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Improving At-Risk Learners' Understanding of Fractions

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The purposes of this study were to investigate the effects of an intervention designed to improve at-risk 4th graders' understanding of fractions and to examine the processes by which effects occurred. The intervention focused more on the measurement interpretation of fractions; the control condition focused more on the part-whole interpretation of fractions and on procedures. Intervention was also designed to compensate for at-risk students' limitations in the domain-general abilities associated with fraction learning. At-risk students (n = 259) were randomly assigned to intervention and control. Whole-number calculation skill, domain-general abilities (working memory, attentive behavior, processing speed, listening comprehension), and fraction proficiency were pretested. Intervention occurred for 12 weeks, 3 times per week, 30 min per session, and then fraction performance was reassessed. On each conceptual and procedural fraction outcome, effects favored intervention over control (effect sizes = 0.29 to 2.50), and the gap between at-risk and low-risk students narrowed for the intervention group but not the control group. Improvement in the accuracy of children's measurement interpretation of fractions mediated intervention effects. Also, intervention effects were moderated by domain-general abilities, but not whole-number calculation skill.

Keywords: fractions, intervention, mathematics

Competence with fractions is considered foundational for learning algebra, for success with more advanced mathematics, and for competing successfully in the American workforce (Geary et al., 2008; National Mathematics Advisory Panel [NMAP], 2008; Siegler et al., in press). Yet, half of middle and high school students in the United States are still not proficient with the ideas and procedures taught about fractions in the elementary grades (e.g., National Council of Teachers of Mathematics [NCTM], 2007; NMAP, 2008). For these reasons, NMAP recommended that high priority be assigned to improving performance on fractions, a theme reflected in the Common Core State Standards (http://www.corestandards.org/). The focus of the present study was improving and understanding the development of fraction competence for fourth graders at risk for poor outcomes. We assessed the efficacy of a 12-week intervention and examined the processes by which effects occurred to increase understanding about the development of competence with fractions. Only a handful of studies have assessed the efficacy of fraction instruction or intervention (Misquitta, 2011). Even fewer studies have examined the processes by which intervention effects occur. In this introduction, we explain our theoretical framework for designing the intervention and for providing insights into the processes by which its effects occur. We then summarize prior research on fraction intervention and explain how the present study extends the literature.

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Theoretical Framework

Our framework for designing intervention and for providing insights into the processes by which effects occur was guided by Geary's (2004) model of mathematics learning. This model has two major components. It discusses conceptual and procedural knowledge as the two primary forms of mathematical knowledge, and it poses that mathematics learning depends on a constellation of foundational mathematical skills and domain-general abilities.

Focusing on Foundational Concepts in Fractions

Although both conceptual and procedural knowledge have been shown to influence the development of the other (e.g., Rittle-Johnson & Siegler, 1998), conceptual knowledge tends to have a larger effect on the development of procedural knowledge than the other way around, and conceptual knowledge has more generalized effects on new learning than procedural knowledge (e.g., Byrnes & Wasik, 1991; Geary et al., 2008; Hecht, Close, & Santisi, 2003; Hecht & Vagi, 2010; Ni & Zhou, 2005; Rittle-Johnson, Siegler, & Alibali, 2001). In the early stages of fraction knowledge, two forms of conceptual interpretation are deemed most relevant (e.g., Hecht & Vagi, 2010). The first is part-whole, with which a fraction is understood as a part of one entire object or a subset of a group of objects. Such understanding is often evident as early as preschool (e.g., Mix, Levine, & Huttenlocher, 1999), based on children's experiences with sharing. In American schools, symbolic fraction notation, typically introduced in first grade, is taught via area models that underpin part-whole understanding. By fourth grade, the dominant emphasis on part-whole interpretation persists, and the focus on concepts versus procedures (i.e., calculations involving fractions) is roughly comparable. Accordingly, in the present study, the school program (i.e., the control condition) emphasized part-whole interpretation, with comparable time allocated to concepts and procedures.

The second type of understanding, the measurement interpretation of fractions, reflects cardinal size (Hecht, 1998; Hecht et al., 2003). This type of interpretation, which is often represented with number lines (e.g., Siegler, Thompson, & Schneider, 2011), is assigned a subordinate role in American schooling (i.e., it is addressed later and with less emphasis) and therefore represents an innovative instructional approach at fourth grade. Measurement interpretation is less intuitive than part-whole understanding and is thought to depend on formal instruction that explicates the conventions of symbolic notation, the inversion property of fractions (i.e., fractions with the same numerator become smaller as denominators increase), and the infinite density of fractions on any given segment of the number line. A focus on the measurement interpretation is in keeping with NCTM (2006) Standards and the fourth-grade Common Core State Standards' explicit emphasis on understanding of fraction equivalence and ordering. Also, Geary et al. (2008) hypothesized that improvement in measurement interpretation is a key mechanism in explaining the development of fraction competence.

For these reasons, we designed intervention to center predominantly on conceptual understanding with the major focus on the measurement interpretation of a fraction. At the same time, because some evidence suggests conceptual and procedural knowledge may be mutually supportive (Hecht & Vagi, 2010; Rittle-Johnson et al., 2001, but see Byrnes & Wasik, 1991, and Siegler et al., 2011), we included procedural instruction, but delayed its introduction until after the conceptual basis for interpreting fractions was established (lesson 22 of 36 lessons). As we addressed fraction calculations, we continued to extend fraction concepts and integrate the concepts with fraction procedures. Therefore, the first major distinctions between the intervention and control conditions were that the control condition focused more on part-whole understanding, more on procedures, and less on the measurement interpretation of fractions.

This framework for conceptualizing the development of fraction competence and designing the intervention led to two hypotheses. First, we expected intervention students' conceptual and procedural outcomes to exceed those of the control group. Second, we hypothesized that improvement in the measurement interpretation of fractions is a causal mechanism by which effects occur.

Addressing Foundational Mathematical Skills and Domain-General Abilities Thought to Influence Fraction Learning

The design of the intervention and the nature of our hypotheses were also informed by the second major component of Geary's (2004) model of mathematics learning: that mathematics learning depends on a constellation of domain-general abilities (e.g., working memory) and foundational mathematical abilities or skills. A type of *foundational mathematical skill* that recurs in the literature as predictive of fraction learning is competence with whole-number calculations (Hecht & Vagi, 2010; Hecht et al., 2003; Jordan et al., 2012; Seethaler, Fuchs, Star, & Bryant, 2011). Whole-number calculations are transparently required to handle fractions, and it may be advantageous to execute whole-number calculations quickly, without taxing mental resources (e.g., retrieving rather than counting to perform simple calculations) that might be used to execute more complex conceptual features of fractions. Also, correlational evidence (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Mazzocco & Myers, 2003; Murphy, Mazzocco, Hanich, & Early, 2007) suggests that students with more versus less severe mathematics difficulty (often designated dichotomously as <15th percentile vs. between the 15th and 25th-35th percentile) have different developmental trajectories and different cognitive profiles, such that students with varying severity of whole-number calculation deficits require different forms of intervention. We therefore hypothesized the children's incoming whole-number calculation skill would moderate response to intervention: that students with stronger pretest calculation scores would be more responsive.

Among the *domain-general resources* thought to affect mathematics learning in Geary's (2004) model, working memory and related attentional processes feature prominently (also see Geary et al., 2008). The measurement interpretation of fractions may especially tax such systems as children simultaneously consider the contribution of numerators and denominators when comparing fractions or placing them on a number line. Studies show that individual differences in working memory and attentive behavior contribute to performance on fraction tasks (Hecht et al., 2003; Hecht & Vagi, 2010; Jordan et al., 2012), even when controlling for other domain-general resources or prior mathematics achievement. Yet, debate exists about whether individual differences in working memory are driven by more fundamental differences in processing speed (Kail, 1991) or whether the attentional focus associated with the central executive speeds information processing (Engle, Tuholski, Laughlin, & Conway, 1999). At-risk children often experience limited working memory capacity, inattentive behavior, and slow processing speed (e.g., Compton, Fuchs, Fuchs, Lambert, & Hamlett, 2012). This signals the need for intervention to incorporate instructional methods that (a) teach students efficient strategies for chunking (recoding a multi-dimensional concept into fewer dimensions) or segmenting (breaking a task into a series of steps, each of which is less resource demanding) measurement interpretation tasks, (b) create automaticity with fractional values in relation to marker fractions (e.g., one-half), and (c) provide a structure to encourage students to exercise attentive behavior and work hard. In our intervention, we adopted such methods, with the goal of reducing demands on working memory, processing speed, and related attentional processes.

The final domain-general ability we considered was listening comprehension. The role of individual differences in listening comprehension has been demonstrated for whole-number word problems but not calculations (e.g., Fuchs et al., 2008; Fuchs, Geary, Compton, Fuchs, Hamlett, & Bryant, 2010; Fuchs, Geary, Compton, Fuchs, Hamlett, Seethaler, et al., 2010; H. L. Swanson & Beebe-Frankenberger, 2004). Yet, prediction studies of fraction competence have revealed a more general role for listening comprehension in fractions, including both calculations (Seethaler et al., 2011) and concepts (Jordan et al., 2012). Some studies show the influence of fraction terminology on mental representations of fractions (Miura, Okamoto, Kim, Steere, & Fayol, 1993), and orally presented explanations of fraction concepts or procedures are typically involved and lengthy, thereby taxing listening comprehension ability. So we designed intervention to provide explanations in simple language (short sentences, active voice, unambiguous pronoun antecedents, etc.), required students to repeat explanations in their own words (while they incorporated terminology), and punctuated explanations with frequent checks for understanding.

Therefore, another major distinction between the intervention and control conditions was that our intervention was designed to compensate for the working memory, processing speed, attentive behavior, and oral language abilities associated with mathematics learning and with which at-risk students typically experience limitations. This also led us to hypothesize that these domain-general resources moderate—or interact with—intervention condition. With such moderation/interaction, the relation between these domain-general cognitive resources and fraction learning differs for intervention versus control students. Because the intervention was designed to compensate for students' limitations in domain-general abilities, we expected intervention students to respond similarly regardless of their domain-general cognitive abilities but expected control students with lower abilities to experience poor outcomes relative to control students with stronger abilities.

Prior Intervention Research on Fractions

As mentioned, few studies have assessed the effects of fraction intervention on at-risk learners; even fewer have examined processes by which effects occur. The most successful studies have adopted an explicit instructional approach (i.e., clear explanations, modeling, guided and independent practice, feedback, cumulative review), which is in line with two national panel conclusions synthesizing the instructional literature for at-risk (AR) students (Gersten et al., 2009; NMAP, 2008). Using a multiple-baseline design, Joseph and Hunter (2001) demonstrated experimental control for a cue-card strategy across three eighth-grade AR students. A teacher initially taught students to use the cue card, which supported a 3-pronged strategy for adding or multiplying fractions and reducing answers. After students showed competence in applying the strategy, they used the cue card while solving problems. In a maintenance phase, the cue card was removed. All three students showed substantial improvement with introduction of the cue card strategy, and maintenance was strong. The study focus was, however, entirely procedural in terms of instruction and outcome.

Kelly, Gersten, and Carnine (1990) also took an explicit approach to fraction instruction, but focused on procedures and concepts. They randomly assigned 28 high-school AR students from three classes to 10 sessions of teacher-mediated videodisc instruction or conventional textbook instruction. Direct instruction was employed in both conditions, but only videodisc instruction provided mixed problem-type instruction, separated highly confusable concepts and terminology in early instructional stages, and provided a broader range of examples to avoid misconceptions. Both groups improved substantially from pretest (40% on a 12-item test) to posttest (96% vs. 82%), with the videodisc group improving significantly more. Yet, despite the instructional focus on concepts and procedures, the fraction measure was largely procedural. A few items required students to name fractions from pictures or distinguish numerators from denominators; the remaining items were procedural.

By contrast, in the next two studies, intervention focused primarily on understanding of fractions and assessed outcomes on concepts as well as procedures. Butler, Miller, Crehan, Babbitt, and Pierce (2003) contrasted two explicit instruction conditions with 50 middle school AR learners. Both conditions carefully transitioned students from a conceptual emphasis, largely based on part-whole understanding, to algorithmic rules for handling fractions, and from visual to symbolic representations. Only one condition included concrete manipulatives. Both groups significantly improved across 10 sessions. On one measure, in which students circled fractional parts of sets, those who received 3 days of manipulatives improved significantly more; on the other four measures, the difference between conditions was not significant, providing minimal evidence regarding the importance of concrete representations. Without random assignment or a control group, however, conclusions are tentative.

Hecht (2011) extended Butler et al. (2003) by employing random assignment, including a control group, and doubling the duration of intervention. He randomly assigned 43 seventh-grade AR students to control or intervention using 23 Rational Number Project lessons (Initial Fraction Ideas; Cramer, Behr, Post, & Lesh, 2009). These lessons rely on area models to teach part-whole relations, the concept of the unit, order, and equivalence, while including a focus on addition and subtraction procedures, word problems, and estimation. Intervention students improved significantly more than control on a range of procedural and conceptual measures.

These studies provide the basis for only tentatively concluding that explicit instruction, based on part-whole understanding of fractions, enhances fraction learning among middle- and high-school AR students. Each study was small; drew students from a limited number of classes; and relied exclusively on experimental outcome measures, aligned with instruction (but see Butler et al., 2003, who included a commercial criterion-referenced measure in addition to experimental tasks). Also, none of these studies addressed the earlier grades, when the foundation for understanding fractions is developed, or provided insight into the processes by which effects occurred. Finally, in these studies, risk was operationalized in terms of participation in remedial classes, school-identified disability, or teacher nomination. Each study therefore lacked clarity about the severity of the sample's incoming mathematics performance and about whether effects of the fraction intervention differ as a function of students' incoming deficits with whole-number calculations.

The Present Study's Extensions to the Literature

To extend this literature, we targeted fourth grade, when the curriculum has a strong focus on understanding of fractions. Thus, our intervention was preventative rather than remedial. We assessed understanding and procedural competence using both experimental tasks and an external, widely-accepted measure of fraction competence: easy, medium, and hard fourth-grade and easy eighth-grade fraction items released from the National Assessment of Educational Progress (NAEP; 18 items from the pool of items released between 1990 and 2009; U.S. Department of Education, 2013). We operationalized risk as pre-intervention whole-number calculation skill below the 35th percentile on a nationally normed test, but to ensure a strong distribution of low skill, we randomly sampled students from two bands (<15th percentile vs. 15–34th percentile) and then randomly assigned students to intervention and control condition within classrooms, while stratifying by bands.[1] To contextualize results, we compared AR students' year-end performance against low-risk (>34th percentile) classmates. This is important because there are few fraction measures available that provide a normative framework or thorough behavior sampling of fourth-grade fraction skill.

To review, we had four hypotheses. We expected (a) intervention students' conceptual and procedural outcomes to exceed the control group's outcomes; (b) improvement in children's measurement interpretation to mediate those effects; (c) working memory, attentive behavior, processing speed, and listening comprehension to moderate the intervention effects; and (d) foundational whole-number calculation skill also to moderate the intervention effects.

Method

Participants

We defined risk as performance below the 35th percentile on a broad-based calculations assessment (Wide Range Achievement Test–4 [WRAT-4]; Wilkinson & Robertson, 2006). In the fourth-grade range, the WRAT almost entirely samples whole-number items. To ensure strong representation across scores below the 35th percentile, we sampled half the AR students from below the 15th percentile (more severe); the other half from between the 15th and 34th percentiles (less severe). Because this study was not about intellectual disability, we excluded students ($n = 18$) with T-scores below the 9th percentile on both subtests of the Wechsler Abbreviated Scales of Intelligence (WASI; Psychological Corporation, 1999). We thus included 290 AR students, half more severe and half less severe, from 53 fourth-grade classrooms in 13 schools. We sampled between two and eight AR students per classroom, stratifying by more versus less severe risk in each classroom. Then we randomly assigned the AR students at the individual level, stratifying by classroom and risk severity, to fraction intervention or control groups (see footnote 1). Another 292 low-risk classmates (>34th percentile) were randomly sampled to represent each of the 53 classrooms in similar proportion to AR students from those classrooms. These low-risk classmates served as a comparison group for interpreting AR progress in response to the same classroom fraction instruction and for gauging the extent to which the intervention closed the fraction achievement gap for AR students.

Of the 290 AR students, 22 moved (10 intervention; 12 control) before the end of the study, and another nine (6 intervention; 3 control) had at least one piece of missing data. These 31 students did not differ statistically from the remaining AR students on pretest measures. Among the 292 low-risk students, 10 moved prior to the end of the study. These 10 students did not differ statistically from remaining low-risk students on pretest measures. WRAT standard scores averaged 84.67 ($SD = 8.03$) for the remaining 129 AR intervention students, 84.44 ($SD = 7.89$) for the remaining 130 AR control students, and 104.75 ($SD = 7.40$) for the remaining 282 low-risk students. There was no difference between the intervention and control conditions, each of which scored reliably lower than the low-risk group. The percentage of males in the intervention and control groups, respectively, was 50 and 54; the percentage of English learners was 12 and 9; the percentage receiving subsidized lunch was 81 and 83. In each condition, 5% received special education. In the intervention condition, the percentages of African American, White, Hispanic, and other students was 51, 26, 19, and 4; in control, 54, 24, 19, and 3 (all Hispanic students were White). Thus, the AR groups were demographically comparable (all $ps > .05$). We did not collect demographic data (or individually administered measures) on low-risk students.[2]

Major Distinctions Between the Intervention and the Control Groups

In terms of content, the major distinctions between the intervention and control conditions were that the control condition focused more on part-whole (less on measurement) interpretation

[1] To increase the possibility of identifying comparable numbers of students with more and less severe risk in the same classrooms, we sampled AR students in each classroom for participation within severity bands. We then stratified by risk severity when randomly assigning students to intervention versus control conditions. In many classrooms, we had an even split between more versus less severe risk, such that an equal number of students with more versus less severe risk were randomly assigned to intervention versus control groups. In some classrooms, we had an uneven number of participants (i.e., one or more extra students with more severe risk or one or more extra students with less severe risk in one or the other study condition), such that the number of students per condition per risk severity status in those classrooms was off by one or more students.

[2] Low-risk classmates were only pre- and posttested on group-administered measures for two reasons. First, because the study was not about low-risk students, we did not think it appropriate to spend school time on the individual assessment battery or to ask teachers to spend time completing demographic forms or attentive behavior ratings on these students. Second, study resources did not permit us to conduct the individual testing battery with an additional 290 children.

and more on procedures. Also, the intervention included a more limited pool of denominators to decrease computational demands (because our major focus was on concepts over procedures). In terms of instructional design, intervention instruction differed from control group instruction in that the intervention was designed to address the working memory, attentive behavior, processing speed, and listening comprehension deficits associated with poor mathematics learning in ways already described.

Control Group Instruction

Control group instruction relied on *Houghton Mifflin Math* (Greenes et al., 2005), which focuses on conceptual understanding and procedural calculations and relies heavily on part-whole understanding by using shaded regions and other manipulatives related to the area model. Number lines are given considerably less emphasis. This instruction was delivered in whole-class arrangement and via math centers, with many AR control students also participating in the schools' small-group remediation period three times per week.

Houghton Mifflin Math conceptual lessons included vocabulary instruction, connections across the curriculum (e.g., social studies, music, writing), guided practice, independent work, and links to real life. Teachers modeled fractions as parts of wholes, as locations on the number line, as parts of a set, and as a way to represent division of whole numbers. Number lines were used to initiate comparing and ordering fractions. Number lines, base ten blocks, Venn diagrams, and hundreds boards were also available to students. Activities included reading, writing, and identifying fractions and mixed numbers; finding equivalent fractions and writing fractions in simplest form; comparing and ordering fractions; finding a fractional part of a whole number or finding a fraction of the number of objects in a set; and drawing pictures to solve problems. Denominators did not exceed 20.

Calculations with fractions focused on procedures for adding and subtracting. Teachers first taught addition and subtraction of fractions and mixed numbers with like denominators. Teaching tools included fraction strips, fraction circles, calculators, function tables, and flashcards. For proper and improper fractions, denominators did not exceed 12. For mixed numbers, denominators did not exceed 15, and the largest whole number component of the mixed number was 9. Then, teachers introduced fractions with unlike denominators using methods and teaching tools similar to those used for like denominators. As in conceptual lessons, teachers introduced relevant vocabulary, made connections across the curriculum, and provided guided and independent practice opportunities. Note that low-risk classmates received the same classroom instruction as the control group but were not eligible for the school's intervention program.

Fraction Intervention

The study's intervention was provided during one of three school instructional periods (depending on teachers' scheduling preferences): during part of the math block (typically 50 min) or math center time (typically 20 min) or the school's intervention period (typically 45 min), such that the amount of math instructional time was similar for AR intervention and control students. The intervention occurred in small groups (3:1). Tutors were full-time or part-time graduate-student employees of the research grant. Some were licensed teachers; most were not. Each was responsible for two to four groups. Tutors were trained in a 2-day workshop, with biweekly 1-hr meetings providing additional updates on upcoming tutoring topics and problem solving concerning difficulty students. The fraction intervention program, *Fraction Challenge* (Fuchs & Schumacher, 2010), was organized in a manual that included all materials and scripts. Scripts provided a model of the lessons and key explanatory language. Tutors reviewed scripts prior to delivering lessons; however, to promote teaching authenticity and responsiveness to student difficulty, tutors did not memorize or read scripts. Tutoring included 36 lessons taught over a 12-week period. Each lesson was 30 min.

Content. Our major focus was measurement interpretation of fractions, with content focused primarily on representing, comparing, ordering, and placing fractions on a 0 to 1 number line. This focus was supplemented by attention to part-whole interpretation (e.g., showing objects with shaded regions) and fair shares representations to build on classroom instruction. So number lines, fraction tiles, and fraction circles were used throughout the 36 lessons, with decreasing emphasis on part-whole interpretation as the lessons progressed. In Lesson 22, fraction computation was introduced. Throughout the program, we focused on proper fractions and fractions equal to one. Improper fractions greater than 1 were introduced with addition and subtraction of fractions in Lesson 22. To reduce computational demands, denominators did not exceed 12 and excluded seven, nine, and eleven.

See Table 1 for more specific information on the sequencing of content. During the first 2 weeks (Lessons 1–6), the focus was understanding of fraction magnitude. We began by addressing "what is a fraction" and taught relevant vocabulary (e.g., *numerator*, *denominator*, *unit*). We relied on a combination of part/whole relations, measurement, and equal sharing to explain the fraction magnitudes. Instruction emphasized the role of the numerator and denominator and how they work together to constitute the fraction, which is one number, even though it comprises two whole numerals.

In the third week (Lessons 7–9), tutors reviewed material presented in the first six lessons. Students practiced naming fractions, reading fractions, and comparing two fractions when the denominators are the same or when the numerators are the same. In this review, two types of flashcards were used to build fluency with the meaning of fractions. The first type showed flashcards with one fraction; students read and stated the meaning of the fraction. For example, for 1/4, students said, "one-fourth, one of four equal parts" (to clarify what 1/4 means). Students took turns over a 2-min period, responding to as many fractions as they could. The tutor kept track of the group total for each lesson; the students' goal was to meet or beat the previous day's score. The second type showed two fractions. Students determined if the fractions pairs fit one of three categories: same numerators (different denominators), same denominators (different numerators), or different numerator and different denominator. Students categorized flashcards in this way for 1 min. Then, the tutor gave each student two fraction cards; for each, students placed the greater than or less than sign between fractions and explained their rationale to the group.

In weeks four and five (Lessons 10–15), students learned about fractions equivalent to 1/2 (2/4, 3/6, 4/8, 5/10, 6/12). They also learned chunking and segmenting strategies for comparing two

Table 1
Fraction Intervention Skill and Sequence

Week	Conceptual focus	Skill(s) addressed
Week 1	Part–whole Equal sharing	Identifying and naming fractions Unit fractions Comparing fractions
Week 2	Measurement Part–whole Equal sharing	Role of numerator and denominator Placing fractions on the number line
Week 3	Measurement Part–whole	Fluency building/cumulative review
Weeks 4–5	Measurement	Comparing 2 fractions with $<$, $>$, or $=$ Ordering 3 fractions least to greatest Place 2 fractions on 0–1 number line (1/2 marked) Fractions equivalent to 1/2
Week 6	Part–whole Measurement	Fraction as collection of items
Week 7	Measurement	Remove 1/2 from 0–1 number line
Weeks 8–9	Measurement	Fraction addition and subtraction
Weeks 10–12		Cumulative review

Note. In the Conceptual focus column, if multiple foci are listed, the greatest emphasis is listed first.

fractions in which the numerators and denominators both differed, using 1/2 as a benchmark for comparison and writing the greater than, less than, or equal sign between the fractions. Then, two activities were introduced: placing two fractions on the 0 to 1 number line, marked with 1/2, and ordering three fractions from smallest to largest.

In week six (Lessons 16–18), tutors introduced fractions representing a collection of items and fractions equivalent to 1, while continuing to work on comparing two fractions, ordering three fractions, and placing fractions on the 0 to 1 number line, now without the 1/2 marker. Students were encouraged, however, to think about where 1/2 goes on the number line in relation to placing other fractions. Week seven (Lessons 19–21) was cumulative review on all concepts and skills.

Weeks eight and nine (Lessons 22–27) focused on simple calculations. Addition with like denominators was introduced in Lesson 22; subtraction with like denominators in Lesson 23; and mixed addition and subtraction in Lesson 24. In Lessons 25 and 26, addition with unlike denominators and then subtraction with unlike denominators were introduced. Lesson 27 reviewed addition and subtraction with like and unlike denominators. Concepts and procedures were addressed. When introducing unlike denominators, tutors limited the pool of problems. In all cases, one fraction was equivalent to 1/2 or 1 so students could write equivalent fractions they had already learned. Weeks 10 through 12 were cumulative review.

Activities. Each lesson comprised four activities: introduction of concepts or skills, group work, the speed game, and individual work. The first activity, introduction of new concepts and skills, lasted 8–12 min. Concrete manipulatives (e.g., fraction tiles, fraction circles), visual representations, and problem-solving strategies were presented. In group work, which lasted 8–12 min (the introduction plus group work lasted 20 min for each lesson), students rehearsed and applied concepts and practiced strategies addressed in the introduction. Students took turns leading the group through problems, while all students showed work for each problem. The third activity, the speed game, was designed to build fluency on one previously taught concept or skill. For instance, to build fluency on fractions equivalent to 1/2, tutors gave each student a paper showing 25 fractions, and students had 1 min to circle fractions equivalent to 1/2. Sometimes, the Speed Game required computation, and students were given specific instructions on which items to solve. For example, students might be told to solve only addition problems or solve only problems with like denominators. In this way, students were required to discriminate between problem types. The fourth activity was individual work for which students independently completed a two-sided practice sheet. One side presented problems taught in the day's lesson; the other side was cumulative review. This activity lasted approximately 8 min, for a total of 30 min per session.

Promoting task-oriented behavior. Because AR students often display attention, motivation, and self-regulation difficulties that may affect learning (e.g., Montague, 2007), we encouraged students to regulate their attention/behavior and to work hard.[3] Tutors taught students that *on-task behavior* means listening carefully, working hard, and following directions and that on-task behavior is important for learning. Tutors set a timer to beep at three unpredictable times during each lesson. If all students were on task when the timer beeped, all students received a checkmark. To increase the likelihood of consistent on-task behavior, students could not anticipate time intervals. Also, on each practice sheet, 2 of 16 problems were bonus problems. As the tutor scored the practice sheet, he or she revealed which problems were bonus items. Students received a checkmark for each correctly answered bonus problem. At the end of the lesson, tutors tallied checkmarks for each student and awarded them with a "half dollar" per check-

[3] To assess the possibility that intervention altered children's attentive behavior in general and thus children learned more fraction information from their classroom teachers, we compared classroom teachers' ratings of attentive behavior on the SWAN (see measures), which were collected approximately 6 weeks after intervention began. Mean ratings were 35.48 (SD = 11.64) for the intervention group and 34.90 (SD = 10.85) for the control group, with no significant difference between conditions, $F(1, 257) = 0.17$, $p = .678$.

mark. At the end of each week, students were allowed to shop at the "fractions store" to spend money earned during tutoring. All items in the store were listed in whole dollar amounts at three price points. Therefore, students needed to exchange half dollars for whole dollars and determine what they could afford. In this way, to use the fraction store, students needed to use their fraction knowledge, while exercising judgment about buying a less expensive item versus saving for a more expensive one. In Lesson 19, we replaced half dollars with quarter dollars.

Fidelity of Intervention

Every intervention session was audiotaped. We randomly sampled 20% of recordings (293 recordings) such that tutor, student, and lesson were sampled comparably. A research assistant listened to each sampled tape, while completing a checklist to identify the essential points the tutor conducted. The mean percentage of points addressed was 97.69 (SD = 3.39). Two research assistants independently listened to 20% (n = 58) of the 293 recordings to assess concordance. The mean difference in score was 1.74% (SD = 2.81).

Screening Measures

The math screening measure was *WRAT-4-Arithmetic* (Wilkinson, 2008), with which students complete calculation problems of increasing difficulty. In the fourth-grade range, WRAT almost entirely samples whole-number items. Alpha on this sample was .85. The IQ screening measure was the 2-subtest Wechsler Abbreviated Scales of Intelligence (WASI; Wechsler, 1999). *Vocabulary* assesses expressive vocabulary, verbal knowledge, memory, learning ability, and crystallized and general intelligence with 37 items; subjects identify pictures and define words. *Matrix Reasoning* measures nonverbal fluid reasoning and general intelligence with 32 items; subjects select 1 of 5 options that best completes a visual pattern. Internal consistency reliability exceeds .92.

Moderator Effect Measures

Whole-number calculation skill. We relied on WRAT to examine the role of whole-number calculation skill as a moderator of intervention effects.

Working memory. To assess the central executive component of working memory, we used the *Working Memory Test Battery for Children* (WMTB-C; Pickering & Gathercole, 2001)–*Listening Recall* and *Counting Recall*. Each subtest includes six dual-task items at span levels from 1–6 to 1–9. Passing four items at a level moves the child to the next level. At each span level, the number of items to be remembered increases by one. Failing three items terminates the subtest. Subtest order is designed to avoid overtaxing any component area and is generally arranged from easiest to hardest. We used the trials correct score. Test-retest reliability ranges from .84–.93. For *Listening Recall*, the child determines if a sentence is true; then recalls the last word in a series of sentences. For *Counting Recall*, the child counts a set of 4, 5, 6, or 7 dots on a card and then recalls the number of counted dots at the end of a series.

Attentive behavior. The Strength and Weaknesses of ADHD-Symptoms and Normal-Behavior (SWAN; J. M. Swanson, 2013) samples items from the *Diagnostic and Statistical Manual of Mental Disorders* (4th ed.) criteria for attention deficit hyperactivity disorder (ADHD) for inattention (9 items) and hyperactivity-impulsivity (9 items), but scores are normally distributed. Teachers rate items on a 1–7 scale. We report data for the inattentive subscale, as the average rating across the nine items. The SWAN correlates well with other dimensional assessments of behavior related to attention (www.adhd.net). Alpha for the inattentive subscale on the present sample was .96.

Processing speed. Woodcock-Johnson III (WJ-III; Woodcock, McGrew, & Mather, 2001) *Cross Out* measures processing speed by asking students to locate and circle five pictures that match a target picture in that row; students have 3 min to complete 30 rows. Internal consistency reliability is .91.

Listening comprehension. *Woodcock Diagnostic Reading Battery* (*WDRB)-Listening Comprehension* (Woodcock, 1997) measures the ability to understand sentences or passages that the tester reads. Students supply the word missing at the end of sentences or passages that progress from simple verbal analogies and associations to discerning implications. Internal consistency reliability is .80.

Outcome Measures

From the 2010 Fraction Battery (Schumacher, Namkung, & Fuchs, 2010), *Comparing Fractions* assesses magnitude understanding with 15 items, each of which shows two fractions. Students write the greater than, less than, or equal sign between the two fractions. The score is the number of correct answers. Items are as follows: 2/6 and 4/6; 1/2 and 7/10; 4/12 and 1/2; 3/6 and 3/8; 8/12 and 1/2; 1/12 and 1/5; 1/2 and 5/10; 4/6 and 1/2; 9/10 and 5/10; 1/2 and 7/8; 1/2 and 3/4; 3/6 and 1/2; 7/8 and 7/12; 1/4 and 3/4; and 1/2 and 4/8. The maximum score is 15. Alpha on this sample was .84.

Fraction Number Line (Siegler et al., 2011) assesses the measurement interpretation of fractions by requiring students to place fractions on a number line. For each trial, a number line with endpoints of 0 and 1 is presented, and a target fraction is shown in a large font below the line. Students practice with a target fraction and then proceed to 10 test items: 1/4, 3/8, 12/13, 2/3, 1/19, 7/9, 4/7, 5/6, 1/2, and 1/7. Items are presented in random order. Accuracy is defined as the absolute difference between the child's placement and the correct position of the number. When multiplied by 100, the scores are equivalent to the percentage of absolute error (PAE), as reported in the literature. Low scores indicate stronger performance. Test-retest reliability, on a sample of 57 students across 2 weeks, was .79.

We also administered 18 released fraction items from 1990–2009 *NAEP* mathematics released items: the pool of items classified by NAEP as easy, medium, or hard from the fourth-grade assessment or as easy from the eighth-grade assessment (U.S. Department of Education, 2013). Testers read each problem aloud (with up to one rereading upon student request). Eight items assess part-whole interpretation; eight assess measurement interpretation; one requires subtraction with like denominators; and one asks how many fourths make a whole. Students select an answer from four choices (11 problems); write an answer (3 problems); shade a portion of a fraction (1 problem); mark a number line (1 problem); write a short explanation (1 problem); or write numbers, shade fractions, and explain the answer (1 problem with multiple parts). The maximum score is 22. Alpha on the sample was .72. We also examined performance on the part-whole interpretation items (alpha = .60) and on the measurement interpretation items (alpha = .62). Two authors independently coded the 16 relevant items into part-whole versus measurement, with 87% agreement; the two disagreements were resolved through discussion. The correlation between part-whole (NAEP-PW) and measurement (NAEP-Meas) items was .37.

From the *2010 Fraction Battery* (Schumacher et al., 2010), *Fraction Addition* includes four addition problems with like denominators and six addition problems with unlike denominators. Five are presented vertically; five are presented horizontally. *Fraction Subtraction* includes five subtraction problems with like denominators and five with unlike denominators; half are presented vertically, and half are presented horizontally. For both subtests, testers terminate administration when all but two students have completed the test. Scoring does not penalize students for not reducing answers. We used the total score across these tests (which correlated .74), with a maximum score of 20. Alpha on this sample was .90.

Procedure

Note that, as per the study design and the Institutional Review Board protocol, we did not administer individual assessments or collect demographic data on low-risk classmates. The study occurred in five steps. In August and September, for *screening*, testers administered the WRAT in large groups and then administered the WASI individually to students who met the WRAT criterion for AR status. In September and October, to assess *pretreatment comparability among study groups on fraction skill*, testers administered Comparing Fractions, NAEP, and Fraction Addition and Subtraction in three large-group sessions and administered Fraction Number Line in an individual session. To assess *pretest cognitive characteristics*, testers administered the following measures in two individual sessions (the second session included Fraction Number Line): WMTB-C Listening Recall, WMTB-C Counting Recall, WJ-III Cross Out, and WDRB Listening Comprehension. SWAN teacher ratings were collected approximately 6 weeks into intervention. *Intervention* occurred for 12 weeks, 3 times per week for 30 min per session in late October to late March (intervention was interrupted on 10 days due to snow and preparation for/administration of statewide testing). In early April (within 2 weeks of intervention ending), testers re-administered Comparing Fractions, NAEP, and Fraction Addition and Subtraction in three large-group sessions and re-administered Fraction Number Line in one individual session to assess *intervention effects*.

All individual testing sessions were audiotaped; 20% of tapes were randomly selected, stratifying by tester, for accuracy checks by an independent scorer. Agreement on test administration and scoring exceeded 98%. Testers were blind to study condition when administering and scoring tests.

Results

See Table 2 for pretest, posttest, and adjusted posttest means on the fraction outcomes for AR intervention, AR control, and low-risk classmates. Table 2 also shows, in the last four columns, the magnitude of the pretest and posttest achievement gaps (with respect to low-risk classmates) for AR intervention students and for AR control students. These are expressed as effect sizes (raw score difference in means, divided by the pooled *SD*). See Table 3 for descriptive information and correlations among the potential moderators and the pre- and posttest fraction scores for AR students.

Preliminary Analyses

We conducted three types of preliminary analyses. First, we assessed fraction pretest comparability as a function of study condition and the risk severity stratification variable. On all measures, pretest performance of the intervention and control AR groups was comparable, and there were no significant effects on any measure for the interaction between treatment condition and risk severity. Second, because we relied on a residualized change approach to analyze intervention effects (i.e., covarying pretest scores to reduce within-group error variance), we assessed the homogeneity of regression assumption, which was met for NAEP, $F(3, 255) = 0.55$, $p = .461$, and NAEP-Meas, $F(3, 255) = 0.21$, $p = .646$, but not met for Comparing Fractions, $F(3, 255) = 16.31$, $p < .001$, Fraction Number Line, $F(3, 255) = 11.10$, $p = .001$, NAEP-PW, $F(3, 686) = 12.21$, $p = .001$, and Fraction Calculations, $F(3, 255) = 4.08$, $p = .044$. We therefore controlled for the interaction between intervention condition and the pretest score on the relevant fraction outcomes by including it in models involving that measure. Third, we estimated the proportion of variance in the fraction outcome measures due to classrooms (using SPSS MIXED, Version 20). These intraclass correlations (ICCs) were negligible to small (see Table 2). We then used SPSS MIXED (Version 20) to run multilevel regression models examining intervention effects on the fraction outcomes (the pretest score on the relevant fraction outcome was the covariate; students were the level-1 unit; level-2 variances quantified the between-classroom variability in their cluster means). Accounting for dependency in this way did not alter any inferences that were based on single-level models; thus we report the more straightforward single-level analyses. (Multilevel results are available from the first author.)

Does Intervention Enhance Conceptual and Procedural Knowledge and Does Whole-Number Calculation Skill Moderate Those Effects?

Following Preacher and Hayes (2008; Hayes, 2012), we used an ordinary least squares path analytical framework to estimate the effects of intervention on conceptual and procedural knowledge and to examine whether pretest whole-number calculation skill moderated those effects. In these models (see Table 4), we controlled for pretest performance on the relevant fraction outcome (see lines labeled “Pretest Covariate”) and the pretest score on whole-number calculations. For comparing fractions, fraction number line, NAEP-PW, and fraction calculations, we also controlled for the interaction between intervention condition and pretest scores on the relevant fraction outcome (see last two lines of Table 4). The interaction between whole-number calculation skill and intervention (see lines labeled “Interaction”) was not significant on any fraction outcome, indicating that whole-number cal-

Table 2
Means and Standard Deviations for At-Risk and Low-Risk Students and Pre- and Postachievement Gaps for At-Risk Intervention and Control Students

		At-risk							
		Intervention (n = 129)		Control (n = 130)		Low-risk (n = 282)			
Variable	ICC	*M*	*SD/SE*[a]	*M*	*SD/SE*[a]	*M*	*SD*	Low-risk vs. intervention achievement gap (ES)	Low-risk vs. control achievement gap (ES)
Comparing fractions	~0								
Pre		6.82	2.04	7.01	2.15	7.16	3.29	−0.12	−0.05
Post		12.72	3.37	7.07	2.84	8.64	4.17	1.04	−0.42
Adjusted post[b]		12.73	0.27	7.06	0.27				
Fraction number line	~0								
Pre		0.35	0.09	0.36	0.09				
Post		0.20	0.09	0.32	0.12				
Adjusted post		0.20	0.01	0.32	0.01				
NAEP-total	.063								
Pre		8.37	3.28	8.32	3.20	12.09	3.57	−1.07	−1.09
Post		14.41	3.11	11.35	3.43	14.69	3.50	−0.08	−0.96
Adjusted post		14.39	0.25	11.36	0.25				
NAEP-PW	.032								
Pre		4.29	1.81	4.28	1.89	5.74	1.52	−0.90	−0.89
Post		5.79	1.12	5.36	1.71	6.25	1.24	−0.38	−0.64
Adjusted post		5.79	0.12	5.36	0.12				
NAEP-meas	~0								
Pre		3.26	1.75	3.20	1.64	5.09	2.37	−0.84	−0.88
Post		7.02	2.21	4.66	2.08	6.74	2.55	0.11	−0.87
Adjusted post		7.00	0.17	4.68	0.17				
Fraction calculations	~0[c]								
Pre		2.81	3.61	2.78	3.61	5.32	4.44	−0.60	−0.61
Post		17.60	3.76	7.50	4.30	10.18	4.84	1.65	−0.57
Adjusted post		17.60	0.35	7.51	0.35				

Note. Comparing fractions is from 2010 Fraction Battery (Schumacher et al., 2010). Fraction number line is Siegler et al. (2011; when multiplied by 100, equivalent to PAE). NAEP is the National Assessment of Educational Progress (18 easy, medium, and hard fourth-grade and easy eighth-grade released fraction items). Fraction calculations is Fraction Addition and Fraction Subtraction from 2010 Fraction Battery (Schumacher et al., 2010). PW = part–whole items; meas = measurement items; PAE = percentage of absolute error; ICC = intraclass correlation; ES = effect size.
[a] For pre and post, *SD*; for adjusted post, *SE*. [b] Adjusted post is posttest score adjusted for pretest score. [c] When classroom was considered as a nesting variable for Fraction calculations, the unconditional model was not positive definite, a result that can occur when the nesting effect is small.

culation skill did not moderate intervention effects. That is, the effects of intervention were similar regardless of students' pretest whole-number calculation skill. The main effect of intervention (see lines labeled "Intervention") was significant for each fraction outcome, with effect sizes (difference in adjusted posttest means, divided by the pooled posttest *SD*) of 1.82 for Comparing Fractions, 1.09 for Fraction Number line, 0.92 for NAEP-Total, 0.29 for NAEP-PW, 1.07 for NAEP-Meas, and 2.50 for Fraction Calculations. As shown in Table 2 (last four columns), on each measure from pre- and posttest, the achievement gap narrowed for AR intervention students, whereas the gap remained similar or increased for AR control students.

Do Domain-General Cognitive Resources Moderate Intervention Effects?

We ran parallel analyses, again following Preacher and Hayes (2008; Hayes, 2012), using an ordinary least squares path analytical framework to examine whether domain-general cognitive resources moderated the effects of intervention. See Table 5. In these models, we controlled for pretest performance on the relevant fraction outcome (see lines labeled "Pretest Covariate") and the pretest score on the moderator were controlled in the model (see lines labeled "Interacting Moderator"). For Comparing Fractions, Fraction Number Line, NAEP-PW, and Fraction Calculations, we also controlled for the interaction between intervention condition and pretest scores on the relevant fraction outcome (see last two lines of Table 5). We tested one moderator at a time for each fraction outcome. On Fraction Number Line, listening recall (working memory) was a significant moderator; on NAEP-Total, attentive behavior was a significant moderator; on Comparing Fractions, counting recall (working memory) and attentive behavior were significant moderators; and on Fraction Calculations, listening comprehension and processing speed were significant moderators. When more than one moderator was significant for given fraction outcome, we re-examined each moderator while controlling for performance on the other significant moderator variable (see lines labeled "Other Cognitive Moderator").

On *Comparing Fractions*, for the model that included counting recall, intervention, the interaction between counting recall and intervention (as well as the pretest comparing fractions covariate, the attentive behavior covariate, and the interaction between condition and pretest scores on Comparing Fractions), $R^2 = .52$, $F(6, 252) = 44.89$, $p < .001$, with R^2change of .03 due to the interaction between counting recall and intervention condition, $F(1, 252) = 13.54$, $p < .001$. This moderator effect is graphed in the

Table 3
Means and Correlations

Variables	Raw score		Standard score[a]		Correlation											
	M	*SD*	*M*	*SD*	LC	LR	CR	AB	W	C1	C2	NL1	NL2	N1	N2	CA1
Listening comprehension (LC)	21.31	4.24	92.22	17.23												
Working memory																
Listening recall (LR)	10.66	3.33	92.25	19.53	.25											
Counting recall (CR)	17.61	4.66	80.88	15.85	.11	.31										
Attentive behavior (AB)	35.19	11.24			.04	.11	.15									
Processing speed (PS)	93.69	11.81	93.69	11.81	.03	.15	.17	.23								
Whole number calculations (W)	24.34	2.09	84.56	7.95	.14	.20	.30	.30								
Fractions																
Compare pre (C1)	7.04	2.79			.03	.03	−.11	−.10	.19							
Compare post (C2)	9.27	4.16			.13	.21	.06	.07	.05	.31						
Number line pre (NL1)	0.35	0.09			−.11	−.02	.04	−.04	−.01	−.13	−.20					
Number line post (NL2)	0.26	0.13			−.18	−.14	−.10	−.12	−.16	−.06	−.50	.22				
NAEP pre (N1)	10.31	3.90			.30	.30	.17	.32	.62	.25	.22	−.15	−.26			
NAEP post (N2)	13.80	3.67			.29	.29	.22	.28	.44	.24	.50	−.18	−.53	.62		
Calculations pre (CA1)	4.10	4.25			.24	.13	.11	.17	.36	.17	.11	−.08	−.20	.44	.31	
Calculations post (CA2)	11.36	5.81			.11	.12	.13	.13	−.01	.21	.54	−.08	−.46	.17	.45	.15

Note. $N = 259$. Listening comprehension is Woodcock Diagnostic Reading Battery–Listening Comprehension (Woodcock, 1997). Working memory is Working Memory Test Battery for Children (Pickering & Gathercole, 2001)—Listening Recall and Counting Recall. Attentive behavior is the Strength and Weaknesses of ADHD—Symptoms and Normal-Behavior (J. M. Swanson, 2013). Processing speed is Woodcock-Johnson III (Woodcock et al., 2001)–Cross Out. Whole number calculations is Wide Range Achievement Test—4 (Wilkinson, 2004). Comparing fractions is from the 2010 Fraction Battery (Schumacher et al., 2010). Fraction number line is Siegler et al. (2011; when multiplied by 100, equivalent to PAE). NAEP is the National Assessment of Educational Progress (18 easy, medium, and hard fourth-grade and easy eighth-grade released fraction items). Fraction calculations is Fraction Addition and Fraction Subtraction from the 2010 Fraction Battery (Schumacher et al., 2010). ADHD = attention-deficit/hyperactivity disorder.

upper left panel of Figure 1, where the effect of intervention decreases as counting recall (a measure of working memory) decreases. The significant intervention effect transitioned to nonsignificance at 2.43 *SD*s below the AR sample mean on counting recall. For the model that included attentive behavior, intervention, the interaction between attentive behavior and intervention (as well as the pretest comparing fractions covariate, the counting recall behavior covariate, and the interaction between condition and pretest scores on Comparing Fractions), $R^2 = .50$, $F(6, 252) = 41.91$, $p < .001$, with R^2change of .01 due to the interaction between counting recall and intervention condition, $F(1, 252) = 4.45$, $p = .036$. This moderator effect is graphed in the middle left panel of Figure 1, where the intervention effect decreases as attentive behavior decreases. There were no regions within the

Table 4
Intervention and Whole-Number Calculation Moderator Effects

Effect	Compare fractions	Fraction number line	NAEP total	NAEP PW	NAEP meas	Fraction calculations
Model summary						
R^2	.50	.34	.42	.19	.44	.65
F(4, 254) (*p*)	49.96 (<.001)	25.82 (<.001)	46.85 (<.001)	11.73 (<.001)	49.45 (<.001)	92.06 (<.001)
Constant *B* (*SE*)	0.01 (0.04)	0.00 (0.05)	0.00 (0.05)	0.00 (0.06)	0.00 (0.05)	0.00 (0.04)
t (*p*)	0.23 (.822)	0.07 (.942)	0.01 (.990)	0.07 (.947)	0.06 (.949)	0.02 (.987)
Whole-number						
Calculations *B* (*SE*)	0.05 (0.04)	−0.14 (0.05)	0.13 (0.05)	0.09 (0.06)	0.17 (0.05)	0.07 (0.04)
t (*p*)	1.17 (.244)	−2.82 (.005)	2.72 (.007)	1.62 (.107)	3.49 (<.001)	1.91 (.057)
Intervention *B* (*SE*)	1.34 (0.09)	−0.94 (0.10)	0.84 (0.10)	0.29 (0.11)	0.94 (0.09)	1.56 (0.07)
t (*p*)	15.10 (<.001)	−9.51 (<.001)	8.79 (<.001)	2.56 (.011)	10.06 (<.001)	20.84 (<.001)
Interaction *B* (*SE*)	0.14 (0.09)	0.01 (0.10)	0.10 (0.10)	0.17 (0.12)	0.08 (0.09)	0.06 (0.08)
t (*p*)	1.61 (.110)	0.05 (.959)	1.07 (.284)	1.43 (.153)	0.84 (.404)	0.77 (.443)
Pretest covariate *B* (*SE*)	0.22 (0.06)	0.34 (0.07)	0.43 (0.05)	0.52 (0.08)	0.38 (0.05)	0.22 (0.05)
t (*p*)	3.66 (<.001)	4.93 (<.001)	8.72 (<.001)	6.54 (<.001)	7.83 (<.001)	4.23 (<.001)
Control for Intervention *B* (*SE*)	−0.35 (.09)	−0.32 (0.10)		−0.45 (0.12)		−0.17 (0.08)
× Pre Interaction *t* (*p*)	−3.87 (<.001)	−3.22 (.001)		−3.82 (<.001)		−2.28 (.024)

Note. For Compare fractions, Fraction number line, NAEP-PW, and Fraction calculations (for which the interaction between intervention and pretest score on the fraction outcome was controlled), *df*s = 5,253. For NAEP total and NAEP meas (which met the homogeneity of regression assumption), *df*s = 4,254. Compare fractions is from the 2010 Fraction Battery (Schumacher et al., 2010). Fraction number line is Siegler et al. (2011). NAEP is the National Assessment of Educational Progress (18 easy, medium, and hard fourth-grade and easy eighth-grade released fraction items). Fraction calculations is Fraction Addition and Fraction Subtraction from the 2010 Fraction Battery (Schumacher et al., 2010). PW is part–whole, and Meas is measurement.

Table 5
Cognitive Moderators of Intervention Effects

	Outcome					
	Compare fractions		Number line	NAEP total	Calculations	
Effect	Listen recall	Attn beh	Count recall	Attn beh	Listen comp	Proc speed
Model summary						
R^2	.52	.50	.34	.43	.67	.67
F^a (p)	44.89 (<.001)	41.91 (<.001)	25.77 (<.001)	47.81 (<.001)	85.65 (<.001)	84.96 (<.001)
Constant *B* (SE)	0.01 (0.04)	0.01 (0.04)	−0.01 (0.05)	0.00 (0.05)	0.00 (0.04)	0.00 (0.04)
t (*p*)	0.32 (.748)	0.25 (.803)	−0.28 (.780)	0.03 (.979)	0.11 (.912)	0.03 (.976)
Interacting moderator						
B (*SE*)	0.03 (0.04)	0.03 (0.05)	−0.09 (0.05)	0.12 (0.05)	0.05 (0.04)	0.16 (0.04)
t (*p*)	0.77 (.440)	0.66 (.511)	−1.69 (.092)	2.41 (.017)	1.30 (.194)	4.45 (<.001)
Intervention *B* (SE)	1.34 (0.09)	1.34 (0.09)	−0.93 (0.10)	0.83 (0.09)	1.57 (0.07)	1.57 (0.07)
t (*p*)	15.36 (<.001)	15.10 (<.001)	−9.38 (<.001)	8.81 (<.001)	21.68 (<.001)	21.63 (<.001)
Interaction *B* (SE)	0.33 (0.09)	0.19 (0.09)	0.24 (0.10)	0.20 (0.10)	−0.16 (0.07)	−0.15 (0.07)
t (*p*)	3.68 (<.001)	2.11 (.036)	2.33 (.021)	2.08 (.038)	−2.16 (.031)	−2.17 (.032)
Pretest covariate *B* (SE)	0.22 (0.06)	0.22 (0.06)	0.34 (0.07)	0.44 (0.05)	0.16 (0.05)	0.17 (0.05)
t (p)	3.69 (<.001)	3.69 (<.001)	4.77 (<.001)	8.71 (<.001)	2.92 (.004)	3.14 (.002)
Other cognitive *B* (SE)	0.03 (0.04)	0.02 (0.05)			0.16 (0.04)	0.04 (0.04)
Moderator *t* (*p*)	0.56 (.578)	0.42 (.677)			4.39 (<.001)	1.16 (.248)
Control for Intervention	−0.31 (0.09)	−0.33 (0.09)	−0.31 (0.10)		−0.10 (0.07)	−0.12 (0.07)
× Pre Interaction	−3.51 (<.001)	−3.68 (<.001)	−3.18 (.002)		−1.32 (.188)	−1.59 (.114)

Note. Listening Comprehension is Woodcock Diagnostic Reading Battery–Listening Comprehension (Listen comp; Woodcock, 1997). Working memory is Working Memory Test Battery for Children (Pickering & Gathercole, 2001)–Listening Recall and Counting Recall. Attentive behavior (Attn beh) is the Strength and Weaknesses of ADHD–Symptoms and Normal-Behavior (J. M. Swanson, 2013). Processing speed (Proc speed) is Woodcock-Johnson III (Woodcock et al., 2001)–Cross Out. Compare fractions is from the 2010 Fraction Battery (Schumacher et al., 2010). Number line is Siegler et al. (2011). NAEP is the National Assessment of Educational Progress (18 easy, medium, and hard fourth-grade and easy eighth-grade released fraction items). Calculations is Fraction Addition and Fraction Subtraction from the 2010 Fraction Battery (Schumacher et al., 2010). ADHD = attention-deficit/hyperactivity disorder.
[a] For Compare fractions and Calculations, *df*s = 6, 252; for Number line, *df*s = 5, 253; and for NAEP total, *df*s = 4, 254.

observed values of attentive behavior, at which the intervention effect fell below significance.

On *Fraction Number Line*, for the model that included listening recall, intervention, the interaction between listening recall and intervention (as well as the pretest fraction number line covariate and the interaction between condition and pretest scores on Fraction Number Line), $R^2 = .34$, $F(5, 253) = 25.77$, $p < .001$, with R^2change of .01 due to the interaction between listening recall and intervention condition, $F(1, 253) = 5.43$, $p = .021$. This moderator effect is graphed in the lower left panel of Figure 1, where the intervention effect decreases as counting recall increases. (Note that on Fraction Number Line, low scores reflect stronger performance.) The significant intervention effect transitioned to nonsignificance at 1.98 *SD*s above the AR sample mean on listening recall.

On *NAEP Total*, for the model that included attentive behavior, intervention, and the interaction between attentive behavior and intervention (as well as the pretest NAEP Total covariate), $R^2 = .42$, $F(4, 254) = 28.62$, $p < .001$, with R^2change of .01 due to the interaction between attentive behavior and intervention condition, $F(1, 254) = 4.33$, $p = .038$. This moderator effect is graphed in the lower right panel of Figure 1, where the intervention effect decreases as attentive behavior decreases. The significant intervention effect transitioned to nonsignificance at 2.05 *SD*s below the AR sample mean on inattentive behavior.

On *Fraction Calculations*, for the model that included listening comprehension, intervention, and the interaction between listening comprehension and intervention (as well as the pretest Fractions Calculations covariate, the pretest processing speed covariate, and the interaction between condition and pretest scores on Fraction Calculations), $R^2 = .67$, $F(6, 252) = 85.65$, $p < .001$, with R^2change of .01 due to the interaction between listening comprehension and intervention condition, $F(1, 252) = 4.69$, $p = .031$. This moderator effect is graphed in the upper right panel of Figure 1, where the intervention effect decreases as listening comprehension increases. There were no regions within the observed values of listening comprehension, at which the intervention effect fell below significance. For the model that included processing speed, intervention, and the interaction between processing speed and intervention (as well as the pretest fractions calculations covariate, the pretest listening comprehension covariate, and the interaction between condition and pretest scores on Fraction Calculations), $R^2 = .67$, $F(6, 252) = 84.96$, $p < .001$, with R^2change of .01 due to the interaction between processing speed and intervention condition, $F(1, 252) = 4.71$, $p = .032$. This moderator effect is graphed in the middle right panel of Figure 1, where the intervention effect increases as processing speed decreases. There were no regions within the observed values of processing speed, at which the intervention effect fell below significance.

Does Improvement in Measurement Interpretation Mediate Intervention Effects?

Again following Preacher and Hayes (2008), we used a path analytical framework for estimating direct and indirect effects to test the hypothesis that children's improvement in measurement interpre-

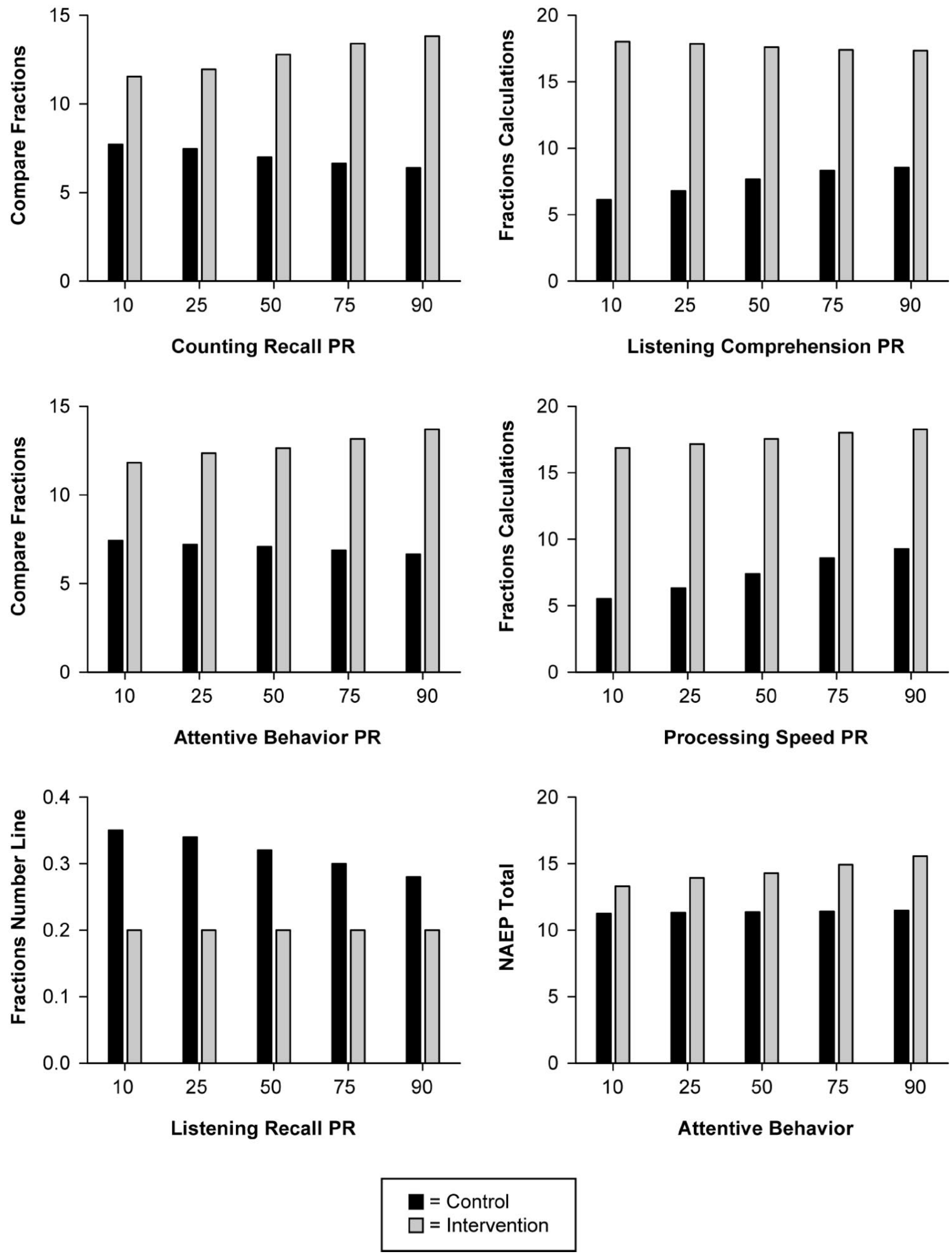

Figure 1. Intervention effect (black bar is control; gray bar is intervention) as a function of percentile rank (PR) of students' incoming cognitive characteristics. NAEP = National Assessment of Educational Progress.

tation mediates the effects of intervention. Each mediation analysis occurs in two steps. The first step examines the effects of intervention, controlling for the covariate(s), on the mediator. The second step examines the effects of intervention on the posttest of interest while controlling for the same covariate(s), with the mediator now added as a predictor in the model. For the indirect (mediation) effect, we used bootstrapping estimation with 5,000 draws to estimate standard errors and 95% confidence intervals; confidence intervals that do not cover zero are statistically significant. We used standard scores and conducted three complementary analyses (in such analyses, note that causal conclusions depend on proper specification of the model).

The first mediation analysis tested whether improvement in fraction number line performance mediated the effects of intervention on the NAEP total score outcome. We focused on the NAEP total score outcome because (a) NAEP is the most multi-faceted, most widely accepted, and most highly valued of our fraction knowledge outcomes; (b) NAEP was the fraction outcome measure least aligned with intervention; and (c) the NAEP items represent the two dominant interpretations of fractions (i.e., measurement and part-whole interpretations) in comparable emphasis. To index the mediator variable (improvement in the measurement interpretation of fractions), we relied on the number line task because (a) it is a widely used and well accepted measure of this construct (e.g., Siegler et al., 2011) and (b) only one of the eight measurement items on NAEP relies on a number line task (results were similar with and without this item included). In this model, we controlled for pretest NAEP scores and for improvement in procedural fraction skill, thereby creating a stringent test of the hypothesis.

Figure 2 shows the major components of this model in the top panel; path coefficients and standard errors (SE) are along the arrows. R^2 for the second-step model, assessing the direct effects of the intervention and the indirect effect of fraction number line improvement on the posttest NAEP score, while controlling for the pretest NAEP score and improvement on fraction calculations, was .46, $F(4, 254) = 52.05$, $p < .001$. Coefficients for the two covariates (not shown in the figure) were .47 (.05) for pretest NAEP and .20 (.07) for improvement in procedural fraction skill. The mediation model partitioned the total intervention effect of 0.59 into direct and indirect effects. The coefficient for the direct effect of 0.42 was significant, $t = 2.83$, $p = .005$ (a 27.8% reduction in the total effect). The coefficient for the indirect effect of 0.18 was also significant (CI = .07 to .25). Thus, improvement in measurement understanding partially mediated the intervention effect on the NAEP outcome.

We could not use an analogous method to assess the mediating role of improvement in part-whole interpretation on the NAEP total score outcome because our only index of the mediator (improvement in part-whole understanding) was based on a subset of NAEP items (i.e., we did not have a measure of part-whole interpretation that was independent from NAEP, as we did for measurement interpretation in the fraction number line task). We therefore conducted two additional, complementary analyses. The first assessed whether improvement in the NAEP-Meas score mediated the effects of intervention on the NAEP-PW posttest score (while controlling for the pretest NAEP-PW score) and then assessed whether improvement in the NAEP-PW score mediated the effects of intervention on the NAEP-Meas posttest score (while controlling for the pretest NAEP-Meas score).

The middle panel of Figure 2 shows major components of the model that assessed whether improvement in the NAEP-Meas score mediated the effects of intervention on the NAEP-PW posttest score, while controlling for the NAEP-PW pretest score. R^2 for the second-step model (assessing the direct effects of intervention and the indirect effect of NAEP-Meas improvement on the NAEP-PW posttest score while controlling for the pretest NAEP-PW score) was .18, $F(3, 255) = 18.84$, $p < .001$. The coefficient for the pretest NAEP-PW covariate (not shown in the figure) was .34 (.06). The mediation model partitioned the total effect of 0.29 into direct and indirect effects. The coefficient for the direct effect of 0.04 was not significant, $t = 0.32$, $p = .751$ (an 86.2% reduction in the total effect). The coefficient for the indirect effect of 0.23 was significant (CI = .12 to .38). Thus, improvement in measurement understanding completely mediated the intervention effect on the NAEP-PW outcome.

The bottom panel of Figure 2 shows major components of the model that assessed whether improvement in the NAEP-PW score, in turn, mediated the effect of intervention on the NAEP-Meas score, while controlling for the NAEP-Meas score. R^2 for the second-step model (assessing the direct effects of intervention and the indirect effect of NAEP-PW improvement on the NAEP-Meas

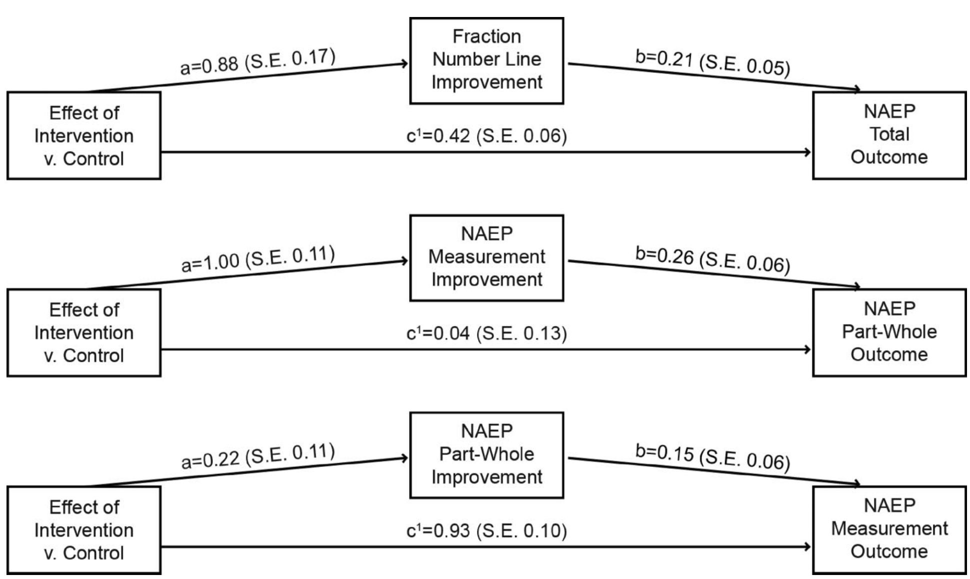

Figure 2. Three mediation models (top) testing whether improvement in fraction number line performance mediated the effects of intervention on the NAEP total score outcome, while controlling for the pretest NAEP score and improvement on fraction calculations; (middle) testing whether improvement in the NAEP-Meas score mediated the effects of intervention on the NAEP-PW posttest score, while controlling for the pretest NAEP-PW score; and (bottom) testing whether improvement in the NAEP-PW score mediated the effects of intervention on the NAEP-Meas posttest score, while controlling for the pretest NAEP-Meas score. Note that the figure does not show covariate effects, which were included in each model for both the mediator and the outcome (see text for that information). NAEP = National Assessment of Educational Progress; Meas = measurement; PW = part–whole.

posttest score while controlling for the pretest NAEP-Meas score) was .42, $F(3, 255) = 61.20$, $p < .001$. The coefficient for the pretest NAEP-Meas covariate (not shown in the figure) was .43 (*SE* .05). The mediation model partitioned the total effect of 0.95 into direct and indirect effects. The coefficient for the direct effect of 0.93 was significant, $t = 9.66$, $p < .001$ (a 2.1% reduction in the total effect). The coefficient for the indirect effect of 0.03 was not significant (CI = .00 to .06). Thus, improvement in part-whole interpretation did not mediate the intervention effect on the NAEP-Meas outcome.

Discussion

Guided in part by Geary's (2004) model of mathematics learning, we designed a fraction intervention that (a) emphasized conceptual over procedural knowledge and (b) attempted to compensate for AR learners' limitations in the domain-general abilities that predict development of fraction competence. Our instructional design to compensate for these limitations involved teaching students efficient strategies for segmenting measurement interpretation tasks, creating automaticity with fractional values in relation to marker fractions (e.g., one-half), providing a structure to encourage students to exercise attentive behavior and work hard, and simplifying the language of explanations.

With respect to conceptual knowledge, our major focus was the measurement interpretation, even though it is a less intuitive form of fraction understanding than the part-whole interpretation and presently plays a subordinate role in American schooling. We centered on the measurement interpretation of a fraction because it is deemed a key mechanism in explaining the development of competence with fractions (Geary et al., 2008). Yet its causal role has not been evaluated in the context of an experimental study. We contrasted this intervention to the typical school program for developing fraction knowledge at fourth grade, which distributed its focus roughly comparably between conceptual and procedural knowledge and assumed a dominant focus on part-whole interpretation. We hypothesized that intervention students' conceptual and procedural outcomes would exceed control group outcomes and that learning would be mediated by student improvement in understanding of the measurement interpretation of fractions. We found support for both hypotheses.

Did the Intervention Promote Conceptual and Procedural Fraction Knowledge Beyond What Might Be Expected With Conventional Instruction?

In terms of conceptual knowledge, we examined effects on three measures. Two of the measures isolated measurement interpretation. On comparing fractions, the effect size (ES) favoring the intervention over the control group was 1.82 *SD*s; on the fraction number line task, it was 1.09. On comparing fractions, intervention students initially performed 0.12 *SD*s behind low-risk classmates but completed intervention 1.04 *SD*s ahead. By contrast, the achievement gap for control students increased from .05 to 0.42 *SD*s. (We did not collect fraction number line data on low-risk classmates, but posttest performance of intervention students was at the 75th percentile for a normative sample of sixth-grade students, as per Siegler et al., in press.)

Because alignment for comparing fractions and fraction number line was greater for the intervention group than the control group, it is important to consider effects on NAEP, which was not aligned with intervention and focused with comparable emphasis on measurement and part-whole interpretations. Here, effects were also significant and strong. The ES favoring intervention over control children was 0.92 *SD*s, and the achievement gap for control students remained large (1.09 at pretest; 0.96 at post), while the gap for intervention students decreased substantially (from 1.07 to 0.08). When restricting focus to NAEP measurement items, the effect again favored the intervention condition, with similar ES (1.07). Even when focusing exclusively on part-whole understanding, which the control condition emphasized more than the intervention condition, the effect favored intervention over control students. In this case, however, the ES was a smaller 0.29. On calculations, effects again favored intervention over control. Here the ES was 2.50; the achievement gap between intervention students and low-risk classmates narrowed, while the gap for control students increased; and intervention students' posttest performance exceeded that of low-risk classmates. Given that classroom instruction allocated substantially more time to calculations during the period in which intervention occurred, this suggests that understanding of the measurement interpretation transfers to procedural skill, at least for adding and subtracting fractions (e.g., Hecht et al., 2003; Mazzocco & Devlin, 2008; Rittle-Johnson et al., 2001; Siegler et al., 2011). This finding has appreciable practical significance and is supported by instructional theory (Siegler et al., 2010).

Substantively, this study extends the fraction intervention literature by focusing an intervention primarily on the measurement interpretation of fractions, targeting younger students to prevent fraction difficulty, and assessing post-intervention achievement gaps with low-risk classmates. Methodological extensions include expanding the range of fraction measures beyond researcher-designed tasks and identifying risk in terms of poor prior mathematics performance (whereas previous studies lacked clarity about the severity of mathematics risk by selecting students based on participation in remedial classes, school-identified disability, or teacher reports).

Did Improvement in Understanding of Measurement Interpretation Mediate Effects?

To test our hypothesis that improvement in understanding of the measurement interpretation of fractions mediates the effects of the intervention, we conducted three analyses. In each analysis, we centered on the NAEP outcome because those items were not aligned with how we designed intervention. This was the case for measurement and part-whole items. In the first mediation analysis, focused on the posttest NAEP total score, we created a stringent test of the hypothesis by controlling for improvement in students' procedural fraction knowledge, on which the effect size was a substantial 2.50 *SD*s. The direct effect of intervention and indirect effect of intervention, via children's measurement understanding, were both significant. This indicates that improvement in measurement understanding partially mediated the effects of intervention on the highly valued NAEP outcome. It also supports the hypothesis that measurement interpretation of fractions is important in the development of students' fraction knowledge.

Of course, it is also possible that part-whole interpretation of fractions is similarly important in the development of students' fraction knowledge. To gain insight into this possibility, we conducted two complementary analyses. We assessed whether improvement in NAEP-Meas mediated the effects of intervention on the NAEP-PW outcome; then we examined whether the reverse was true. Results showed that measurement understanding completely mediated improvement in part-whole understanding; by contrast, part-whole understanding did not play a causal role in measurement interpretation. This strengthens the conclusion that measurement interpretation of fractions is important in the development of students' fraction knowledge. It also suggests that part-whole interpretation is less central to development, at least at fourth. At the same time, we remind readers that possible misspecification of models limits causal conclusions.

Were Intervention Effects Moderated by Domain-General Cognitive Resources?

At the same time, in line with Geary's (2004) model of mathematics learning, the intervention was designed to compensate for the kinds of limitations AR students experience in the domain-general cognitive resources associated with fraction learning. This brings us to our third hypothesis: that intervention effects are moderated by (interact with) domain-general cognitive resources. In these interactions, we expected intervention students to score similarly on the fraction outcomes regardless of their performance on domain-general measures. By contrast, we expected control students who scored low on domain-general measures to perform more poorly on fraction outcomes than control students with higher domain-general capacity. We found mixed support for this hypothesis.

Two patterns characterized the significant moderating effects we identified. The first pattern, which is in line with our hypothesis, indicates that student response to the intervention did not depend on children's cognitive resources (such as working memory), whereas student response to control group instruction did. For example, the bottom left panel of Figure 1 illustrates the interaction between listening recall (a form of working memory) and intervention condition on fraction number line performance (note that lower scores denote more accurate placement on the number line). As shown (see white bars), students completed intervention with almost identical (and better) number line scores, regardless of working memory capacity. By contrast, control students' (generally weaker) performance worsened (i.e., scored increased) with poorer working memory capacity (i.e., their scores increased; see black bars). We see the same pattern for the interaction between listening comprehension and the intervention condition on fraction calculations (top right panel in Figure 1) and the interaction between processing speed and the intervention condition on fraction calculations (middle right panel in Figure 1). (Note than on these two fraction calculation graphs, higher scores denote stronger performance. That is, in the fraction number line graph [middle left], we see black bars getting lower as listening recall increases because low scores on the fraction number line mean students placed fractions closer to the correct location on the number line. But in the fraction calculation graphs [top and middle right], we see black bars increasing as listening comprehension or processing speed increases because high scores on fraction calculations mean students completed more problems accurately.)

These interactions corroborate correlational evidence showing that working memory, listening comprehension, and processing speed typically play a role in the development of fraction learning (Hecht et al., 2003, 2007; Hecht & Vagi, 2010; Jordan et al., 2012). But importantly, the present study also illustrates that interventions may be designed to help students compensate for limitations in the cognitive resources associated with fraction learning. In the present study, we compensated for limitations in the above three domain-general abilities by (a) teaching students efficient strategies for chunking and segmenting measurement interpretation tasks (to reduce working memory demands), (b) building automaticity with fractional values in relation to marker fractions like one-half (again to reduce working memory demands and also to compensate for otherwise slow processing speed), and (c) providing explanations in simple language, requiring students to repeat explanations in their own words, and checking for understanding frequently (to reduce the language demands of instruction).

Even so, a second pattern, revealed in the other three significant interactions, suggest a need to strengthen the instructional design further. In these interactions, control student outcomes were similarly low regardless of cognitive resources, whereas the intervention student scores improved as cognitive resources increased. This was the case for two interactions involving attentive behavior and one involving counting recall (another form of working memory). For example, the middle left panel of Figure 1 shows that, on comparing fractions, control student outcomes (black bars) are similarly low regardless of students' attentive behavior; by contrast, intervention student outcomes (white bars) improve as attentive behavior increases. Both interactions involving attentive behavior (on comparing fractions and NAEP Total) conformed to this pattern, suggesting that our intervention component designed to promote on-task behavior and hard work did not adequately address the challenges AR learners experience with inattentive behavior. The same was true for the role of working memory on the comparing fractions outcome.

Additional work extending the compensatory strategies we designed with respect to attentive behavior and working memory appear necessary. In addition to compensatory strategies, instructional procedures may be required to build working memory capacity or attentive behavior—although such efforts have to date failed to transfer to mathematics outcomes, at least for AR learners (Melby-Lervag & Hulme, 2012). These previous efforts have trained working memory or attentive behavior outside the context of mathematics tasks. A more effective approach, especially for AR learners who experience transfer difficulty, may be to embed training activities within fraction tasks to build fraction knowledge as these cognitive resources are strengthened.

Together, these moderating effects corroborate the individual differences literature in identifying a role for working memory, attentive behavior, processing speed, and listening comprehension in fraction knowledge. At the same time, it is important to note that these moderating effects were modest, adding 1%–3% of variance beyond the much larger contribution of the intervention. Moreover, the advantage for intervention over control remained significant across the range of observed cognitive resource values for three of the six significant moderator effects (attentive behavior on comparing fractions; listening comprehension on fraction calculations; processing speed on fraction calculations). For the other three moderator effects, the value of the cognitive resource at which the intervention effect became nonsignificant was extreme (approximately 2 *SD*s from the mean).

Did Skill With Whole-Number Calculations Moderate Intervention Effects?

We hypothesized that skill with whole-number calculations moderates intervention effects based on two types of evidence. First, descriptive research (e.g., Geary et al., 2007; Mazzocco & Myers, 2003; Murphy et al., 2007) indicates that students with more severe mathematics difficulty, often defined by whole-number calculation skill, demonstrate distinct cognitive profiles and developmental trajectories, leading to the hypothesis in the literature that these students require different forms of intervention. Second, correlational research shows that whole-number calculation skill predicts fraction knowledge (Hecht et al., 2003; Hecht & Vagi, 2010; Jordan et al., 2012; Seethaler et al., 2011). Given that whole-number calculations are required to handle fractions, it may be advantageous to execute whole-number calculations quickly, without taxing mental resources that might instead be used to execute more complex conceptual features of fractions. However, we found no evidence to support this hypothesis on fractions. The interaction between whole-number calculation skill and intervention was not significant on any fraction outcome: The effects of the intervention were comparable for students with varying levels of whole-number calculation skill.

It is important to note, therefore, that fractions have distinctive features that may inhibit transfer from whole number addition and subtraction and create opportunities for early fraction intervention to reverse the trajectory of early whole-number difficulty. Most obviously, a fraction comprises two digits that operate together to determine the magnitude of one number (the fraction), and larger denominators denote smaller parts. Also, some invariant properties of whole numbers do not apply to fractions. For example, whole numbers, but not fractions, have unique successors, and fraction but not whole-number division can produce quotients greater than either operator. In fact, many students who are competent with whole numbers struggle with fractions (NMAP, 2008). Even so, finding that proficiency with whole-number addition and subtraction calculations did not moderate intervention effects is surprising given prior descriptive research (Hecht & Vagi, 2010; Seethaler et al., 2011), and some evidence indicates that fractions and whole numbers are processed in similar areas of the brain (Jacob, Vallentin, & Nieder, 2012). Given that fraction knowledge is more highly correlated with whole number division than addition or subtraction (Siegler et al., in press), additional study is warranted beyond fourth grade, as the curricular focus on multiplication and division grows.

Limitations and Conclusions

Readers should note three limitations in the present study. First, to address the two major components of Geary's (2004) model of mathematics development, the intervention differed from the control group in two major ways: (a) a stronger focus on conceptual knowledge, specifically the measurement interpretation of fractions and (b) instruction designed to compensate for AR students' limitations in domain-general cognitive resources. This creates challenges for understanding which component(s) account for the intervention effects. To address this challenge, we planned the study to investigate the role of measurement interpretation as a causal mechanism (via mediation analysis) and to explore the hypothesis that the intervention compensated for domain-general limitations (via moderation analysis). Even so, additional study that systematically manipulates these instructional components, with two active researcher-designed and tightly implemented intervention conditions, is warranted. Second, we had no observations of the school's classroom or intervention program, so we had to rely on descriptions of what occurred provided in the curriculum and by teachers. Third, we did not follow students to examine the durability of intervention effects. Future studies might incorporate direct observations of the school's (control) program and follow students into subsequent grades.

With these caveats in mind, we tentatively offer four conclusions. First, as indicated in the mediation analyses, measurement understanding appears to be a key mechanism in fraction learning. This, in combination with the strong effects of the intervention that focuses strongly on the measurement interpretation of fractions, suggests that schools should infuse a stronger focus on the measurement interpretation into the fraction curriculum at fourth grade, as reflected in the recent Common Core State Standards. Second, as revealed in the moderation analyses, interventions can be designed to compensate for the types of limitations AR learners experience with predictors associated with fraction learning: working memory, attentive behavior, processing speed, and listening comprehension. Even so, additional work is required, perhaps extending the framework for intervention to include a focus on strengthening these abilities in the context of fraction tasks. Finally, although these moderator effects are interesting to consider and carry implications for the design of intervention to reach the full range of learners, they also reveal the robustness of the intervention. By any standard, the effects of intervention designed to foster measurement interpretation of fractions for AR fourth graders were strong, with the achievement gap for AR learners substantially narrowed or eliminated. Moreover, the schools' failure, not only to address the needs of a substantial majority of AR learners in a more successful manner but also to promote stronger learning among low-risk classmates, raises questions about the quality and nature of business-as-usual fraction instruction. This in part explains widespread difficulty with fractions (e.g., NCTM, 2007; Ni, 2001) and highlights the pressing need to improve the quality of fraction instruction and learning in the United States (NMAP, 2008).

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