Looking at Students' Work: Assessment Strategies that Inform Instruction

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Assessment Strategies that Inform Instruction

- Common Core State Standards in Mathematics
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- Evaluating Understanding and Reasoning
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- Looking at Students’ Work Samples
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High school graduates today are increasingly expected to...evaluate arguments, and understand and convey complex information in the college classroom, in the workplace...

The ability to reason allows for the systematic development of ideas, the ability to make sound choices, and the ability to make and understand persuasive arguments.

American Diploma Project, 2004, p. 29
Overview of the Mathematics Standards

- Grade Level Standards
  - K-8, grade-by-grade standards organized by domain
  - 9-12 high school standards organized by conceptual categories

- Standards for Mathematical Practice
  - Describe mathematical “habits of mind”
  - Standards for mathematical proficiency:
    - reasoning, problem solving, modeling, decision making, and engagement
  - Connect with content standards in each grade
Organizational Structure

The High School Standards

- Expect students to practice applying mathematical ways of thinking to real world issues and challenges
- Require students to develop a depth of understanding and ability to apply mathematics to novel situations, as college students and employees regularly are called to do
- Emphasize mathematical modeling, the use of mathematics and statistics to analyze empirical situations, understand them better, and improve decisions
- Identify the mathematics that all students should study in order to be college and career ready.
Assessment

Race to the Top:

- PARCC Consortium
- Smarter Balanced Consortium
Smarter Balanced Assessment Consortium
Building a pathway to college and career readiness

- Aligned to the college- and career-ready Common Core State Standards.

- Students who score proficient on the assessments will know they are on track for the next steps in their education, creating a more meaningful target.

- High School results will send an early signal about whether students are ready for entry-level, non-remedial courses at higher education institutions.

- Scores will provide reliable state-to-state comparability, with standards set against national and international benchmarks.
Creating better assessments ...

The assessment systems will include:

- A mix of item types – multiple choice, short answer, longer open response and performance-based:
  - Encourage teachers to focus on helping each student develop a deep understanding of the subject matter, rather than just narrowing their instruction in order to “teach to the test”

- Testing at multiple points throughout the year
  - Provide teachers, parents and students information about whether students are “on track” or need some additional support in particular areas during the school year, not just at the end of the year
Knowledge and Skills that are important to measure:

Students can

- explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

- frame and solve a range of complex problems in pure and applied mathematics.

- clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

- analyze complex, real-world scenarios and use mathematical models to interpret and solve problems.
Looking at Students’ Work—
How do we evaluate understanding and reasoning skills?
Solve the following problem.

Show all work. Explain your answer.

A college sold tickets to a play at $4 per ticket in advance. Anyone who attended and purchased a ticket at the door had to pay $5 a ticket. A total of 480 people attended the play, and the revenue from the ticket sales was $2100.

How many people bought tickets in advance, and how many people bought tickets at the door?
Floodlights

Eliot is playing football. He is 6 feet tall. He stands exactly halfway between two floodlights. The floodlights are 12 yards high and 50 yards apart. The floodlights give Eliot two shadows, falling in opposite directions.

1. Draw a diagram to represent this situation. Label your diagram with the measurements.

2. Find the total length of Eliot’s shadows. Explain your reasoning in detail.

3. Suppose Eliot walks in a straight line towards one of the floodlights. Figure out what happens to the total length of Eliot’s shadows. Explain your reasoning in detail.
Using Assessments thoughtfully

Looking at students’ work...

• Gives the ability to measure more complex skills that go beyond those that are measurable by multiple choice items.

• Multiple choice questions can determine whether students can solve many kinds of problems, but cannot determine if they can construct a mathematical proof.

• Constructed responses do not allow for “guessing the correct answer among the multiple choices presented”
Scoring the responses

Rubric
• a set of guidelines that tells the scorer what features of the response to focus on

• how to decide how many points to award to the response.
Scoring the responses

Analytic scoring

• lists specific features of the response
• specifies the number of points to award for each feature
• consistent from scorer to scorer
• explicitly specifies features of the response which should be awarded points, or lose points
• Important features of the response can be evaluated individually.
• Quality of the response does not depend on interactions among those features of the response. (Almost like a checklist)
• Some analytic scoring systems allow for “partial credit”
Scoring the responses

Holistic scoring

• The holistic rubric identifies characteristics of a typical response at each score level.

• A single judgement of the quality of the response assigns a numerical score.

• A single numerical score is given to the whole response

• Used in conjunction with exemplars (samples of responses for each possible score. Can include “borderline cases” – barely 5, almost 4...
New Jersey...
Scoring Rubrics
Rubrics - SCORING STUDENT RESPONSES

Rubric with Analytic Components According to Webster's: rubric, n. 5. An authoritative direction or rule.

Scoring rubrics provide the criteria for evaluating and scoring student performance. There is an item specific rubric supplied for each Open-Ended item in mathematics. These item specific rubrics are based upon the generic mathematics rubric and are developed by a committee of mathematicians and teachers. Rubrics ensure that there is consistency, fairness, and accuracy in scoring open-ended questions.
<table>
<thead>
<tr>
<th>3 - Point Response</th>
<th>2 - Point Response</th>
<th>1 - Point Response</th>
<th>0 - Point Response</th>
</tr>
</thead>
</table>
| • **complete** understanding of the problem's essential mathematical concepts  
• executes procedures completely and gives relevant responses to all parts of the task. contains few minor errors, if any.  
• contains a clear, effective explanation detailing how the problem was solved so that the reader does not need to infer how and why decisions were made. | • **nearly complete** understanding of the problem's essential mathematical concepts.  
• executes nearly all procedures and gives relevant responses to most parts of the task.  
• response may have minor errors. explanation detailing how the problem was solved may not be clear, causing the reader to make some inferences. | • **limited** understanding of the problem's essential mathematical concepts.  
• response and procedures may be incomplete and/or may contain major errors.  
• incomplete explanation of how the problem was solved may contribute to questions as to how and why decisions were made. | • **insufficient** understanding of the problem's essential mathematical concepts. procedures, if any, contain major errors. no explanation of the solution or the reader may not be able to understand the explanation.  
• The reader may not be able to understand how and why decisions were made. |
Assessing Levels of Expertise in Mathematics:

• Items and tasks will assess student knowledge and skill at the *novice, apprentice, or expert* level.

• The level of expertise refers to the student’s ability to call upon and use the mathematics they understand in a range of contexts (*adaptive expertise*).

• Items will range from those with extensive scaffolding (novice) to those requiring the student to reason through a number of steps on their own and measure the ability to make strategic decisions on how to approach and solve the problem (expert).
Solve the following problem.

Show all work. Explain your answer.

A college sold tickets to a play at $4 per ticket in advance. Anyone who attended and purchased a ticket at the door had to pay $5 a ticket. A total of 480 people attended the play, and the revenue from the ticket sales was $2100.

How many people bought tickets in advance, and how many people bought tickets at the door?
Student Work

At your tables are 6 sample responses.

- Choose one of the samples and become familiar with the work the student did.
  - Was it correct?
- As an assessment, what does the work tell you?
- Within your group discuss all 6 samples.
- How would you assess the work shown in each sample?
- Explain what was done.
- How could this student work be improved?
<table>
<thead>
<tr>
<th>Number of $4 tickets</th>
<th>Number of $5 tickets</th>
<th>Number of People Attended</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>420</td>
<td>= 420</td>
</tr>
<tr>
<td>525</td>
<td>0</td>
<td>= 525</td>
</tr>
<tr>
<td>100</td>
<td>340</td>
<td>= 440</td>
</tr>
<tr>
<td>200</td>
<td>260</td>
<td>= 460</td>
</tr>
<tr>
<td>300</td>
<td>180</td>
<td>= 480</td>
</tr>
<tr>
<td>400</td>
<td>100</td>
<td>= 500</td>
</tr>
<tr>
<td>275</td>
<td>200</td>
<td>= 475</td>
</tr>
<tr>
<td>150</td>
<td>300</td>
<td>= 450</td>
</tr>
<tr>
<td>250</td>
<td>220</td>
<td>= 470</td>
</tr>
<tr>
<td>350</td>
<td>140</td>
<td>= 490</td>
</tr>
<tr>
<td>337.5</td>
<td>150</td>
<td>= 487.5</td>
</tr>
<tr>
<td>212.5</td>
<td>250</td>
<td>= 462.5</td>
</tr>
<tr>
<td>87.5</td>
<td>350</td>
<td>= 437.5</td>
</tr>
</tbody>
</table>

300 people bought tickets in advance
180 people bought tickets at the door
**Guess & Check Table**

<table>
<thead>
<tr>
<th>Guess # of people who bought ticket in advance</th>
<th># of people that bought tickets</th>
<th>Total people tickets</th>
<th>Cost of advance tickets ($4)</th>
<th>Cost of tickets at the door ($5)</th>
<th>Total cost of tickets</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>480 - 200 = 280</td>
<td>280 + 200 = 480</td>
<td>200 * $4 = $800</td>
<td>280 * $5 = $1400</td>
<td>$2200</td>
</tr>
<tr>
<td>190</td>
<td>480 - 190 = 290</td>
<td>190 + 290 = 480</td>
<td>190 * $4 = $760</td>
<td>290 * $5 = $1450</td>
<td>$2210</td>
</tr>
<tr>
<td>220</td>
<td>480 - 220 = 260</td>
<td>220 + 260 = 480</td>
<td>220 * $4 = $880</td>
<td>260 * $5 = $1300</td>
<td>$2180</td>
</tr>
<tr>
<td>240</td>
<td>480 - 240 = 240</td>
<td>240 + 240 = 480</td>
<td>240 * $4 = $960</td>
<td>240 * $5 = $1200</td>
<td>$2160</td>
</tr>
<tr>
<td>300</td>
<td>480 - 300 = 180</td>
<td>300 + 180 = 480</td>
<td>300 * $4 = $1200</td>
<td>900 + $200 = $2100</td>
<td></td>
</tr>
</tbody>
</table>

Check 2200: too high
$2100 2210: too high
$2180: too high
$2160: too high
$2100: correct

300 people bought tickets in advance and 180 people bought tickets at the door.
Extra Credit

A = advanced
D = door

4A + 5D = $2,100
A + D = 480

4A + 5D = 2100
-A -D = -480
3A + 4D = 1620
-A -D = -480
2A + 3D = 1140
-A -D = -480

A + 2D = 660
-A -D = -480

D = 180

480
-180
300

A = 300 x $4
D = 180 x $5

Words:
First I made the problem into an equation.
Then I subtracted 1 away from A and D
everytime and 480,
which was the amount of tickets sold from the
amount of money there was. I keep doing this
so I could get the D by itself to find out
how much money the
tickets got. Then
to find out the amount of
the advanced tickets, I subtracted
the amount of the door tickets
from the original amount of
tickets sold. Then I multiplied
the amount of the advanced tickets
by $4 because that is how much
they sold for and I multiplied
the amount of the door tickets
by $5 because that is how much they
sold for. 300 x $4 = 1200 and
180 x $5 = 900 and 1200 and 900
added together equals $2100, the
original amount of the total
price of all of the tickets.
My answer was: 300 people bought at the door ($5), + 180 people bought in advance ($4).

- First, I divided 480 (people) and 2100 (dollars) by 60 so I can calculate more easily. Now I got 8 (people) and 35 (dollars).

- Then I wrote a chart:

<table>
<thead>
<tr>
<th>set</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>people who bought by $4</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>total</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$5</td>
<td>$5</td>
<td>$5</td>
<td>$5</td>
<td>$5</td>
<td>$5</td>
<td>$5</td>
</tr>
</tbody>
</table>

- If I calculate one set (the set I circled) by one, only the third one ($\frac{5}{3}$) equals to 35 dollars. The rest equals to: $33, 34, 36, 37, 38.

- So it means 5 people bought for $4, + 3 people bought for $5.

- Lastly, I multiplied 5 + 3 by 60, which equaled to: $5 \times 60 = 300; 3 \times 60 = 180.$

300 people bought for $5
180 = for $4
1. From the problem I know: → tickets in advance are 4 $ → tickets at the door are 5 $ → 480 people attended the play → the entire profit was 2100 $
2. I am being asked to find out how many people paid at the door.
3. To get my solution I did the following:
   \[ \frac{480 \text{ people}}{4 \text{ $}} \times 1,920 \text{ $} = 1,920 \text{ $} \]
   \[ \frac{180 \text{ people}}{5 \text{ $}} \times 1,200 \text{ $} = 1,800 \text{ $} \]
4. The answer is 180 people buy tickets at the door.
5. To check my answer I did the following:
   \[ \frac{300 \text{ people}}{4 \text{ $}} \times 1,200 \text{ $} = 900 \text{ $} \]
   \[ \frac{180 \text{ people}}{5 \text{ $}} \times 900 \text{ $} = 1,200 \text{ $} \]

<table>
<thead>
<tr>
<th></th>
<th># spent</th>
<th>people</th>
</tr>
</thead>
<tbody>
<tr>
<td>buy tickets at door</td>
<td>$900</td>
<td>180</td>
</tr>
<tr>
<td>pay in advance</td>
<td>$1200</td>
<td>300</td>
</tr>
<tr>
<td>total</td>
<td>$2,100</td>
<td>480</td>
</tr>
</tbody>
</table>
First we wrote down the facts. $x$ became the number of tickets sold in advance, and $d$ became the number sold at the door. Equation number one was $4x + 5d = 2,100$. Two distinct variables in an equation is not something we know how to deal with yet, so we wrote a few more equations. Next equation was $x + d = 480$, which also means (by the subtraction property of equality) $480 - x = d$. Using that we substituted: $4x + 5(480 - x) = 2,100$. The Distributive property was what came next: $4x + 2,400 - 5x = 2,100$. Then we combined like terms $-x + 2,400 = 2,100$. Now we needed to isolate the variable, so we subtracted the constant 2,400 from both sides: $-x = -300$. Last we subtracted 300 from 480 which is 180. Our solution was found. 300 people bought tickets in advance and 180 bought tickets at the door.

There is another, more advanced way to do this equation. Taking the equation $x + d = 480$, we multiplied both sides by $-4$. Second we took the old equation; $4x + 5d = 2,100$, and added it to the revised equation of $-4x - 4d = -1920$. $-4x$ and $4x$ cancelled and $-4d$ plus $5d$ became $d$. $2,100 - 1920 = 180$. The reason for multiplying by 4 was to cancel out the $4x$ in the second equation and isolate the variable $d$. Both sides could be multiplied by $-5$ to cancel out $5d$, but either works.
Floodlights

Eliot is playing football.
He is 6 feet tall.
He stands exactly half way between two floodlights.
The floodlights are 12 yards high and 50 yards apart. The floodlights give Eliot two shadows, falling in opposite directions.

1. Draw a diagram to represent this situation. Label your diagram with the measurements.

2. Find the total length of Eliot’s shadows. Explain your reasoning in detail.

3. Suppose Eliot walks in a straight line towards one of the floodlights. Figure out what happens to the total length of Eliot’s shadows. Explain your reasoning in detail.
Sample Response: Wendy

1 square = 2 yards.

Eliot is 6 feet = 2 yards tall.

<table>
<thead>
<tr>
<th>Place Eliot stands</th>
<th>Left Shadow</th>
<th>Right Shadow</th>
<th>Total Shadow</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3 1/2</td>
<td>1</td>
<td>4 1/2</td>
</tr>
<tr>
<td>B</td>
<td>2 1/2</td>
<td>1 1/2</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>2 1/2</td>
<td>2 1/2</td>
<td>5</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>E</td>
<td>1 1/2</td>
<td>4 1/2</td>
<td>6</td>
</tr>
</tbody>
</table>

As Eliot goes to the right, the left shadow gets shorter and the right shadow gets longer. Total shadow stays about the same.
AB and CD are vertical. The football field is horizontal (BC) and so is Elliot. RQ and QT are shadows. I want to find QT.

\[ QT^2 = PT^2 - 4 \text{(Pythag)} \]

Triangle ABT and triangle PQT are similar so

\[ \frac{PT}{PT} = \frac{AT - AP}{AT - AQ} \]
\[ AP^2 = 10^2 + 25^2 \]
\[ AT^2 = 12^2 + (25^2 + QT^2) \]

and \[ \frac{QT}{2} = \frac{BT}{12} \]
\[
\hat{TAD} = \hat{RTP} = \alpha \\
\hat{RAP} = \hat{RTP} = \beta \\
\hat{APD} = \hat{RTP} = 180 - (\alpha + \beta) \\
\triangle RPT \text{ is similar to } \triangle APD \\
SP \text{ is the perpendicular height of } \triangle APD \\
P\Phi \text{ is the perpendicular height of } \triangle PRT \\
SP = 12 - 2 = 10 \text{ yds} , \quad P\Phi = 2 \text{ yds} \\
SP : P\Phi = 10 : 2 = 5 : 1
\]
For More Information and General Resources

- Common Core State Standards: [www.corestandards.org](http://www.corestandards.org)
- College Board Standards for College Success: [http://professionals.collegeboard.com/k-12/standards](http://professionals.collegeboard.com/k-12/standards)
For More Information and General Resources

- Mathematics Assessment Project (MAPS)  
  http://map.mathshell.org/materials/index.php

- Exemplars: Standards Based Assessment and Instruction:  
  www.exemplars.com

- Constructed-Response test questions: Why we use them; How we score them.  
Contact information

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