Use of Time Information in Models behind Adaptive System for Building Fluency in Mathematics

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ABSTRACT

In this work we introduce the system for adaptive practice of foundations of mathematics. Adaptivity of the system is primarily provided by selection of suitable tasks, which uses information from a domain model and a student model. The domain model does not use prerequisites but works with splitting skills to more concrete sub-skills. The student model builds on variation of Elo rating system which provide good accuracy and easy application in online system. The main feature of the student model is use of response times which can carry useful information about mastery.

1. INTRODUCTION

Our aim is to develop a practice system focused on basic mathematics which uses concepts of Computerized adaptive practice [8], i.e. to provide children with tasks that are most useful to them. We focus especially on detecting mastery and fluency using both correctness and timing information about children's responses.

Mathematics is usually associated with procedural knowledge. However, for achieving mastery of advanced topics it is necessary to solve some basic mathematical tasks at the level of fluency and automaticity. Good example of this is multiplication of small numbers which starts as procedural knowledge (child knows that $3 \cdot 5$ is $5 + 5 + 5$ and is able to complete calculation) but ends as declarative knowledge (child knows $3 \cdot 5$ is 15 without further thoughts) [15]. In both cases child gives correct response with high probability and the system is not able to distinguish between these scenarios based only on the correctness of the answer. Thus we want incorporate into our student model the information about response time, which is necessary to detect mastery, the state when the child is correct and fast.

Because our goal is to lead a child to automaticity we want to analyse strengths and weaknesses of the child at the level of individual items. Thus we need to track child's skills in great detail and we treat every item in the system independently. Also the fact that various graphical representations of the same task influence difficulty of the item, highlight need to track their difficulty individually. To estimate correctly difficulties of the items requires a lot of expertise, it is time consuming and is not always reliable. Therefore we do not want to make any assumptions about difficulties of the items and we rather use model which can estimate the difficulty of the solving data from the system. As a consequence we will be able to easily analyse which items are more difficult and why.

Proposed system is called MatMat and is currently available online in beta version at `matmat.cz` for all children (the system is so far implemented only in Czech) and it is free to use. The goal of the system is to provide adaptive practice of arithmetic operations which guide children from basic work with numbers (e.g. counting objects) to mastery of basic mathematical operations.

In contrast with complex intelligent systems for learning mathematics as Carnegie Learning's Tutors [14, 9] or ASSISTments [4] we focus only on small part of learning mathematics and we work only with atomic tasks. Therefore the system does not work with explanations of curriculum or hints and focuses on adaptive selection of tasks and appropriate feedback. Between related systems belongs Dybuster Calcularis [6] which works with basic math especially in context of dyscalculia; Math Garden [8] which has similar focus, works with similar student model and also incorporates time information; or FASTT Math [3] which also focus on building computational fluency.

2. MODELS

In this section we describe working draft of the domain model, which describes how is the content of the system organized, and the student model, which is built on the domain model and provides information about children who interact with the system. We have several requirements for the design of our models. We are in the situation when we use models in online environment and we rely more on collected data instead of expertise or other outside information. Hence we require models which can work on the fly and can quickly adapt to new data in the system. The goal of the student model is to provide estimation of child's abilities which are used for creation of feedback and selection of suitable tasks to practice.

2.1 Domain Model

Mathematics is very complex domain full of diverse components and relationships. Even in our very simplified case, when we considered only basics, situation can still be relatively complicated. One way how to build a domain model for mathematics is based on Knowledge space theory [1]. This approach splits the curriculum to skills a defines relations of prerequisites between them. This oriented graph can then be treated as dynamic Bayesian network [7].

We used different approach which allows us to capture information about very specific abilities, e.g. how good is child in multiplication of 5 and 7. The relations between such concrete skills, are not always prerequisites, e.g. the abilities to compute $5 \cdot 7$ and $5 \cdot 9$ are not one prerequisite to another but they are clearly dependent. We organized the skills into the tree structure (Figure 1) where every node corresponds to skill and its successors to more concrete sub-skills. Similarity of skills then can be expressed as level of the nearest common ancestor. Denote the fact that a skill d is ancestor of a skill c as $d > c$.

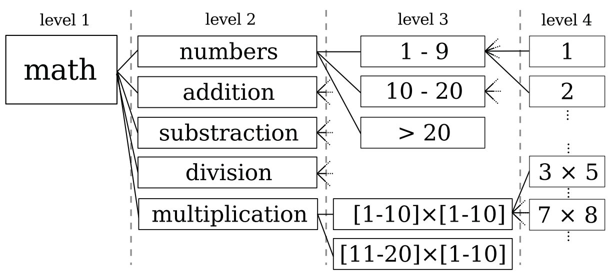

Figure 1: The tree structure of the skills

The root of the tree is a global skill which represents overall knowledge of mathematics. Under that are skills which correspond to basic units in system (level-2) — numbers, addition, subtraction, multiplication and division. In level-3 are sub-skills which represent concepts (inspired by [6]) within parent skill, e.g. under 'numbers' skill are 'numbers in range from 1 to 9', 'numbers in range from 10 to 20', 'numbers greater then 20', ...; or under 'addition' are 'addition in range from 1 to 9 (without bridging to 10)', 'addition in range from 10 to 20 with bridging to 10', ... And finally level-4 skills correspond to the tasks for which mastery on the level of declarative knowledge is expected. Example of these are skills that correspond to numbers (1, 2, 3, ...), simple addition tasks ($1+2$, $5+7$) or multiplication of numbers smaller than 10 ($3 \cdot 5$, $7 \cdot 8$). There are no level-4 skills for more complicated task (e.g. $11 \cdot 13$) for which procedural knowledge is more involved. The items representing these tasks belong typically to more general level-3 skills.

In current model every item in the system is mapped to exactly one skill (typically a leaf skill). So under a skill are multiple items. In case of the more general level-3 skills it can be tens or hundreds. In case of the level-4 skills there are from 2 to 10 items which are various forms of the task ($5+7$ and $7+5$) and different graphical representations of task (numbers, objects, number line ...).

2.2 Student Model

Rather then the discrete representation of ability (known or unknown) we used the continues representation, which is more suitable for our situation when we need to track abilities also for relatively general skills. The relation between these abilities and expected probability of correct answer is defined by a logistic function.

For the skill from s and the child c model estimates the value v_{sc} which represents difference of ability relative to parent skill. Overall value of ability is then $\theta_{sc} = \sum_{s \leq \bar{s}} v_{\bar{s}c}$. This approach allows to capture relations between leaf skills. Information obtained from observation about one ability can be naturally propagated to other related abilities. This is especially important for new children in the system with small number of responses (relatively to large number of abilities). The model also estimates the difficulties β_i of the items i, which can be interpreted as a required ability to have 50% chance of solving item correctly. Expected response is then $e_{ci} = \frac{1}{1+e^{\beta_i - \theta_{sc}}}$.

To estimate abilities and difficulties we used a model based on Elo rating system [2] and PFA [12] which is inspired by models which have been successfully used in other projects [8, 11]. The main idea is to update all related abilities and item difficulty based on unexpectedness of response after every answer. To empathize the fact that the correct answer (even repetitive) does not mean mastery we need to take into account the response time t_{ci}. This can be achieved by extension of discrete response r_{ci} (correct or incorrect) to continuous one where values between 0 and 1 mean the correct answer but with longer time than the targeted time τ_i. Example of this extension is decay of the response value exponentially relatively to the ratio of t_{ci} and τ_i (Figure 2).

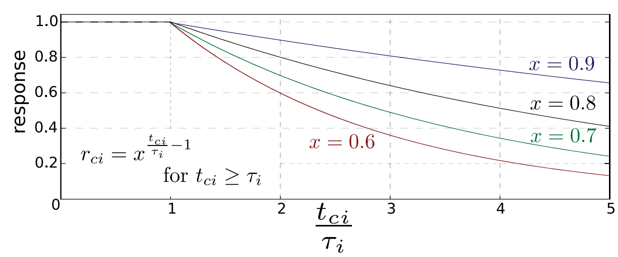

Figure 2: The response value for the correct answers

After the answer, all abilities θ_{sc} belonging to ancestors' skills s are updated. Updates of abilities are performed sequentially from the root of the skill tree. If the answer is the first answer of the child to the item, the difficulty of item β_i is also updated.

$$\beta_i = \beta_i + \frac{\alpha}{1 + \beta \cdot n_i} \cdot (e_{ci} - r_{ci}),$$
$$\theta_{sc} = \theta_{sc} + \gamma_s \cdot K_r \cdot (r_{ci} - e_{ci}).$$

The parameters α and β define shape of the decay function [13] which prevents excessive influence of recent responses. The decay function takes argument n_i – number of previous updates of that difficulty. Parameter K_r corresponds to PFA updates and depends on correctness of answer. Parameter $\gamma_s \in [0, 1]$ tells how much response to the item testifies about a skill and consequently, how much information obtained from response is propagated to sub-skills. Reasonable values of γ_s are near 1 for the most concrete skills and near 0 for the global skill.

2.3 Item Selection

The selection of an appropriate item that suits the ability of a child is a key feature of the system and has to balance several aspects. The system should not select the same or similar item in a short time, it should select diverse items for better exploration of child's abilities and, foremost, the system should select items with appropriate difficulty – not already mastered (high probability of success) and not too difficult (small probability of success). Currently used algorithm is very similar to the one described in [11]. Only difference is in bringing into account also similarity of items (e.g. $5 + 7$ is similar with $7 + 5$).

3. FUTURE WORK

Most of adaptive educational systems currently work only with correctness of responses. Our goal is to find out if this classical approach can be robustly extended by taking into account timing information and if this extension can be useful in building fluency in the basic mathematical tasks. To target this questions we proposed the system described in this work. This system is still in testing phase but the first analysis of 28 thousand collected answers, show that the ability and difficulty values estimated by the student model make intuitive sense, the system can adapt quickly and the item selection algorithm works reasonably. However, there is a lot of space for improvement.

The domain model can be enriched with prerequisites which can be useful for both ability estimation and for item selection. The current choice of the skills used in the domain model should be reviewed by a domain expert or compared with automatic methods which use collected data [10]. The proposed student model is incorporating response time but current approach is quite simplified and explicitly does not distinguish between accuracy and speed, which can be modeled separately. Also it is not clear how to set, or rather automatically estimate, targeted response times τ_i. Next characteristic of the model is propagation of information about abilities across all skills, which is useful in first phases but later can be undesirable. The propagation is closely connected to parameters γ_s and their influence to the model behaviour should be investigated.

To evaluate our approach the proposed models will be compared to alternative models (e.g. Bayesian network model [7] which works with prerequisites) or simpler versions of Elo model (e.g. model which uses only one global skill and independent local skills [11]). The comparison of the models can be done offline with respect to the quality of predictions or online by comparison of an improvement rate or behaviour of children groups using different models and item selection strategies. These comparisons should bring some light an whether the proposed methods are useful.

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