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Early Predictors of High School Mathematics Achievement

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Abstract

Identifying the types of mathematics content knowledge that are most predictive of students' long-term learning is essential for improving both theories of mathematical development and mathematics education. To identify these types of knowledge, we examined long-term predictors of high school students' knowledge of algebra and overall mathematics achievement. Analyses of large, nationally representative, longitudinal data sets from the United States and the United Kingdom revealed that elementary school students' knowledge of fractions and of division uniquely predicts those students' knowledge of algebra and overall mathematics achievement in high school, 5 or 6 years later, even after statistically controlling for other types of mathematical knowledge, general intellectual ability, working memory, and family income and education. Implications of these findings for understanding and improving mathematics learning are discussed.

Keywords

mathematics achievement, cognitive development, childhood development, fractions, division

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Knowledge of mathematics is crucial to educational and financial success in contemporary society and is becoming ever more so. High school students' mathematics achievement predicts college matriculation and graduation, early-career earnings, and earnings growth (Murnane, Willett, & Levy, 1995; National Mathematics Advisory Panel, 2008). The strength of these relations appears to have increased in recent decades, probably because of a growing percentage of well-paying jobs requiring mathematical proficiency (Murnane et al., 1995). However, many students lack even the basic mathematics competence needed to succeed in typical jobs in a modern economy. Children from low-income and minority backgrounds are particularly at risk for poor mathematics achievement (Hanushek & Rivkin, 2006).

Marked individual and social-class differences in mathematical knowledge are present even in preschool and kindergarten (Case & Okamoto, 1996; Starkey, Klein, & Wakeley, 2004). These differences are stable at least from kindergarten through fifth grade; children who start ahead in mathematics generally stay ahead, and children who start behind generally stay behind (Duncan et al., 2007; Stevenson & Newman, 1986). There are substantial correlations between early and later knowledge in other academic subjects as well, but differences in children's mathematics knowledge are even more stable than differences in their reading and other capabilities (Case, Griffin, & Kelly, 1999; Duncan et al., 2007).

These findings suggest a new type of research that can contribute both to theoretical understanding of mathematical development and to improving mathematics education. If researchers can identify specific areas of mathematics that consistently predict later mathematics proficiency, after controlling for other types of mathematical knowledge, general intellectual ability, and family background variables, they can then determine why those types of knowledge are uniquely predictive, and society can increase efforts to improve instruction and learning in those areas. The educational payoff is likely to be strongest for areas that are strongly predictive of later achievement and in which many children's understanding is poor.

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In the present study, we examined sources of continuity in mathematical knowledge from fifth grade through high school. We were particularly interested in testing the hypothesis that early knowledge of fractions is uniquely predictive of later knowledge of algebra and overall mathematics achievement.

One source of this hypothesis was Siegler, Thompson, and Schneider's (2011) integrated theory of numerical development. This theory proposes that numerical development is a process of progressively broadening the class of numbers that are understood to possess magnitudes and of learning the functions that connect those numbers to their magnitudes. In other words, numerical development involves coming to understand that all real numbers have magnitudes that can be assigned specific locations on number lines. This idea resembles Case and Okamoto's (1996) proposal that during mathematics learning, the central conceptual structure for whole numbers, a mental number line, is eventually extended to rational numbers. The integrated theory of numerical development also proposes that a complementary, and equally crucial, part of numerical development is learning that many properties of whole numbers (e.g., having unique successors, being countable, including a finite number of entities within any given interval, never decreasing with addition and multiplication) are not true of numbers in general.

One implication of this theory is that acquisition of fractions knowledge is crucial to numerical development. For most children, fractions provide the first opportunity to learn that several salient and invariant properties of whole numbers are not true of all numbers (e.g., that multiplication does not necessarily produce answers greater than the multiplicands). This understanding does not come easily; although children receive repeated instruction on fractions starting in third or fourth grade (National Council of Teachers of Mathematics, 2006), even high school and community-college students often confuse properties of fractions and whole numbers (Schneider & Siegler, 2010; Vosniadou, Vamvakoussi, & Skopeliti, 2008).

This view of fractions as occupying a central position within mathematical development differs substantially from other theories in the area, which focus on whole numbers and relegate fractions to secondary status. To the extent that such theories address development of understanding of fractions at all, it is usually to document ways in which learning about them is hindered by whole-number knowledge (e.g., Gelman & Williams, 1998; Wynn, 1995). Nothing in these theories suggests that early knowledge of fractions would uniquely predict later mathematics proficiency.

Consider some reasons, however, why elementary school students' knowledge of fractions might be crucial for later mathematics—for example, algebra. If students do not understand fractions, they cannot estimate answers even to simple algebraic equations. For example, students who do not understand fractions will not know that in the equation $1/3X = 2/3Y$, X must be twice as large as Y, or that for the equation $3/4X = 6$, the value of X must be somewhat, but not greatly, larger than 6. Students who do not understand fraction magnitudes also would not be able to reject flawed equations by reasoning that the answers they yield are impossible. Consistent with this analysis, studies have shown that accurate estimation of fraction magnitudes is closely related to correct use of fractions arithmetic procedures (Hecht & Vagi, 2010; Siegler et al., 2011). Thus, we hypothesized that 10-year-olds' knowledge of fractions would predict their algebra knowledge and overall mathematics achievement at age 16, even after we statistically controlled for other mathematical knowledge, information-processing skills, general intellectual ability, and family income and education.

Method

To identify predictors of high school mathematics proficiency, we examined two nationally representative, longitudinal data sets: the British Cohort Study (BCS; Butler & Bynner, 1980, 1986; Bynner, Ferri, & Shepherd, 1997) and the Panel Study of Income Dynamics-Child Development Supplement (PSID-CDS; Hofferth, Davis-Kean, Davis, & Finkelstein, 1998). Detailed descriptions of the samples and measures used in these studies and of the statistical analyses that we applied are included in the Supplemental Material available online; here, we provide a brief overview.

The BCS sample included 3,677 children born in the United Kingdom in a single week of 1970. The tests of interest in the present study were administered in 1980, when the children were 10-year-olds, and in 1986, when the children were 16-year-olds. Mathematics proficiency at age 10 was assessed by performance on the Friendly Maths Test, which examined knowledge of whole-number arithmetic and fractions. Mathematics proficiency at age 16 was assessed by the APU (Applied Psychology Unit) Arithmetic Test, which examined knowledge of whole-number arithmetic, fractions, algebra, and probability. General intelligence was assessed at age 10 by performance on the British Ability Scale, which included measures of verbal and nonverbal intellectual ability, vocabulary, and spelling. Parents provided information about their education and income and their children's gender, age, and number of siblings.

The PSID-CDS included a nationally representative sample of 599 U.S. children who were tested in 1997 as 10- to 12-year-olds and in 2002 as 15- to 17-year-olds. At both ages, they completed parts of the Woodcock-Johnson Psycho-Educational Battery-Revised (WJ-R), a widely used achievement test. The 10- to 12-year-olds performed the Calculation Subtest, which included 28 whole-number arithmetic items (8 addition, 8 subtraction, 7 multiplication, and 5 division items) and 9 fractions items. The 15- to 17-year-olds completed the test's Applied Problems Subtest, which included 60 items on whole-number arithmetic, fractions, algebra, geometry, measurement, and probability. Applied Problems items 29, 42, 43, 45, and 46 were used to construct the measure of fractions knowledge, and items 34, 49, 52, and 59 were used to construct the measure of algebra knowledge. Also obtained at ages 10 to 12 were measures of working memory (as indexed by backward digit span), demographic characteristics (gender, age, and number of siblings), and family background (parental education in years and log mean income averaged over 3 years). Two measures of literacy from the WJ-R, passage comprehension and letter-word identification (a vocabulary test), were obtained both at age 10 to 12 and at age 15 to 17.

Results

The results yielded by bivariate and multiple regression analyses are presented for the British sample in Table 1 and for the U.S. sample in Table 2. In both tables, results are presented for algebra scores (Models 1 and 2) and total math scores (Models 3 and 4).

Our main hypothesis was that knowledge of fractions at age 10 would predict algebra knowledge and overall mathematics achievement in high school, above and beyond the effects of general intellectual ability, other mathematical knowledge, and family background. The data supported this hypothesis.

In the United Kingdom (U.K.) data, after effects of all other variables were statistically controlled, fractions knowledge at age 10 was the strongest of the five mathematics predictors of age-16 algebra knowledge and mathematics achievement (Table 1, Models 2 and 4). A 1-*SD* increase in early fractions knowledge was uniquely associated with a 0.15-*SD* increase in subsequent algebra knowledge and a 0.16-*SD* increase in total math achievement ($p < .001$ for both coefficients). In the U.S. data, after effects of other variables were statistically controlled, the relations between fractions knowledge at ages 10 to 12 and high school algebra and overall mathematics achievement at ages 15 to 17 were of approximately the same strength as the corresponding relations in the U.K. data (Models 2 and 4 in Tables 1 and 2). As documented in the Supplemental Material (see Tables S5 and S6), in both data sets, the predictive power of increments to fractions knowledge was equally strong for children lower and higher in fractions knowledge.

If fractions knowledge continues to be a direct contributor to mathematics achievement in high school, as opposed to having influenced earlier learning but no longer being directly influential, we would expect strong concurrent relations between high school students' knowledge of fractions and their overall mathematical knowledge. High school students' knowledge of fractions did correlate very strongly with their overall mathematics achievement, in both the United Kingdom, $r(3675) = .81$, $p < .001$, and the United States, $r(597) = .87$, $p < .001$. Their fractions knowledge also was closely related to their knowledge of algebra in both the United Kingdom, $r(3675) = .68$, $p < .001$, and the United States, $r(597) = .65$, $p < .001$. Although algebra is a major part of high school mathematics and fractions constitute a smaller part, the correlation between high school students'

Table 1. Early Predictors of High School Mathematics Achievement: British Cohort Study Data (*N* = 3,677)

	Algebra score		Total math score	
Predictor	Model 1 (bivariate regression)	Model 2 (multiple regression)	Model 3 (bivariate regression)	Model 4 (multiple regression)
Age-10 math skills				
Fractions	0.42*** (0.02)	0.15*** (0.02)	0.46*** (0.02)	0.16*** (0.02)
Addition	0.20*** (0.02)	0.00 (0.02)	0.26*** (0.02)	0.05** (0.02)
Subtraction	0.22*** (0.02)	0.04* (0.02)	0.24*** (0.02)	0.03 (0.02)
Multiplication	0.32*** (0.02)	0.06*** (0.02)	0.37*** (0.02)	0.08*** (0.02)
Division	0.37*** (0.02)	0.13*** (0.02)	0.40*** (0.02)	0.12*** (0.02)
Age-10 abilities				
Verbal IQ	0.39*** (0.02)	0.11*** (0.02)	0.42*** (0.02)	0.10*** (0.02)
Nonverbal IQ	0.41*** (0.02)	0.17*** (0.02)	0.46*** (0.02)	0.19*** (0.02)
Demographic characteristics				
Female gender	–0.02 (0.02)	0.00 (0.02)	–0.01 (0.02)	0.00 (0.01)
Age	0.01 (0.02)	–0.03* (0.02)	0.01 (0.02)	–0.03* (0.01)
Log mean household income	0.38*** (0.04)	0.08* (0.03)	0.40*** (0.04)	0.09* (0.04)
Parents' education	0.27*** (0.02)	0.10*** (0.02)	0.29*** (0.02)	0.10*** (0.02)
Number of siblings	–0.05** (0.02)	–0.01 (0.01)	–0.09 (0.02)	–0.05*** (0.01)
Mean R^2		.29		.35

Note: This table presents results from regression models predicting algebra and total math scores at age 16 from math skills, cognitive ability, and child and family characteristics at age 10. All predictors and dependent variables were standardized; therefore, although the coefficients reported are unstandardized, they can be interpreted much like standardized coefficients. Parameter estimates and standard errors (in parentheses) are based on 20 multiply imputed data sets. The British Cohort Study data on which these analyses were based are publicly available from the Centre for Longitudinal Studies, Institute of Education, University of London Web site: http://www.cls.ioe.ac.uk/bcs70.
*$p < .05$. **$p < .01$. ***$p < .001$.

Table 2. Early Predictors of High School Mathematics Achievement: Panel Study of Income Dynamics Data (N = 599)

Predictor	Algebra score: Model 1 (bivariate regression)	Algebra score: Model 2 (multiple regression)	Total math score: Model 3 (bivariate regression)	Total math score: Model 4 (multiple regression)
Early math skills				
Fractions	0.41*** (0.06)	0.17* (0.08)	0.49*** (0.05)	0.18** (0.06)
Addition	0.26*** (0.06)	0.09 (0.06)	0.30*** (0.06)	0.05 (0.05)
Subtraction	0.26*** (0.05)	0.04 (0.05)	0.39*** (0.05)	0.12* (0.05)
Multiplication	0.31*** (0.05)	0.00 (0.06)	0.43*** (0.05)	0.02 (0.05)
Division	0.40*** (0.05)	0.19*** (0.06)	0.53*** (0.05)	0.26*** (0.06)
Early abilities				
Backward digit span	0.29*** (0.06)	0.10 (0.06)	0.33*** (0.05)	0.08 (0.05)
Passage comprehension	0.38*** (0.05)	0.11 (0.06)	0.51*** (0.05)	0.20*** (0.05)
Demographic characteristics				
Female gender	–0.06 (0.05)	–0.09 (0.05)	–0.08 (0.05)	–0.13*** (0.04)
Age	0.04 (0.05)	–0.18*** (0.05)	0.06 (0.05)	–0.22*** (0.04)
Log mean family income (1994–1996)	0.31*** (0.06)	0.05 (0.06)	0.38*** (0.06)	0.12* (0.06)
Parents' education	0.39*** (0.05)	0.19*** (0.05)	0.41*** (0.06)	0.11 (0.06)
Number of siblings	–0.17** (0.06)	–0.03 (0.05)	–0.18** (0.06)	–0.03 (0.04)
Mean R^2		.35		.52

Note: This table presents results from regression models predicting algebra and total math scores at age 15 to 17 from math skills, cognitive abilities, and child and family characteristics at age 10 to 12. All predictors and dependent variables were standardized; therefore, although the coefficients reported are unstandardized, they can be interpreted much like standardized coefficients. Parameter estimates and standard errors (in parentheses) are based on 20 multiply imputed data sets. The data on which these analyses were based came from the Panel Study of Income Dynamics public-use data set, available at http://psidonline.isr.umich.edu.
*p < .05. **p < .01. ***p < .001.

knowledge of fractions and their overall mathematics achievement was stronger than the correlation between their algebra knowledge and their overall mathematics achievement in both the U.K. data, $r(3675) = .81$ versus .73, $\chi^2(1, N = 3{,}677) = 66.49$, $p < .001$, and the U.S. data, $r(597) = .87$ versus .80, $\chi^2(1, N = 599) = 15.03$, $p < .001$.

Early knowledge of whole-number division also was consistently related to later mathematics proficiency. Among the five mathematics variables derived from the elementary school tests, early division had the second-strongest correlation with later mathematics outcomes in the U.K. data (Table 1) and the strongest correlation with later mathematics outcomes in the U.S. data (Table 2). Concurrent correlations between high school students' knowledge of division and their overall mathematics achievement were also substantial both in the United Kingdom, $r(3675) = .59$, and in the United States, $r(597) = .69$, ps < .001. To the best of our knowledge, relations between elementary school children's division knowledge and their mathematics proficiency in high school have not been documented previously.

Regressions like those in Tables 1 and 2 place no constraints on the estimated coefficients. Therefore, we reestimated our regression models, first imposing an equality constraint on the coefficients for fractions and division, and then imposing an equality constraint on the coefficients for addition, subtraction, and multiplication (see Table S4 in the Supplemental Material). Finally, we tested whether the pooled coefficients for these two sets of skills differed from each other. The predictive relation was stronger for fractions and division than for the other mathematical skills in both the U.K. and the U.S. data—U.K.: $F(1, 3664) = 36.92$, $p < .001$, for algebra and $F(1, 3664) = 28.79$, $p < .001$, for overall mathematics; U.S.: $F(1, 558) = 7.12$, $p < .01$, for algebra and $F(1, 558) = 9.72$, $p < .01$, for overall mathematics (see Table S4).

The greater predictive power of knowledge of fractions and knowledge of division was not due to their generally predicting intellectual outcomes more accurately. When Models 2 and 4 in Tables 1 and 2 were applied to predicting high school students' literacy (spelling and vocabulary from the British Ability Scale in the BCS; passage comprehension and letter-word identification from the WJ-R in the PSID-CDS), only two of the eight predictive relations between fractions and division knowledge, on the one hand, and literacy, on the other, were significant: Fractions knowledge predicted vocabulary in the U.K. data, and division knowledge predicted letter-word identification in the U.S. data (see Tables 3 and 4). Moreover, in all cases but one, the pooled predictive effect of fractions and division knowledge on literacy was no greater than the pooled predictive effect of addition, subtraction, and multiplication knowledge on literacy (see Table S4 in the Supplemental Material). The one exception was that fractions and division knowledge more accurately predicted vocabulary in

Table 3. Results From Regression Models Predicting Literacy at Age 16 From Math Skills and Child and Family Characteristics at Age 10: British Cohort Study Data (N = 3,677)

	Spelling		Vocabulary	
Predictor	Model 1 (bivariate regression)	Model 2 (multiple regression)	Model 3 (bivariate regression)	Model 4 (multiple regression)
Age-10 math skills				
Fractions	0.16*** (0.02)	0.02 (0.02)	0.38*** (0.02)	0.09*** (0.02)
Addition	0.10*** (0.02)	0.00 (0.02)	0.21*** (0.02)	0.03 (0.02)
Subtraction	0.12*** (0.02)	0.04 (0.02)	0.17*** (0.02)	0.00 (0.02)
Multiplication	0.17*** (0.02)	0.05* (0.02)	0.27*** (0.02)	0.03 (0.02)
Division	0.16*** (0.02)	0.03 (0.02)	0.28*** (0.02)	0.04 (0.02)
Age-10 abilities				
Verbal IQ	0.19*** (0.02)	0.09*** (0.02)	0.49*** (0.02)	0.30*** (0.02)
Nonverbal IQ	0.20*** (0.02)	0.08*** (0.02)	0.38*** (0.02)	0.10*** (0.02)
Demographic characteristics				
Female gender	0.14*** (0.02)	0.14*** (0.02)	0.00 (0.02)	0.03 (0.02)
Age	0.01 (0.02)	0.00 (0.02)	0.02 (0.02)	–0.02 (0.02)
Log mean household income	0.19*** (0.03)	0.05 (0.04)	0.39*** (0.03)	0.05 (0.03)
Parents' education	0.13*** (0.02)	0.04* (0.02)	0.32*** (0.02)	0.14*** (0.05)
Number of siblings	–0.09*** (0.02)	–0.07*** (0.02)	–0.12*** (0.02)	–0.05*** (0.02)
Mean R^2		.09		.30

Note: All predictors and dependent variables were standardized; therefore, although the coefficients reported are unstandardized, they can be interpreted much like standardized coefficients. Parameter estimates and standard errors (in parentheses) are based on 20 multiply imputed data sets. The British Cohort Study data on which these analyses were based are publicly available from the Centre for Longitudinal Studies, Institute of Education, University of London Web site: http://www.cls.ioe.ac.uk/bcs70.
*p < .05. ***p < .001.

the U.K. data, $F(1, 3664) = 5.53$, $p < .05$. (See Tables 3 and 4 for a summary of predictors of literacy in the two data sets.)

Discussion

These findings demonstrate that elementary school students' knowledge of fractions and whole-number division predicts their mathematics achievement in high school, above and beyond the contributions of their knowledge of whole-number addition, subtraction, and multiplication; verbal and nonverbal IQ; working memory; family education; and family income. Knowledge of fractions and whole-number division also had a stronger relation to math achievement than did knowledge of whole-number addition, subtraction, and multiplication; verbal IQ; working memory; and parental income. These results were consistent across data sets from the United Kingdom and the United States. The fact that the relations of the predictor variables to algebra knowledge and overall mathematics achievement were similar in strength in the two samples, despite differences in the samples, the tests, and the times at which the data were obtained, is reason for confidence in the generality of the findings.

The correlation between knowledge of fractions in elementary school and achievement in algebra and mathematics overall in high school was expected, but the relation between early division knowledge and later mathematical knowledge was not. Fractions and division are inherently related (N/M means N divided by M), but the finding that early knowledge of fractions and early knowledge of division accounted for independent variance in later algebra knowledge and overall mathematics achievement indicated that neither relation explained the other.

There are several likely reasons why knowledge of division uniquely predicted later mathematics achievement. Mastery of whole-number division, like mastery of fractions, is required to solve many algebra problems (e.g., to apply the quadratic equation). Also, as is the case with fractions, high percentages of students fail to master division; thus, when high school students in the PSID-CDS were presented with a seemingly easy problem in which a boy wants to fly on a plane that travels 400 miles per hour in order to visit his grandmother who lives 1,400 air miles away, only 56% of the students correctly indicated how long the flight would take. More speculatively, poor knowledge of both division and fractions might lead students to give up trying to make sense of mathematics, and instead to rely on rote memorization in subsequent mathematics learning.

An alternative interpretation is that the unique predictive value of knowledge of fractions and knowledge of division stems from those operations being more difficult than addition, subtraction, and multiplication, and thus measuring more advanced thinking. Some of our results are inconsistent with this interpretation, however. First, knowledge of fractions and knowledge of division were not uniquely predictive of most subsequent literacy skills (see Tables 3 and 4), as should have been the case if their predictive value was due solely to their

Table 4. Results From Regression Models Predicting Literacy at Age 15 to 17 From Math Skills and Child and Family Characteristics at Age 10 to 12: Panel Study of Income Dynamics Data (N = 599)

	Letter-word identification		Passage comprehension	
Predictor	Model 1 (bivariate regression)	Model 2 (multiple regression)	Model 3 (bivariate regression)	Model 4 (multiple regression)
Early math skills				
Fractions	0.40*** (0.05)	0.03 (0.05)	0.41*** (0.05)	0.04 (0.06)
Addition	0.22*** (0.05)	–0.05 (0.04)	0.19*** (0.06)	–0.06 (0.05)
Subtraction	0.38*** (0.06)	0.07 (0.06)	0.35*** (0.05)	0.05 (0.05)
Multiplication	0.47*** (0.06)	0.14* (0.06)	0.43*** (0.05)	0.11* (0.05)
Division	0.49*** (0.06)	0.14* (0.07)	0.44*** (0.05)	0.08 (0.05)
Early abilities				
Backward digit span	0.30*** (0.05)	0.03 (0.04)	0.36*** (0.05)	0.10* (0.04)
Passage comprehension	0.65*** (0.06)	0.46*** (0.06)	0.65*** (0.04)	0.43*** (0.05)
Demographic characteristics				
Female gender	0.15** (0.05)	0.05 (0.04)	0.13* (0.05)	0.03 (0.04)
Age	0.17*** (0.05)	–0.08 (0.05)	0.13** (0.05)	–0.08 (0.04)
Log mean family income (1994–1996)	0.33*** (0.05)	0.08 (0.05)	0.39*** (0.06)	0.05 (0.05)
Parents' education	0.32*** (0.07)	0.05 (0.06)	0.47*** (0.05)	0.23*** (0.05)
Number of siblings	–0.06 (0.06)	0.06 (0.04)	–0.15*** (0.05)	0.03 (0.04)
Mean R^2		.51		.53

Note: All predictors and dependent variables were standardized; therefore, although the coefficients reported are unstandardized, they can be interpreted much like standardized coefficients. Parameter estimates and standard errors (in parentheses) are based on 20 multiply imputed data sets. Models were weighted by 2002 child-level weights and adjusted for the clustering of children within the same family. The data on which these analyses were based came from the Panel Study of Income Dynamics public-use data set, available at http://psidonline.isr.umich.edu.
*p < .05. **p < .01. ***p < .001.

greater difficulty. Second, spline tests (see Tables S5 and S6 in the Supplemental Material) showed that the predictive strength of early knowledge of fractions and division did not differ between students with greater and lesser mathematics achievement in high school. Thus, the unique predictive value of early fractions and division knowledge seems to be due to many students not mastering fractions and division and to those operations being essential for more advanced mathematics, rather than simply to fractions and division being relatively difficult to master.

Over 30 years of nationwide standardized testing, mathematics scores of U.S. high school students have barely budged (National Mathematics Advisory Panel, 2008). The present findings imply that mastery of fractions and division is needed if substantial improvements in understanding of algebra and other aspects of high school mathematics are to be achieved. One likely reason for students' limited mastery of fractions and division is that many U.S. teachers lack a firm conceptual understanding of fractions and division. In several studies, the majority of elementary and middle school teachers in the United States were unable to generate even a single explanation for why the invert-and-multiply algorithm (i.e., $a/b \div c/d = ad \times bc$) is a legitimate way to solve division problems with fractions. In contrast, most teachers in Japan and China generated two or three explanations in response to the same question (Ma, 1999; Moseley, Okamoto, & Ishida, 2007). These and the present results suggest that improved teaching of fractions and division could yield substantial improvements in students' learning, not only of fractions and division but of more advanced mathematics as well.

Declaration of Conflicting Interests

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

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Supplemental Material

Additional supporting information may be found at http://pss.sagepub.com/content/by/supplemental-data

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Corrigendum

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On page 696 of this article, near the bottom of the first column, the equation for the invert-and-multiply algorithm ($a/b \div c/d = ad \times bc$) was incorrect. The equation should read as follows:

$a/b \div c/d = ad \div bc$.

Supporting Online Material for

Precursors of High School Mathematics Achievement

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This PDF file includes:

Materials and Methods
SOM Text
Tables S1 to S6

Materials and Methods

Data

The present data come from two nationally representative, longitudinal datasets containing detailed mathematics assessments at two different time points: the UK 1970 British Cohort Study (BCS) and the US Child Development Supplement of the Panel Study of Income Dynamics (PSID-CDS).

British Cohort Study (BCS). The 1970 British Cohort Study is a longitudinal study following into adulthood all individuals born in Great Britain during a single week in April, 1970 (*1*). Data collection sweeps for BCS have taken place when the cohort members were aged 5, 10, 16, 26, 30, 34, and most recently 38 years (*2*). The birth sample of 17,196 infants was approximately 97% of the target birth population. The responding sample at age 10 was 14,350 (83%) and at age 16 was 11,206 (65%). A teachers' strike at the same time as the age 16 sweep reduced the number of cohort members for which we have achievement data: math test scores are only available for 21% of the cohort for that age group. The current sample is therefore made up of the 3,677 individuals (52% male) whose mathematics knowledge was assessed at ages 10 and 16.

Analyses of response bias in these data have shown that the achieved samples did not differ from their target samples across a number of critical variables (social class, parental education, and gender), despite a slight under-representation of the most disadvantaged groups (*3*). Bias due to attrition of the sample during childhood has also been shown to be minimal (*3*). Other analyses using these data find that the mathematics test score available for the reduced age 16 sample is as good a predictor of subsequent labor market outcomes as a more general achievement measure available for the whole age 16 cohort, further increasing our confidence that the lower response rate did not adversely affect our results (*4*). Finally, these same analyses also demonstrated high comparability in the distribution and predictive power of mathematics test scores with another U.K. longitudinal birth cohort, the 1958 National Child Development Survey, which did not suffer the same attrition problem.

Data for the BCS were collected from a variety of sources, including the mother, health care professionals, teachers, school health service personnel, and the individual child, and in a number of ways, including paper and electronic questionnaires, clinical records, medical examinations, tests of ability, educational assessments, and diaries. Data and documentation are available at http://www.cls.ioe.ac.uk/

Panel Study of Income Dynamics – Child Development Supplement (PSID-CDS). The Panel Study of Income Dynamics began in 1968 by drawing a nationally representative sample of over 18,000 individuals living in 5,000 families in the United States. Information on these individuals and their descendants has been collected continuously (annually through 1993, biennially since then). The information includes data on employment, income, wealth, expenditures, health, marriage, childbearing, child development, philanthropy, education, and numerous other topics. In 1997, all PSID families who had children between birth and 12 years of age were recruited to participate in the Child Development Supplement of the PSID (*5*), which provides the data used in this article. The CDS includes up to two children selected randomly from each PSID family that agreed to participate (CDS-I). The CDS-I collected data on 2,394 families (88% of eligible families) and their 3,563 children. The families who remained active in the PSID were reassessed in 2002 and 2003 (CDS-II). The CDS-II collected data on the 2091 families (91% of those in the CD-I) and their 2,907 children.

Like the BCS, the CDS includes interviews, assessments, and home observations that provide information on a broad range of developmental outcomes in the areas of health, psychological well being, social and cognitive development, and education, as well as a range of measures of the family neighborhood and school environment, among other variables. Data and documentation are available at http://psidonline.isr.umich.edu/

For the present study, 2,523 children were included who, in the CDS-II, had complete data in three subtests of the Woodcock Johnson Revised Tests of Achievement (WJ-R): Applied Problems (Word Math Problems), Letter-Word Identification (Vocabulary), and Passage Comprehension (Reading Comprehension). This sample was used to conduct multiple (20) data imputations. The sample was further reduced to 599 when children were selected for the 10 to 12 years target age range for our study.

Analysis procedures

Our analyses of data from the BCS and PSID-CDS use multiple regression to assess the predictive importance of different domains of math measured at age 10 (in the BCS) or ages 10-12 (for the PSID-CDS) for advanced mathematic achievement at age 16 (in the BCS) or ages 15-17 (in the PSID-CDS). In both datasets, baseline achievement tests were used to form our key measures of knowledge of fractions and whole number addition, subtraction, multiplication and division. Each of these five subscales was measured by the proportion of items correct on that subscale. To facilitate comparisons across the subscales, all five are standardized to a mean of zero and a standard deviation of one.

To avoid attributing to these math components predictive power that is more properly attributed to general cognitive ability or family background, both sets of analysis control for measures of the child's intellectual ability, age, parents' social class and highest level of education/maternal literacy, as well as family income and family size. As explained below, we also conducted falsification tests using age 16 (in the BCS) or 15-17 (in the PSID) literacy achievement measures as dependent variables. To account for missing data in both data sets, we used multiple imputations by chained equations (ICE) as implemented in STATA (*6*). The PSID-CDS data are weighted using weights supplied by the study to account for differential sampling fractions and attrition.

Measures**:**

In the BCS, age 10 mathematics achievement was measured by the "Friendly Maths Test" developed specifically for the BCS in collaboration with specialists in primary mathematics (*7*). The test consists of 72 multiple choice questions and assesses knowledge of the rules of arithmetic, number skills, and fractions. The reliability of the test is α=.93. For the purposes of this study, the math score was divided into items measuring knowledge of fractions and whole number addition, subtraction, multiplication, and division.

Sixteen year olds were given a timed arithmetic test consisting of 60, increasingly difficult, problems beginning with basic arithmetic expressions and simple word problems, and moving on to questions on fractions, percentages, algebra, estimation of area, and probability. Literacy at age 16 was assessed by separate tests in spelling and vocabulary.

Children's general intellectual ability was measured at age 10 using the British Ability Scales (*8*), a cognitive test battery for children between 3 and 17 years of age that assesses verbal and non-verbal reasoning. Verbal sub-scales comprised word definitions (37 items, α=.83) and word similarities (42 items, α=.80). Non-verbal sub-scales comprised recall of digits (34 items, α=.84) and matrices (28 items, α=.87).

Additional child level characteristics included in our analysis include child gender (0 = male, 1 = female) and age (in years) at the time of testing. Given the logistical complications associated with the size of the BCS cohort and the difficulties of administering the age 10 tests in schools, children's ages ranged from 9.3 to 11.4 years, with an average of 10.2 years.

Highest household education is measured in terms of parents' estimated years of schooling, ranging from 10 to 17 years, with an average of 12.5. This measure has the advantage of including vocational as well as academic qualifications and indicates that less than a third (29%) of this cohort's parents went on to post-compulsory education. Gross weekly family income, before deductions, was measured in bands when children were age 10. Our analysis uses the natural logarithm of the midpoint of each band. Baseline information on family size was provided by the mother at the time of study enrolment, i.e. birth, and was updated with further detail provided at the age 10 assessments.

Missing data on these variables was multiply imputed, based on 20 data imputations. All variables were standardized to z-scores. Descriptive statistics for all variables used in our analyses are shown in Table S1. Correlations among age 10 and 16 math and literacy measures in the BCS are provided in Table S2.

In the PSID-CDS, 10 to 12 year-old children were assessed using the Calculation subtest of the Woodcock-Johnson Psycho-Educational Battery – Revised (WJ-R) in CDS- I and the Applied Problems, Passage Comprehension, and Letter-Word Identification subtests of the WJ-R in CDS-II (at 15-17 years of age) (*9*). The Woodcock-Johnson has been widely used in national longitudinal studies, and has good psychometric properties. The split-half reliabilities reported for the group of 10-17 year-old children ranges between .78 and .94 (*9*). For the purpose of this study, the Calculation and Applied Problems subscales were divided into the subcomponents of specific mathematics skill areas — addition, subtraction, multiplication, division, fractions, and algebra. These specific mathematics skills were measured in proportion of correct responses. The child's total mathematics score was measured as the total score of Applied Problems at 15-17 years old. Literacy at ages 15-17 years of age were assessed by separate tests of vocabulary (WJ-Letter Word) and reading comprehension (WJ-Passage Comprehension).

Children's general intellectual ability was measured in the PSID by their short-term memory scores on the Digit Span subtest of the WISC-R (*10*). Children were given a series of numbers and asked to repeat them either forward or backward. The proportion correct for the 14 items of the backward digit span measure at Wave I (CDS-I) was used.

As in the BCS, additional child level characteristics (gender and age) were included in the analysis. For gender, males were assigned a code of 0 and females a code of 1. Age was measured at the time of children's assessment (CDS-I) and, it is expressed as the number of years at the time of the interview. The average age at CDS-I was 11.5 years of age and range between 10.0 and 12.9 years of age.

The educational level of the family was measured by the highest education of the head of household and his/her spouse. The values range from 0-17, with a mean of 13.24, which indicates a mean of slightly more than a high school education for the sample. Family income was measured using the natural logarithm of average income of the family from 1994, 1995 and 1996. The number of siblings represents the number of children living in the house at the time of the 1997 interview.

All variables were standardized to z-scores using weighted mean and standard deviation for the 10-12 year olds in the CDS-I from the imputed data. Because the CDS is intended to be a nationally representative sample of the children and their primary caregivers in the U.S, sample weights were used to account for differential probabilities of selection. The CDS weights also adjust for attrition across interviewing waves. The child-level weight was used in all of the analyses presented in this paper (*11*).

Descriptive statistics for all variables in our analyses are shown in Table S1. Correlations among age 10 and 16 mathematics and literacy measures in the PSID-CDS are provided in Table S3.

<u>Nonlinear Effects</u>

To explore the generality of these relations over achievement levels, we fit piecewise linear (spline) functions that allowed for different slopes for children in the bottom and top halves of the baseline distributions of fractions and division knowledge. Although slope estimates for fractions knowledge were somewhat steeper for children in the top as opposed to bottom half of the fractions knowledge distributions, in only one of the four cases was the slope difference statistically significant at $p<.05$. There were no clear patterns for slope differences in division knowledge. Overall, the spline analyses showed that the predictive power of early fractions and division knowledge to later algebra and overall mathematics achievement was similar for children with low and high levels of fractions and division knowledge (Tables S5 and S6).

Table S1: Descriptive characteristics of the study samples

British Cohort Study	*Mean*	*Std. Dev.*	*Min*	*Max*
Age 16 mathematics domain:				
Fractions	.72	(.23)	0	1
Division	.89	(.18)	0	1
Algebra	.55	(.27)	0	1
Overall Math score	.64	(.21)	0	1
Age 16 literacy domains:				
Spelling	.78	(.21)	0	1
Vocabulary	.55	(.16)	.01	1
Age 10 mathematics domain:				
Fractions	.62	(.27)	0	1
Addition	.95	(.10)	0	1
Subtraction	.91	(.13)	.2	1
Multiplication	.78	(.23)	0	1
Division	.72	(.27)	0	1
Age 10 ability:				
Verbal IQ	100.00	(15.00)	46.27	151.56
Non-verbal IQ	100.00	(15.00)	46.34	158.13
Child characteristics:				
Girl	.54	(.50)	0	1
Child's age (years)	10.15	(.21)	9.40	11.37
Background characteristics:				
(Logged) Income	4.77	(.51)	2.86	5.70
Highest household education (number of years)	12.48	(2.50)	10	17
No. Siblings	1.42	(1.00)	0	8

Table S1: Descriptive characteristics of the study samples (continued)

Panel Study of Income Dynamics	***Mean***	***Std. Dev.***	***Min***	***Max***
Age 15-17 domains of math:				
Algebra	.36	(.25)	0	1
Fractions	.58	(.28)	0	1
Total Math score	.73	(.11)	.37	1
Age 15-17 literacy domains:				
Letter Words	.89	(.08)	.44	1
Passage Comprehension	.72	(.10)	.28	1
Age 10-12 domains of math:				
Fractions	.20	(.21)	0	1
Addition	.85	(.10)	.38	1
Subtraction	.93	(.12)	0	1
Multiplication	.74	(.19)	0	1
Division	.65	(.32)	0	1
Age 10-12 ability:				
Digit Span Backward	5.60	(2.05)	0	14
Passage Comprehension	.60	(.11)	0	0.95
Child characteristics:				
Girl	.52	(.50)	0	1
Child's age (years)	11.50	(.89)	10.00	12.99
Background characteristics:				
Log Mean Family Income 94-96	10.60	(.80)	7.48	13.63
Parent Education (highest)	13.24	(3.28)	0	17
No. Siblings	1.63	(1.14)	0	7

Table notes: BCS results are based on 20 multiple imputations (N=3,677 each). PSID results based on 20 multiple imputations (N=599 each) and weighted by 2002 child level weights.

Table S2: Correlations among mathematics, literacy and cognitive ability variables, British Cohort Study

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	Age 16 domains of math:													
(1)	Fractions	1												
(2)	Division	.52	1											
(3)	Algebra	.68	.41	1										
(4)	Total Math	.81	.59	.73	1									
	Age 16 literacy domains:													
(5)	Spelling	.29	.24	.27	.35	1								
(6)	Vocabulary	.52	.36	.50	.58	.27	1							
	Age 10 domains of math:													
(7)	Fractions	.44	.26	.43	.47	.16	.39	1						
(8)	Addition	.24	.20	.20	.26	.10	.20	.28	1					
(9)	Subtraction	.21	.17	.22	.24	.12	.17	.28	.29	1				
(10)	Multiplication	.34	.25	.32	.37	.17	.27	.43	.28	.32	1			
(11)	Division	.37	.26	.37	.40	.15	.28	.48	.26	.30	.47	1		
	Age 10 ability:													
(12)	Verbal IQ	.40	.24	.39	.43	.19	.49	.53	.26	.23	.34	.35	1	
(13)	Non-verbal IQ	.42	.30	.42	.46	.20	.38	.50	.30	.26	.39	.41	.52	1

Table notes: All correlations significant at $p<.01$. Results are based on 20 multiple imputations (N=3,677 each).

Table S3: Correlations among math, literacy and cognitive ability variables, Panel Study of Income Dynamics

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
	Age 15-17 domains of math:													
(1)	Fractions	1												
(2)	Division	.59	1											
(3)	Algebra	.65	.48	1										
(4)	Total Math	.87	.69	.80	1									
	Age 15-17 literacy domains:													
(5)	Letter Words	.50	.49	.43	.60	1								
(6)	Passage Comprehension	.56	.51	.54	.67	.71	1							
	Age 10-12 domains of math:													
(7)	Fractions	.46	.34	.41	.49	.40	.41	1						
(8)	Addition	.25	.25	.26	.30	.22	.19	.37	1					
(9)	Subtraction	.31	.36	.26	.39	.38	.35	.26	.38	1				
(10)	Multiplication	.36	.37	.31	.43	.47	.43	.51	.36	.45	1			
(11)	Division	.48	.41	.40	.53	.49	.44	.58	.34	.40	.64	1		
	Age 10-12 ability:													
(12)	Digit Span Backward	.27	.24	.29	.33	.30	.36	.27	.21	.23	.32	.31	1	
(13)	Passage Comprehension	.47	.42	.38	.51	.65	.64	.44	.32	.41	.45	.46	.35	1

Table notes: All correlations significant at p<.01. Results based on 20 multiple imputations (N=599 each) and weighted by 2002 child level weights.

Table S4: F-tests from model 2 multiple regression of math and literacy scores on earlier math skills and child and family characteristics

British Cohort Study	Algebra	Total maths score	Spelling	Vocabulary
N	3,677	3,677	3,677	3,677
Equality test from Model 2				
fraction=addition	*F(1, 3664) = 27.93****	*F(1, 3664) = 16.60****	*F(1, 3664) = 0.36*	*F(1, 3664) = 4.41**
fraction=subtraction	*F(1, 3664) = 14.00***	*F(1, 3664) = 19.42****	*F(1, 3664) = 0.49*	*F(1, 3664) = 8.72***
fraction=multiplication	*F(1, 3664) = 7.95**	*F(1, 3664) = 7.47***	*F(1, 3664) = 0.94*	*F(1, 3664) = 3.89**
fraction=division	*F(1, 3664) = 0.46*	*F(1, 3664) = 1.45*	*F(1, 3664) = 0.24*	*F(1, 3664) = 2.74*
mean(fra div)=mean(add subt mult)	*F(1, 3664) = 36.92****	*F(1, 3664) = 28.79****	*F(1, 3664) = 0.09*	*F(1, 3664) = 5.53**
Panel Study of Income Dynamics (PSID)	Algebra	Total maths score	Letter Words	Passage Comprehension
N	599	599	599	599
Equality test from Model 2				
fraction=addition	*F(1, 558) = 0.65*	*F(1, 558) = 2.80*	*F(1, 558) = 1.37*	*F(1, 558) = 1.49*
fraction=subtraction	*F(1, 558) = 2.23*	*F(1, 558) = 0.78*	*F(1, 558) = 0.38*	*F(1, 558) = 0.05*
fraction=multiplication	*F(1, 558) = 3.26*	*F(1, 558) = 3.53*	*F(1, 558) = 1.84*	*F(1, 558) = 0.73*
fraction=division	*F(1, 558) = 0.04*	*F(1, 558) = 0.85*	*F(1, 558) = 1.28*	*F(1, 558) = 0.18*
mean(fra div)=mean(add subt mult)	*F(1, 558) = 7.12***	*F(1, 558) = 9.72***	*F(1, 558) = 0.30*	*F(1, 558) = 0.21*

Table notes: * *p*<.05, ** p<.01, *** p<.001

Table S5: (Standardized) Coefficients and standard errors from spline regression analysis of math assessments on earlier math skills and child and family characteristics, BCS

Age 10 domains of math:	Algebra		Total math score	
Fractions	.05		.09	*
	(.04)		(.04)	
Addition	.00		.05	**
	(.02)		(.02)	
Subtraction	.04		.03	
	(.02)		(.02)	
Multiplication	.06	***	.08	***
	(.02)		(.02)	
Division	.14	***	.17	***
	(.03)		(.03)	
Spline analysis:				
Top half of fractions score (binary indicator)	.04		.11	*
	(.06)		(.06)	
Top half of fractions * Fractions interaction	.20	**	.06	
	(.06)		(.06)	
Top half of division score (binary indicator)	-.21		-.26	
	(.14)		(.14)	
Top half of division * Division interaction	.19		.15	
	(.14)		(.14)	
Age 10 ability:				
Verbal IQ	.10	***	.10	***
	(.02)		(.02)	
Non-verbal IQ	.17	***	.19	***
	(.02)		(.02)	
Child characteristics:				
Girl	-.01		.00	
	(.02)		(.01)	
Child's age (years)	-.03	*	-.03	*
	(.02)		(.01)	
Background characteristics:				
(Logged) Income	.08	*	.09	*
	(.04)		(.04)	
Highest household education	.10	***	.10	***
	(.02)		(.02)	
No. Siblings	-.01		-.05	***
	(.01)		(.01)	
N	3,677		3,677	
Mean R^2 for model 2	.29		.35	

Table notes: * *p*<.05, ** p<.01, *** p<.001

Table S6: (Standardized) Coefficients and standard errors from spline regression analysis of math assessments on earlier math skills and child and family characteristics, PSID

Age 10 domains of math:	Algebra		Total math score	
Fractions	-.11		.05	
	(.29)		(.30)	
Addition	.08		.04	
	(.06)		(.05)	
Subtraction	.05		.13	**
	(.05)		(.05)	
Multiplication	.00		.03	
	(.06)		(.05)	
Division	.24	**	.27	**
	(.09)		(.10)	
Spline analysis:				
Top half of fractions score (binary indicator)	-.09		-.14	
	(.26)		(.27)	
Top half of fractions * Fractions interaction	.47		.27	
	(.30)		(.31)	
Top half of division score (binary indicator)	.01		.12	
	(.24)		(.19)	
Top half of division * Division interaction	-.05		-.08	
	(.25)		(.21)	
Age 10-12 ability:				
Digit Span Backward	.10		.07	
	(.06)		(.05)	
Passage Comprehension	.10		.20	***
	(.06)		(.05)	
Child characteristics:				
Girl	-.09		-.13	***
	(.05)		(.04)	
Child's age (years)	-.18	***	-.22	***
	(.05)		(.04)	
Background characteristics:				
Log Mean Family Income 94-96	.05		.12	*
	(.06)		(.05)	
Parent Education (highest)	.20	***	.12	*
	(.05)		(.06)	
No. Siblings	-.03		-.04	
	(.04)		(.04)	
N	599		599	
Mean R^2 for spline model	.36		.53	

Table notes: * *p*<.05, ** p<.01, *** p<.001

Appendix A. References

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