Effects of Intervention to Improve At-Risk Fourth Graders' Understanding, Calculations, and Word Problems with Fractions

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Abstract

The purposes of this study were to (a) investigate the efficacy of a core fraction intervention program on understanding and calculation skill and (b) isolate the effects of different forms of fraction word-problem (WP) intervention delivered as part of the larger program. At-risk 4th graders (n = 213) were randomly assigned at the individual level to receive the school's business-as-usual program or 1 of 2 variants of the core fraction intervention (each 12 weeks, 3 sessions/week). In each session of the 2 variants, 28 min were identical, focused mainly on the measurement interpretation of fractions. The other 7 min addressed fraction WPs: multiplicative WPs versus additive WPs. Children were pre- and posttested on fraction understanding, calculations, and WPs. On understanding and calculations, both intervention conditions outperformed the control group, and the effect of intervention versus control on released fraction items from the National Assessment of Education Progress were mediated by children's improvement in the measurement interpretation of fractions. On multiplicative WPs, multiplicative WP intervention was superior to the other 2 conditions, but additive WP intervention and the control group performed comparably. By contrast, on additive WPs, there was a step-down effect in which additive WP intervention was superior to multiplicative WP intervention, which was superior to control.

KEY WORDS: fractions, intervention, efficacy, word problems, measurement interpretation, risk

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Competence with fractions is important for more advanced mathematics achievement and success in the American workforce (National Mathematics Advisory Panel [NMAP], 2008; Geary et al., 2012; Siegler et al., 2012). Yet fraction knowledge, including understanding about fractions and skill in operating with fractions, is a persistent source of difficulty for many students (e.g., National Council of Teachers of Mathematics, 2007; NMAP; Ni, 2001). The NMAP thus recommended that high priority be assigned to improving fraction knowledge, especially understanding of fractions – due to its importance for learning and maintaining accurate fraction procedures (e.g., Hecht, Close, & Santisi, 2003; Mazzocco & Devlin, 2008; Ni & Zhou, 2005; Rittle-Johnson, Siegler, & Alibali, 2001).

At fourth grade, understanding includes the part-whole interpretation of fractions as well as the measurement interpretation of fractions. The part-whole relationship involves understanding that a fraction is one or more equal parts of a single object (e.g., two of eight equal parts of a candy bar) or a subset of a group of objects (e.g., two of five candy bars). Such understanding is typically represented using an area model, in which a region of a shape is shaded or a subset of objects is distinguished from other objects. This form of fraction is based on children's experiences with sharing; is more intuitive than competing interpretations; and is observed as early as 4 years-of-age (e.g., Mix, Levine, & Huttenlocher, 1999). Formal schooling is, however, required to achieve full understanding of the part-whole interpretation, and this type of interpretation dominates representation of fractions in American schooling.

The measurement interpretation of fractions, by contrast, reflects cardinal size (Hecht, 1998; Hecht et al., 2003). Often represented with number lines (e.g., Siegler, Thompson, &

Schneider, 2011), the measurement interpretation of fractions can be linked to children's experiences with measuring (e.g., with a ruler or a measuring cup), but it depends more than part-whole understanding on formal instruction that explicates the conventions of symbolic notation, the inversion property of fractions (i.e., fractions with the same numerator become smaller as denominators increase), and the infinite density of fractions on any segment of the number line.

The NMAP (2008) hypothesized that improvement in the measurement interpretation of fractions is a key mechanism in explaining the development of overall fraction knowledge. Fuchs et al. (2013, 2014) found support for this idea, documenting that fraction intervention focused on the measurement interpretation improves fraction knowledge among at-risk fourth graders and that improvement in children's measurement interpretation of fractions mediates those effects. In the present study, we attempted to replicate those findings for the same core fraction intervention program by randomly assigning at-risk children to two variants of that core fraction intervention (both focused mainly on the measurement interpretation of fractions) and a control group (in which the typical school program focused mainly on part-whole understanding).

To extend previous work, we designed the two variants of the core program to isolate the effects of two types of fraction word-problem (WP) intervention. Prior to the present study, WP intervention had not been a component of the core program, and we located no randomized control trial examining the effects of fraction WP intervention at fourth grade. We focused on WPs because, although fraction knowledge is foundational for more advanced mathematics, the best school-age predictor of employment and wages in adulthood is WPs (e.g., Every Child a Chance Trust, 2009; Parsons & Bynner, 1997; Murnane et al., 2001). A combined focus on fractions and WPs therefore represents an important instructional target.

The two intervention conditions were configured as follows. Of each 35-min intervention session, 28 min were identical across both intervention conditions. These 28 min were focused on understanding fractions, especially the measurement interpretation of fractions but included smaller foci on the part-whole interpretation of fractions (to build on students' prior knowledge and classroom instruction) and on adding and subtracting fractions. WPs were not addressed in the 28-min core program. The remaining 7 min of each session focused on WPs. In one condition, WP intervention was designed to enhance performance on multiplicative WPs (M-WPs); in the other condition, on additive WPs (A-WPs).

We were primarily interested in M-WPs because multiplicative thinking is central to fraction knowledge, as reflected in the fact that finding equivalent fractions requires multiplying or dividing the numerator and denominator in one fraction by the same quantity. As framed by Jacob and Willis (2001), multiplicative thinking develops in stages. After basic counting insights are achieved, children come to appreciate additive composition, with which they recognize equal groups within sets. This permits skip counting or repeated addition, with which the focus is on the multiplicand, not the multiplier. Eventually, children learn to count groups of objects while tracking the number of repetitions, hence keeping track simultaneously of the multiplier and the multiplicand. Gradually, they achieve full understanding of the relationship between the number in each group and the number of groups, as well as the total in multiplicative situations. This permits understanding of the inverse relationship between multiplication and division.

This developmental process can be difficult to achieve with whole numbers. Yet, extending that understanding to real-life multiplicative situations that involve fractions is a major stumbling block for many students, in part because multiplying fractions typically results in smaller quantities just as dividing fractions produces larger amounts. This violates multiplicative expectations based on whole numbers.

We therefore designed the present study's focus on "splitting" and "grouping" WPs. Examples of splitting and grouping WPs, respectively, are: "Matthew has 2 watermelons. He cuts each watermelon into fifths. How many pieces of watermelon does Matthew have?" and "Keisha wants to make 8 necklaces for friend. For each necklace, she needs ½ of a yard of string. How many yards of string does Keisha need?" The hope was to develop competence with fraction WPs that relate more to multiplicative reasoning than the competing condition and thereby extend fourth graders' understanding of fractions (while creating the foundation for fifth-grade Common Core Standards involving multiplying and dividing with fractions). The contrast (additive) WP condition focused on fraction "increase" and "decrease" WPs. Examples of increase and decrease WPs, respectively, are: "Maria bought 1 4/10 pounds of candy. Later she bought another 3/10 of a pound of candy. How many pounds of candy does Maria have?" and "Jessica had 5/6 of a cake. She gave 2/6 of that cake to her friend. How much cake does Jessica have now?" These WP types represent fourth-grade Common Core State Standards and, in the present study, also controlled for instructional time in assessing the effects of the M-WP condition.

Our major outcomes, on which we pre- and posttested students, were the measurement interpretation of fractions, calculation skill (adding/subtracting fractions), released fraction items from the National Assessment of Educational Progress (NAEP; U.S. Department of Education, 2010; easy, medium, and hard fourth-grade items as well as easy eighth-grade items), and both types of WPs. We expected both intervention groups to outperform the typical school program on each outcome. Also in line with the NMAP's (2008) hypothesis, we expected that improvement on the core program's major proximal outcome, the measurement interpretation of fractions, would mediate effects on the study's far-transfer measure of general fraction knowledge (NAEP), but that improvement in part-whole understanding would not mediate those effects.

In terms of fraction WP performance, we expected advantages between the two intervention conditions to be specific to the WP types targeted by the intervention conditions. At the same time, because fraction competence depends on multiplicative reasoning (Harel & Confrey, 1994), we also expected children in the M-WP condition to perform better than children in the A-WP condition on the measurement interpretation of fractions (the near-transfer fraction understanding measure). Moreover, because A-WPs provided additional practice in adding and subtracting fractions (whereas solutions to M-WPs were derived using multiplicative arrays), we expected A-WP to show an advantage over the M-WP condition on our fraction calculations (adding/subtracting) measure.

Method

Participants

We defined risk as performance below the 35th percentile at the start of fourth grade on a broad-based calculations test (Wide Range Achievement Test–4 [WRAT]; Wilkinson & Robertson, 2006). We sampled half the at-risk (AR) students from < the 15th percentile; the other half between the 15th and 34th percentiles. Because this study was not about intellectual disability, we administered the 2-subtest Wechsler Abbreviated Scales of Intelligence (WASI; Wechsler, 1999) to students who met the risk criterion and excluded 18 children with T-scores below the 9th percentile on both subtests. We sampled 3-9 AR students per class, stratifying by more versus less severe risk in each classroom. After WASI exclusions and children moving to non-study schools during pretesting, the sample comprised 231 AR students from 45 classrooms in 14 schools. We randomly assigned these students at the individual level, stratifying by class and risk severity, to the three study conditions: the core program with M-WP (*n* = 77), the core program with A-WP (*n* = 78), and business-as-usual control (*n* = 76). Of the 231 AR students, 18 moved (5 M-WP; 7 A-WP; 6 control) after random assignment to schools beyond the study's reach. These students did not differ statistically from the remaining AR students on pretest measures and did not differ significantly on any pretest measure by condition. We omitted these children, leaving 213 students in the final AR sample: 72 M-WP, 71 A-WP, and 70 control.

To understand the extent to which intervention versus control helped close achievement gaps, we compared AR students' year-end performance on group-administered fraction measures against performance of not-at-risk (NAR) classmates. We randomly sampled 339 NAR classmates (>34th percentile) to represent each class in similar proportion to AR students. Among these NAR students, the 19 who moved did not differ from remaining students on pretest measures. We omitted them, leaving 320 in the NAR comparison group.

On the screening measure, NAR students performed reliably higher than each of the AR groups, which performed comparably. The mean WRAT standard score was 105.33 (*SD* = 7.03) for NAR; 85.08 (*SD* = 7.86) for M-WP; 85.58 (*SD* = 7.00) for A-WP; and 86.73 (*SD* = 7.66) for control. There were no significant differences among AR conditions on WASI IQ, with mean standard scores of 95.31 (*SD* = 10.92) for M-WP; 93.56 (*SD* = 11.14) for A-WP; and 96.44 (*SD* = 12.40) for control. The AR groups were demographically comparable. In M-WP, A-WP, and control, respectively, percentage of females was 54, 54, and 61; the percentage of English learners was 11, 23, and 16; percentage on subsidized lunch was 93, 86, and 86; and percentage receiving special education was 17, 17, and 10. In M-WP, percentages of African-American, white, Hispanic, and other students were 57, 18, 21, and 4; in A-WP, 58, 10, 24, and 8; and in control, 59, 19, 17, and 5 (all Hispanic students were white). We did not collect demographic data or individually administered test data on NAR students. (Because the study was not about NAR students, we judged it inappropriate to spend their school time on the individual assessment battery or to ask teachers to complete demographic forms on these students. Also, resources did not permit us to administer the individual test battery to an additional 290 children.)

Screening Measures

The mathematics screening measure was *WRAT-4-Arithmetic* (Wilkinson & Robertson, 2006), with which students complete calculation problems of increasing difficulty. Alpha on this sample was .74. The IQ screening measure was the *WASI* (Weschler, 1999), which includes two tests. With *Vocabulary,* students identify pictures and define words. With *Matrix Reasoning,* students select 1 of 5 options that best completes a visual pattern. Reliability exceeds .92.

Fraction Measures

To assess the measurement interpretation of fractions, we used *Fraction Number Line* (Hamlett, Schumacher, & Fuchs, 2011, adapted from Siegler et al., 2011), which requires students to place proper fractions, improper fractions, and mixed numbers on a number line labeled with endpoints (0 and 2). In each trial, a target fraction appears in a large font above the number line. Students practice with two fractions and then estimate the location of 20 items: 12/13, 7/9, 5/6, ¼, 2/3, ½, 1/19, 3/8, 7/4, 3/2, 4/3, 7/6, 15/8, 1 1/8, 1 1/5, 1 5/6, 1 2/4, 1 11/12, 5/5, and 1. Items are presented in random order. The score for each item is the absolute difference between the child's placement and the correct position. Scores are averaged across the 20 items and divided by 2. When multiplied by 100, scores are equivalent to the percent of absolute error (PAE), as reported in the literature. Lower scores indicate stronger performance (but in some analyses, we multiplied scores by -1). Test-retest reliability, on a sample of 63 students across 2 weeks, was .80.

To index skill with fraction procedures, we administered two subtests from the 2012 *Fraction Battery* (Schumacher, Namkung, Malone, & Fuchs, 2012). *Fraction Addition* includes five addition problems with like denominators and seven addition problems with unlike denominators, six presented vertically and six horizontally. *Fraction Subtraction* includes six subtraction problems with like denominators and six with unlike denominators, six presented vertically and six horizontally. For each subtest, administration is terminated when all but two students are finished. One point is awarded for finding the correct numerical answer; 2 if the item is appropriately reduced one time (7 items on addition; 8 items on subtraction); 3 if the item is appropriately reduced two times (1 subtraction item). We used the total score across the tests, which correlated .83. The maximum score is 41. Alpha on this sample was .91 - .94.

To index generalized learning about fractions, we administered 19 released items from 1990-2009 *NAEP* (U.S. Department of Education, 2010): easy, medium, or hard fraction items from the fourth-grade assessment and easy from the eighth-grade assessment. Testers read each problem aloud (with up to one rereading upon request). Eight items assess the part-whole interpretation (e.g., given a rectangle divided into six equal parts, students are directed to shade 1/3); nine assess measurement interpretation (e.g., given four lists of three fractions, students identify which are arranged from least to greatest); one requires subtraction with like denominators; and one asks how many fourths make a whole. Students select an answer from four choices (11 items); write an answer (3 items); shade a portion of a fraction (1item); mark a number line (2 items); write a short explanation (1 item); or write numbers, shade fractions, and explain answers (1 problem with multiple parts). The maximum score is 25. Alpha on this sample was .84 - .87.

To index M-WP skill, we used *Multiplicative Word Problems* from the *2012 Fraction Battery* (Schumacher et al., 2012), which includes six WPs requiring students to make fractions from units (the "splitting" problem type), six WPs requiring students make units from fractions (the "grouping" problem type), and two distractor problems requiring students to compare two fraction quantities (e.g., Ruby ate ¼ of the pizza, and Bob ate 1/8 of the pizza. Who ate less pizza?). None of the WPs on this or the A-WP measure was used for instruction. Two splitting WPs rely on the vocabulary/question *structure* used in instruction (e.g., Lauren has 3 yards of ribbon. She cuts each yard of ribbon into sixths. How many pieces of ribbon does Lauren have now?); four include novel vocabulary and/or questions (e.g., Jamie has 5 cups of batter to make cupcakes. Each cupcake needs 1/2 cup of batter. How many cupcakes can Jamie make? [novel vocabulary and question because *5 cups of batter* is the unit and *cupcakes* are the "pieces," when students generally think of *cupcakes* as a unit]). Four grouping WPs incorporate unit fractions, as in instruction; two include non-unit fractions. The tester reads each item aloud while students follow along on their copy. The tester progresses to the next problem when all but two students are finished. Students can ask for the problem to be reread. For each problem, 1 point is awarded for finding the correct numerical value and 1 point for finding the correct label (e.g., pieces of ribbon). The maximum score is 26 (for each of the two distractor problems, only 1 point can be earned for finding the numerical value). Alpha on this sample was .85 - .90.

To index A-WP skill, we used *Additive Word Problems* from the *Fraction Battery-2012-*revised (Schumacher et al., 2012), which includes six WPs of the change-increase problem type (e.g., Deshaun rode his bike 6/8 of a mile last Saturday. He rode another 5/8 of a mile this

Saturday. How many miles did Deshaun ride his bike?), six WPs of the change-decrease problem type (e.g., Paul bought 9/10 pound of jellybeans. He ate 3/10 pound of those jelly beans at the movies. How many pounds of jelly beans does Paul have left?), and two distractor problems requiring students to compare fraction quantities (see preceding paragraph). Of six increase WPs and of six decrease WPs, two include fractions with same denominators, two with different denominators, and two with mixed numbers. See preceding paragraph for administration. For each problem, 1 point is awarded for finding the correct numerical value; 1 point if the item is appropriately reduced (possible to earn for three increase WPs and five decrease WPs); and 1 for writing a correct label. The maximum score is 34. Alpha on this sample was .81 - .87.

Classroom Fraction Instruction

To describe classroom fraction instruction, we relied on two sources: our analysis of the fraction components of the district's fourth-grade mathematics program, enVisionMATH (Scott Foresman-Addison Wesley, 2011), and a questionnaire we administered to classroom teachers, each of whom had students in the intervention and control conditions.

enVisionMATH addresses fractions at fourth grade in two units: Understanding Fractions and Adding/Subtracting Fractions, with 70% of lessons allocated to understanding fractions. For understanding fractions, the program relies mainly on part-whole understanding by using shaded regions and other area model manipulatives, while encouraging students to write and draw when explaining fraction concepts. In a single lesson, benchmark fractions and equivalent fractions address magnitude decisions (number lines are not used). Adding/Subtracting Fractions are taught via procedural rules. Fraction WPs are addressed by focusing dominantly on additive WPs and equal sharing WPs (e.g., Eight friends divide 3 pizzas equally. How much does each friend eat?), with a smaller emphasis on multiplicative WPs (e.g., Danielle bought 3 ¼ yards of ribbon. How many pieces of ¼-yards of ribbon did Danielle buy?).

According to What Works Clearinghouse (WWC; U.S. Department of Education, 2013), enVisionMATH has "potentially positive effects." WWC reported an improvement index of six percentile points. In our teacher survey, 2.2% of teachers reported relying exclusively on this program, but the majority (71.8%) reported relying on a combination of enVisionMATH and the Common Core State Standards (CCSS), while 26.1% indicated they relied exclusively on CCSS.

Mathematics Instructional Time for Intervention versus Control

On the questionnaire, teachers reported the amount of supplemental mathematics (beyond the classroom mathematics program) AR control students received: an average of 57.43 min (*SD* = 25.39) per week. This included instruction during the school's intervention period (33.93, *SD* = 21.02), after-school one-to-one tutoring (17.29, *SD* = 12.16), school-day intervention as part of individual educational plans (6.00, *SD* = 4.83), and other (0.21, *SD* = 1.79). This, combined with 300 min of weekly classroom instruction (an average of 60 min per day), sums to 357.43 min of mathematics instruction per week. Teachers also reported that intervention students typically received the present study's intervention (105 min per week) during part of the classroom teacher's math instructional period. On average, intervention students' weekly mathematics instructional time, including the present study's intervention, was 349.84 min, with no meaningful difference across conditions (350 vs. 357).

Distinctions in Fraction Instruction among Conditions

See Table 1 for responses on teacher questionnaires concerning the nature of fraction instruction. For the types of fraction representations used, teachers distributed 100 points across response options. Part-whole representations (tiles, circles, pictures with shaded regions, blocks) constituted 74.71% of their emphasis; number lines (reflecting measurement interpretation) accounted for ~20%. By contrast, number lines reflected 70% of the intervention program's emphasis, with only 30% allocated to part-whole representations.

For helping students understand the relative magnitude of fractions, teachers relied most strongly on finding common denominators, an activity that can be but, as practiced in the present study's classrooms, was not typically focused on understanding why fractions have differing magnitudes. When combined with cross multiplying, a strategy that typically circumvents fraction understanding, teachers indicated ~50% of the instructional emphasis for understanding fraction magnitudes was procedural. Yet, teachers also reported almost comparable emphasis (43%) on activities/strategies that address meaningful ideas: thinking about relative placement on number lines, comparing fractions to benchmark fractions, using manipulatives, and considering the meaning of the numerator and denominator. Even so, 90% of intervention program's activities focused on these conceptual strategies for comparing fractions.

Teachers reported a variety of strategies to help students understand WP narratives: drawing pictures, writing an equation, using words to explain thinking, and making tables to represent WP stories. They did, however, also indicate 16% of their instructional emphasis was key words, which can discourage deep thinking about WP narratives and often produce incorrect solutions (Schumacher & Fuchs, 2011). By contrast, the A-WP condition did not address key words and instead relied largely on writing equations to represent the underlying WP structure, using words to explain thinking, and identifying problems as belonging to problem types. The M-WP condition shared the A-WP condition's emphasis on using words to explain thinking and identifying problems as belonging to problem types. However, instead of writing equations to represent the underlying WP structure, the M-WP condition taught students to use arrays to represent the multiplicative structure of WPs.

Therefore, there were four major distinctions between the control group versus the two intervention conditions. First, the control group focused mainly on part-whole understanding, whereas the intervention conditions emphasized the measurement interpretation of fractions. Second, the control group addressed some advanced skills not covered in the intervention conditions (e.g., estimation). Third, the control group did not restrict the range of fractions, whereas intervention conditions limited the pool of denominators to 2, 3, 4, 5, 6, 8, 10, and 12 and the pool of equivalent fractions and reducing activities to 1/2, 1/3, ¼, 1/5, and 1/1. Fourth, control group WP instruction focused more on drawing pictures, making tables, and key words, while the intervention conditions focused more on using words to explain thinking, identifying problems as belonging to WP types, and representing the structure of WPs. To represent the structure of WPs, the M-WP and A-WP conditions differed: M-WP instruction relied on arrays; A-WP instruction on number sentences.

The Two Intervention Conditions

Of each 35-min intervention session, 28 min were identical in the two intervention conditions. Each intervention condition was delivered in groups of two children. Tutors were full- or part-time employees of the research grant. Some were licensed teachers; most were not. Each was responsible for 2-4 groups, distributed across the M-WP and A-WP conditions. To avoid contamination, we color coded materials, regularly monitored fidelity of implementation tapes (see below), and provided guidance in bi-weekly meetings (see below).

Tutors were initially trained in a week-long workshop. Follow-up trainings occurred bi-weekly for 1 hour to provide opportunities for (a) dynamic feedback as the fraction lessons progressed in difficulty and (b) solving problems related to students' challenging behavior and skill-level differences in dyads. The intervention program, *Fraction Face-Off!* (Fuchs, Schumacher, Malone, & Fuchs, 2015), was organized in a manual that included materials and lessons guides for the 36 lessons. *Fraction Face-Off!* is a revision of Fuchs et al. (2013; 2014). The lesson guides provide a model for each lesson and the language of explanations. Tutors reviewed but did not read from or memorize guides. Prior to delivering lessons, tutors practiced delivering lessons to fellow tutors. All this helped promote a high level of implementation fidelity while preserving teaching authenticity and responsiveness to student misunderstandings.

Content. The focus on the measurement interpretation of fractions was achieved primarily through instruction and activities involving comparing, ordering, placing fractions on number lines, and equivalencies. To build on classroom instruction, this focus was preceded by attention to part-whole interpretation (e.g., showing objects with shaded regions) and equal sharing examples. Number lines, fraction tiles, and fraction circles were used throughout the 36 lessons. Initial instruction relied on a combination of part/whole relations and equal sharing; then, the focus emphasized measurement understanding. See Table 2 for the sequence in which topics were introduced. Note that (a) after a topic was introduced, it was cumulatively reviewed; (b) ~85% of content was allocated to understanding fractions and WPs (rather than calculations); and (c) A-WPs (but not M-WPs) required adding/subtracting fractions to find solutions.

Lesson activities. Each 35-min lesson comprised six activities. Activity names reflected a sports theme, as in *Fraction Face-off!* In "Word-Problem Warm-Up" (7 min; introduced in lesson 7), students received instruction on M-WPs or A-WPs. In "Training" (8-12 min), tutors introduced concepts, skills, problem-solving strategies, and procedures, while relying on manipulatives and visual representations. "The Relay" (8-12 min) involved group work on concepts and strategies taught during that day's Training. Students took turns completing problems while explaining their work to the group. All students simultaneously showed work for each problem on their own papers. The Training and Relay activities together lasted 20 min. "Sprint" (2 min; introduced in lesson 10) provided strategic, speeded practice on four measurement interpretation topics: identifying whether fractions are equivalent to ½; comparing the value of proper fractions; comparing the value of a proper and an improper fraction; identifying whether numbers are proper fractions, improper fractions, or mixed numbers. In "The Individual Contest" (5 min) and "The Scoreboard" (1 min), students independently completed paper-pencil problems on that day's Training topics with cumulative review. Tutors scored work and provided corrective feedback. In the first three weeks, the Training and Relay were extended to account for the full 35 min. In the last 2 weeks, the Training and Relay were replaced with "The Fraction Championship," in which students competed by solving fraction problems of varying difficulty, with differing pre-determined point values.

Commonalities and Distinctions between the M-WP and A-WP Intervention Conditions

Instructional methods and fraction content in Training, The Relay, Sprint, The Individual Contest, and The Scoreboard were identical across the two intervention conditions. WP instruction occurred during the first (7-min) activity, The Word-Problem Warm-Up, when instruction focused on M-WPs or A-WPs, and one side of the Individual Contest differed to provide practice on M-WPs or A-WPs. The instructional approach to WPs in the two intervention conditions was rooted in schema-based instruction (e.g., Fuchs et al., 2003, 2009, 2010; Jitendra & Star, 2012; Jitendra, Star, Rodriguez, et al., 2011; Jitendra, Star, Starosta, et al., 2009), which teaches students to (a) identify WPs as belonging to WP types that share structural features and (b) represent the underlying structure of the WP type with a number sentence (e.g., Fuchs et al., 2003, 2009, 2010) or visual display (e.g., Jitendra & Star, 2012, Jitendra, Star, Rodriguez, et al., 2011; Jitendra, Star, Starosta, et al., 2009). In both conditions, once both WP types in that condition had been taught, we embedded distractor WPs in practice. These distractor WPs required students to identify the larger or smaller fraction. The goal was to increase students' ability to recognize non-examples of the taught WPs types and decrease the tendency to overgeneralize the taught WP-solving strategies.

Multiplicative Reasoning (M-WP) condition. The two M-WP types taught were "Splitting" and "Grouping" WPs. Splitting WPs were introduced first, with an intact story (no missing value) describing a "Splitting" problem. For example, "Melissa had two lemons. She cut each lemon in half. Now she has 4 pieces of lemon." Tutors presented the intact story and used fraction circles (units and halves) to show the meaning of the narrative: two lemons, cut in half, resulting in 4 pieces. Then, the tutor presented the same story, but substituting the final sentence with a question asking, "How many pieces of lemon did she have now?" Tutors completed a worked example, providing a rationale for each step of the WP-solving strategy and used fraction circles to explain solution methods. To wrap up the lesson, tutors explained that WPs with the same structure as the "Melissa" problem are called "Splitting" WPs: a WP describing a unit being cut, divided, or split into equal parts. Tutors also explained that whenever students identify a Splitting WP, they should use the Splitting WP solution strategy they will learn and practice.

On the second day of WP instruction, tutors reviewed the underlying structure of the Splitting WP type. Then, they presented a novel Splitting problem and asked students to name it and explain why it was a Splitting problem. Tutors taught a series of strategic steps to help students organize their papers, synthesize information in the WP, and solve the problem. First, students underlined the unknown. Second, they identified the units and the size of each piece and labeled these “U” and “S” in the WP. Third, students created an array to represent the underlying structure of the splitting WP type. Fourth, students entered the array information provided in the WP to show each unit divided into fractional pieces (e.g., for each unit divided into fifths, 1/5 was written to represent each piece for each unit). Finally, students solved the WP and wrote their numerical answer and word label.

Instruction focused on Splitting problems for the first three weeks of the Word-Problem Warm-Up activity (Lessons 7-15 of the larger program). After two days, students practiced one WP per day during Warm-Up. To challenge and extend students’ identification of Splitting WPs and encourage flexibility in the WP-solving strategies we had taught, we incorporated transfer features that altered the typical language in splitting WPs. We taught students to recognize different vocabulary, where we taught students synonyms for *pieces* (e.g., *wedges*, *slices*). We taught students to recognize a different, unfamiliar question format. For example, consider this Splitting WP: The relay race is 4 miles. Each leg of the race is ½ mile. How many kids do we need to run the relay race? It has novel vocabulary and a novel question because there are no familiar vocabulary words initiating a Splitting or dividing action and because *kids,* the “pieces” in this WP, are not typically thought of as pieces.

In Lesson 16 of the larger program, tutors introduced “Grouping” problems (e.g., Keisha wants to make 8 necklaces for friend. For each necklace, she needs ½ of a yard of string. How many yards of string does Keisha need?), using parallel methods but with two major distinctions that clarified the underlying structure of Grouping problems. First, students identified “items” (instead of units), which refers to how many fractional pieces are needed. Second, the array was structured to accommodate a different set of WP-solving strategies representing the underlying structure of Grouping problems. Beginning in Lesson 22 of the larger program, the M-WP condition focused on discriminating between the two WP types and applying WP-solving strategies correctly. For Grouping problems, non-unit fractions were introduced in Lesson 27, which required an additional WP-solving step and increased difficulty.

Distinctions: A-WP condition. Students were taught to solve two WP types, “Increase” and “Decrease,” using methods parallel methods. For Increase WPs, the WP-solving sequence was (a) underline the unknown, (b) circle the start and increase value, (c) write a number sentence to represent the structure of increase WPs, and (d) solve for the unknown and write a numerical answer and word label. Instruction focused on Increase problems for the first two weeks of Word-Problem Warm-Up (Lessons 7-15 of the larger program). After two days, students practiced one WP each day during Warm-Up.

Decrease problems were introduced in Lesson 13, again using parallel methods. Tutors taught students to distinguish Increase from Decrease WPs by focusing on the action in the story. We used similar cover stories to highlight the underlying structural differences. In Lesson 19 of the larger program, instruction focused on discriminating between the two WP types. We increased difficulty by incorporating mixed numbers (rather than by addressing vocabulary or language transfer features). We incorporated the same distractor problems to increase students’ ability to recognize non-examples of Increase and Decrease problems. After both WP types were taught and practiced, the warm-up activity focused on cumulative review.

The M-WP and A-WP conditions were distinguished by the mathematical structure of the WPs. Students followed a similar set of WP-solving strategies in the competing conditions. These strategies taught students to synthesize WP information in ways that focused on the underlying structure of the WP, name the WP type, organize their papers, center WP-solving on an array (for M-WPs) or number sentence (for A-WPs) that represented in the underlying structure of the WP type, and show their work numerically while using words to underscore their WP-solving thinking process. Teaching in this way aligns to the CCSS and prompts students to demonstrate understanding. One major difference between conditions was that instruction for the M-WP condition taught students to represent the underlying structure of the WPs by constructing an array, whereas A-WP instruction taught students to use a number sentence. This distinction was necessary given the nature of the underlying structure of the WP types and the processes required to solve the different WP types.

Promoting Task-Oriented Behavior in Both Intervention Conditions

We encouraged students to regulate attention/behavior and work hard. Tutors taught students that *on-task behavior* means listening carefully, working hard, and following directions and that on-task behavior is important for learning. Tutors set a timer to beep at three unpredictable times during each lesson, so students could not anticipate intervals. If all students were on task when the timer beeped, all students received a checkmark (if any one student was off task, no one earned a checkmark). Students also earned check marks for correct work as follows. For The Individual Contest, two bonus point problems were pre-designated to tutors (one on fraction content and one on WPs). Students were not told which problems would earn a bonus point until all students completed work. During the Scoreboard activity, tutors tallied checkmarks and awarded students a “half dollar” for each checkmark earned and each correct bonus problem. During Sprint, students earned a dollar or whole dollar, depending on the number of days it matched or exceeded the previous week’s score. On the last session of each week, tutors opened the “Fraction Store,” where students spent earnings on small prizes ($1, $7, $13, or $20). Students exchanged half dollars for whole dollars to determine which prizes they could afford or saved for more expensive prizes in the future. In Lesson 19, we introduced quarter dollars by increasing the number of bonus problems for The Individual Contest.

Fidelity of Implementing the Intervention

Every intervention session was audiotaped. We randomly sampled 20% of 2,630 recordings such that tutor, student, and lesson were sampled comparably. A research assistant listened to each sampled tape, while completing a checklist to identify the essential points the tutor implemented. The mean percentage of points addressed was 98.41 (*SD* = 0.96) in the M-WP condition and 97.54 (*SD* = 1.75) in the A-WP condition. For the 7-min WP component, the mean percentage of points addressed was 98.74 (*SD* = 1.29) for the M-WP condition and 97.07 (*SD* = 2.77) for the A-WP condition. Two research assistants independently listened to 20% (*n* = 113) of the 562 recordings to assess concordance. The mean difference in score was 1.63%.

Procedure

As per the study design and the Institutional Review Board, we did not administer individual assessments or collect demographic data on NAR classmates. The study occurred in four steps. In August/September, for *screening*, testers administered the WRAT in large groups and then administered the WASI individually to students who met the WRAT criterion for AR status. In September/October, to assess *pretreatment comparability among study groups on fraction knowledge*, testers administered NAEP, Fraction Addition and Subtraction, and the two WP measures in three large-group sessions and administered Fraction Number Line in an individual session. *Intervention* occurred for 12 weeks, 3 times per week for 35 min per session from late October to late February. In early March, we assessed *intervention effects* by re-administering NAEP, Fraction Addition and Subtraction, and the WP measures in three large-group sessions and re-administering Fraction Number Line in one individual session. All individual testing sessions were audiotaped; 20% of tapes were randomly selected, stratifying by tester, for accuracy checks by an independent scorer. Agreement on test administration and scoring exceeded 98%. Testers were blind to conditions when administering and scoring tests.

Results

See Table 3 for pretest, posttest, and posttest means adjusted for pretest scores on the fraction outcome measures for the AR conditions and for NAR classmates. In the last three columns, Table 3 also shows the magnitude of the pretest and posttest achievement gaps (with respect to NAR classmates) for each AR condition. These gaps are expressed as effect sizes (raw score difference in means, divided by the NAR students' *SD*). Table 4 shows results of analyses testing intervention effects and effect sizes comparing the three AR conditions (differences in adjusted posttest scores divided by the pooled posttest *SD*). Table 5 shows means and *SD*s for and correlations among measures used in the mediation analyses for AR students.

Preliminary Analyses

We first conducted preliminary analyses to evaluate the nested structure of the data (i.e., a cross-classified partially nested design in which nesting occurred at the classroom level for all three AR conditions and at the tutoring-group level for the two tutoring conditions). We began by estimating the proportion of variance in each fraction outcome measure due to classrooms and to tutoring groups. For classrooms and tutoring groups, respectively, these intraclass correlations were negligible to small (~.00 and ~.00 for number line; .04 and .01 for calculations; .01 and .05 for NAEP; ~.00 and ~.00 for M-WPs, and ~.00 and .02 for A-WPs). Then, we examined the random effects model for only the two active tutoring conditions while accounting for the nesting involved in the tutoring groups. Results indicated the random effects due to tutoring clusters could be ignored. Next, having dropped the tutoring clusters and addressing all three AR groups together, we ran multilevel regression models examining intervention effects on each fraction outcome while accounting for nesting at the classroom level. Results did not alter any inferences based on single-level models. Given these results along with the fact that random assignment was conducted at the individual student level, we report single-level analyses.

We then confirmed that pretest performance of the three AR groups on each fraction measure was comparable. Next, because we relied on a residualized change approach to analyze effects of study condition (i.e., covarying pretest performance to reduce within-group error variance), we assessed the homogeneity of regression assumption, which was met for all measures except M-WPs, $F(2,207) = 21.80$, $p < .001$. Therefore, in the model involving that measure, we controlled for the interaction between pretest M-WP scores and condition.

Does Intervention Enhance the Measurement Interpretation of Fractions, Fraction Calculations, and Generalized Fraction Knowledge over Control Group Performance?

To test our first hypothesis, that intervention results in superior learning compared to the business-as-usual condition, we conducted 1-way analyses of covariance (with treatment condition as the factor) on each fraction outcome, while controlling for the pretest score on the relevant measure. (For the M-WP outcome, we also controlled for the interaction between pretest and condition.) As shown in Table 4 and Figure 1, the intervention effect was significant on each outcome. To evaluate pairwise comparisons for significant effects, we used the Fisher least significant difference post hoc procedure (Seaman, Levin, & Serlin, l991). Posttest scores on number line, NAEP, and calculations, controlled for pretest scores, were stronger for each of the two intervention conditions compared to the control group. Effects sizes (ESs; difference between adjusted means, divided by the pooled unadjusted *SD*) favored M-WP over control group and ranged from 0.44 to 1.22; ESs favoring A-WP over control ranged from 0.33 to 1.70.

So in support of the first hypothesis, the fraction learning of each intervention condition exceeded that of the control group on each measure.

Does Improvement in the Measurement Interpretation of Fractions Mediate the Effects of Intervention versus Control?

To assess our second hypothesis, that improvement in the measurement understanding of fractions mediates the effects of intervention over control, we followed Preacher and Hayes (2008) and Hayes and Matthes (2009), using an ordinary least squares path analytical framework. For the indirect effect, we used bootstrapping estimation with 5000 draws to estimate standard errors and 95% confidence intervals; confidence intervals that do not cover zero are statistically significant. For this analysis, we contrasted control against intervention (i.e., combined the two intervention conditions into one group) and used standard scores. We focused on the NAEP outcome because NAEP is the most multi-faceted, distal, generalized outcome of fraction knowledge in the present study. To index the mediator variable, improvement (i.e., pretest to posttest gain) in the measurement interpretation of fractions, we relied on the number line task because it is a widely used and accepted measure of this construct (e.g., Siegler et al., 2011; Siegler & Pyke, 2013). We multiplied number line scores by -1 to simplify interpretation.

For path analytic mediation analysis, the causal steps involve the following series of statistical relations. First, the independent variable (the effect of intervention vs. control) must be associated with the dependent variable (NAEP). This is the *c* path. It establishes an effect to mediate. Second, the independent variable must be associated with the mediator. This is the *a* path, which provides a test of the action theory. Third, the mediator must affect the dependent variable. This *b* path substantiates that the mediator is related to the dependent variable. Fourth, the indirect (or mediated) effect, which is the product of the *a* and *b* paths ($a*b$), must be significant. This is equivalent to testing whether adding the mediator changes the relation between the independent and dependent variable (*c* is the relation before the mediator is added; c' is the relation after the mediator is added). If so, showing the direct effect is no longer significant in the face of a significant indirect effect provides evidence for complete mediation; if the direct effect remains significant in the face of a significant indirect effect, the mediation effect is partial. Identifying whether the direct effects of intervention are mediated by the measurement interpretation of fractions provides insights into the process by which intervention effects occur (but we remind readers that mediation analyses are correlational; so causation should not be inferred).

Figure 2, Panel A, depicts results. The a, b, and c' path coefficients and standard errors (S.E.s) are along the arrows. R^2 for this model was .38, F (3,209) =42.87, $p < .001$. The effect of intervention versus control on number line improvement (the *a* path) was 0.80 (S.E. = .13), $t = 6.24$, $p < .001$. The effect of number line improvement on the NAEP outcome (the *b* path) was 0.18 (S.E. = .06), $t = 2.87$, $p = .005$. The model partitioned the total intervention effect of 0.37 into direct and indirect effects. The coefficient for the direct effect (after the mediator was added to the model) was 0.23 (S.E. = .13), t=1.84, $p = .067$ (a 38% reduction in the total effect). The coefficient for the indirect effect (the c' path) of 0.14 (S.E. = .05) was significant (CI = .0557 to .2811). Thus, improvement in measurement understanding fully mediated the effect of intervention versus control on NAEP. Other coefficients were 0.16 (S.E. = .10), t=1.55, $p = .123$ for the constant and .51 (S.E. = .06), t=9.02, $p < .001$ for pretest.

We could not use an analogous method to assess the mediating role of improvement in part-whole understanding on the NAEP outcome, because our only index of part-whole understanding was based on a subset of the NAEP items. We therefore contrasted two additional analyses, in which we first assessed whether improvement in the NAEP-Measurement score mediated the effect of intervention versus control on the total NAEP scores and then assessed whether improvement in the NAEP-Part/Whole score mediated the effect of intervention versus control on the total score.

Figure 2 Panel B shows the model that assessed improvement in NAEP-Measurement as the mediator. R^2 for this model was .20, F (3,209) =17.18, $p < .001$. The effect of intervention versus control on improvement in NAEP-Measurement (the *a* path) was 0.57 (S.E. = .14), $t = 3.98$, $p < .001$. The effect of improvement in NAEP-Measurement on the NAEP total score outcome (the *b* path) was 0.58 (S.E. = .04), $t = 14.40$, $p < .001$. The model partitioned the total intervention effect of 0.37 into direct and indirect effects. The coefficient for the direct effect (after the mediator was added to the model) was 0.04 (S.E. = .09), t=0.51, $p = .609$ (an 89% reduction in the total effect). The coefficient for the indirect effect (the c' path) of 0.33 (S.E. = .08) was significant (CI = .1789 to .5116). Thus, improvement in NAEP-Measurement fully mediated the effect of intervention versus control on the NAEP total score. Other coefficients were 0.03 (S.E. = .07), t=0.43, $p = .671$ for the constant and .75 (S.E. = .04), t=18.11, $p < .001$ for pretest.

Figure 2 Panel C shows the model that assessed improvement in NAEP-Part/Whole as the mediator. R^2 for this model was .50, F (3,209) =68.92, $p < .001$. The effect of intervention versus control on improvement in NAEP-Part/Whole (the *a* path) was 0.04 (S.E. = .16), $t = 0.25$, $p = .806$. The effect of improvement in NAEP-Part/Whole on the NAEP total score outcome (the *b* path) was 0.33 (S.E. = .04), $t = 7.65$, $p < .001$. The model partitioned the total intervention effect of 0.37 into direct and indirect effects. The coefficient for the direct effect (after the mediator was added to the model) was 0.36 (S.E. = .10), t=3.46, $p < .001$ (a 3% reduction in the total effect). The coefficient for the indirect effect (the c' path) of 0.01 (S.E. = .06) was not significant (CI = -.0996 to .1241). Thus, improvement in NAEP-Part/Whole did not mediate the effect of intervention versus control on the NAEP total score. Other coefficients were 0.24 (S.E. = .09), t=2.84, p = .005 for the constant and .68 (S.E. = .05), t=13.20, p < .001 for pretest.

What Are the Effects of M-WP Versus A-WP Intervention?

See Table 3 and Figure 1. On number line and NAEP, the two WP intervention conditions performed comparably. On calculations, however, the A-WP condition performed significantly stronger than the M-WP condition, as hypothesized (ES = 0.23). By contrast, on the WP measures, results were condition specific. On the M-WP measure, M-WP intervention students performed significantly better than both contrasting conditions. The ES favoring the M-WP condition over the control group was 1.06, and the ES favoring the M-WP condition over the A-WP condition was 0.89. The comparison between the A-WP condition versus control was not significant (ES = 0.16). By contrast, on the A-WP measure, although A-WP students performed significantly better than both contrasting conditions, the M-WP condition also performed significantly better than the control condition. The ES favoring the A-WP condition over the control group was 1.40; the ES favoring the M-WP condition over the control group was 1.10; and the ES favoring the A-WP condition over the M-WP condition was 0.29.

Discussion

In the present study, we investigated the efficacy of a core intervention program for improving at-risk fourth graders' fraction performance. We designed the intervention and the study to gain insight whether an instructional focus on the measurement interpretation of fractions produces stronger learning than the dominant approach in the U.S., which focuses mainly on the part-whole interpretation of fractions (as in the control condition). Given this interest, we also assessed whether student improvement in the measurement interpretation or in the part-whole interpretation of fractions mediated the effects of intervention versus control.

We also extended our previous work by isolating the effects of fraction intervention designed to enhance performance on multiplicative WPs by randomly assigning intervention students to two versions of the core program: one with a module on multiplicative WPs; the other with a module on additive WPs. Previously, the topic of fraction WPs had been neglected in research conducted at fourth grade. Moreover, by including a contrast condition focused on additive WPs, we created a more stringent test of fraction WP intervention than previously had been conducted at the higher grades.

Intervention Efficacy and the Mediator of Those Effects

On the number line task, our proximal measure of students' measurement interpretation of fractions, the two intervention conditions, each focused on the measurement interpretation of fractions, outperformed the control group, which allocated greater emphasis to the part-whole interpretation of fractions. ESs were large for each intervention condition over control (0.88 and 1.10). The number line task is an important outcome in its own right, given that the accuracy of placing fractions on a number line is a strong predictor of fraction learning between grades 3 and 5 (Jordan et al., 2013; Vukovic et al., 2014) and more advanced mathematics achievement including algebra (e.g., Siegler et al., 2012).

Although the computer task used for pre- and posttesting was similarly novel in all conditions, intervention students engaged in number line activities more than control group students. So it is important that both intervention conditions also demonstrated large advantages over the control group on calculations (ESs = 1.06 and 1.10), even though control group instruction allocated more time to calculations than did the intervention conditions. As in earlier work (Fuchs et al., 2013, 2014; Hecht et al., 2003; Mazzocco & Devlin, 2008; Ni & Zhou, 2005; Rittle-Johnson et al., 2001), this suggests that understanding of fractions is important for learning accurate fraction procedures. Moreover, on the calculation measure (where we also had data on NAR classmates), the achievement gap of intervention students decreased. At pretest, the gap was almost a full *SD* below that of low-risk classmates; at posttest, it was somewhat above NAR classmates. By contrast, the achievement gap for control group students remained approximately constant in that time frame, with ESs of -0.94 at pretest and -0.84 at posttest (see Table 3).

Closing the achievement gap for intervention students, while the achievement gap remained large for control students, is especially noteworthy for two reasons. First, as mentioned, control group students received more instruction on fraction calculations than the intervention groups. Second, control group instruction was guided by current and well thought of instructional design: a combination of the ambitious CCSS and a basal program the WWC considers a “potentially effective program” – with stronger demonstrated outcomes than contrasting basal programs (Resendez & Azrin, 2009).

Therefore, across our two proximal measures, the accuracy of number line placement and adding/subtraction fractions, the present study corroborates previous findings supporting the efficacy of the core program (Fuchs et al., 2013; Fuchs et al., 2014). More generally, results provide support for an instructional approach focused on the measurement interpretation of fractions. Such support is further strengthened by two additional findings. First, effects were reliably stronger for each of the intervention conditions over the control group on the present study’s more distal and multi-faceted outcome, the NAEP (we return to this point later). Second, mediation analyses indicated that improvement on the number line measure completely mediated the effect of intervention versus control on the total NAEP score. In exploratory fashion, we also “competed” improvement in the measurement interpretation against improvement in part-whole understanding as mediators of the intervention’s effect on total NAEP, which included items tapping both forms of fraction understanding. Results showed that improvement on NAEP-Measurement items completely mediated the effects of intervention versus control on total NAEP; by contrast, improvement on the NAEP-Part/Whole items failed to mediate those effects.

Mediation effects are correlational. But combined with results favoring intervention over control on each study outcome in the context of a randomized control trial, findings provide evidence for the causal role of the measurement interpretation in fraction learning. The measurement interpretation of fractions is less intuitive than part-whole interpretation, which has dominated American schooling to date (and was thus emphasized in the study's control group). The measurement interpretation, by contrast, reflects cardinal size; is often (but not exclusively) represented with number lines (e.g., Siegler et al., 2011); and depends on formal instruction. The NMAP (2008) hypothesized that improvement in measurement interpretation is an important mechanism explaining the development of fraction knowledge and recommended that fraction instruction be reoriented in this direction. Our findings support that hypothesis, while corroborating recent prior work (Fuchs et al., 2013; 2014).

At the same time, because NAEP was the most distal measure in the present study, covering multi-faceted forms of fraction knowledge as well as response formats not addressed during intervention, it is not surprising that ESs (0.44 and 0.33) were smaller compared to ESs on more proximal measures (which ranged between 0.88 and 1.10). Even so, NAEP effects were significant and were in the moderate range according to the WWC's guidelines. Moreover, the NAEP achievement gap decreased markedly for intervention students (a decrease in ES of 0.23 *SD* units for M-WP and 0.17 for A-WP), while the gap increased by 0.18 for at-risk control group students. This is an advantage of 0.41 to 0.35 *SD*s in narrowing the achievement gap (for M-WP and A-WP, respectively) over the control group's achievement gap.

Does M-WP Intervention Provide Added Value over A-WP Intervention?

Our second purpose was to consider whether a multiplicative WP module, which we incorporated within the core program, provided added value over a WP component requiring additive thinking. M-WPs were of central interest because multiplicative thinking is foundational to fraction knowledge and can be difficult to achieve. This is the case for whole numbers, but extending that understanding to multiplicative situations involving fractions is a major source of difficulty for many students. This is because multiplying fractions typically produces smaller quantities and dividing fractions results in larger amounts, which violates expectations based on whole numbers. In focusing on "splitting" and "grouping" WPs, which involve multiplicative thinking, our hope was not only to develop competence with fraction WPs that involve multiplicative reasoning, but also to extend fourth graders' understanding of fractions.

To control for the instructional time allocated to the M-WP condition, the A-WP condition addressed fourth-grade CCSS focused on additive thinking (i.e., fraction "increase" and "decrease" WPs). Beyond the distinction between multiplicative versus additive thinking, a second major difference between conditions was that M-WP instruction taught students to represent the underlying structure of the WPs by constructing an array, whereas A-WP instruction taught students to use a number sentence. This distinction was necessary given the nature of the underlying structure of the WP types and the processes required to solve the different WP types, but we note that this second distinction may also contribute to results.

We expected an advantage on multiplicative WPs for the M-WP condition over the A-WP condition, but we also hypothesized stronger fraction understanding, at least on our proximal number line measure. After all, placing fractions on the number line involves comparing fractions to benchmark values (e.g., ½ or ¼), and such transformations require multiplicative thinking (e.g., to place 3/8, students find an equivalent fraction to ½, a benchmark fraction, with 8 in the denominator; then they place 3/8 somewhat less than half way between 0 and 1).

The instructional approach to WPs in the two intervention conditions was rooted in schema-based instruction (e.g., Fuchs et al., 2003, 2009, 2010; Jitendra & Star, 2012; Jitendra, Star, Rodriguez, et al., 2011; Jitendra, Star, Starosta, et al., 2009), an approach that teaches students to identify WPs as belonging to WP types that share structural features (splitting and grouping WP types in M-WP; increase and decrease WP types in A-WP). As with schema-based instruction, students are also taught to represent the underlying structure of the WP type with a number sentence (e.g., Fuchs et al., 2003, 2009, 2010) or visual display (e.g., Jitendra & Star, 2012, Jitendra, Star, Rodriguez, et al., 2011; Jitendra, Star, Starosta, et al., 2009). These schema-based design principles constituted 64% of the WP instructional emphasis in the two intervention conditions. By contrast, the control group allocated no attention to these schema-based instructional design principles.

Effects on WP outcomes were as we hypothesized. On M-WPs, the multiplicative WP condition outperformed the control group (ES = 1.06) as well as the additive WP condition (ES = 0.89). On A-WPs, the additive WP condition outperformed the control group (ES = 1.40) as well as the multiplicative WP condition (ES = 0.29). It is, however, noteworthy that the ES comparing the two active conditions was dramatically smaller on A-WPs than the ES comparing to the two active conditions on M-WPs: 0.89 vs. 0.29. Moreover, whereas the multiplicative WP condition outperformed the control group on A-WPs (ES = 1.10), the additive WP condition and control group conditions performed comparably on multiplicative thinking WPs, with an ES of only 0.16. Thus, schema-based intervention on multiplicative WPs produced positive overall effects on fraction WPs—including M-WPs and A-WPs. By contrast, the effects of schema-based intervention on additive WPs were limited to A-WPs. This suggests that intervention on multiplicative WPs may be a more efficient instructional target for improving fractions WPs generally – at least at fourth grade.

We had also hoped that intervention on multiplicative WPs would be reflected in superior outcomes on the proximal measure of fraction understanding: the accuracy of placing fractions on the number line. The ES favoring M-WP over control was 1.10, sizably larger than the ES favoring A-WP over control (0.81). Although this advantage of 0.22 *SDs* for the M-WP condition over the A-WP condition did not achieve statistical significance, the ES meets the WWC's criterion for a moderate ES. This suggests the need for additional research, with stronger power, to investigate the potential advantage of multiplicative WP intervention on students' understanding of fraction magnitudes. Future research should also examine longitudinal effects of intervention to assess whether students who start fourth grade with risk for poor fraction learning require additional intervention as the fraction curriculum addresses multiplication and division fraction concepts.

More generally, findings suggest the efficacy of schema-based intervention for improving students' WP performance. This corroborates previous work conducted on whole-number WPs (e.g., Fuchs et al., 2003, 2009, 2010) as well as research at higher grades focused on rational numbers (e.g., Jitendra & Star, 2012; Jitendra, Star, Rodriguez, et al., 2011; Jitendra, Star, Starosta, et al., 2009). Fraction WPs are especially important because on the one hand fraction knowledge is foundational for more advanced mathematics and because on the other hand WPs are the best school-age predictor of employment and wages in adulthood (e.g., Every Child a Chance Trust, 2009; Parsons & Bynner, 1997; Murnane et al., 2001).

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Table 1
Curriculum Emphases in Classroom the School's Fourth-Grade Fraction Interventional Program (*n* =46)

Emphasis	Fraction Faceoff! M-WPs	Fraction Faceoff! A-WPs	School Program M (*SD*)
% Types of Fraction Representations Used			
Fraction tiles	10	10	16.44 (13.51)
Fraction circles	15	15	10.58 (12.10)
Pictorial representations with shaded regions	5	5	35.58 (12.29)
Fraction blocks	0	0	12.11 (12.55)
Number lines	70	70	21.02 (10.49)
Other	0	0	4.00 (9.45)
% Activities/Strategies for Comparing Fraction Magnitudes			
Cross multiplying	0	0	21.51 (15.22)
Thinking about relative placement on number lines	30	30	14.07 (11.73)
Comparing fractions to benchmark fraction	30	30	10.07 (11.68)
Finding common denominator	5	5	27.07 (19.07)
Using manipulatives	5	5	16.33 (8.31)
Considering meaning of numerator &denominator	30	30	7.53 (8.34)
Other	0	0	2.56 (8.70)
% Word-Problem Strategies			
Draw a picture	4	4	29.49 (15.17)
Make a table	0	0	9.38 (9.75)
Make an array	32	0	0.00 (0.00)
Write a number sentence	0	32	21.60 (9.26)
Use words to explain thinking	32	32	21.24 (8.68)
Relies on key words	0	0	16.24 (11.42)
Identifies problem within a problem type	32	32	0.00 (0.00)
Other	0	0	0.89 (5.26)

Table 2
Topics Introduced by Week[a]

Week	Topic
1-2	Fraction foundations
	Key vocabulary: numerator, denominator, unit, equivalent, equal parts
	Meaning of fractions (equal sharing, part-whole, quotients)
	Role of numerators (N) vs. denominators (D)
	Naming fractions
	Comparing fractions with like Ns, like Ds, and fractions equivalent to one whole
	Proper and improper fractions equal to one.
3-5	Magnitude reasoning when comparing 2 fractions and ordering 3 fractions
	½ and equivalencies (2/4, 3/6, 4/8, 5/10, 6/12) as benchmarks to compare fractions with unlike Ns and Ds
	Placing 2 fractions on 0 - 1 number lines marked with 0, ½, and 1
6	Improper fractions and mixed numbers > 1 and < 2
	0-2 number lines
	Converting between and equivalent properties of improper fractions and mixed numbers
7-8	Word Problems (M-WP types or A-WP types, depending on condition)
	Comparing, ordering, and number line activities integrating proper fractions, improper fractions, and mixed numbers
9	Adding/subtracting proper and improper fractions, first with like Ds, then unlike Ds
	Adding mixed numbers.
10	Adding/subtracting mixed numbers
	Removal of ½ from 0-1 and 0-2 number lines
	Removal of 1 from 0-2 number lines
	Equivalencies for 1/3, ¼, and 1/5 via multiplication
11-12	Review

[a]After a topic was introduced, cumulative review occurred thereafter.
Note that none of the number line activities relied on computers, as done in the pre/posttest assessment task.

Table 3
Pretest and Posttest Fraction Measure Scores for At-Risk Students by Condition and for Low-Risk Classmates, with Achievement Gaps for At-Risk Students

	At-Risk Condition								Achievement Gap (Effect Size[a])		
	M-WP (n=72)		A-WP (n=71)		Control (n=70)		NAR (n=320)		NAR vs.		
Variable	Mean	(*SD/SE*[b])	Mean	(*SD/SE*)	Mean	(*SD/SE*)	Mean	(*SD*)	MT-WP	AT-WP	Control
Number Line											
Pre	0.29	(0.07)	0.28	(0.06)	0.30	(0.06)	--				
Post	0.18	(0.08)	0.19	(0.09)	0.26	(0.07)	--		--	--	--
Adjusted Post	0.17	(0.01)	0.19	(0.01)	0.26	(0.01)	--				
NAEP											
Pre	9.36	(3.36)	9.08	(2.76)	8.91	(3.16)	14.48	(4.18)	-1.22	-1.29	-1.33
Post	15.38	(4.35)	14.68	(4.02)	13.21	(4.04)	18.95	(3.81)	-0.99	-1.12	-1.51
Adjusted Post	15.20	(0.40)	14.70	(0.40)	13.37	(0.41)	--				
Calculations											
Pre	3.60	(4.31)	3.68	(4.07)	3.79	(4.10)	8.35	(4.85)	-0.98	-0.96	-0.94
Post	19.64	(8.49)	21.39	(6.68)	11.43	(5.09)	19.21	(9.25)	+0.08	+0.24	-0.84
Adjusted Post	19.67	(0.80)	21.40	(0.80)	11.39	(0.81)	--				
M-Word Problems											
Pre	6.17	(4.19)	5.24	(3.72)	4.81	(3.60)	9.48	(5.38)	-0.62	-0.79	-0.87
Post	13.89	(6.59)	8.21	(5.14)	7.37	(4.74)	13.52	(6.46)	+0.06	-0.82	-0.95
Adjusted Post	13.48	(0.61)	8.31	(0.61)	7.69	(0.62)	--				
A-Word Problems											
Pre	4.94	(3.90)	3.89	(3.08)	3.99	(3.09)	8.17	(4.25)	-0.76	-1.01	-0.98
Post	13.88	(5.94)	15.08	(6.44)	7.94	(3.86)	14.10	(5.66)	-0.04	+0.17	-1.09
Adjusted Post	13.49	(0.62)	15.31	(0.62)	8.11	(0.62)	--				

[a] Effect size achievement gaps are difference in posttest scores, divided by low-risk *SD*s.
[b] *SE* is standard error, reported for adjusted posttest scores; *SD* is standard deviation.
Fraction Number Line is Schumacher et al. (2011), modeled after Siegler et al. (2011) for 0-2 number lines. NAEP is the National Assessment of Educational Progress items (19 easy, medium, and hard fourth-grade and easy eighth-grade released fraction items). M-WPs is Multiplicative Word Problems (Schumacher et al., 2012). A-WPs is Arithmetic Word Problems (Schumacher et al., 2012).

Table 4
Intervention Effects, Follow-Up Tests, and Effect Sizes

	Number Line		NAEP		Calculations		M-WPs		A-WPs	
Corrected Model	35.44	(<.001)	39.00	(<.001)	32.35	(<.001)	16.73	(<.001)	35.47	(<.001)
Intercept	4.19	(.042)	104.80	(<.001)	677.96	(<.001)	171.08	(<.001)	291.78	(<.001)
Pretest	51.01	(<.001)	101.96	(<.001)	10.03	(.002)	17.91	(<.001)	29.99	(<.001)
Condition	25.57	(<.001)	5.44	(.005)	43.96	(<.001)	9.56	(<.001)	36.60	(<.001)
Pretest x Condition	NA		NA		NA		0.50	(.605)	NA	
Follow-Up[b]	1=2>3		1=2>3		2=1>3		1>2=3		2>1>3	
Effect Sizes										
M-WP v. Control	1.10		0.44		1.22		1.06		1.10	
A-WP v. Control	0.81		0.33		1.70		0.16		1.40	
M-WP v. A-WP	0.22		0.12		-0.23		0.89		-0.29	

[a]For all measures except M-WPs, *F*(3,209) for corrected model; *F* (1,209) for intercept; *F* (1,209) for pretest; and *F* (2,209) for condition. For M-WPs, *F* (5,207) for corrected model); *F* (1,207) for intercept; (1,207) for pretest; *F* (2,207) for pretest x condition.
[b]1=M-WP; 2=A-WP; 3=Control.
Fraction Number Line is Schumacher et al. (2011), modeled after Siegler et al. (2011) for 0-2 number lines. NAEP is the National Assessment of Educational Progress items (19 easy, medium, and hard fourth-grade and easy eighth-grade released fraction items). M-WPs is Multiplicative Word Problems (Schumacher et al., 2012). A-WPs is Arithmetic Word Problems (Schumacher et al., 2012).

Table 5
Descriptive Information and Correlations among Measures in Mediation Analyses (n=213)

	Raw Scores		Correlations			
Variables	X	(SD)	N1	N2	NL	M
NAEP-Pre (N1)	9.12					
- Post (N1)	14.43		**.57**			
Improvement in Number Line (NL)	13.84		.07	**.32**		
NAEP-Measurement (M)	2.70		**-.30**	**.40**	-.09	
NAEP-Part-Whole	1.00		**-.30**	**.19**	.09	**.54**

Bold indicates $p<.001$ except correlation between Part-Whole and NAEP-Post, which is $p=.005$.
Fraction Number Line is Schumacher et al. (2011), modeled after Siegler et al. (2011) for 0-2 number lines. NAEP is the National Assessment of Educational Progress items (19 easy, medium, and hard fourth-grade and easy eighth-grade released fraction items).

Figure 1. *Effect sizes by Contrast by Measure (Asterisk indicates statistical significance; positive value indicate first condition in contrast, see key, was higher; negative value indicates second condition, see key, was higher)*

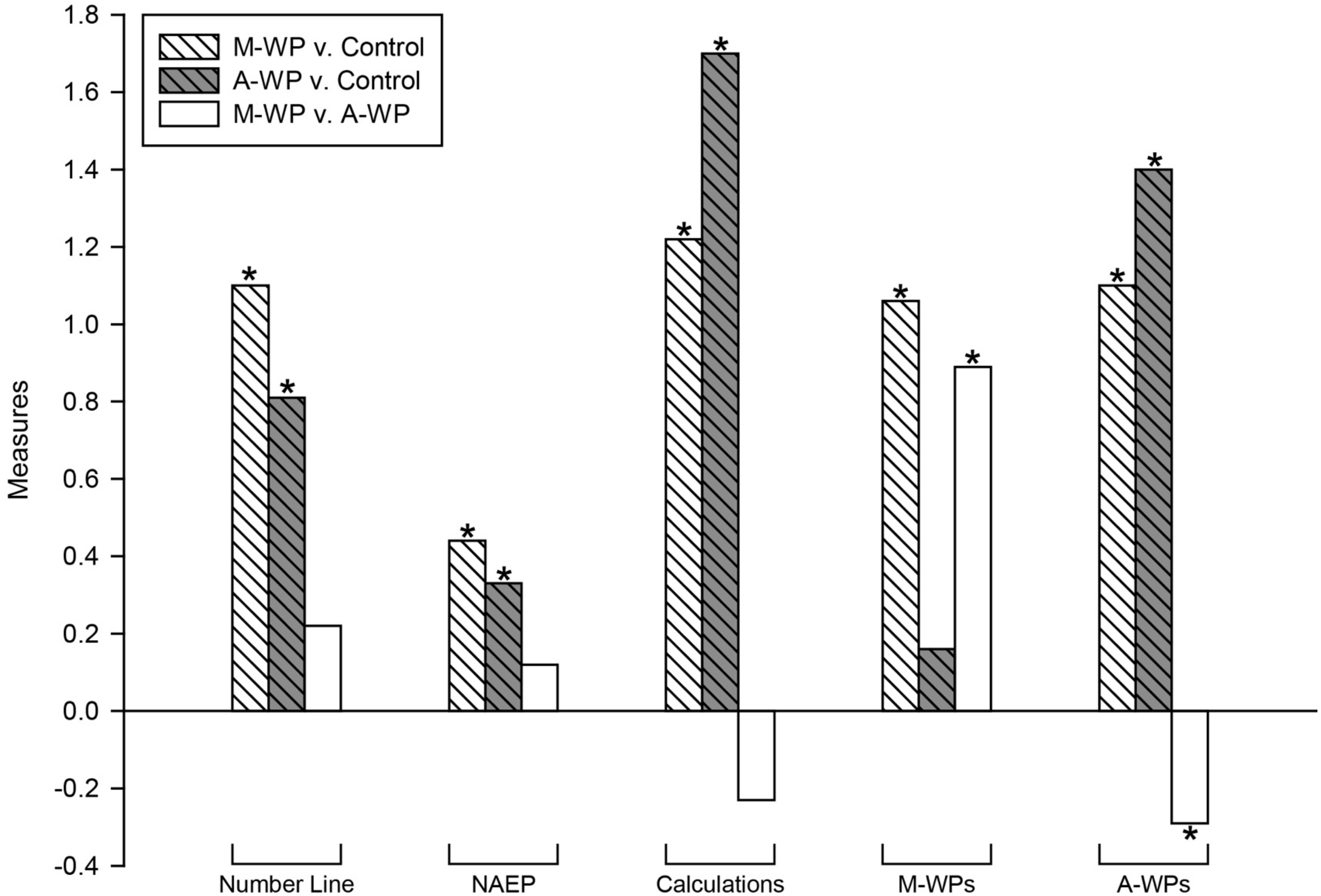

Figure 2. *Mediation models: Panel A assess improvement on Number Line as a mediator of the effects of intervention versus control on the total NAEP score; Panel B assesses improvement on NAEP Measurement as a mediator of effects of intervention versus control on the total NAEP score; and Panel C assesses improvement on NAEP Part-Whole as a mediator of the effects of intervention versus control on the total NAEP score (also see text for additional effects for these models).*