” (that is, greater than 2.0 on a numeric grading scale) had higher CST scores but lower course grades on average when they repeated the course. And students who initially scored “proficient” on the algebra I CST had higher grades but lower CST scores on average when they repeated the course. Students who initially did well in both course grades and the CST, defined as grades averaging at least a “C” and scoring at least “proficient” on the algebra I CST, had declines in CST performance and no statistically significant change in grades when they repeated the course.

Conclusions:

The information on how students of varying achievement levels perform when they repeat the course can aid educators who are making math placement decisions. More specifically, by giving a sense of how a student might perform if he or she were to repeat algebra I this information can help educators decide whether a student should repeat the course. Educators might also examine the course options available to students of different achievement levels and the processes by which students are placed in math courses. This study replicates a key finding from previous research (Waterman, 2010) that many students who initially perform well in algebra I earn lower grades when they repeat the course. However, analysis of variation in improvement levels disaggregated by course grades and CST scores suggests that Waterman’s finding could be due to regression to the mean. Although these results may weaken Waterman’s conclusion that students who initially earn high grades should not repeat algebra I, only a more rigorous study (such as a randomized controlled trial) can provide a definitive answer. While this study answered some questions, other remain. To better understand why students—especially high-performing students—repeated algebra I, interviews could be conducted with educators at each of the high schools analyzed.¹¹ For instance, repeating students may not have grasped certain content standards that the educators considered critical for success in future math classes. A further study could analyze student performance on these content standards when students repeated the course.

Appendices

Not included in page count.

Appendix A. References

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Appendix B. Tables and Figures
Not included in page count.