

Cognitive Predictors of Calculations and Number Line Estimation  
with Whole Numbers and Fractions among At-Risk Students

Jessica M. Namkung<sup>1</sup> and Lynn S. Fuchs<sup>2</sup>

<sup>1</sup>University at Albany, State University of New York

<sup>2</sup>Vanderbilt University

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Inquiries should be sent to Jessica M. Namkung, ED 226. 1400 Washington Ave., Albany, NY 12203; [mnamkung@albany.edu](mailto:mnamkung@albany.edu)

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### Abstract

The purpose of this study was to examine the cognitive predictors of calculations and number line estimation with whole numbers and fractions. At-risk 4<sup>th</sup>-grade students ( $N = 139$ ) were assessed on 7 domain-general abilities (i.e., working memory, processing speed, concept formation, language, attentive behavior, and nonverbal reasoning) and incoming calculation skill at the start of 4<sup>th</sup> grade. Then, they were assessed on whole-number and fraction calculation and number line estimation measures at the end of 4<sup>th</sup> grade. Structural equation modeling and path analysis indicated that processing speed, attentive behavior, and incoming calculation skill were significant predictors of whole-number calculations whereas language, in addition to processing speed and attentive behavior, significantly predicted fraction calculations. In terms of number line estimation, nonverbal reasoning significantly predicted both whole-number and fraction outcome, with numerical working memory predicting whole-number number line estimation and language predicting fraction number line estimation. Findings are discussed in terms of distinctions between whole-number and fraction development and between calculations and number line learning.

*Keywords:* whole numbers, fractions, calculations, number line estimation, cognitive predictors, mathematics

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Fraction knowledge is one of the foundational forms of competence required to perform successfully in more complex and advanced mathematics, such as algebra (Booth & Newton, 2012; NMAP, 2008). In a longitudinal study examining the types of mathematical knowledge that predict later mathematical achievement in the United States and United Kingdom, students' fraction knowledge in fifth grade uniquely predicted their algebraic knowledge and overall mathematics achievement in high school, even after controlling for other types of mathematical knowledge, general intellectual ability, working memory, family income, and education (Siegler et al., 2012). Its predictive value compared favorably to whole-number addition, subtraction, and multiplication.

Yet, fractions is one of the most difficult mathematical topics to master (e.g., Bright, Behr, Post, & Waschmuth, 1988; Lesh, Post, & Behr, 1987; Test & Ellis, 2005). Difficulty in understanding fractions is not new. In a national survey, algebra teachers identified fractions as an area with which students have the poorest preparation (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). Furthermore, more than 40 years of data from the National Assessment of Educational Progress (NAEP) consistently indicate that students struggle with fractions. For example, results from 1996 NAEP indicated that only 49% of fourth -grade students correctly identified how many fourths are in one whole. In 2013 NAEP, only 60% of fourth-grade students correctly identified the greatest unit fraction.

### **Developmental Pathways**

Such difficulty with fractions is often attributed to the fundamental differences between whole numbers and fractions. For example, there is no predecessor and successor of a fraction, and adding and subtracting fractions require a common denominator. Also, quantities decrease

with multiplication and increase with division in fractions (Stafylidou & Vosniadou, 2004). Thus, learning fractions has been considered different from and discontinuous with students' understanding of whole numbers, leading to the potential conflict between students' prior knowledge about whole numbers and new information about fractions (Cramer, Post, & delMas, 2002; Cramer & Wyberg, 2009; Siegler, Thompson, & Schneider, 2011). Accordingly, common mistakes with fractions may stem from students' overgeneralization of their understanding of whole numbers to fractions. This is referred as *whole-number bias* (Cramer et al., 2002; Ni & Zhou, 2005; Stafylidou & Vosniadou, 2004). Although there is controversy over the origin of the whole-number bias, one hypothesis is that children develop numerical cognition with an innate domain-specific mechanism that privileges whole numbers, which are discrete, over fractions, which are continuous (Ni & Zhou, 2005). For example, Hiebert, Wearne, and Taber (1991) found that low-achieving fourth graders were significantly more responsive to intervention that focused on a discrete representation of decimals (i.e., base-10 blocks) compared to intervention that focused on a continuous representation of decimals (i.e., number line and circle stopwatch). According to this view, whole-number and fraction competence may represent two distinct constructs and may follow different developmental paths.

Alternatively, evidence suggests that whole-number and fraction competence may follow similar developmental paths. For example, using a nonverbal procedure of assessing calculation ability, Mix, Levine, and Huttenlocher (1999) found that three to seven years old children's competence with whole-number and fraction calculations followed the same gradual rise in performance, not an abrupt shift of performance at a particular age. They also found that understanding of important ideas about fractions is evident in children as young as four years old. Additionally, according to a recently proposed integrated theory of numerical development, fraction understanding develops as students broaden their understanding of whole numbers to

include magnitudes of fractions with specific locations on a number line. That is, Siegler and Pyke (2012) found that as with whole numbers, sixth and eighth graders' accuracy of fraction magnitude representation was strongly correlated with fraction calculation fluency and overall mathematics achievement. They argued that fractions and whole numbers should, therefore, be considered within a single numerical developmental framework.

Given insufficient evidence at the present time to support one view over the other, it is important to examine whether the development of these mathematical skills relies on the same or distinct cognitive abilities. This would shed light on underlying developmental processes. In examining how whole-number and fraction competence develop, we focused on two domains: calculations and number line estimation. We chose these domains because they are major indicators, respectively, of procedural and conceptual mathematics knowledge. Additionally, whole-number calculation skill is a major component of the primary-grade mathematics curriculum and represents a deficit for many students (NMAP, 2008). Also, difficulty with fraction calculations is persistent and stable, as evidenced by low-achieving students' accuracy in solving fraction calculation problems remaining low across sixth through eighth grades while high-achieving students improved in accuracy (Siegler & Pyke, 2012).

We focused on number line estimation as a contrasting outcome because students' ability to approximate numbers on a number line is another important form of mathematical development. Accuracy on number line representations has been found to be a significant predictor of mathematics achievement and whole-number calculations (e.g., Booth & Siegler, 2006, 2008; Schneider, Grabner, & Paetsch, 2009; Siegler & Booth, 2004). As with whole numbers, accuracy of fraction magnitude representations is related to fraction calculation competence and overall mathematics achievement (Siegler et al., 2011; Siegler & Pyke, 2012).

Another advantage is that the format of the tasks is highly similar for both whole numbers and fractions.

While calculations and number line estimation each represents a fundamental form of mathematics competence, prior research suggests that they follow different developmental patterns (e.g., Levine, Jordan & Huttenlocher, 1992; Feigenson, Dehaene, & Spelke, 2004; Geary, Hoard, Nugent, Byrd-Craven, 2008). For example, children develop calculation competence hierarchically as they acquire quantitative abilities prior to entering school (Levine et al., 1992). Then, they develop procedural efficiency and fluency, and eventually achieve automatic recall of number combinations during early elementary years. Next, they learn to perform multi-digit addition and subtraction calculations without and then with regrouping as they skill increases. Multiplication and division and eventually calculations with rational numbers are introduced through the upper elementary grades. In this way, calculation competence develops as a continuum of successive skills that are dependent on each other.

By contrast, making placements on a number line is a basic numerical representation, which taps into children's numerical magnitude understanding. Students at first rely on a logarithmic representation, exaggerating the distance between the magnitudes of small numbers and minimizing the distance between magnitudes of larger numbers (Feigenson et al., 2004; Siegler & Booth, 2004). With schooling, children acquire a linear representation with equal distances between two consecutive numbers at any point in the sequence. This shift occurs early, with most second graders generating fairly accurate linear estimates of numerical magnitudes (Booth & Siegler, 2008). Then, they broaden their understanding of numerical magnitude to include fractions (Siegler et al., 2011). In estimating the location of a given number on a number line, students engage in analytical and reasoning processes instead of rather than executing a series of procedural steps. Students infer the connection between the given number and endpoints

of the number line, so they can deduce the location of the given number relative to the endpoints. For example, students locate 50 midway between 0 and 100 because 50 is half of 100. In an analogous way, they place  $\frac{1}{2}$  midway between 0 and 1 because  $\frac{1}{2}$  is half of 1.

Taken together, whole-number and fraction calculation and number line estimation competence may draw on a shared set of cognitive resources, but also may require distinct cognitive abilities given the different developmental processes. Our conceptual framework for identifying potential predictors grew out of Geary's (2004) model of mathematics learning. At the most general level, this model distinguishes between mathematics concepts and mathematics procedures. Acquisition of mathematical knowledge in any given area requires both accurate and fluent execution of procedures and concepts. Conceptual and procedural knowledge is mutually supportive, with increasing competence of each type contributing to increasing competence in the other (Hecht & Vagi, 2010, 2012; NMAP, 2008; Rittle-Johnson & Siegler, 1998).

Within Geary's model, general cognitive processes, such as working memory and attention, support learning of both math concepts and procedures. The central executive controls the cognitive processes needed for learning and executing procedures. Geary also identified both symbolic and nonverbal cognitive systems as important for representing and manipulating mathematical information. Language systems are involved in learning number names and the verbal count sequence; nonverbal reasoning is crucial for representing and comparing numerical magnitudes. This model guided the processes that were assessed in the present study.

In addition to the theoretical model, empirical studies examining cognitive abilities that underlie whole-number and fraction competence provide some insight on underlying cognitive mechanisms. Such analysis can shed light on whether whole-number and fraction competence represent a unitary construct or distinct forms of competence. Yet, we identified only three prior studies that examined the underlying cognitive mechanisms of number line estimation. Jordan et

al. (2013) found that language, nonverbal reasoning, attention, working memory, reading fluency, and calculation fluency were significant predictors of whole-number line estimation skill at third grade. Geary, Hoard, Nugent, and Byrd-Craven (2008) identified intelligence scores and central executing working memory as significant predictors of accuracy with whole-number line estimation in first and second graders. Bailey, Siegler, and Geary (2014), who focused on fractions, found that first graders' whole-number line estimation, whole-number calculation competence, and central executive working memory significantly predicted eighth-grade fraction magnitude competence, indexed by both fraction comparison and fraction number line estimation. The literature on predictors of development in the calculation domain is more extensive. Studies provide support for working memory, attentive behavior, processing speed, phonological processing, and nonverbal reasoning as predictors, while language uniquely supported fraction calculations. We consider findings on each of these predictors in the next section.

### **Prior Work on Potential Cognitive Predictors of Whole-Number and Fraction Calculations**

The first cognitive resource that may support the development of calculation skill is working memory, which provides temporal storage of information to support ongoing cognitive tasks (Baddeley, 1986). Whole-number calculation procedures require regulating and maintaining arithmetic combinations derived either through retrieval from long-term memory or by relying on counting while simultaneously attending to regrouping demands and place values. Therefore, students with low working memory would have difficulty holding sufficient information to complete a task (e.g., keeping track of where they are in a task; Alloway, Gathercole, Kirkwood, & Elliot, 2009). With fraction calculations, prior work suggests that working memory may influence whole-number arithmetic calculations, which in turn influences fraction calculations as in Hecht, Close, and Santisi (2003). This may reflect the hierarchical nature of whole-number and fraction calculations. At the same time, working memory may also



influence fraction calculations beyond its effects via whole-number calculations (Jordan et al., 2013; Seethaler, Fuchs, Star, & Bryant, 2011). That is, besides supporting whole-number calculation tasks embedded within fraction calculations, working memory may help students regulate the interacting role of numerators and denominators as well as the planning and executing multiple steps (e.g., finding common denominators and equivalent fractions).

Prior work also identifies attentive behavior as an important cognitive predictor of whole-number calculations (Fuchs et al., 2005, 2006, 2008, 2010a, 2013; Swanson, 2006). Given that considerable attention is necessary to execute calculation procedures and monitor errors simultaneously, it is not surprising that attentive behavior is a key determinant of whole-number calculations. Attentive behavior also appears to influence fraction calculations in two ways. More attentive students may perform better at whole-number calculations, which in turn has a positive effect on fraction calculations into which whole-number calculation tasks are embedded (Hecht et al., 2003). Attentive behavior may also influence fraction calculations above and beyond its effects through whole-number calculations (Hecht & Vagi, 2010). This may be because even greater attention is required to carry out complex fraction calculation procedures, such as attending to the interacting role of numerators and denominators and converting fractions to have the same denominators.

Processing speed, which refers to the efficiency with which cognitive tasks are executed (Bull & Johnston, 1997), is another potential resource involved in the development of calculation skill. In whole-number calculations, processing speed may facilitate the simple processes, such as counting or retrieving arithmetic facts from long-term memory (Bull & Johnston, 1997; Geary, Brown, & Samaranayake, 1991). Faster processing supports more automated mathematics performance, which permits more efficient processing of the mathematics, and this in turn improves performance (Bull & Johnston, 1997). Findings are less consistent with fractions. In

Seethaler et al. (2011), the effects of processing speed on fraction calculations were not significant. On the other hand, in Fuchs et al. (2013), processing speed moderated students' responsiveness to intervention on fraction calculations: Intervention compensated for students' low processing speed, resulting in similar outcomes for students across the process speed distribution whereas control students with superior processing speed benefited more from classroom instruction.

Another potentially important cognitive factor to consider is nonverbal reasoning, which refers to the ability to identify patterns and relations and to infer and implement rules (Nutley et al., 2011). It allows students to analyze and form accurate representations of quantitative and qualitative relations among numbers (Primi, Ferrao, & Almeida, 2010). However, mixed findings exist for both whole-number and fraction calculations. Although researchers failed to find significant effects of nonverbal reasoning on whole-number calculations in four Fuchs et al. studies (2005, 2006, 2010a, 2010b), one study identified nonverbal reasoning as a unique contributor of whole-number calculations (Seethaler et al., 2011), and nonverbal reasoning was found to moderate responsiveness to first-grade calculations intervention (Fuchs et al., 2013). Similarly, whereas prior research failed to find significant effects of nonverbal reasoning on fraction calculations (Jordan et al., 2013; Fuchs et al., 2013), one study identified nonverbal reasoning as a unique predictor of rational number calculations, including fractions, percentages, and decimals (Seethaler et al., 2011). With inconsistent evidence, nonverbal reasoning is important to consider because it may play an important role in expanding and reorganizing students' initial knowledge of whole numbers to include fractions.

At the same time, prior studies (e.g., Fuchs et al., 2005, 2006) have found phonological processing to be a unique predictor of whole-number calculations. Phonological processing is required whenever phonological name codes of numbers are used (Geary, 1993). For example,

students first convert numbers and operators of a calculation problem to a verbal code. Then, they must process the phonological information and either retrieve a phonologically-based answer from long-term memory or use counting strategies to derive an answer. Both strategies require phonological processing abilities (Hecht, Torgesen, Wagner, & Rashotte, 2001).

Phonological processing may also play a role in forming an association between the arithmetic fact and phonological representations of the words, such as “four times three is 12.” Reliable connections facilitate both memorization and recall of the facts (Robinson, Menchetti, & Torgesen, 2002). Yet, we identified no studies examining the role of phonological processing in fraction calculations. Phonological processing may be an important factor to consider because strong phonological processing may help students establish representations of fractions and fraction names.

Last, although oral language ability (vocabulary, listening comprehension) is not associated with development of whole-number calculation skill (e.g., Fuchs et al., 2005, 2006, 2008, 2010a, 201b, 2013; Seethaler et al., 2011), it has been found to support fraction calculations (Fuchs et al., 2013; Seethaler et al., 2011). Unlike whole-number calculations, fraction calculations require processing of the interacting role of numerators and denominators beyond adding and subtracting whole numbers (i.e., numerators). These processes, such as finding the same denominators and converting fractions with the same denominator, require understanding about fractions in addition to the ability to carry out rote calculation procedures. Prior research demonstrated that understanding of fractions is supported by language (Miura, Okamoto, Vlahovic-Stetic, Kim, & Han, 1999; Paik & Mix, 2003), suggesting that students with strong language ability may gain deeper understanding compared to those with weak language ability. Better understanding of fraction concepts may in turn facilitate fraction calculations as

conceptual and procedural understandings have been found to influence each other (e.g., Rittle-Johnson & Siegler, 1998; Rittle-Johnson, Siegler, & Alibali, 2001).

Taken together, evidence may be converging on which cognitive characteristics are shared or distinct for whole-number and fraction calculations. Further investigation is, however, warranted for several reasons. First, only a few studies have investigated the cognitive predictors of fraction calculations. Second, conflicting findings exist for each cognitive factor because some studies have not considered all cognitive abilities in their analysis (e.g., verbal working memory in Alloway, 2009; numerical working memory and attentive behavior in Hecht et al., 2003 and Hecht & Vagi, 2010) and due to methodological differences (e.g., different outcome measures and study participants) that exist across the literature. In fact, only Seethaler et al. (2011) considered cognitive predictors of whole-number and fraction calculations within the same study, thus with the same predictors and methodological features for both outcomes. Even so, because two separate regression analyses were used for whole-number and fraction calculation outcomes, comparing the predictors across both outcomes is difficult.

### **Present Study**

To address these limitations, the purpose of the present study was to examine the cognitive predictors associated with calculations and number line estimation with whole numbers and fractions. Our goal was to gain insight into whether the developmental paths are similar or different. We were specifically interested in the at-risk students (i.e., performing below the 35<sup>th</sup> percentile) because the development of at-risk students differs from that of typically achieving students. For example, at-risk students are characterized by having deficits in understanding number knowledge and relationships (e.g. Jordan, Hanich, & Kaplan, 2003; Jordan, Kaplan, Oláh, & Locuniak, 2006), counting knowledge and using efficient counting strategies (e.g., Butterworth, 2005; Geary, Bow-Thomas, & Yao, 1992; Geary, Hoard, Byrd-Craven, & DeSoto,

2004; Geary et al., 2007), mastering arithmetic facts and using efficient calculation strategies to solve arithmetic problems (e.g., Geary, 1990; Geary, Brown, & Samaranayake, 1991; Jordan & Montani, 1997;), and solving word problems (e.g., Fuchs & Fuchs, 2005).

Yet, little is known about the development of whole-number versus fraction competence for at-risk students. Most studies include students spanning typical development and mathematics difficulties. Although including samples that span the range of performance offers the advantage of increasing variance to identify sources of individual differences in mathematics outcomes, the importance of key variables may differ for lower-performing students. We also note that because of the limited literature on cognitive predictors of number line estimation, we examined whether the cognitive predictors that have been identified as potentially important for development of calculation skill also predict whole-number and fraction number line estimation.

The present study extends the literature in three ways. First, we extended Geary's (2004) model of mathematical learning by including other cognitive variables, such as attentive behavior, processing speed, and concept formation, which have been previously identified to affect calculation development. Thus, we assessed the contributions of a more comprehensive set of cognitive and linguistic predictors (numerical working memory, working memory-sentences, language, attentive behavior, processing speed, nonverbal reasoning, and concept formation), therefore integrating more recent theoretical and empirical findings. Simultaneously considering a fuller set of important cognitive abilities offers the advantage of providing a more accurate and stringent test of each ability's contribution because each variable competes for variance against other constructs. Second, we analyzed the relation between cognitive predictors and both whole-number and fraction outcomes within the same model allowing for direct comparisons across the two outcomes. Third, although number line estimation has often been examined as a correlate

and predictor of mathematics achievement, few studies have examined the underlying cognitive mechanisms of number line estimation.

To our knowledge, the present study was the first to examine cognitive characteristics that underlie fraction number line estimation alone. Examining cognitive predictors of both whole-number and fraction domains should provide insights on the cognitive mechanisms that underlie each form of competence. Such knowledge may help guide understanding the development of each form of competence and the nature of intervention for improving these mathematics outcomes in at-risk students. It may also be theoretically useful for developing screening targets by which students are identified for early intervention.

## **Method**

### **Participants**

Data in the present study were collected as part of a larger study investigating the efficacy of a fraction intervention. As part of this larger study, 315 fourth-grade at-risk students were sampled from 53 classrooms in 13 schools in a southeastern metropolitan school district. We sampled two to eight at-risk students per classroom. When screening yielded more students in a class than could be accommodated in the study, we randomly selected students for participation. We defined risk as performance on a broad-based calculations assessment (Wide Range Achievement Test–4 or WRAT-4; Wilkinson, 2008) below the 35<sup>th</sup> percentile. We excluded students ( $n = 18$ ) with T-scores below the 9<sup>th</sup> percentile on both subtests of the Wechsler Abbreviated Scales of Intelligence (WASI; Psychological Corporation, 1999) because this study was not about intellectual disability.

Those 297 at-risk students were randomly assigned at individual level to fraction tutoring ( $n = 145$ ) or a control condition ( $n = 152$ ), stratifying by classroom. In the present study, we used data only from the control at-risk group because intervention was designed to disturb the

predictive value of cognitive abilities and early mathematics competencies. Of 152 control students, 12 moved before the end of the study, and one student had incomplete pretest data. These 13 students did not differ from remaining students. We therefore omitted these 13 cases, with 139 students comprising the final sample. Their scores on the pretest WRAT averaged 9.01 ( $SD = 2.04$ ). Their mean age was 9.49 ( $SD = 0.39$ ). Of these 139 students, 58 (41.7%) were male, 12 (8.6%) were English learners, 114 (82.0%) received a subsidized lunch, and 12 (8.6%) had a school-identified disability. Race was distributed as 77 (55.4%) African American, 32 (23.0%) White, 25 (17.9%) Hispanic, and 5 (3.6%) “Other.”

### **Mathematics and Cognitive Predictors**

**Calculation skill.** WRAT-4 Math Computation (Wilkinson, 2008) includes solving simple oral problems (e.g., counting) and written calculation problems, but none of the students scored low enough on the written computation section to require the oral section of the test. Students have 10 min to complete calculation problems of increasing difficulty. In the beginning-of-fourth-grade range of performance, WRAT almost entirely samples whole-number items. Of 40 calculation items, 23 involve whole-number calculations and this dominates the skill set assessed at fourth grade. Cronbach’s alpha on this sample was .77. We used fall WRAT scores for screening students into the study and to estimate incoming calculation skill as a predictor of outcomes.

**Nonverbal reasoning.** WASI Matrix Reasoning (Wechsler, 1999) measures nonverbal fluid reasoning, spatial ability, and perceptual organization with pattern completion, classification, analogy, and serial reasoning tasks on 32 items. Students complete a matrix, from which a section is missing, by selecting from five response options. Reliability is .94.

**Language.** We used two tests of language, from which we created a unit-weighted composite variable using a principal components factor analysis. Because the principal

components factor analysis yielded only one factor, no rotation was necessary. WASI Vocabulary (Wechsler, 1999) measures expressive vocabulary, verbal knowledge, and foundation of information with 42 items. The first four items present pictures; the student identified the object in the picture. For the remaining items, the tester says a word for the student to define. Responses are awarded a score of 0, 1, or 2 depending on quality. Split-half reliability is .86. Woodcock Diagnostic Reading Battery (WDRB) - Listening Comprehension (Woodcock, 1997) measures the ability to understand sentences or passages that the tester reads. With 38 items, students supply the word missing at the end of sentences or passages that progress from simple verbal analogies and associations to discerning implications. Reliability is .80.

**Concept formation.** With Woodcock Johnson-III Tests of Cognitive Abilities (WJ-III; Woodcock, McGrew, & Mather, 2001)-Concept Formation, students identify the rules for concepts when shown illustrations of instances and non-instances of the concept. Students earn credit by correctly identifying the rule that governs each concept. Cut-off points determine the ceiling. Reliability is .93.

**Working memory.** Mixed findings exist depending on what type of working memory was assessed, but prior work has found consistent evidence for the central executive component of working memory (Fuchs et al., 2005, 2008, 2010b). Therefore, we assessed the central executive component of working memory using The Working Memory Test Battery for Children (WMTB-C; Pickering & Gathercole, 2001)-Listening Recall and Counting Recall. Each subtest includes six dual-task items at span levels from 1-6 to 1-9. Passing four items at a level moves the child to the next level. At each span level, the number of items to be remembered increases by one. Failing three items terminates the subtest. Subtest order is designed to avoid overtaxing any component area and is generally arranged from the easiest to hardest. We used the trials correct score. Test-retest reliability ranges from .84-.93. For Listening Recall, the child



determines if a sentence is true; then recalls the last word in a series of sentences. For Counting Recall, the child counts a set of 4, 5, 6, or 7 dots on a card and then recalls the number of counted dots at the end of a series. We opted to include both subtests, rather than creating a composite variable based on prior work (a) showing that listening recall may tap the verbal demands of word problems whereas calculations may derive strength from the specific ability to handle numbers within working memory (Fuchs et al., 2010b) and (b) suggesting individual differences in working memory for numbers versus words (Dark & Benbow, 1991; Siegel & Ryan, 1989).

**Processing speed.** WJ-III (Woodcock et al., 2001) Cross Out measures processing speed by asking students to locate and circle five identical pictures that match a target picture in each row. Students have 3 min to complete 30 rows and earn credit by correctly circling the matching pictures in each row. Reliability is .91.

**Attentive behavior.** The Strength and Weaknesses of ADHD Symptoms and Normal-Behavior (SWAN; Swanson et al., 2004) samples items from the *Diagnostic and Statistical Manual of Mental Disorders* (4th ed.) criteria for attention deficit hyperactivity disorder (ADHD) for inattention (9 items) and hyperactivity impulsivity (9 items), but scores are normally distributed. Teachers rate items on a 1–7 scale. We report data for the inattentive subscale, as the average rating across the nine items. The SWAN correlates well with other dimensional assessments of behavior related to attention ([www.adhd.net](http://www.adhd.net)). Reliability for the inattentive subscale at fourth grade is .96.

### **Whole-Number Outcome Measures**

**Whole-number calculations.** We administered two subtests of Double-Digit Calculation Tests (Fuchs, Hamlett, & Powell, 2003). The first subset, Double-Digit Addition, includes 20 2-digit by 2-digit addition problems with and without regrouping. The second subtest, Double-Digit Subtraction, includes 20 2-digit by 2-digit subtraction problems with and without

regrouping. Students have 3 min to complete each subtest. We used the number of correct answers across both subtests as the score. Alpha at fourth grade is .91. We also used spring scores from whole-number calculation items from the WRAT-4 Math Computation (Wilkinson, 2008) to index whole-number calculation competence (see description above).

**Whole-number line estimation.** Whole-Number Lines Estimate (Siegler & Booth, 2004) assesses children's representations of numerical magnitudes. Following Siegler and Booth (2004), students estimate the placement of numbers on a number line. Students are presented with a 25-cm number line displayed across the center of a standard computer screen, with a start point of 0 and an endpoint of 100. A target number is printed approximately 5 cm above each number line, and students place the target number on the number line. Target numbers are 3, 4, 6, 8, 12, 17, 21, 23, 25, 29, 33, 39, 43, 48, 52, 57, 61, 64, 72, 79, 81, 84, 90, and 96. Stimuli are presented in a different, random order for each child. The tester first explains a number line that includes the 0 and 100 endpoints and is marked in increments of 10. When the tester determines that the child recognizes the concept, a number line that includes the 0 and 100 endpoints only is presented, and the child points to where 50 should go. A model number line with the endpoints and the location of 50 marked is shown, and the child compares his/her response to the model. The tester explains how "the number 50 is half of 100, so we put it halfway in between 0 and 100 on the number line." Next, the tester teaches the child to use the arrow keys to place a red pointer on the line where 50 should fall on the computer screen. Then, the measure is administered, with only the end points of 0 and 100 marked. For each item, the tester asks, "If this is zero (pointing), and this is 100 (pointing), where should you put N?" There is no time constraint. The computer automatically calculates the absolute value of the difference between the correct placement and the child's placement of the target number (i.e., estimation of accuracy); this is averaged across trials to produce the score. This estimation accuracy score correlates with mathematics

achievement (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Siegler & Booth, 2004), and as Siegler and Booth showed, the source of improvement in estimation accuracy is increasing linearity of estimates. Cronbach's alpha as per Fuchs et al. (2010a) was .91.

### **Fraction Outcome Measures**

**Fraction calculations.** We administered Addition (Hecht, 1998), in which students have 1 min to answer 12 fraction addition problems presented horizontally. Two items include adding a whole number and a fraction, six items with like denominators, of which two items involve adding a mixed number and a fraction, and four items with unlike denominators, of which two items involve adding a mixed number and a fraction. The score is the number of correct answers. Cronbach's alpha on this sample was .93.

From the 2010 Fraction Battery (Schumacher, Namkung, & Fuchs, 2010), Fraction Subtraction (Schumacher et al., 2010) includes five subtraction problems with like denominators and five with unlike denominators; half are presented vertically and half horizontally. Testers terminate administration when all but two students have completed the test. Scoring does not penalize students for not reducing answers. The score is the number of correct answers. Cronbach's alpha on this sample was .88.

**Fraction number line estimation.** Fraction Number Line (Siegler et al., 2011) assesses magnitude understanding by requiring students to place fractions on a number line with two endpoints, 0 and 1. For each trial, a number line with endpoints is presented, along with a target fraction shown in a large font above the line. Students practice with the target fraction  $\frac{4}{5}$  and then complete 10 test items:  $\frac{1}{4}$ ,  $\frac{3}{8}$ ,  $\frac{12}{13}$ ,  $\frac{2}{3}$ ,  $\frac{1}{19}$ ,  $\frac{7}{9}$ ,  $\frac{4}{7}$ ,  $\frac{5}{6}$ ,  $\frac{1}{2}$ , and  $\frac{1}{7}$ . Items are presented in random order. Accuracy is defined as the absolute difference between the child's placement and the correct position of the number. When multiplied by 100, the scores are equivalent to the percentage of absolute error, as reported in the literature. Low scores indicate

stronger performance. Test-retest reliability, on a sample of 57 fourth-grade students across 2 weeks, was .79.

### **Procedure**

In August and September, testers administered the WRAT-4 in large groups. In September and October, testers administered Double-Digit Addition, Double-Digit Subtraction, and Fraction Addition and Subtraction in three large-group sessions. Testers administered cognitive measures (WDRB Listening Comprehension, WMTB-C Listening Recall, WMTB-C Counting Recall, WJ-III Concept Formation, WJ-III Processing Speed), Whole-Number Lines Estimate, and Fraction Number Line in two individual sessions. In early April, testers re-administered WRAT-4, Double-Digit Addition, Double-Digit Subtraction, and Fraction Addition and Subtraction in three large-group sessions and re-administered Whole-Number Lines Estimate and Fraction Number Line in one individual session. All test sessions were audiotaped; 20% of tapes were randomly selected, stratifying by tester, for accuracy checks by an independent scorer. Agreement on test administration and scoring exceeded 98%.

### **Data Analysis and Results**

Data analysis progressed in three stages. First, more than one measure was available for whole-number calculations and for fraction calculations allowing latent variables to be formed. A measurement model for these outcome variables was estimated using confirmatory factor analysis to determine the factor structure among the calculation variables. Second, for whole-number fractions and for fraction calculations, the covariance structure of the data was modeled using structural equation modeling with seven cognitive predictors and one incoming calculation skill variable predicting latent variables representing whole-number and fraction calculations. Third, for whole-number and fraction number line estimation, in which only one outcome measure was available, path analysis was used to model the covariance structure among the eight

predictors and number line estimation. In all analyses, because only one measure was available for each cognitive predictor variable, those were entered as manifest variables. All analyses were carried out using the Mplus statistical software (Muthen & Muthen, 1998).

Prior to conducting model estimation, we conducted preliminary analysis to identify outliers and univariate and multivariate normality. Univariate plots revealed no significant outliers (plus or minus three standard deviations from the mean for each variable used in the study). However, several variables were significantly skewed. These variables were normalized using transformations outlined by Howell (2007) and Tabachnick and Fidell (2007). Fraction Addition and Double-Digit Addition were substantially skewed and were log transformed. Matrix Reasoning was slightly skewed and square-root transformed. Whole-Number Lines Estimate was moderately skewed and was given reciprocal transformation. Scores on Fraction Number Line were reversed by multiplying by -1, so higher scores mean higher performance. After normalizing the data, further analysis revealed that these variables were not multivariate normal. Therefore, models were constructed using a scaled chi-square estimated with robust standard errors using the robust maximum likelihood (MLR) estimator command in Mplus. Scaling correction factors ranged from 1.08 to 1.14 across models, suggesting little difference between the standard and scaled chi-square values.

Table 1 presents means and standard deviations on raw scores, as well as standard scores when available, on the cognitive predictors at the start of fourth grade and on the math outcomes at the end of fourth grade. Table 2 presents correlations among measures used in the study.

### **Whole-Number and Fraction Calculations**

**Outcome measurement model.** The measurement model for whole-number and fraction calculations outcome included two correlated dimensions. The latent whole-number calculations variable comprising three manifest variables: WRAT-4 Math Computation, Double-Digit

Addition, and Double-Digit Subtraction. The second latent variable, fraction calculations, was represented by two manifest variables: Fraction Addition and Fraction Subtraction. A good model fit is indicated by (a) small values of chi-square relative to degrees of freedom, (b) large  $p$ -value associated with the chi-square, (c) root mean square error of approximation (RMSEA) approaching or equal to 0.0, (d) comparative fit index (CFI) approaching or equal to 1.0, (e) Tucker-Lewis index (TLI) approaching or equal to 1.0, and (f) standardized root-mean-square residual (SRMR) approaching or equal to 0.0. All manifest variables loaded significantly and reliably onto their respective factors (standardized coefficients: .65-.78,  $ps < .001$ ). The overall fit of the two-factor model was excellent,  $\chi^2(4, N = 139) = 3.23, p = .519$ ; RMSEA = 0.000, CFI = 1.000, TLI = 1.012, SRMR = 0.021. The correlation between two factors was significant,  $r(137) = .49, p = .000$ .

We contrasted this base measurement model with an alternative one-factor measurement model to confirm that both dimensions of calculations were necessary. Table 3 shows model fits and model comparisons for the measurement models. An adjusted chi-square difference tests (i.e.,  $\Delta\chi^2$ ) using the Satorra-Bentler scaling correction yielded a significantly worse fit of the one-factor measurement model,  $\Delta\chi^2(1, N = 139) = 23.11, p = .000$ . Therefore, both whole-number and fraction calculations were incorporated into structural model.

**Structural model.** Structural model, in which all cognitive predictors and incoming calculation skill had paths to both whole-number and fraction calculations, was tested. Figure 1 shows the results, with statistically significant paths in bold. Table 4 shows the correlations between the predictors. Standardized path coefficient values are shown along the arrows. The chi-square was not statistically significant,  $\chi^2(28, N = 139) = 39.74, p = .070$ , and the model fit was adequate, RMSEA = .055, CFI = .953, TLI = .916, SRMR = .034. The correlation between whole-number and fraction calculation factors was moderate, but not significant,  $r(137) = .40, p$

= .107. The model accounted for 51% and 32% of the variance in whole-number calculations and fraction calculations, respectively. For whole-number calculations, significant predictors were processing speed, attentive behavior, and incoming calculation skill. For fraction calculations, significant predictors were language, processing speed, and attentive behavior. We further tested equality of path coefficients for the significant and non-significant paths from language and incoming calculation skill to whole-number and fraction calculations using unstandardized path coefficients. These tests rejected the null of no difference for both predictors ( $p = .042$  for language,  $p = .003$  for incoming calculation skill).

### **Whole-Number and Fraction Number Line Estimation**

Because only one measure was available for the whole-number line construct and for fraction number line estimation construct, two-factor (whole-number and fraction number line factors) versus one-factor measurement (general number line factor) models could not be tested. The correlation between whole-number and fraction number line estimation measure was low and not significant,  $r(137) = .16$ ,  $p = .070$ , suggesting that the two represent different estimation skills. Path analysis was used to estimate the relations among the cognitive predictors and incoming calculation skill, with whole-number and fraction number line estimation. Each measure was entered as a manifest variable, allowing whole-number and fraction number line outcomes to correlate. Because this was a saturated model, one non-significant path (Attention to Whole-Number Number Line Estimation) was set to 0. The chi-square was not statistically significant, and the model fit the data structure adequately,  $\chi^2(2, N = 139) = 0.02$ ,  $p = .992$ ; RMSEA = 0.000, CFI = 1.000, TLI = 1.494, SRMR = .001. As expected, the correlation between two variables was not significant,  $r(137) = .03$ ,  $p = .705$ . The model accounted for 14% and 17% of the variance in whole-number estimation and fraction number line estimation, respectively. Figure 2 shows the results, with statistically significant paths in bold. Standardized path

coefficient values are shown along the arrows. Table 5 shows the correlations between the predictors. For whole-number number line estimation, the significant predictors were nonverbal reasoning and numerical working memory. For fraction number line estimation, the significant predictors were language and nonverbal reasoning. We further tested equality of path coefficients for the significant and non-significant paths from language and numerical working memory to whole-number and fraction number line estimation using unstandardized path coefficients. The test rejected the null of no difference for numerical working memory ( $p = .026$ ), but failed to reject the null for language to whole-number and fraction number line estimation ( $p = .546$ ).

### **Discussion**

The purpose of this study was to examine cognitive predictors associated with calculations and number line estimation using whole numbers versus fractions. Results indicated that processing speed, attentive behavior, and incoming calculation skills were significant predictors of whole-number calculation skill; by contrast, language, in addition to processing speed and attentive behavior, significantly predicted fraction calculations. On the other hand, nonverbal reasoning significantly predicted number line performance with whole numbers and fractions. Specific predictors were numerical working memory for whole-number line competence and language for fraction number line competence.

Thus, on one hand, whole-number and fraction competence seem to draw upon some shared cognitive abilities: for calculation skill, processing speed and attentive behavior; for number line competence, nonverbal reasoning. This suggests that whole-number and fraction competence develop similarly, relying on the same kinds of cognitive resources. On the other hand, some abilities underlying whole-number and fraction competence differ. Language appears to be a key ability for fraction calculations, but not for whole-number calculations, and incoming



calculations was a distinctive predictor of whole-number calculations while numerical working memory was a distinctive predictor of whole-number number line estimation. In these ways, whole-number and fraction competence may take different developmental paths. Thus, our findings suggest that at-risk students' whole-number and fraction development appear to share some commonalities even as they show distinct patterns of development in other ways. In the sections below, we discuss the processes by which these shared and distinct cognitive abilities may affect whole-number and fraction competence. Finally, we identify limitations of the present study and discuss instructional implications.

### **Shared Cognitive Processes for Whole-Number and Fraction Competence**

It is interesting to consider the three cognitive processes that appear common to whole-number from fraction competence. Attentive behavior uniquely predicted whole-number and fraction calculations, and the strength of its predictive power for attentive behavior for whole-number ( $\beta = 0.22$ ) and fraction calculations ( $\beta = 0.23$ ) was similar. Executing both types of calculation tasks require keeping track of multiple numbers and steps and therefore require considerable attention. During these calculation processes, inattentive students may commit more arithmetic and procedural errors than attentive peers (Raghubar et al., 2009) and may demonstrate less perseverance with tasks (Fuchs et al., 2005, 2006). Even so, attentive behavior was not predictive of either number line outcome. On the number line tasks, students may rely on their reasoning skills, rather than executing multiple strategic steps. This would reduce demands on attention and help explain why attentive behavior predicts both whole-number fraction calculations, but not whole-number or fraction number line estimation.

The second ability that operated in similar ways for whole numbers and fractions was processing speed. It significantly predicted both whole-number and fraction calculations, with similar magnitude (both coefficients .32), but neither number line estimation outcome. In

comparison to whole-number and fraction calculation competence, which depends on efficient processing of each sub-step, number line estimation competence draws on students' background knowledge about number magnitudes and reasoning skills to derive the answer. Therefore, as with attention, it makes sense that processing speed is not predictive of either number line estimation task. Findings corroborate previous research, in which processing speed has been shown to correlate with some but not all forms of mathematics performance (e.g., Fuchs et al., 2010b; Kail, 1992; Kail & Hall, 1994). Bull and Johnston (1997) found that students with calculation difficulties were slow in speed of executing operations, identifying numbers, and matching number and shapes. Executing multiple tasks embedded is required to derive answers to calculation problems. Because processing speed facilitates simple processes necessary to carry out calculation procedures, such as counting or retrieving arithmetic facts from long-term memory, students with faster processing speed may be able to find answers more quickly and pair the problems with their answers in working memory before decay sets in (Bull & Johnston, 1997; Geary et al., 1991; Lemaire & Siegler, 1995).

The third cognitive ability that operated similarly for whole number and fractions was nonverbal reasoning, with coefficients of .27 and .11. Yet, in contrast to attentive behavior and processing speed, reasoning predicted both number line outcomes, but neither calculation outcome. Reasoning reflects the analytical capacity that is important in drawing inferences and applying concepts when solving problems (Primi, Ferrao, & Almeida, 2010). Whereas calculations are generally taught procedurally, for which processing speed and attentive behavior appear to play a significant role, students must apply their knowledge about number magnitudes when placing numbers on number lines. This appears to draw on reasoning abilities, which may facilitate learning the logical structure of the number line (Geary et al., 2008). For example, students must think logically and systemically to identify the location of the target number

relative to the endpoints of number line, e.g., 75 is 75% of the way from 0 to 100 (Bailey et al., 2014). Therefore, it is not surprising that reasoning helps predict both whole-number and fraction number line estimation, but not calculations.

### **Distinct Predictors of Whole-Number and Fraction Competence**

We also identified three predictors that appear to operate in distinctive ways for whole numbers versus fractions. First, language significantly predicted development of fraction calculations, but not whole-number calculations. This is in line with previous findings, in which language played a significant role in fraction calculation development (Fuchs et al., 2013; Seethaler et al., 2011), but not in whole-numbers (e.g., Fuchs et al., 2005, 2006, 2008, 2010a, 2011b, 2013; Seethaler et al., 2011). This finding also corroborates studies that demonstrate the importance of language for acquiring conceptual understanding of fractions (Miura et al. 1999; Paik & Mix, 2003). That is, students must understand the interacting role of numerators and denominators and the concept of having the same denominators beyond being able to carry out rote procedures. In particular with fractions, Miura et al. (1999) suggested that East Asian languages with transparent verbal labels of fractions that represent part-whole relations facilitate conceptual understanding of fractions. Better understanding of fraction concepts may in turn facilitate fraction calculations. Such relation between conceptual and procedural understandings is demonstrated in the literature, in which they were found to influence each other iteratively (e.g., Rittle-Johnson & Siegler, 1998; Rittle-Johnson et al., 2001).

With respect to number line estimation, although the path analysis indicated that language significantly predicted fraction number line estimation, but not whole-number line estimation, further testing of the equality of these path coefficients indicated no differential effects of language on whole-number and fraction number line estimation. Given this study's relatively small sample size ( $N = 139$ ) along with the magnitude of the path coefficients for the

effects of language on whole-number and fraction number line estimation ( $\beta = 0.18$  vs.  $\beta = 0.08$ ), we suggest that future studies be conducted with larger samples to evaluate whether language ability may be especially important for understanding fraction magnitudes. Baturo and Cooper (1999) found that when asked to place improper and mixed number fractions on number lines, sixth- and eighth-grade students often associated the numerators with a whole-number marker on the number line and counted whole numbers instead of fractional parts. So, in comparison to whole-number line estimation competence, which most students master early on, fraction number line estimation may be harder to achieve, in part because this form of learning depends on oral language ability. After all, teachers rely heavily on language to explain mathematical ideas, and fractions also involve novel vocabulary (e.g., *equivalent*, *common denominator*, and *improper fractions*). These demands may be especially challenging for students with delayed language development.

The second predictor that appears to operate in distinctive ways for whole numbers versus fractions is incoming calculation skill, which made the largest contribution to whole-number calculations (as would be expected). By contrast, whole-number calculations did not predict fraction calculations or number line estimation outcome. It makes sense that whole-number calculations did not help predict either number line estimation. As noted, although our measure of incoming calculation skill (i.e., WRAT-4 Math Computation) includes fraction, decimal, and percent calculations, it almost entirely samples whole-number items in the beginning-of-fourth-grade range of performance. Given the nature of incoming calculation tasks, in which students solely worked on deriving answers to whole-number calculation problems, it makes sense that incoming calculation skill is not predictive of number line estimation, which assesses number magnitude understanding. However, it is surprising that incoming calculation skill did not predict fraction calculations given the hierarchical nature of calculations. This

finding suggests that fraction calculations may be distinct from whole-number calculations and that fluency with whole-number calculations may not automatically transfer to fraction calculations. This makes sense given evidence that even those who are competent with whole numbers struggle with fractions (NMAP, 2008). The distinctive features of fraction versus whole-number calculations may be due to fundamental differences between whole-number and fractions, which include the fact that, infinite quantities exist between two fractions and that calculation tasks require common denominators.

The third predictor that appears to operate in distinctive ways for whole numbers versus fractions was working memory. Specifically, numerical working memory uniquely predicted whole-number number line estimation, but not fraction number line estimation. Nonsignificant effects of numerical working memory were found for both calculation outcomes, and no significant effects of working memory-sentences were found for either form of number line estimation and calculations. Whereas nonsignificant effects of working memory-sentences corroborate previous studies, in which working memory-sentences has been shown to uniquely predict word problem-solving but not calculations (e.g., Fuchs et al., 2005, 2010b), it is interesting that the effects of numerical working memory on whole-number and fraction calculations were nonsignificant. After all, both types of calculations require controlling, regulating, and maintaining numerical information while simultaneously carrying out calculation procedures and keeping track the multi-step calculation procedures. However, mixed findings also exist in the literature regarding the contribution of numerical working memory. Fuchs et al. (2006, 2010a) did not find significant effects of numerical working memory on arithmetic and procedural calculations, whereas Fuchs et al. (2008, 2010b) found effects for numerical working memory on whole-number calculations with similar participants, outcome measures, and the same working memory measures. Additional studies are needed to understand how and what

components of working memory affect calculation competence.

With respect to number line estimation, in which numerical working memory significantly predicted whole-number but not fraction number line estimation, it is possible that students use the whole-number counting number sequence (e.g., by 10s or 20s), which is involved in the numerical working memory task, to place whole-numbers on a number line. Such counting is not applicable to fractions. This suggests a potential domain-specificity for numerical working memory. Previous studies provide evidence that numerical working memory may be specific to tasks involving numbers whereas working memory-sentences may be specific to tasks that also involve language (Fuchs et al., 2008; Hitch & McAuley, 1991; McLean & Hitch, 1999; Peng, Sun, Li, & Tao, 2012; Siegel & Ryan, 1989). Even so, it appears that numerical working memory may be even more specific to whole-number tasks.

### **Limitations and Future Directions**

We note that although we included a more complete set of predictors than has been incorporated in most prior studies involving fraction outcomes, it is possible that other important cognitive factors were omitted from our models. For calculations, our predictors accounted for 51% of the variance in whole-number competence and 32% of variance for fractions; for number line, they accounted for 14% of variance in whole-numbers and 17% for fractions. This indicates that other cognitive resources (e.g., phonological loop, inhibition) or environmental factors (e.g., socioeconomic status; quality of classroom instruction) are yet to be identified.

Moreover, the percentages of variance accounted for whole-number and fraction number line estimation were significantly lower than for calculations. Although this was expected, given that the predictors included in the present study were selected primarily due to their role in calculation competence, the small percentages of variance explained for the number line outcomes indicate that additional study focused on other or additional cognitive processes is

warranted. The present findings provide the basis for generating hypotheses for such studies. One potential cognitive predictor that future studies should consider is visuospatial abilities. Visuospatial working memory was found to mediate the relation between number line estimation accuracy and mathematics achievement among students with mathematics disabilities (Geary et al., 2007). Prior research has also found that brain regions associated with number and magnitude processing are located near areas that support visuospatial processing and that damage to these regions disrupts forming spatial representations and imagining a mental number line (de Hevia, Vallar, & Girelli, 2008; Zorzi, Priftis, Meneghello, Marenzi, & Umiltà, 2006; Zorzi, Priftis, & Umiltà, 2002). Also, spatial ability at the beginning of first and second grades, in the form of forming, maintaining, and mentally rotating a visual representation of various shapes, significantly predicted improvement in linear number line representation at the end of first and second grades (Gunderson, Ramirez, Beilock, & Levine, 2012).

Another study limitation pertains to how we assessed each cognitive factor. We used measures that are similar to those used in previous studies, but there are other ways to measure these cognitive constructs. For example, the processing speed task involved finding five identical pictures that matched the target picture in a row of 19 pictures. Students need to maintain the representation of the target picture internally as they encode information for each picture. This may place demands on working memory. Therefore, the contribution of working memory may have been captured by the processing speed measure, leading to the lack of significant effects for working memory in the present study. We note, however, that prior work has identified working memory as a significant predictor even when the same processing speed was controlled in the model (e.g., Fuchs et al., 2008; Fuchs et al., 2010b; Fuchs et al., 2013; Seethaler et al., 2011). Therefore, further research is warranted.

We also remind readers our sample was limited to at-risk students that findings may not generalize to students with typical development. As discussed, prior research indicates that the cognitive processes of students with mathematics difficulties are weaker than those of typically achieving students (e.g., Fuchs et al., 2008; Geary et al., 2007; Geary et al., 1999). Yet, in related prior work, cognitive processes are related to mathematics outcomes in similar ways across broader samples (i.e., relations apply across the distribution). That said, typically achieving students may exhibit different developmental patterns than those of at-risk students. Therefore, further studies are needed to investigate this issue by directly comparing the development of alternative forms of whole-number and fraction competence for at-risk versus typically achieving students.

Finally, although our proposed models were theoretically based on previous findings, we note that other models may fit the data adequately, and future studies should explore alternative ways of modeling development. In a related way, some of the predictors in our models may serve as mediators of the relation between other predictors and the outcomes. For example, the effect of working memory on later math outcomes may be mediated by processing speed (e.g., Lee, Bill, & Ho, 2013; McAuley & White, 2011). Investigating mediation questions requires that potential mediators be assessed at a testing occasion subsequent to the assessment of the predictors. Future should incorporate such a design to specify cognitive mechanisms, under which whole-number and fraction competence develops via a combination of direct and mediation effects.

### **Instructional Implications**

With these limitations in mind, the findings provide insight on the nature of interventions that may reduce the cognitive demands associated with whole-number and fraction calculations and number line estimation – even as we acknowledge that the findings for number line



estimation are more limited than for calculations. Therefore, we caution our readers that our implications for instruction on number line estimation are preliminary. We also remind our readers that because our study population was at-risk students, our recommendations for instruction pertain to this subset of the population.

With respect to whole-number and fraction calculations, interventions should incorporate effective strategies to improve students' attention and academic engagement, such as providing positive reinforcement for on-task behavior and implementing self-monitoring of attention (e.g., Edwards, Salant, Howard, Brougher, & McLaughlin, 1995; Harris, Friedlander, Saddler, Frizzelle, & Graham, 2005; Shimabukuro, Prater, Jenkins, & Edelen-Smith, 1999). Providing instructional strategies that can compensate for slow processing may also be helpful in improving calculation skills. For example, students with mathematical difficulties often rely on counting the entire set of numbers when adding and subtracting. Teaching addition and subtraction strategies, such as counting up and counting down, may help them compensate for slow processing.

In terms of fraction calculations, instruction should be designed to reduce demands on language. For example, explicitly teaching fraction vocabulary, using simple language, and checking for students' understanding frequently may be helpful in reducing demands on language abilities. The present findings also suggest that practice on the whole-number calculation procedures that are embedded within fraction calculations alone may not lead to successful development of fraction calculations. Conceptual understanding of fractions that is supported by language appears to be a determinant of success with both fraction calculations and number line estimation. Therefore, fraction instruction should focus on improving students' conceptual understanding of fractions. Such instruction should address teaching fractions as numbers, providing multiple representations with number lines being the central representational

tool, and helping students understand why procedures for fraction calculations make sense as outlined by the Institute of Education Science (Siegler et al., 2010) and as demonstrated as efficacious in randomized control trials (Fuchs et al., 2013; Fuchs et al., 2014; Fuchs et al., in press).

For whole-number and fraction number line estimation, interventions should be designed to compensate for limitations in children's reasoning ability. One instructional strategy that may be effective in reducing demands on reasoning ability is fluency practice. Geary et al. (2008) suggested that fluency practice may especially enhance at-risk students' learning by building automaticity with foundational skills and therefore compensating for their limitations in cognitive resources. In line with Geary's hypothesis, Fuchs et al. (2013) demonstrated that speeded strategic practice helped at-risk students compensate for weak reasoning ability. With respect to whole-number line estimation, strategies that reduce demands for holding and manipulating numerical information may be helpful. For example, teaching students efficient strategies for chunking (recoding a multidimensional concept into fewer dimensions) or segmenting (breaking a task into a series of steps) may reduce cognitive load (Fuchs et al., 2014). Instruction should incorporate teaching students the location of benchmark numbers (i.e., 25, 50, 75;  $\frac{1}{4}$ ,  $\frac{1}{2}$ ,  $\frac{3}{4}$ ) to compare the relative location of the target number when making placements on a number line.

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Table 1

*Means and Standard Deviations*

Variable	Raw score	Standard score
	<i>M (SD)</i>	<i>M (SD)</i>
Language factor	.00 (1.00)	--
WASI Vocabulary	32.00 (6.33)	47.08 (8.93)
WDRB Listening Comprehension	21.05 (4.01)	91.12 (16.41)
Nonverbal reasoning	17.46 (6.16)	47.19 (10.11)
Concept formation	16.06 (5.18)	88.66 (9.16)
WMTB Listening Recall	10.37 (3.21)	91.35 (19.71)
WMTB Counting Recall	17.45 (4.76)	80.33 (16.23)
Processing speed	15.35 (2.74)	94.16 (11.29)
Attentive Behavior	34.94 (10.74)	--
Incoming calculation skill	24.34 (2.15)	84.55 (7.88)
WRAT whole-number calculations	12.07 (2.90)	--
Double Digit Subtraction	10.15 (4.82)	--
Double Digit Addition	16.93 (4.21)	--
Number Line Estimation	95.80(64.13)	--
Fraction Subtraction	4.06 (2.52)	--
Fraction addition	3.65 (2.41)	--
Fraction Number Line	0.32 (0.12)	--

*Note.* WASI = Wechsler Abbreviated Scale of Intelligence; WDRB = Woodcock Diagnostic Reading Battery; WMTB = Working Memory Test Battery; WRAT = Wide Range Achievement Test.

Table 2

*Correlations among All Measures*

Measure	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1. Language factor	--																
2. WASI Vocabulary	.88*	--															
3. WDRB Listening Comprehension	.88*	.56*	--														
4. Nonverbal reasoning	.30*	.28*	.25*	--													
5. Concept formation	.40*	.39*	.33*	.31*	--												
6. WMTB Listening Recall	.26*	.30*	.17*	.06	.20*	--											
7. WMTB Counting Recall	.03	.02	.03	.16	.00	.20*	--										
8. Processing speed	.09	.12	.03	.18*	.27*	.17*	.12	--									
9. Attentive Behavior	.11	.21*	-.00	.19*	.06	.03	.09	.10	--								
10. Incoming calculation skill	.17*	.24*	.06	.11	.22*	.17*	.24*	.14	.29*	--							
11. WRAT whole-number calculations	.16	.24*	.05	.22*	.25*	.12	.21*	.28*	.23*	.46*	--						
12. Double Digit Subtraction	.05	.09	.00	.11	.01	.05	.18*	.34*	.36*	.43*	.60*	--					
13. Double Digit Addition	.15	.23*	.03	.13	.05	.02	.23*	.30*	.32*	.38*	.49*	.53*	--				
14. Number Line Estimation	.17*	.20*	.11	.30*	.09	.15	.21*	.04	.08	.10	.34*	.15	.15	--			
15. Fraction Subtraction	.25*	.24*	.19*	.30*	.21*	.05	.16	.35*	.25*	.06	.32*	.22*	.23*	.16	--		
16. Fraction addition	.16	.15	.13	.18*	.12	.00	.04	.08	.15	-.00	.29*	.26*	.17*	.17*	.48*	--	
17. Fraction Number Line	.32*	.31*	.25*	.29*	.24*	.21*	.10	.01	.08	.15	.20*	.11	.14	.16	.14	.22*	--

*Note.* WASI = Wechsler Abbreviated Scale of Intelligence; WDRB = Woodcock Diagnostic Reading Battery; WMTB = Working Memory Test Battery; WRAT = Wide Range Achievement Test. \* $p < .05$



Table 3

*Model Fits and Model Comparisons for the Measurement Models*

Model	df	$\chi^2$	$p$	RMSEA	CFI	TLI	SRMR	$\Delta\chi^2$ Base Model
Two-factor model	4	3.23	.519	0.000	1.000	0.012	0.021	
One-factor model	5	26.17	.000	0.175	0.867	0.734	0.079	23.11*

\* $p < .001$

Table 4

*Calculations: Correlations among Cognitive and Incoming Calculation Manifest Variables*

Variable	1	2	3	4	5	6	7	8
1. Language	--							
2. Nonverbal reasoning	.30	--						
3. Concept formation	.41	.32	--					
4. Working memory-Sentences	.25	.05	.21	--				
5. Numerical working memory	.04	.15	.01	.20	--			
6. Processing speed	.11	.20	.27	.19	.12	--		
7. Attentive behavior	.11	.19	.06	.03	.09	.10	--	
8. Incoming calculation skill	.18	.12	.22	.18	.24	.13	.29	--

Table 5

*Number Line Estimation: Correlations among Cognitive and Incoming Calculation Manifest*

*Variables*

Variable	1	2	3	4	5	6	7	8
1. Language	--							
2. Nonverbal reasoning	.29	--						
3. Concept formation	.41	.31	--					
4. Working memory-Sentences	.26	.06	.22	--				
5. Numerical working memory	.04	.16	.01	.20	--			
6. Processing speed	.10	.19	.26	.20	.12	--		
7. Attentive behavior	.11	.19	.06	.03	.10	.10	--	
8. Incoming calculation skill	.17	.11	.21	.19	.25	.12	.29	--

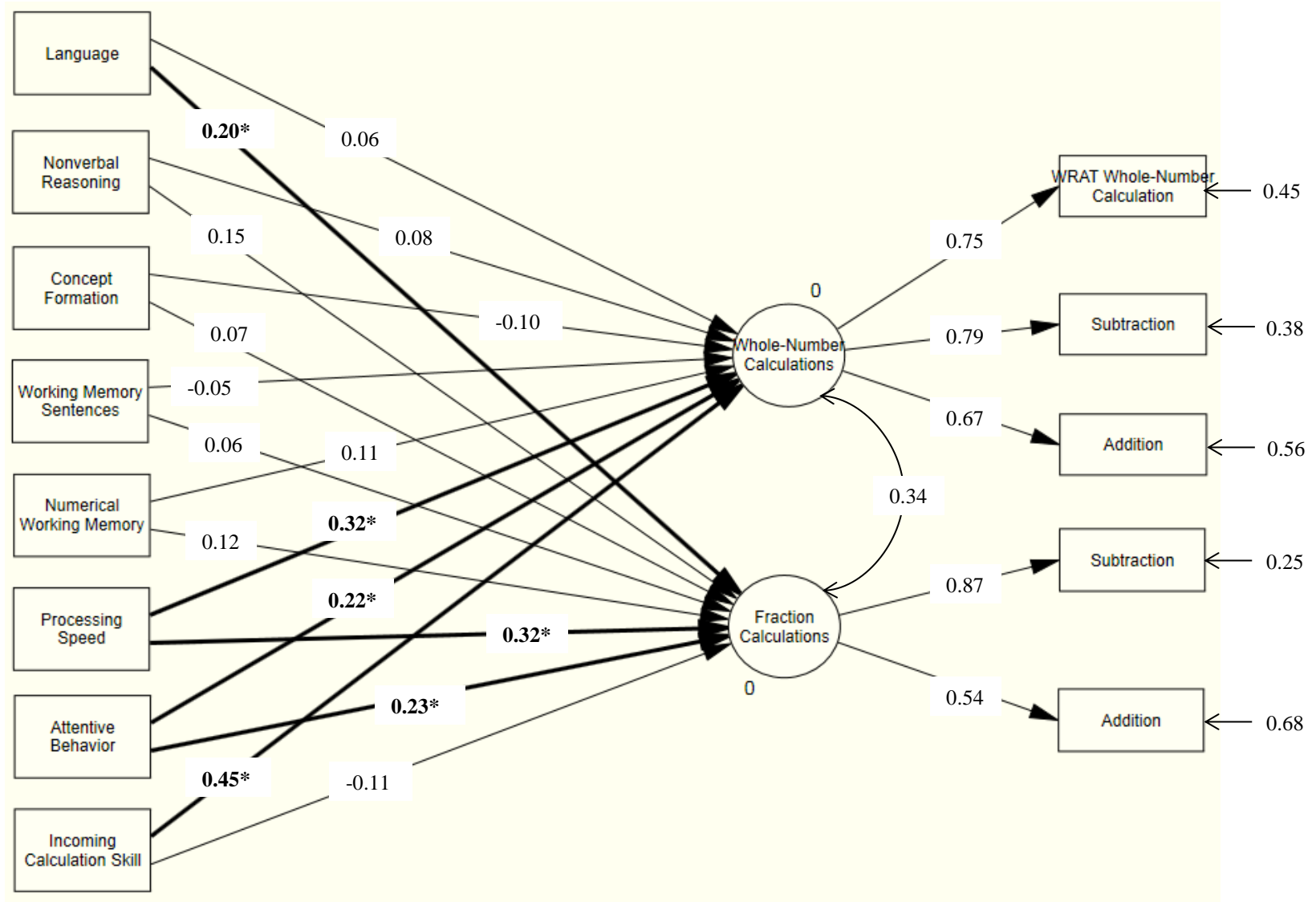


Figure 1. Whole-number and fraction calculation structural model with all coefficients specified. \* $p < .05$

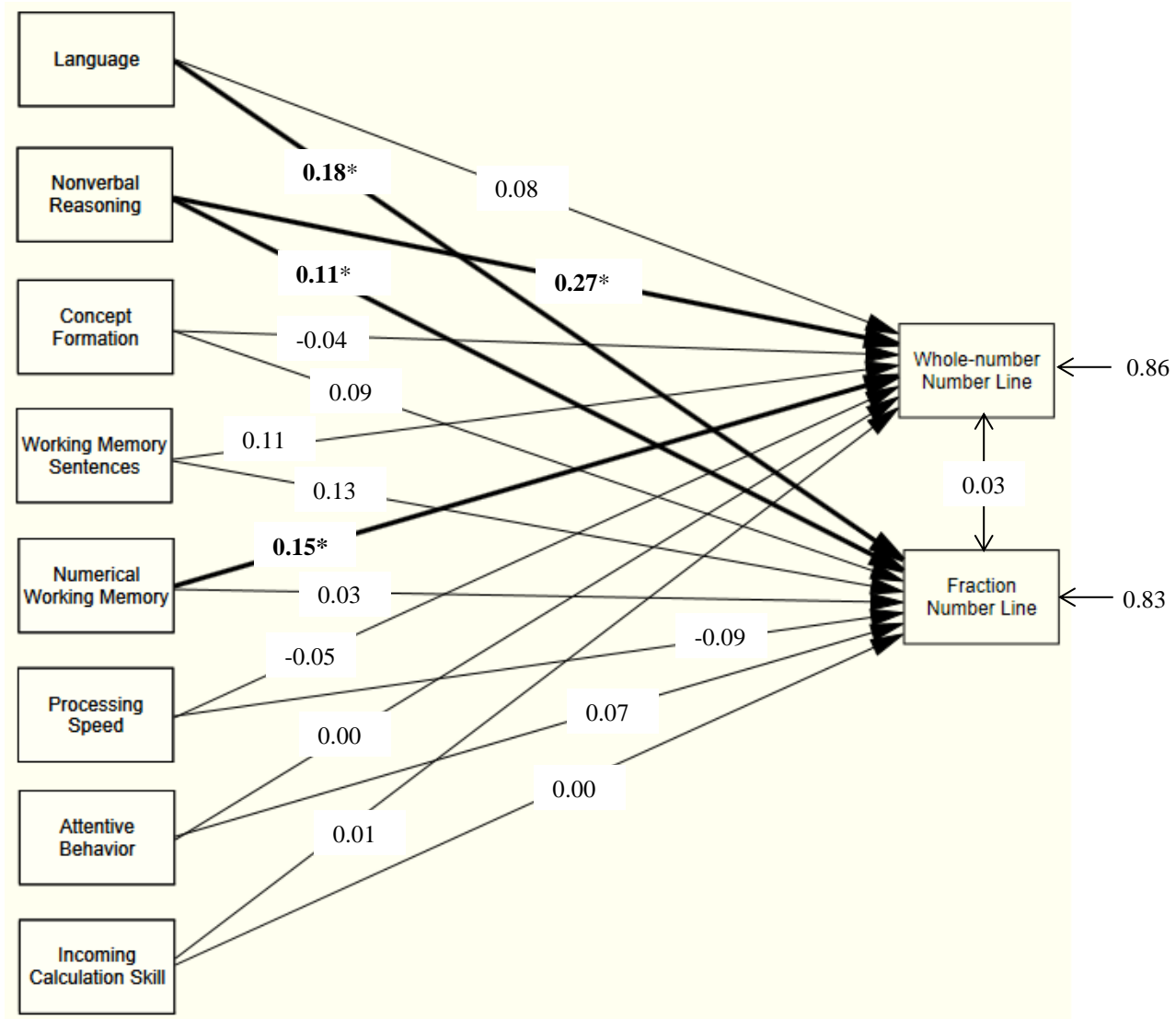


Figure 2. Whole-number and fraction number line path model with all coefficients specified. \* $p < .05$