Title: Updating Algebra for All?: Evidence of a middle-grades math acceleration policy

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Abstract Body

Limit 4 pages single-spaced.

Background / Context:

In the US, mathematics achievement is often regarded as essential for individual educational and economic success as well as national global competitiveness (Chazan, 2008; The College Board, 2000). Indeed, recent evidence suggests that increasing the number of years of mathematics required of students raises earnings, especially among disadvantaged minorities (Goodman, 2013). Beginning with Sputnik in 1957 and continuing with A Nation at Risk (1983) and more recent reports (see for example, NCES, 2013; National Mathematics Advisory Panel, 2008), policymakers have called for increased proficiency in math as a national imperative (Gardner, 1983; Tate, 1997). Efforts thus have focused on increasing the amount and rigor of mathematics course taking, with a particular focus on exposure to algebra (Adelman, 2006). As a key pre-requisite for a sequence of courses culminating in college-level classes such as calculus and statistics, Algebra I is considered a critical “gatekeeper” course (Adelman, 2006; Education Commission of the States, 2008). Research has documented the relationship between algebra enrollment and a variety of educational and economic outcomes (Adelman, 2006; NCTM, 1989; Gamoran & Hannigan, 2000; Ham & Walker, 1999, Stein et al, 2011). Yet, much of this research suffers from selection bias (Stein et al, 2011) and therefore does not support causal claims about the impact of algebra.

In studies of the impact of universal algebra policies, researchers unsurprisingly find that such policies increase algebra enrollment (Allensworth, et al, 2009; Burris et al, 2006; Everson & Dunham, 1996; Stein et al, 2011). Impacts on student achievement, however, are mixed (Stein et al, 2011). Clotfelter and colleagues (2011) find negative effects of accelerating low-skilled students into Algebra I in 9th grade. Ninth grade universal algebra in Chicago negatively impacted the mathematics achievement of high-skilled students placed in heterogeneous classes (Nomi, 2012). While the Chicago policy increased overall algebra credit accumulation, it also increased failure rates across ability groups, and it did not lead to improved standardized test scores (Allensworth et al, 2009). For low performing students assigned to a “double dose” of algebra, the policy yielded positive short-term impacts on GPA and standardized test scores (Nomi & Allensworth, 2009) but also a positive increase in course failure rates. In the longer term, the double-dose strategy yielded positive effects on ACT performance, high school graduation, and college entrance (Cortes, Goodman & Nomi, 2012). Thus, there may be promise for algebra enrollment policies when combined with appropriate support for underprepared students, although care must be taken with how such policies are implemented (Nomi & Allensworth, 2013).

In the face of ongoing debates and mixed evidence on universal placement strategies, practitioners and policymakers have begun to seek alternative, objective mechanisms to advance students’ mathematics trajectories and to identify policies that might best encourage early and equitable exposure for students who are prepared. It is therefore important to understand the benefits and consequences of alternative mathematics course assignment strategies.

Purpose / Objective / Research Question / Focus of Study:

Description of the focus of the research.
The Wake County Public School System (WCPSS) in North Carolina recently addressed the issue of advancement in and equitable access to advanced mathematics. Under a recent policy, WCPSS uses a SAS-generated predicted probabilities of students’ success in obtaining a passing score on the NC Algebra I End-of-Course (EOC) exam, to determine recommended course placement in grades 6 through 8. For example, 8th grade students with a 0.70 or higher probability of passing the Algebra I EOC are recommended for Algebra I. We examine the question: do students who are just eligible for advancement in mathematics in middle school and who participate in advanced mathematics lead to improved academic outcomes compared to students who just miss being eligible for advancement and who do not enroll in advanced mathematics. By focusing on the students on the margin of eligibility under this policy we use a regression-discontinuity design to estimate the causal impact of this mathematics acceleration policy on subsequent test scores, grades, and course-taking behavior of middle-school students in Wake County who were exposed to this policy.

**Setting:**
*Description of the research location.*

We focus on middle school students in WCPSS. WCPSS is the largest school district in North Carolina and the 15th largest in the nation with over 150,000 students. The district is both racially and economically diverse. We utilize a longitudinal student information system provided by WCPSS. This system assigns students unique identifiers that allow the district to follow their progress from primary school through secondary school. For our purposes, the longitudinal structure of this data makes this research possible by tracking students from the end of fifth grade, when they are assigned EVAAS scores that determine whether they will be accelerated in math, through middle and high school, where many of the most important outcomes of interest are measured. We can track students as long as they stay within WCPSS, and will explore whether acceleration affected the rate of attrition from the district. We use data for students who were in grades six through eight during the academic years 2010-2011 through 2012-2013.

**Population / Participants / Subjects:**
*Description of the participants in the study: who, how many, key features, or characteristics.*

Our study includes student who were in the 6th through 8th grades in the academic years 2010-2011 through 2012-2013 and were subject to this policy.

**Intervention / Program / Practice:**
*Description of the intervention, program, or practice, including details of administration and duration.*

In contrast to blunt algebra-for-all policies, WCPSS sought to increase advanced mathematics course-taking by ensuring that all students who are prepared to be successful in Algebra I are enrolled as early as possible in their academic trajectory. Beginning in 2010, WCPSS implemented a targeted enrollment strategy through which they utilize a numeric criterion developed by the SAS Institute’s Education Value-Added Assessment System (EVAAS) to determine student eligibility for placement in advanced math coursework in the middle grades, including Algebra I in grade 8. We investigate the impact of this policy on student end-of-year
assessments in mathematics in grades 6, 7, and 8 during the policy’s first two years of implementation.

**Research Design:**

Description of the research design.

Students are eligible for advanced mathematics if they have a 0.70 or higher estimated probability of passing the Algebra I EOC examination, but not all eligible students actually enroll in the recommended advanced course. These discrepancies between “eligibility” and “take-up” mean that the probability of a student enrolling in the advanced mathematics course appropriate for his grade level does not jump sharply from zero to one at the EVAAS probability threshold of 0.70. As a result, we model the relationship between our outcome and predicted probability as a “fuzzy” regression discontinuity (Imbens & Lemieux, 2008; Murnane & Willett, 2011). To do so, we employ a two-stage instrumental-variable analytic approach to estimate the effect of participating in advanced mathematics on our outcome, Y. Under the “fuzzy” approach, we first use each student’s position relative to the probability cutoff as an instrument for take up of the offer of advanced mathematics (ADVANCE). At the first stage, we fit the following model:

\[
P(ADVANCE_{igr}) = \alpha_0 + \alpha_1(CEVAAS)_{igr} + \alpha_2ELIG_{igr} + \alpha_3((CEVAAS) \times ELIG)_{igr} + X'_i\theta_1 + \varphi_g + \omega_r + \delta_{igr},
\]

for student i in cohort g in school r, where \(\varphi\) and \(\omega\) represent the fixed effects of cohort and school, and \(\delta\) is a residual. The second-stage model is then:

\[
Y_{igr} = \pi_0 + \pi_1(CEVAAS)_{igr} + \pi_2ADVANCE_{igr} + \pi_3(CEVAAS) \times ADVANCE_{igr} + X'_i\theta + \varphi_g + \omega_r + \delta_{igr},
\]

where \(Y_{igr}\) represents a generic outcome. Because the second-stage predictor \(ADVANCE_{igr}\) is potentially endogenous, we replace it at the second stage with its fitted values from the first-stage equation. The parameter of interest is \(\pi_2\), the population causal effect of participating in advanced mathematics in middle school for students who are just eligible to participate compared to those who just missed eligibility. We interpret positive (negative) and statistically significant estimates of \(\pi_2\) as an indication that enrollment in an advanced mathematics course impacted students’ outcomes positively (negatively); we interpret a zero estimate as an indication that enrollment in an advanced course had negligible impact on students’ outcomes.

We perform all regression-discontinuity analyses within a local-linear regression framework in conjunction with an optimal choice of bandwidth (Imbens and Lemieux, 2008). We choose an optimal bandwidth by employing an automated procedure consistent with Imbens & Kalyanaraman (2012), which uses a triangular kernel. By taking this approach to choosing optimal bandwidth and by modeling the relationship between the outcome and forcing variable as locally linear, we address issues of bandwidth and functional form in ways that are consistent with the literature.
Data Collection and Analysis:
Description of the methods for collecting and analyzing data.

We rely entirely on longitudinal administrative data from the WCPSS school system from the academic years 2010-2011 through 2012-2013 inclusive. These data include student demographics, school and course enrollment, course grades, and state standardized test scores from grades five through high school. The design of the acceleration policy is such that we employ a regression-discontinuity design to isolate the causal effect of the policy on students who were just eligible and enrolled in accelerated mathematics compared to those who just missed being eligible and did not participate. This is the local average treatment effect, or the effect of the policy on policy compliers.

Findings / Results:
Description of the main findings with specific details.

Our analyses yield evidence of a significant, negative impact of assignment to advanced mathematics on student end-of-grade mathematics scores in the first two years of implementation (see Table 2). In particular, students of color and 6th graders seem to bear the majority of the adverse impact (see Table 2). Though the sign and magnitude of the effects are relatively stable, the statistical significance differs somewhat by choice of bandwidth. Results from our first-stage analysis indicate strong adherence to the policy’s rule for assigning students to these advanced courses (see Figure A1 and Table A1), and we find no evidence that the effects are heterogeneous by gender or race. These findings are consistent with earlier work from Clotfelter, Ladd, and Vigdor (2011) and Nomi (2012) who also find a negative impact of similar policies.

Conclusions:
Description of conclusions, recommendations, and limitations based on findings.

Our current conclusions are that students on the margin of eligibility and who participate in advanced mathematics in middle school may experience negative effects as a result of being in a peer group of more advanced ability, on average. We hypothesize that the negative effects may transfer as a result of a suboptimal match of ability or through negative psychological impacts of perceiving oneself as less able than your peers. That these effects are experienced disproportionately by groups that are historically less well represented in advanced mathematics classrooms is consistent with prior work. Further analyses will investigate additional outcomes, and investigate potential mechanisms for the negative impact of the policy, such as teacher quality and classroom peer effects, at the margin of assignment.
Appendices
Not included in page count.

Appendix A. References
References are to be in APA version 6 format.


Appendix B. Tables and Figures
Not included in page count.

Figure A1. Probability of enrollment in an advanced mathematics by EVAAS probability, all grades.
### Table A1. First-stage Estimates

<table>
<thead>
<tr>
<th>Bandwidth</th>
<th>Grades 6-8</th>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
</tr>
</thead>
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<tr>
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<td>0.254***</td>
<td>0.416***</td>
<td>0.228**</td>
<td>0.117</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.101)</td>
<td>(0.103)</td>
<td>(0.121)</td>
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<tr>
<td>F</td>
<td>20.3</td>
<td>17.1</td>
<td>4.9</td>
<td>0.9</td>
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<tr>
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<td>251</td>
<td>333</td>
<td>334</td>
</tr>
<tr>
<td>5</td>
<td>0.244***</td>
<td>0.260***</td>
<td>0.261***</td>
<td>0.191**</td>
</tr>
<tr>
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<td>(0.051)</td>
<td>(0.070)</td>
<td>(0.077)</td>
<td>(0.090)</td>
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<tr>
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<td>11.4</td>
<td>4.5</td>
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<td>638</td>
<td>556</td>
<td>540</td>
</tr>
<tr>
<td>10</td>
<td>0.193***</td>
<td>0.165**</td>
<td>0.220***</td>
<td>0.199***</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.067)</td>
<td>(0.064)</td>
<td>(0.068)</td>
</tr>
<tr>
<td>F</td>
<td>19.3</td>
<td>6.0</td>
<td>12.0</td>
<td>8.6</td>
</tr>
<tr>
<td>N</td>
<td>3,564</td>
<td>1,346</td>
<td>1,123</td>
<td>1,095</td>
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<tr>
<td>20</td>
<td>0.191***</td>
<td>0.143**</td>
<td>0.197***</td>
<td>0.248***</td>
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<td>(0.036)</td>
<td>(0.055)</td>
<td>(0.057)</td>
<td>(0.050)</td>
</tr>
<tr>
<td>F</td>
<td>28.6</td>
<td>6.8</td>
<td>12.1</td>
<td>24.4</td>
</tr>
<tr>
<td>N</td>
<td>8,096</td>
<td>3,092</td>
<td>2,610</td>
<td>2,394</td>
</tr>
</tbody>
</table>

Bandwidth = 20, controls

<table>
<thead>
<tr>
<th></th>
<th>Grades 6-8</th>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.182***</td>
<td>0.142***</td>
<td>0.184***</td>
<td>0.266***</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.050)</td>
<td>(0.054)</td>
<td>(0.048)</td>
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<tr>
<td>F</td>
<td>23.7</td>
<td>7.9</td>
<td>11.7</td>
<td>30.7</td>
</tr>
<tr>
<td>N</td>
<td>8,096</td>
<td>3,092</td>
<td>2,610</td>
<td>2,394</td>
</tr>
</tbody>
</table>

Notes: Heteroskedasticity robust standard errors clustered by middle school are in parentheses (* p<.10 ** p<.05 *** p<.01). Each row shows first stage estimates of the impact of eligibility for advanced coursework on the probability of enrollment in an advanced course. The coefficients shown are generated by local linear regression using an edge kernel with the listed bandwidth. No additional controls are included except in the final row, which conditions on gender, race, LEP and disability status, prior math and reading score, grade level and middle school fixed effects. Below each coefficient is the F-statistic associated with the excluded instrument.
Table A2. Instrumental-variable estimates including heterogeneity of effect by race and gender.

<table>
<thead>
<tr>
<th></th>
<th>(1) Math score, grades 6-8</th>
<th>(2) Math score, grade 6</th>
<th>(3) Math score, grade 7</th>
<th>(4) Math score, grade 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>All students</td>
<td>-0.292**</td>
<td>-0.582*</td>
<td>0.043</td>
<td>-0.286*</td>
</tr>
<tr>
<td></td>
<td>(0.130)</td>
<td>(0.325)</td>
<td>(0.207)</td>
<td>(0.162)</td>
</tr>
<tr>
<td>N</td>
<td>7,844</td>
<td>2,991</td>
<td>2,521</td>
<td>2,332</td>
</tr>
<tr>
<td>Female</td>
<td>-0.355**</td>
<td>-0.989**</td>
<td>0.226</td>
<td>-0.263</td>
</tr>
<tr>
<td></td>
<td>(0.155)</td>
<td>(0.491)</td>
<td>(0.241)</td>
<td>(0.227)</td>
</tr>
<tr>
<td>N</td>
<td>4,095</td>
<td>1,574</td>
<td>1,334</td>
<td>1,187</td>
</tr>
<tr>
<td>Male</td>
<td>-0.275</td>
<td>-0.190</td>
<td>-0.180</td>
<td>-0.388</td>
</tr>
<tr>
<td></td>
<td>(0.227)</td>
<td>(0.479)</td>
<td>(0.302)</td>
<td>(0.305)</td>
</tr>
<tr>
<td>N</td>
<td>3,749</td>
<td>1,417</td>
<td>1,187</td>
<td>1,145</td>
</tr>
<tr>
<td>Black or Hispanic</td>
<td>-0.402*</td>
<td>-0.639</td>
<td>-0.015</td>
<td>-0.431**</td>
</tr>
<tr>
<td></td>
<td>(0.205)</td>
<td>(0.391)</td>
<td>(0.230)</td>
<td>(0.200)</td>
</tr>
<tr>
<td>N</td>
<td>4,135</td>
<td>1,567</td>
<td>1,361</td>
<td>1,207</td>
</tr>
<tr>
<td>White or Asian</td>
<td>-0.171</td>
<td>-0.486</td>
<td>0.194</td>
<td>-0.202</td>
</tr>
<tr>
<td></td>
<td>(0.167)</td>
<td>(0.498)</td>
<td>(0.401)</td>
<td>(0.220)</td>
</tr>
<tr>
<td>N</td>
<td>3,709</td>
<td>1,424</td>
<td>1,160</td>
<td>1,125</td>
</tr>
</tbody>
</table>

Notes: Heteroskedasticity robust standard errors clustered by middle school are in parentheses (* p<.10  ** p<.05  *** p<.01). Each row shows instrumental variables estimates for the listed sub-group of the impact of advanced coursework on end of grade math scores. The coefficients shown are generated by local linear regression using an edge kernel with a bandwidth of 20. All regressions condition on gender, race, LEP and disability status, prior math and reading score, grade level and middle school fixed effects.