

**Abstract Title Page**  
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**Title:** Doubling Up: Intensive Math Instruction and Educational Attainment

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## **Abstract Body**

*Limit 4 pages single-spaced.*

### **Background / Context:**

*Description of prior research and its intellectual context.*

Our work contributes to three strands of the research literature. First, given that the intervention studied here doubled the amount of time students were exposed to freshman algebra, our work adds to the literature on the importance of instructional time to student achievement. Some education reformers have pushed U.S. schools to lengthen school days and years, noting that students in many academically successful nations, particularly in Asia, spend substantially more time in school than do American students. Proponents of this view point to evidence on summer learning loss (Cooper et al. 1996), the impact of snow days (Marcotte and Hemelt 2008), the association between charter school effectiveness and instructional time (Dobbie et al. 2011, Hoxby and Murarka 2009), and other such patterns linking student achievement to hours spent learning (Lavy 2010, Fitzpatrick et al. 2011). Another set of studies suggests this evidence is weaker than it first appears, with Fryer Jr and Levitt (2004) observing little differential summer learning loss, Goodman (2012a) showing little impact of snow days on achievement, Angrist et al. (2011) showing little relation between instructional time and charter school effectiveness, and Abt Associates (2011) showing little effect of an intervention that substantially increased schools' instructional times. The emerging consensus from this literature is that increasing instructional time is no guarantee of better outcomes if such time is not well spent. The results we present below are consistent with heterogeneous impacts of increased instructional time by math and reading skill.

Second, our work adds to the literature concerning the short-run impact of curricular interventions, particularly for students struggling in math. Recent years have seen three main approaches tried by American schools. Remediation, which diverts students into basic courses prior to taking regular courses, has generally had little discernible impact on student achievement, particularly at the college level where it has most often been studied (Jacob and Lefgren 2004, Lavy and Schlosser 2005, Calcagno and Long 2008, Bettinger and Long 2009, Martorell and McFarlin Jr 2011, Boatman and Long 2010, Scott-Clayton and Rodriguez 2012). Algebra "for all", which pushes students to take algebra courses in earlier grades than they otherwise would, actually harms student achievement by forcing students into subjects for which they are not sufficiently prepared (Clotfelter et al. 2012, Nomi 2012). Double-dosing, which places students in regular courses but supplements those courses with additional instructional time, has generated modest short run gains in some settings and no gains in others (Nomi and Allensworth 2009, Nomi and Allensworth 2010, Fryer 2011, Taylor 2012, Dougherty 2012). Perhaps because of perceived effectiveness at raising short-run achievement levels, the double dose strategy has become increasingly common, with half of large urban districts reporting it as their most common form of support for struggling students.

Third, and perhaps most important, we contribute to the literature on the long-run impacts of curriculum on student outcomes. Nearly all such research points to a close association between coursework completed in high school and later outcomes such as college enrollment and labor market earnings (Altonji 1995, Levine and Zimmerman 1995, Rose and Betts 2004, Attewell and Domina 2008, Long et al. 2009, Long et al. 2012). Most such papers attempt deal with bias generated by selection into coursework by controlling for a rich set of covariates, either through OLS or propensity score matching. Such methods leave open the possibility that

remaining unobservables are still important factors. The few papers that use quasi-experimental methods to convincingly eliminate such selection bias also, however, find strong associations between completed coursework and long run outcomes, suggesting that such selection bias is not generating the central findings (Joensen and Nielsen 2009, Goodman 2012b). This paper is, to our knowledge, the most convincingly identified link between high school coursework and educational attainment.

**Purpose / Objective / Research Question / Focus of Study:**

*Description of the focus of the research.*

The purpose of the research is to study the long-run impacts (i.e. on educational attainment) of a freshman math intervention called “double-dose algebra”.

**Setting:**

*Description of the research location.*

The intervention was conducted in 2003 and 2004 within the Chicago Public Schools, a large, poor urban school district.

**Population / Participants / Subjects:**

*Description of the participants in the study: who, how many, key features, or characteristics.*

We follow two cohorts of students from the Chicago Public Schools from their freshman high school fall (in 2003 and 2004) through the end of high school and the beginning of college. Our main analytic sample contains upwards of 11,000 students, 95% of whom are poor and black or Hispanic.

**Intervention / Program / Practice:**

*Description of the intervention, program, or practice, including details of administration and duration.*

In response to these low passing rates in 9th grade algebra, CPS launched the double-dose algebra policy for students entering high school in the fall of 2003. Instead of reinstating the traditional remedial courses from previous years, CPS required enrollment in two periods of algebra coursework for all first-time 9th graders testing below the national median on the math portion of the 8th grade Iowa Tests of Basic Skills (ITBS). Such students enrolled for two math credits, a full-year regular algebra class plus a full-year algebra support class.

Prior to the double-dose policy, algebra curricula had varied considerably across CPS high schools, due to the fairly decentralized nature of the district. Conversely, CPS offered teachers of double-dose algebra two specific curricula called Agile Mind and Cognitive Tutor, stand-alone lesson plans they could use, and thrice annual professional development workshops where teachers were given suggestions about how to use the extra instructional time. Though it is difficult to know precisely what occurred in these extra classes, Nomi and Allensworth (2010) surveyed students to learn more about the classroom learning environment. They found that students assigned to double-dose algebra reported much more frequently: writing sentences to explain how they solved a math problem; explaining how they solved a problem to the class; writing math problems for other students to solve; discussing possible solutions with other students; and applying math to situations in life outside of school. The additional time thus

focused on building verbal and analytical skills may have conferred benefits in subjects other than math.

The treatment under consideration here thus had many components. Assignment to double-dose algebra doubled the amount of instructional time and exposed students to the curricula and activities discussed above. As we will show, the recommendation that students take the two classes with the same set of peers caused tracking by skill to increase, thus reducing classroom heterogeneity. All of these factors were likely to, if anything, improve student outcomes (Duflo et al. 2011). We will also show, however, that the increase tracking by skill placed remediated students among substantially lower skilled peers than non-remediated students. Anecdotal evidence suggests that remedial classes were taught by less experienced teachers. Both of these factors were likely to, if anything, hurt student outcomes. Our estimates will capture the net impact of all of these components.

### **Research Design:**

*Description of the research design.*

Comparison of the outcomes of students who are and are not assigned to double-dosed algebra would likely yield biased estimates of the policy's impacts given potentially large differences in unobserved characteristics between the two groups of students. To eliminate this potential bias, we exploit the fact that students scoring below the 50th percentile on the 8th grade ITBS math test were supposed to enroll in double-dose algebra. This rule allows us to identify the impact of double-dose algebra using a regression discontinuity design applied to the two treated cohorts. We use the assignment rule as an exogenous source of variation in the probability that a given student will be double-dosed.

Our preferred specification will fit straight lines on either side of the threshold using a bandwidth of 10 percentiles, and will also control for gender, race, special education status, socioeconomic and poverty measures, and 8th grade reading scores. We will show that our central results are robust to alternative choices of controls and bandwidths. In all specifications, heteroskedasticity robust standard errors will be clustered by each student's initial high school to account for within high school correlations in the error terms.

### **Data Collection and Analysis:**

*Description of the methods for collecting and analyzing data.*

Chicago Public Schools provided us data on students' 8<sup>th</sup> grade test scores, high school transcripts and college enrollment status through links with National Student Clearinghouse data.

### **Findings / Results:**

*Description of the main findings with specific details.*

Double-dosing increased the proportion of students earning at least a B in freshman algebra by 9.4 percentage points, a more than 65% increase from a base of 13.8 percentage points. Though passing rates for freshman algebra increased by 4.7 percentage points, the increase is statistically insignificant. Double-dosed students were also no more likely to pass geometry. They were, however, substantially more likely to pass trigonometry, a course typically taken in the third year of high school. Mean GPA across all math courses taken after freshman year increased by a marginally significant 0.14 grade points on a 4.0 scale. As a whole, these

results imply that the double-dose policy greatly improved freshman algebra grades for the upper end of the double-dosed distribution, but had relatively little impact on passing rates for the lower end of the distribution. This latter fact is one of the primary reasons that CPS has since moved away from this strategy. There is, however, some evidence of improved passing rates and GPA in later math courses, suggesting the possibility of longer run benefits beyond freshman year.

Double-dosing increases algebra scores in fall of grade 11 by a statistically significant 0.15 standard deviations and has a slightly smaller though statistically insignificant impact on geometry. Double-dosing thus raises overall math scores in the fall of students' third years by 0.16 standard deviations. Perhaps more importantly, a nearly identical effect is seen on the math portion of the ACT, with double-dose algebra raising such scores by a statistically significant 0.15 standard deviations on an exam used by many colleges as part of the admissions process. These effects are nearly identical across the two cohorts. We should also note here that these results are not driven by selection into exam-taking. This suggests that double-dosed students experienced medium-run gains that persisted at least two years after the end of double-dose classes.

Double-dosing increases 4- and 5-year high school graduation rates by 8.7 and 7.9 percentage points respectively. This represents a 17% improvement over the 51% of non-double-dosed students at the threshold who graduate within 4 years. Double-dosing also dramatically improves college enrollment outcomes. Double-dosed students are 8.6 percentage points more likely to ever enroll in college within five years of starting high school, a 30% increase over the base college enrollment rate of 29.0%. Nearly all of this college enrollment increase comes from two-year colleges, with the majority of that coming from part-time enrollment in such colleges. Given the relatively low academic skills and high poverty rates of CPS students at the double-dose threshold, it is unsurprising that double-dosing improved college enrollment rates at relatively inexpensive and non-selective two-year postsecondary institutions. Within eight years of starting high school and thus within four years of expected high school graduation, double-dosed students have enrolled in 0.6 additional semesters of college, 0.4 of which are in two-year colleges.

## **Conclusions:**

*Description of conclusions, recommendations, and limitations based on findings.*

We provide evidence of positive and substantial long-run impacts of one particular form of intensive math instruction on college entrance exam scores, high school graduation rates and college enrollment rates. We show that this intensive math instruction was most successful for students with average math skills but relatively low reading skills, and not successful for the average treated student. This highlights the importance of carefully targeting such interventions to students most likely to benefit from them. Also, like other recent studies, we find that the test score impacts of this policy dramatically understate its long-run benefits as measured by educational attainment (Deming, 2009; Chetty et al., 2011).

One final piece of evidence suggests that CPS's double-dose policy impacts could be replicated in other urban school districts across the U.S. We find little evidence that adherence to CPS' detailed implementation guidelines for double-dose is related to the program's impact. Districts looking to adopt the double-dose strategy could likely reap its benefits without needing to radically restructure their school days.

## Appendices

*Not included in page count.*

### Appendix A. References

*References are to be in APA version 6 format.*

Allensworth, E. and J. Easton (2005). The on-track indicator as a predictor of high school graduation. Consortium on Chicago School Research, University of Chicago.

Allensworth, E. and J. Easton (2007). What matters for staying on track and graduating in Chicago Public High Schools. Chicago, IL: Consortium on Chicago school research. 17.

Altonji, J. (1995). The effects of high school curriculum on education and labor market outcomes. *The Journal of Human Resources* 30(3), 409–438.

Angrist, J. D., P. A. Pathak, and C. R. Walters (2011). Explaining charter school effectiveness. Working Paper 17332, National Bureau of Economic Research. Associates, A. (2011). Evaluation of the expanded learning time initiative. Annual report.

Attewell, P. and T. Domina (2008). Raising the bar: Curricular intensity and academic performance. *Educational Evaluation and Policy Analysis* 30(1), 51–71.

Bettinger, E. and B. Long (2009). Addressing the needs of underprepared students in higher education. *The Journal of Human Resources* 44(3), 736–771.

Boatman, A. and B. Long (2010). Does remediation work for all students? How the effects of postsecondary remedial and developmental courses vary by level of academic preparation.

Calcagno, J. C. and B. T. Long (2008, July). The impact of postsecondary remediation using a regression discontinuity approach: Addressing endogenous sorting and noncompliance. Working Paper 14194, National Bureau of Economic Research.

Clotfelter, C. T., H. F. Ladd, and J. L. Vigdor (2012, June). The aftermath of accelerating algebra: Evidence from a district policy initiative. Working Paper 18161, National Bureau of Economic Research.

Cooper, H., B. Nye, K. Charlton, J. Lindsay, and S. Greathouse (1996). The effects of summer vacation on achievement test scores: A narrative and meta-analytic review. *Review of Educational Research* 66(3), 227–268.

Dobbie, W., R. Fryer, and G. Fryer Jr (2011). Are high-quality schools enough to increase achievement among the poor? Evidence from the harlem children’s zone. *American Economic Journal: Applied Economics* 3(3), 158–187.

Dougherty, S. (2012). Bridging the discontinuity in adolescent literacy: Evidence of effectiveness from one district. Unpublished manuscript.

Duflo, E., P. Dupas, and M. Kremera (2011). Peer effects, teacher incentives, and the impact of tracking: Evidence from a randomized evaluation in Kenya. *The American Economic Review* 101(5), 1739–1774.

Fitzpatrick, M., D. Grissmer, and S. Hastedt (2011). What a difference a day makes: Estimating daily learning gains during kindergarten and first grade using a natural experiment. *Economics of Education Review* 30(2), 269–279.

Fryer Jr, R. and S. Levitt (2004). Understanding the black-white test score gap in the first two years of school. *The Review of Economics and Statistics* 86(2), 447–464.

Goodman, J. (2012a). Flaking out: Snowfall, disruptions of instructional time, and student achievement. Working Paper Series.

Goodman, J. (2012b). The labor of division: Returns to compulsory math coursework. Working Paper Series.

Heckman, J. and P. LaFontaine (2010). The American high school graduation rate: Trends and levels. *The Review of Economics and Statistics* 92(2), 244–262.

Herlihy, C. (2007). State and district-level support for successful transitions into high school. National High School Center, 14.

Horwitz, A. and J. Snipes (2008). Supporting successful transitions to high school. Council of Great City Schools 6.

Hoxby, C. M. and S. Murarka (2009). Charter schools in New York City: Who enrolls and how they affect their students' achievement. Working Paper 14852, National Bureau of Economic Research.

Jacob, B. and L. Lefgren (2004). Remedial education and student achievement: A regression discontinuity analysis. *The Review of Economics and Statistics* 86(1), 226–244.

Joensen, J. and H. Nielsen (2009). Is there a causal effect of high school math on labor market outcomes? *The Journal of Human Resources* 44(1), 171–198.

Lavy, V. (2010). Do differences in schools' instruction time explain international achievement gaps in math, science, and reading? Evidence from developed and developing countries. Working Paper 16227, National Bureau of Economic Research.

Lavy, V. and A. Schlosser (2005). Targeted remedial education for underperforming teenagers: Costs and benefits. *Journal of Labor Economics* 23(4), pp. 839–874.

Levine, P. and D. Zimmerman (1995). The benefit of additional high-school math and science classes for young men and women. *Journal of Business & Economic Statistics*, 137–149.

- Long, M., P. Iatarola, and D. Conger (2009). Explaining gaps in readiness for college-level math: The role of high school courses. *Education Finance and Policy* 4(1), 1–33.
- Long, M. C., D. Conger, and P. Iatarola (2012). Effects of high school course-taking on secondary and postsecondary success. *American Educational Research Journal* 49(2), 285–322.
- Marcotte, D. and S. Hemelt (2008). Unscheduled school closings and student performance. *Education Finance and Policy* 3(3), 316–338.
- Martorell, P. and I. McFarlin Jr (2011). Help or hindrance? The effects of college remediation on academic and labor market outcomes. *The Review of Economics and Statistics* 93(2), 436–454.
- Nomi, T. (2012). The unintended consequences of an algebra-for-all policy on high-skill students: Effects on instructional organization and students’ academic outcomes. *Educational Evaluation and Policy Analysis*.
- Nomi, T. and E. Allensworth (2009). “double-dose” algebra as an alternative strategy to remediation: Effects on students’ academic outcomes. *Journal of Research on Educational Effectiveness* 2(2), 111–148.
- Nomi, T. and E. Allensworth (2010). The effects of tracking with supports on instructional climate and student outcomes in high school algebra. Working paper, University of Chicago Consortium on Chicago School Research.
- Roderick, M. and E. Camburn (1999). Risk and recovery from course failure in the early years of high school. *American Educational Research Journal* 36(2), 303–343.
- Roland G. Fryer, J. (2011, October). Injecting successful charter school strategies into traditional public schools: Early results from an experiment in Houston. Working Paper 17494, National Bureau of Economic Research.
- Rose, H. and J. Betts (2004). The effect of high school courses on earnings. *The Review of Economics and Statistics* 86(2), 497–513.
- Scott-Clayton, J. and O. Rodriguez (2012). Development, diversion, or discouragement? a new framework and new evidence on the effects of college remediation.
- Stacy A. Wenzel, Kareem Lawal, B. C. C. R. F. S. R. S. (2005). Data brief: Algebra problem solving teachers talk about their experiences, December 2004. Brief report, UIC CMSI Evaluation Project.
- Starkel, R., M. J. and K. Price (2006). Two-period algebra in the 05-06 school year: Implementation report. Technical report.
- Taylor, E. (2012). Allocating more of the school day to math: Regression-discontinuity estimates of returns and costs. Unpublished manuscript.





**Appendix B. Tables and Figures**  
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Figure 1: Double-Dosing Rates

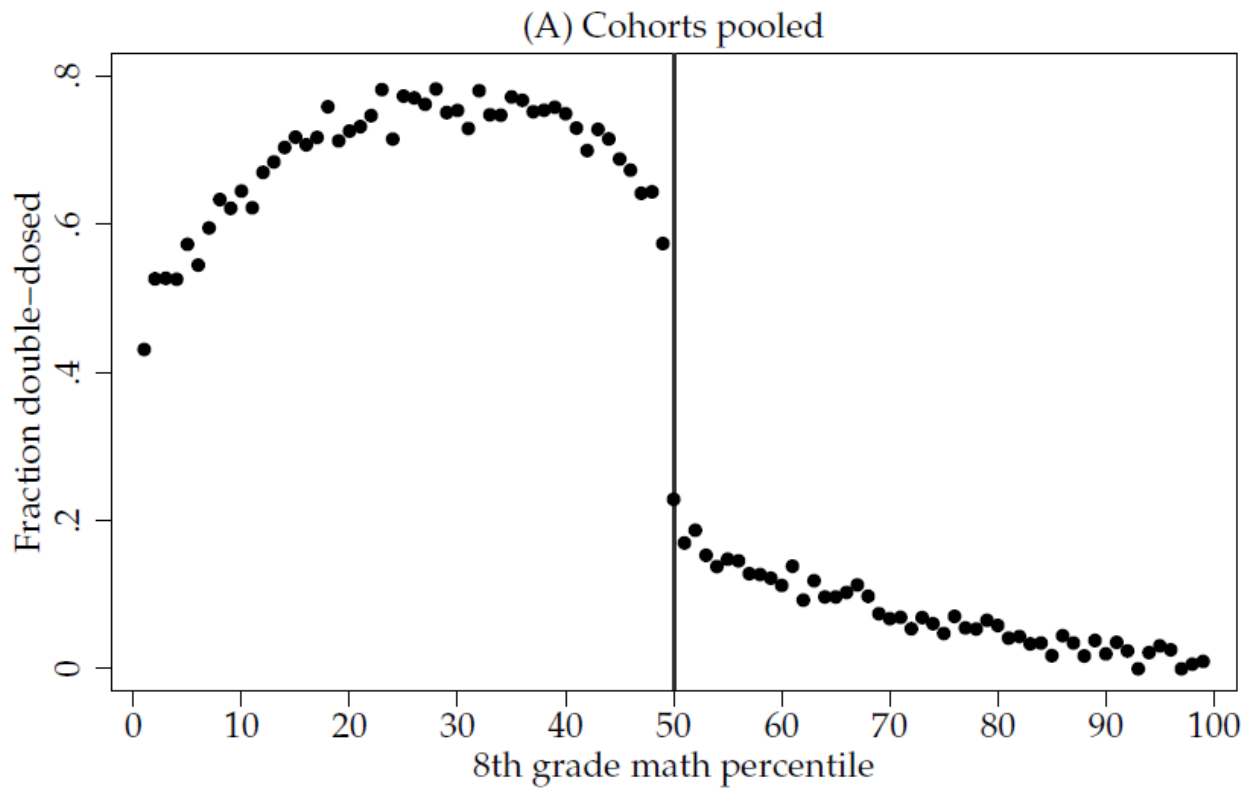


Figure 6: Math Achievement

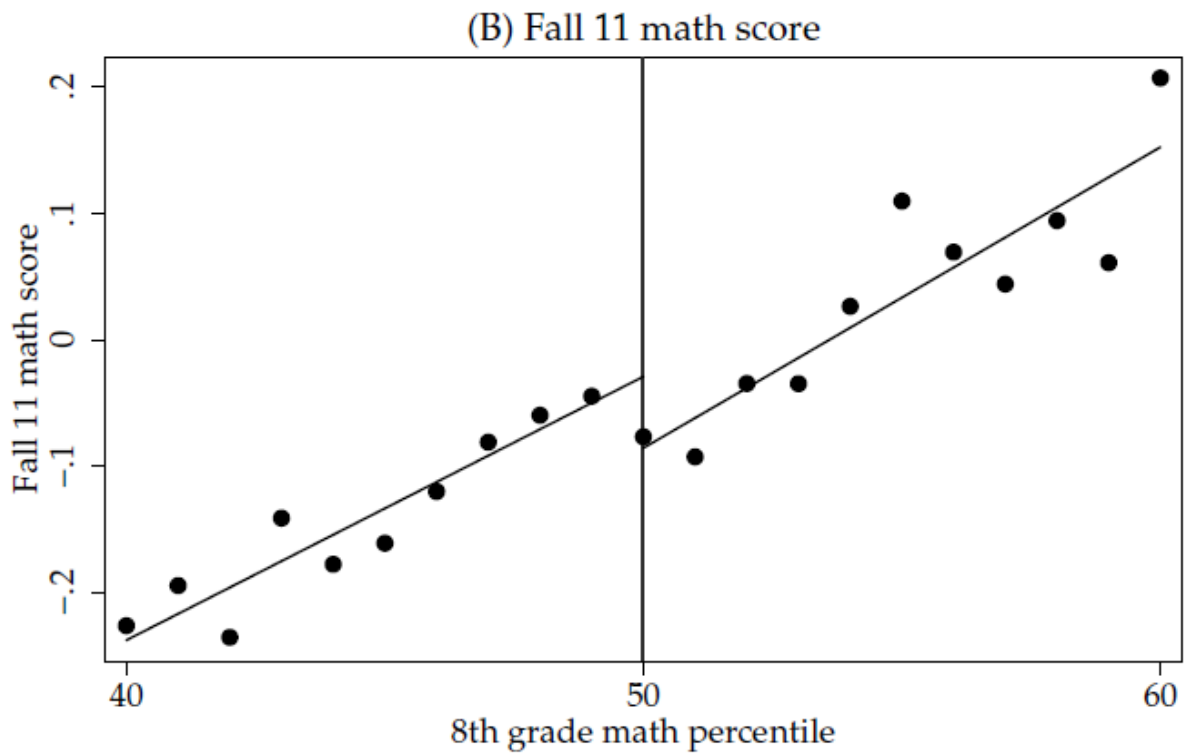
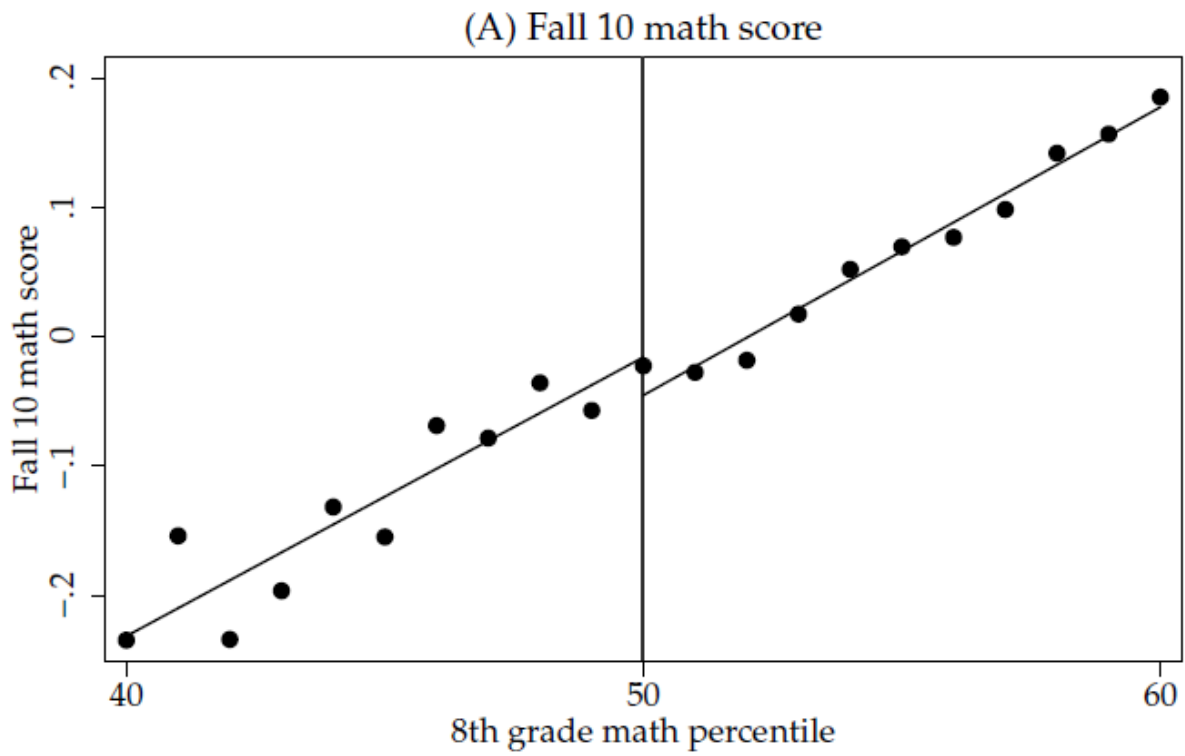


Figure 7: Educational Attainment

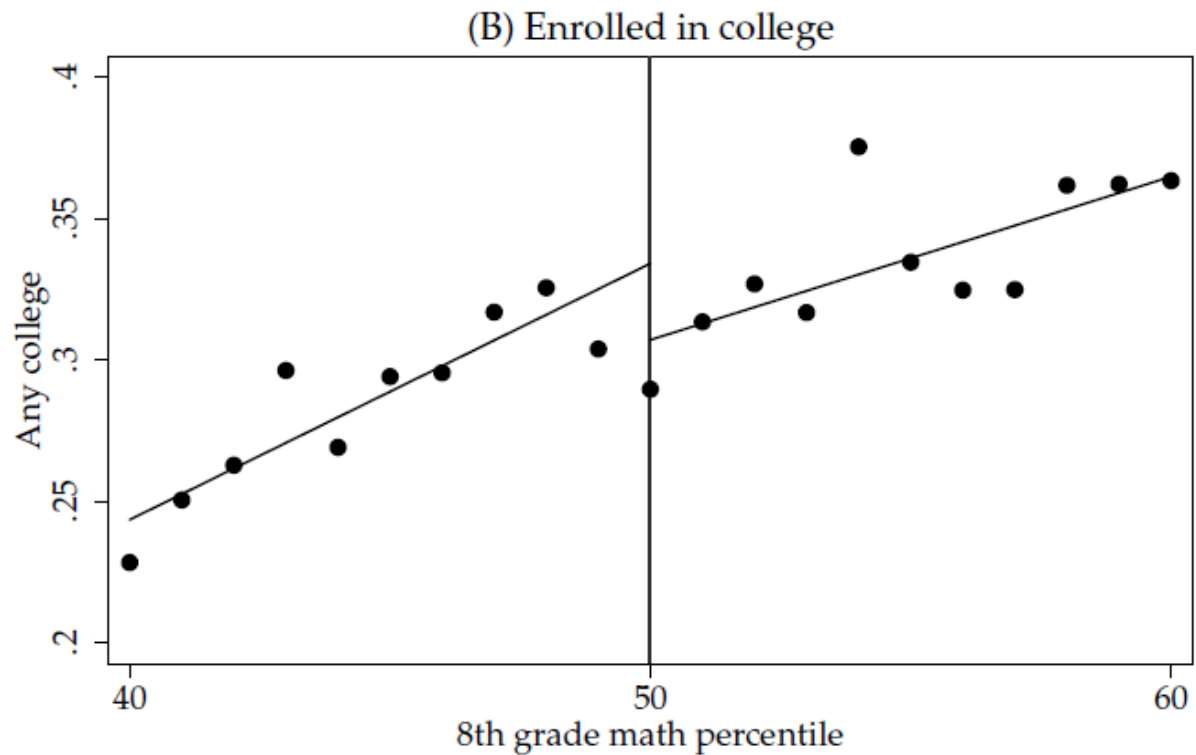
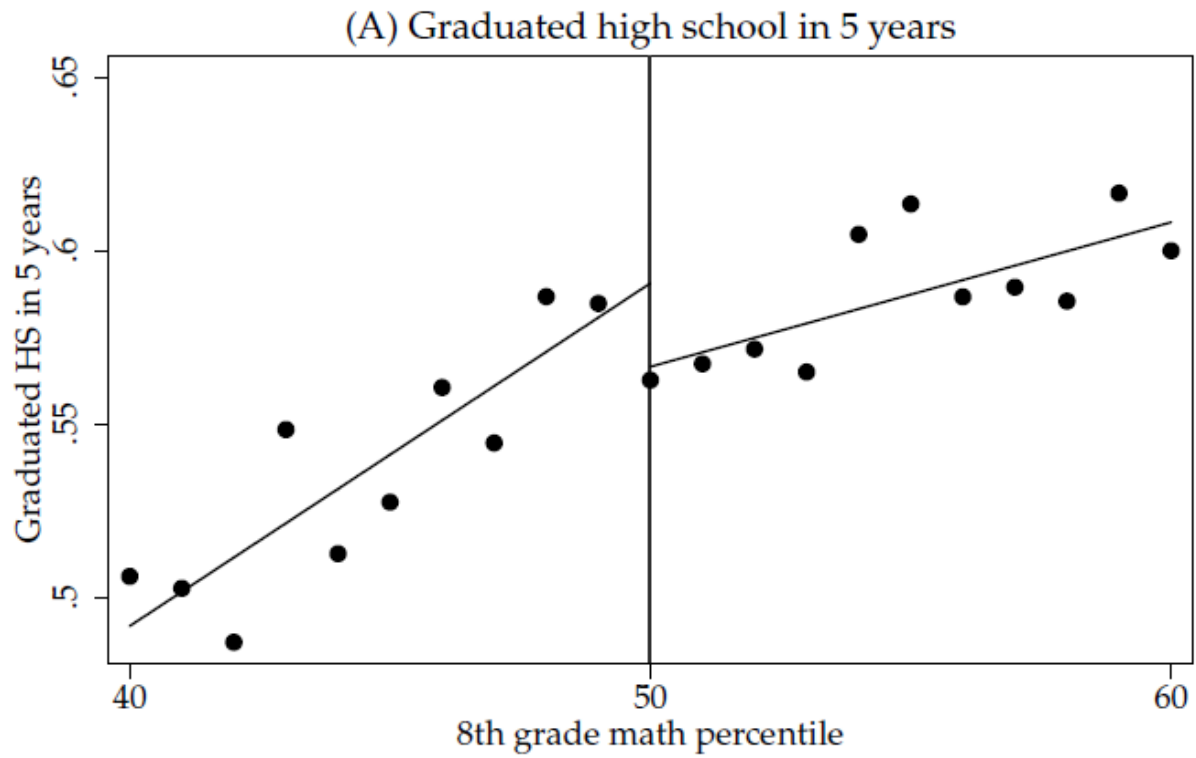


Table 5: The Impact of Double-Dose Algebra on Math Coursework

	(1) A or B in algebra	(2) Passed algebra	(3) Passed geom.	(4) Passed trig.	(5) Math GPA 10-12
<b>(A) Cohorts pooled</b>					
Double-dosed	0.094** (0.042)	0.047 (0.046)	-0.022 (0.049)	0.084** (0.041)	0.143* (0.082)
<b>(B) Cohorts separated</b>					
Double-dosed * 2003	0.095** (0.038)	0.035 (0.045)	-0.010 (0.043)	0.075* (0.039)	0.149* (0.079)
Double-dosed * 2004	0.093* (0.053)	0.067 (0.057)	-0.044 (0.067)	0.101* (0.055)	0.132 (0.107)
$p(\beta^{2003} = \beta^{2004})$	0.930	0.417	0.432	0.512	0.832
$\bar{Y}$ at threshold	0.138	0.624	0.576	0.544	1.398
N	11,507	11,507	11,507	11,507	10,439

Notes: Heteroskedasticity robust standard errors clustered by initial high school are in parentheses (\*  $p < .10$  \*\*  $p < .05$  \*\*\*  $p < .01$ ). Each column in each panel represents the instrumental variables regression of the listed outcome on semesters of double-dose algebra, where double-dose is instrumented by eligibility according to the first-stage given in table 2. Panel (A) pools the two treatment cohorts. Panel (B) interacts the instrument and endogenous regressor with cohort indicators. Below panel (B) is the p-value from an F-test of the equality of the two listed coefficients. Also listed is the mean value of each outcome at the eligibility threshold.

Table 6: The Impact of Double-Dose Algebra on Math Achievement

	(1) Fall 10 math	(2) Fall 10 algebra	(3) Fall 10 geometry	(4) Fall 11 math	(5) Fall 11 algebra	(6) Fall 11 geometry	(7) Spring 11 ACT math	(8) Took ACT
<b>(A) Cohorts pooled</b>								
Double-dosed	0.086 (0.067)	0.085 (0.070)	0.008 (0.082)	0.159** (0.064)	0.149** (0.059)	0.101 (0.085)	0.153** (0.071)	-0.017 (0.039)
<b>(B) Cohorts separated</b>								
Double-dosed * 2003	0.077 (0.059)	0.079 (0.064)	0.023 (0.073)	0.145** (0.059)	0.140** (0.056)	0.053 (0.077)	0.140** (0.064)	0.004 (0.036)
Double-dosed * 2004	0.100 (0.090)	0.097 (0.090)	-0.017 (0.107)	0.184** (0.080)	0.165** (0.078)	0.182* (0.110)	0.173** (0.088)	-0.055 (0.054)
$p(\beta^{2003} = \beta^{2004})$	0.670	0.726	0.504	0.375	0.663	0.043	0.455	0.146
$\bar{Y}$ at threshold	-0.023	-0.107	-0.101	-0.076	-0.119	-0.131	-0.211	0.576
N	8,210	8,210	8,210	7,428	7,428	7,428	6,700	11,507

Notes: Heteroskedasticity robust standard errors clustered by initial high school are in parentheses (\*  $p < .10$  \*\*  $p < .05$  \*\*\*  $p < .01$ ). Each column in each panel represents the instrumental variables regression of the listed outcome on semesters of double-dose algebra, where double-dose is instrumented by eligibility according to the first-stage given in table 2. Panel (A) pools the two treatment cohorts. Panel (B) interacts the instrument and endogenous regressor with cohort indicators. Below panel (B) is the p-value from an F-test of the equality of the two listed coefficients. Also listed is the mean value of each outcome at the eligibility threshold.

Table 7: The Impact of Double-Dose Algebra on Educational Attainment

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Graduated high school in 4 years	Graduated high school in 5 years	Enrolled, any college	Enrolled, two-year college	Part-time, two-year college	Semesters, any college	Semesters, two-year college
<hr/> (A) Cohorts pooled <hr/>							
Double-dosed	0.087* (0.047)	0.079** (0.039)	0.086*** (0.033)	0.079** (0.036)	0.066*** (0.026)	0.599** (0.239)	0.423** (0.167)
<hr/> (B) Cohorts separated <hr/>							
Double-dosed * 2003	0.105** (0.044)	0.094*** (0.036)	0.094*** (0.031)	0.070** (0.034)	0.057** (0.024)	0.656*** (0.236)	0.324** (0.157)
Double-dosed * 2004	0.055 (0.060)	0.052 (0.052)	0.072 (0.046)	0.093* (0.048)	0.083** (0.037)	0.498 (0.329)	0.598** (0.241)
p ( $\beta^{2003} = \beta^{2004}$ )	0.173	0.238	0.512	0.514	0.358	0.559	0.157
$\bar{Y}$ at threshold	0.509	0.563	0.290	0.159	0.077	1.795	0.873
N	11,507	11,507	11,507	11,507	11,507	11,507	11,507

Notes: Heteroskedasticity robust standard errors clustered by initial high school are in parentheses (\*  $p < .10$  \*\*  $p < .05$  \*\*\*  $p < .01$ ). Each column in each panel represents the instrumental variables regression of the listed outcome on semesters of double-dose algebra, where double-dose is instrumented by eligibility according to the first-stage given in table 2. Panel (A) pools the two treatment cohorts. Panel (B) interacts the instrument and endogenous regressor with cohort indicators. Below panel (B) is the p-value from an F-test of the equality of the two listed coefficients. Also listed is the mean value of each outcome at the eligibility threshold.