

## **Time Series in Education: The Analysis of Daily Attendance in Two High Schools**

**Matthijs Koopmans (Strategic Education Research Partnership Institute)**

**Presented at the Annual Convention of the American Educational Research Association, New Orleans, LA, April 8-12**

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### **ABSTRACT**

This presentation discusses the use of a time series approach to the analysis of daily attendance in two urban high schools over the course of one school year (2009-10). After establishing that the series for both schools were stationary, they were examined for moving average processes, autoregression, seasonal dependencies (weekly cycles), outliers and heteroscedasticity. Seasonal dependencies were significant in both schools. In addition, contrary to what the traditional attendance statistics (mean, median, range) would lead one to expect, attendance differs appreciably in the two schools, with expanding variance and predictable Friday plunges in one school, and a clear trend toward lower attendance in days preceding holidays and vacations in the other.

The purpose of this presentation is to report on the analysis of daily high school attendance over the course of one school year (2009-10) in two urban high schools, using time series analysis. While significant conceptual work was done to appreciate the potential of time series analysis in education (Glass, 1972), there have not been very many instances in which the technique has been utilized to analyzing real education data. The urban districts that have begun to publicize daily attendance rates on their websites have created an opportunity to conduct these types of analyses to help us appreciate the usefulness of the technique in educational research, as well as obtain a better understanding of how patterns of daily high school attendance behave over time.

### **Attendance**

High school attendance and high school dropout are among our greatest concerns in educational policy. When considering how well students attend school, the common practice of relying on average daily attendance measures (e.g., National Center for Education Statistics, 2008) carries with it the implicit assumption that the mean is the best way of characterizing the distribution of attendance over time and that the distribution of errors over time is random. Moreover, reliance on an average conceals any information about the nature of fluctuations over time.

The lack of attention to the effect of time on daily attendance patterns in high schools leaves a set of very interesting questions unanswered. Do attendance levels tend to go up, go down or stay the same in the course of a school year? Are there differences between schools in this respect? Is variance constant over time, or should we expect heteroscedastic patterns, with perhaps greater variability as the end of the school year approaches? Is attendance on a given day a good indicator of what attendance will be on the next day? Are there correlations between attendance on given weekdays and that on the same day in other weeks? With how much confidence can we predict school attendance going forward, based on what we know about existing patterns of attendance? How far into the future can we predict patterns of attendance at a given school? Is high fluctuation in attendance a predictor of a future decline in attendance rates? The educational literature is largely silent on these questions, which is remarkable considering how prominent low attendance and dropout are in our educational policy discussions.

The purpose of this paper is to discuss the basics of time series modeling in situations where it cannot be assumed that observations are independent, and to explore how the approach can help us understand the contribution of time to patterns of school attendance. A single case design is used to analyze separately the attendance in each of the two schools. The analyses should therefore be seen as descriptive, with no claims to generalizability to other schools or school districts.

### **Data Sources**

The urban district whose data are used in these analyses weekly publicizes the daily attendance data for each of its schools. Attendance data were recorded for the entire 2009-10 school year for a convenience sample of 25 schools. For the illustrative purposes of this paper, the attendance trajectories of two of these schools will be analyzed. Daily attendance data were recorded for the entire 2009-10 school year in these schools starting on September 9 and finishing at the close of the last full week of school (June 11), yielding 183 observations for each school. For the days that schools were closed, the district

entered zeros on the attendance record. For this analysis, a nearest neighbor imputation was done substituting the value of  $Y_{t+1}$  for each  $Y_t = 0$ , unless  $Y_{t-1}$  was more typical (i.e., closer to the mean) of the series. If schools were closed for an entire week, that week was eliminated from the analysis, thus preserving the weekly cycles in the time series.

### Analytical Approach

To understand how daily attendance behaves over time, an ordinary least squares regression of attendance outcomes on time is not sufficient, because it cannot be assumed that errors are independent. A time series approach enables one to model the dependencies in those errors and obtain a more reliable estimation of how time, in the course of one school year, affects attendance rates in a given school. In addition, it can be decided to what extent fluctuation patterns replicate over time in predictable ways.

The analysis proceeds as follows. After establishing that the series for both schools are stationary, using the Dickey-Fuller Unit Root test (Fuller, 1996), the series are examined for moving average (MA) processes, autoregression (AR), as well as combined ARMA and seasonal MA and AR patterns, focusing on weekly cycles. It was also determined to what extent the series were impacted by the presence of outliers, and heteroscedasticity. Below, these procedures are described in greater detail. All notation and descriptions are based on Cryer and Chan (2008).

#### **Autoregressive (AR) Processes**

In an autoregressive process, the value of  $Y_t$  is predicted as a linear combination of its own past values, plus the error term  $e_t$  that embodies variability occurring at time  $t$  that is not explained by the past values. Hence,  $e_t$  is often referred to as an innovation term. In its general form, this process is a  $p$ th order autoregression process,  $AR(p)$ , that can be summarized as follows:

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots + \phi_p Y_{t-p} + e_t$$

This model estimates  $Y_t$  using  $p$  lags, with a parameter  $\phi$  associated with each past value included in the model. It is assumed that for every  $t$ ,  $e_t$  is independent of  $Y_{t-1}, Y_{t-2}, Y_{t-3}, \dots$ .

A first order autoregressive model,  $AR(1)$ , would be:

$$Y_t = \phi_1 Y_{t-1} + e_t$$

Assuming stationarity and a series mean of 0, the variance in the  $AR(1)$  case is equal to:

$$\gamma_0 = \frac{\sigma_e^2}{1 - \phi^2}$$

In the more general case of  $AR(p)$ , the variance is computed as:

$$\gamma_k = \phi^k \frac{\sigma_e^2}{1 - \phi^2} \quad \text{for } k = 1, 2, 3, \dots$$

The autocorrelation in the general case can then be defined as:

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \phi^k$$

### ***Moving Average (MA) Processes***

A moving average process describes time dependencies as a pattern of disturbances, where the estimation  $Y_t$  is based on the weighted sum of previous and current errors in the series. A general MA ( $q$ ) model uses  $q$  lags for this estimation, and can be formulated as follows:

$$Y_t = e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \dots - \theta_q e_{t-q}$$

A parameter  $\theta$  is associated with each innovation included in the model. For a first order moving average, MA (1) process, this model simplifies to:

$$Y_t = e_t - \theta_1 e_{t-1},$$

with the following mean function:

$$E(Y_t) = 0.$$

Variance in an MA (1) process can be expressed in terms of the moving average parameter estimate as follows:

$$Var(Y_t) = \sigma_e^2 (1 + \theta^2),$$

and the autocorrelation function for the first lag ( $k = 1$ ) as follows:

$$\rho_1 = \frac{-\theta_1}{1 + \theta_1^2}$$

For subsequent lags in an MA(1) model,  $\rho_k = 0$ .

### ***Autoregressive Moving Average (ARMA) Processes***

If time series seems to contain both autoregressive and moving average processes, a model can be fitted that combines the AR and MA components. A first order ARMA model can be defined as follows:

$$Y_t = \phi_1 Y_{t-1} + e_t - \theta_{t-1}$$

This equation can be extended as described above for AR and MA separately to include higher order terms for both components.

### ***Seasonal Autoregressive and Moving Average Patterns***

An important part of many time series analyses is the detection and analysis of cyclical patterns, that is, patterns that repeat in a predictable manner (e.g., weekly, monthly or yearly). In the analysis of school

attendance, it makes intuitive sense to consider weekly cycles by estimating AR and MA processes across a five-day lag. Thus, we consider a seasonal AR (1)<sub>5</sub> model:

$$Y_t = \Phi_1 Y_{t-5} + e_t.$$

as well as a seasonal MA (1)<sub>5</sub> model:

$$Y_t = e_t - \Theta_1 e_{t-5},$$

We also estimated were the second order seasonal variants, i.e. AR (2)<sub>5</sub>,

$$Y_t = \Phi_1 Y_{t-5} + \Phi_2 Y_{t-10} + e_t,$$

and MA (2)<sub>5</sub>,

$$Y_t = e_t - \Theta_1 e_{t-5} - \Theta_2 e_{t-10},$$

respectively, for the MA and the AR components of the seasonal process. Also considered was a mixed seasonal ARMA (1,1)<sub>5</sub> model, defined as:

$$Y_t = \Phi_1 Y_{t-5} + e_t - \Theta_1 Y_{t-5}$$

### **Outliers**

Outliers were identified in the series for each of the two schools, and it was assessed whether those outliers were additive or innovative. Outliers are considered additive if they perturb the system at one time point  $t = T$ , but not beyond that point. Outliers are considered innovative if the parameters associated with the estimation of their effects continue influence the estimation of  $Y_t$  in subsequent lags. If outliers are *additive*, the perturbed series can be described as:

$$Y'_t = Y_t + \omega_A P_t^T$$

where:

$Y'_t$  is the perturbed attendance

$Y_t$  is the unperturbed attendance

$P_t^T = 1$  at impact; 0 otherwise ; i.e., the pulse

$\omega_A$  is the estimated impact of the pulse on  $Y_t$

*Innovative* outliers produce the following perturbation pattern:

$$Y'_t = [e_t + \Psi_1 e_{t-1} + \Psi_2 e_{t-2} + \dots] + \Psi_{t-T} \omega_I$$

or

$$Y'_t = Y_t + \Psi_{t-T} \omega_I.$$

If  $t > T$ ,  $\omega_I$  expresses the added perturbation after  $T$ . As the weights  $\Psi$  are moved from  $t$  to  $t + 1$ ,  $t + 2$ , ..., etc, the impact of this perturbation term on the series gradually diminishes. The procedures described by Chang, Tiao and Chan (1988) and Cryer and Chan (2008) were used to detect both types of outliers in the series and they were incorporated them into the models accordingly.

### **Heteroscedasticity**

To deal with heteroscedasticity the conditional variance was estimated using a generalized autoregressive conditional heteroscedasticity GARCH(1,1) model which includes two lags of conditional variance in the model. The conditional variance is estimated as

$$r_t = \sigma_{t|t-1} + \varepsilon_t$$
$$\sigma_{t|t-1}^2 = \omega + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1|t-2}^2$$

or

$$\sigma_{t|t-1}^2 = (1 - \alpha_1 - \beta_1)\sigma^2 + \alpha_1 r_{t-1}^2 + \beta_1 \sigma_{t-1|t-2}^2$$

where  $\alpha$ ,  $\beta$  and  $\omega$  are parameters associated with the estimation of the conditional variance at lag 1 and lag2, and  $\{\varepsilon_t\}$  represent the innovations to the system. Assuming that  $\{\varepsilon_t\}$  has a unit variance,  $r_t = \sigma_{t|t-1}$ .

Also considered was the estimation of heteroscedasticity estimating conditional variance at one lag only GARCH(1,0):

$$\sigma_{t|t-1}^2 = \omega + \alpha_1 r_{t-1}^2.$$

## **Results**

Table 1 summarizes some of the main characteristics of the two schools whose attendance patterns are analyzed. The most remarkable differences between these two schools are their sizes (School 1 enrolled 3,254 students in 2009-10 and School 2 enrolled 212), and grade configuration. School 1 served grades 9-12, while School 2, a recently created small high school, served only grades 9 and 10 in 2009-10. There are also some demographic differences between the two schools, with 16% of the students identified as Asian/Pacific Islander in School 1 and only 4.7% in School 2; 36.3% of the students in School 2 were African American, compared to 12.9% in School 1. The percent of teachers with five years of experience or more is higher in School 1 (71.4%) than in School 2 (42.9%).

Table 2 provides the summary statistics for the daily attendance rates in both schools. It can be seen that attendance in the two schools looks approximately the same on these summary measures: mean attendance equals 77.4% in School 1 and 78.6% in School 2; the medians are 78.9 and 81.7%, respectively. The range is somewhat narrower in School 1 (from 41.7% to 84.8%) than in School 2 (from 43.8% to 90.7%) A discussion of daily attendance measures over time in each of the two schools follows.

### **School 1**

Figure 1a shows the daily attendance at School 1 for the entire 2009-10 school year; Figure 1b shows the same series with the days of the week labeled (R is Thursday). Some prominent outliers can be seen in the series. In addition, it appears that attendance tends to be lower on Fridays, suggesting a seasonal pattern. Figure 2 shows the autocorrelation function (ACF) plot for this school. The dotted lines in the plot mark a 95% confidence interval. The spikes at lag 5 in both plots point to a seasonal pattern that could be either autoregressive or moving average, but most likely not both. The plot does not indicate any non-seasonal autoregressive or moving average processes (nor does the partial ACF plot). The Dickey-Fuller Unit Root test (Fuller, 1996) shows no evidence of non-stationarity ( $ADF=-5.28$ ,  $k=4$ ,  $p<.10$ ), indicating that we can assume that the series is linear with a zero mean.

Table 3 summarizes the model fitting process for School 1. Consistent with what the ACF plot indicates, first order AR and MA processes do not improve the fit of the model (see Model II and III). The AR (1) parameter  $\phi_1$  and MA (1) parameter  $\theta_1$  estimates are not different from zero and the goodness of fit indicators (log likelihood and Aiken's Information Criterion) are indistinguishable from the baseline model (Model I). Adding seasonal terms to the model improves matters somewhat, showing a slightly better fit for the seasonal AR (1)<sub>5</sub> model (Model V with  $\Phi_1 = .23$ ) than for the seasonal MA (1)<sub>5</sub> model (Model IV with  $\Theta_1 = .18$ ). An ARMA (1,1)<sub>5</sub> model (Model VI) further improved the estimation of the series, with  $\Phi_1 = .98$  and  $\Theta_1 = -.91$ . Since second order seasonal AR and MA models did not further improve goodness of fit, Model VI was used as the basis for outlier detection. Detected outliers were included in a next model, and an assessment was again made as to whether outliers could be found in the new models, to be incorporated into a subsequent model. Table 3 summarizes this iterative process (Models VII through X) showing the considerable improvements after outlier removal, with substantial increases in the log-likelihood function (from -576.61 for Model VI to -409.56 for Model X), and substantial reductions in the information criterion (from 1159.22 to 845.10).

All but one of the outliers detected through this process were additive outliers. The parameter estimates associated with each outlying attendance date show the size of the drop, or the pulse response on each particular day. Inspecting those estimates in Model X shows specifically that October 14, October 30 and March 19 deviate from the series by more than 30 percentage points. Some of these dates invite speculation about the reason for the discrepancies. October 30 is the Friday before Halloween, November 25 is the Wednesday before Thanksgiving, February 11 followed a snow day on the 10<sup>th</sup>, and February 25 preceded one on the 26<sup>th</sup>; June 11 was the last Friday concluding a full week of school. October 9<sup>th</sup> was an innovative outlier with a gradually diminishing influence on the series as time progressed. Its positive value (5.52) suggests a relatively good Friday turnout on that particular day.



## School 2

Figure 3a shows the daily attendance rates for School 2; Figure 3b shows the same series with labels for the days of the week. It can be seen that daily attendance rates over time in this school display a pronounced Friday-effect. The autocorrelation and partial autocorrelation plots shown in Figure 4a and Figure 4b suggest a seasonal AR (2) process with the ACF tailing off slowly at  $t = 5$ ,  $t = 10$ ,  $t = 15$  and  $t = 20$ , while the PACF cuts off after two seasonal lags ( $t = 5$  and  $t = 10$ ). The plots in Figure 3 and 3b also show clear signs of heteroscedasticity, with a progressive increase in variability as the school year progresses. The plot of the first difference of the series ( $Y_t - Y_{t-1}$ ) shows this trend more clearly (see Figure 5). However, since the Dickey-Fuller Unit Root test shows no evidence of non-stationarity (ADF=-3.65,  $k=9$ ,  $p < .10$ ), we assume a constant mean function and do not analyze the series based on the first difference.

Table 4 summarizes the model fitting process for School 2. First and second order AR and MA parameters (not shown) were not statistically significant. Seasonal effects, i.e., five day cycles, are estimated in Models II through V. It can be seen that the best fitting model is the seasonal AR(2)<sub>5</sub> model with highest log-likelihood values and lowest AIC values; both coefficients depicting the autoregressive seasonal process are statistically significant ( $\Phi_{t-5} = .31$  and  $\Phi_{t-10} = .37$ ).

Model VI summarizes the outlier detection process and indicate a substantial improvement in the goodness of fit as indicated by log-likelihood and AIC values. The dates with outlying values have significantly lower attendance, as indicated by the negative values of the coefficients, and estimated plunge at the perturbation points are 24.48 (October 30, pre-Halloween), -31.03 (February 11, post-snow day) and -17.24 (March 19). The ACF plots for Model VI show no remaining autocorrelation. However, the Ljung-Box test statistic (Cryer & Chan, 2008) shows that if more than lags or more are included in the test, there are clear signs of heteroscedasticity, as the statistic turns significant at  $k > 9$ .

To address the heteroscedastic tendency in the series, table 5 summarizes the properties of the models that estimate the conditionality of the variance in the series on previous lags, using the general autoregressive conditional heteroscedasticity approach (GARCH) outlined above. The model estimates the contribution of the conditional variances at lag 1 and lag 2 of the series using the residuals of Model IV (no outliers) and Model VI (with outliers). Model IVa and IVb show the improvements to the goodness of fit of Model IV with lag 1 and lag 1 + 2 conditional variances. Models VIa, VIb and VIc show such improvement for the models that include the outliers (Model VI). The GARCH models were fitted to the residuals of Model IV and Model VI. The improvements to both models IV and VI indicate that including conditional variances for the first two lags, improves the goodness of fit for both Model IV and VI, but that it does not matter a great deal whether the conditional variances are modeled over one or two lags. Contrary to the other models considered, the GARCH (2,1) model, which includes a coefficient for  $\beta_2 \sigma_{t-2|t-3}^2$  in the estimation of  $\sigma_{t|t-1}^2$  passes the test for normally distributed residuals (Shapiro's  $W = .99$ ,  $p > .05$ ), although the differences in error distribution between Models VIa, VIb and VIc are superficial.

## Discussion

A lot of valuable information can be added to the traditional summary statistics of daily attendance by looking closely at how attendance patterns behave over the course of the school year. The analyses presented in this paper reveal some of these patterns. In both schools, there is a seasonal trend in attendance rates indicating a weekly cyclical nature. This trend is particularly pronounced in School 2, where fewer students attend school on Friday than on other days of the week. Also noteworthy is the prominent role of outliers in the time series. The distribution of attendance is negatively skewed, and significant model improvements are obtained in both schools, by detecting those outliers and estimating their impact on the series. While the time series in both schools contain outlying values, the two schools are nonetheless dissimilar in this respect. School 1 has eight days with outlying attendance rates, while School 2 has only three. In addition, in School 1, all outliers appear to be additive indicating that they do not impact the series beyond the particular date itself, while School 2 has one innovative outlier (March 19), whose influence gradually diminishes as time progresses. The similarities between the two schools are also noteworthy. The three outlying dates in School 2 are all outlying dates in School 1 as well. One marks the anticipation of Halloween with lower attendance (October 30) and another follows what was a snow-day in the Northeast (February 11). In School 1, there are more examples of these kinds of outlying days (the anticipation of Thanksgiving on November 25, and the Christmas Vacation on December 23), while in School 2, there are not. On the other hand, in School 2, attendance rates are persistently lower on Fridays than on other days of week, a trend that is found only to a very modest extent in School 1. Apparently, where it comes to attendance, different high schools may have different 'relaxation points.' It is beyond the scope of this investigation to decide whether these dips emanate from the community, local peer culture or the schools themselves. Notable differences between the two schools include size (School 1 is a large high school serving 3,254 students and School 2 is a small one serving 212, Table 1) and grade configuration (School 1 serves grades 9-12 + special education, while School 2 served grade 9 and 10 and special education in 200-10, Table 1).

It is of interest to note that there is much less variability in daily attendance rates in School 1 than in School 2. Moreover, in School 2, variability in attendance rates increases as the school year progresses, while in School 1, it does not. While this shift over time does not rise to the level of non-stationarity of the series, attendance in School 2 is clearly heteroscedastic, and modeling conditional variances into the time series models does indeed improve the goodness of fit of the models.

For feedback purposes, teachers and administrators at the school and school district level could benefit from the availability of daily attendance data such as those summarized here because they reveal where the irregularities lie in school wide daily attendance records and to what extent there are shifts in collective attendance behavior in the course of a school year that it would be helpful to know. The availability of this information enables schools and districts to implement a more focused attendance policy than would be possible based on average daily attendance measures, which conceal most of the trends discussed in this paper (see Table 2).

The analysis presented here has several important limitations. The rates analyzed here are school wide and therefore do not reveal differences by grade level, nor do they allow disaggregation of student

groups by gender and socioeconomic and ethnic categories. From a policy perspective, this information may be highly valuable as well. Ultimately, information of high school attendance is of interest in the context of college preparation and dropout prevention efforts. It would therefore be of interest also to analyze the attendance rates of cohorts of students who are followed for a four-year period, starting in grade 9 to attempt to describe whether and how growing instability in school-wide attendance may initiate a downward trend (self-organized criticality, Bak, 1996).

There are also inherent limitations to the use of single case designs, as they remain particular to the schools whose attendance is analyzed. While interesting in their own right, case studies such as these can also be used as a tool to generate hypotheses that can be tested on a larger sample of schools. Such an analysis would involve pooling the time series of the individual schools to estimate between school variability in processes such as those described in this paper.

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**Table 1. Summary Student and School Characteristics: School 1 and School 2**

School	School 1	School 2
Grade configuration	9-12 & SE	9-10 & SE
<b>Students</b>		
N enrolled	3,254	212
% Male	51.4	61.3
% Native Am.	.2	1.4
% Asian/Pacific	16	4.7
% Hispanic	58.8	53.8
% African Am.	12.9	36.3
% White	12.1	2.8
% Not responding	0.2	0.9
% ELL	14.4	13.2
% General Ed.	87.3	82.6
% below poverty	60.2	60
<b>Teachers</b>		
% > 5 yrs. exp.	71.4	42.9
% w/ Master's	90	86
Students per Teacher	16.6	14.9

**Table 2. Summary Statistics Attendance Rates School 1 and School 2**

	M (SD)	MD	Range	N
School 1	77.4 (5.9)	78.9	41.7 – 84.8	183
School 2	78.6 (8.9)	81.7	43.8 – 90.7	183

**Table 3. Summary of Model Selection Process School 1: Coefficients and Goodness of Fit Statistics**

MODEL	I	II	III	IV	V	VI	VII	VIII	IX	X
Parameters	Baseline	MA (1)	AR (1)	MA (1) <sub>5</sub>	AR(1) <sub>5</sub>	ARMA (1,1) <sub>5</sub>	ARMA (1,1) <sub>5</sub> with outliers (Step 1)	ARMA (1,1) <sub>5</sub> with outliers (Step 2)	ARMA (1,1) <sub>5</sub> with outliers (Step 3)	ARMA (1,1) <sub>5</sub> with outliers (Step 4)
Intercept (se)	77.4 (.44)	77.4 (.45)	77.4 (.45)	77.36 (.5)	77.33 (.55)	77.23 (1.07)	77.58 (.77)	77.4 (.9)	77.33 (.97)	77.26 (1.0)
$\theta_1$ (se)		.04 (.08)								
$\phi_1$ (se)			.04 (.08)							
$\Theta_1$ (se)				.18 (.07)		-.91 (.09)	-.49 (.1)	-.4 (.09)	-.37 (.09)	-.36 (.09)
$\Phi_1$ (se)					.23 (.07)	.98 (.04)	.87 (.06)	.9 (.05)	.9 (.04)	.91 (.04)
Oct 9 (t=23) (se)										5.52 (1.93)
Oct 14 (t=26) (se)							-30.48 (2.75)	-30.77 (2.11)	-30.83 (1.98)	-30.85 (1.93)
Oct 30 (t=38) (se)							-31.39 (2.75)	-31.53 (2.11)	-31.55 (1.98)	-31.34 (1.93)
Nov 25 (t=56) (se)									-8.54 (1.98)	-8.55 (1.93)
Dec 23 (t=76) (se)								-14.65 (2.11)	-14.83 (1.98)	-14.84 (1.93)
Jan 25 (t=94) (se)							-16.13 (2.74)	-16.2 (2.11)	-16.21 (1.98)	-16.22 (1.93)
Feb 11 (t=107) (se)							-20.08 (2.83)	-24.06 (2.21)	-24.09 (2.09)	-24.11 (2.03)
Feb 25 (t=112) (se)								-16.2 (2.21)	-16.25 (2.09)	-16.27 (2.03)
Mar 19 (t=128) (se)							-32.99 (2.74)	-32.77 (2.11)	-32.73 (1.99)	-32.71 (1.93)
June 11 (t=183) (se)							-15.17 (3.02)	-14.66 (2.43)	-14.54 (2.31)	-14.48 (2.26)
$\sigma^2$	34.79	34.73	34.74	33.4	33.05	31.5	8.89	5.76	5.23	5.0
LL	-584.43	-584.28	-584.28	-580.79	-579.86	-576.61	-460.94	-422.23	-413.53	-409.56
AIC	1170.86	1172.56	1172.59	1165.59	1163.73	1159.22	939.87	866.46	851.05	845.1

**Table 4. Summary of Model Selection Process School 2: Coefficients and Goodness of Fit Statistics**

MODEL	I	II	III	IV	V	VI
Parameter	Baseline	AR (1) <sub>5</sub>	MA (1) <sub>5</sub>	AR (2) <sub>5</sub>	MA (2) <sub>5</sub>	AR (2) <sub>5</sub> with outliers
Intercept	78.65 (.66)	78.42 (1.09)	78.58 (.78)	78.22 (1.5)	78.48 (.94)	78.45 (1.52)
$\theta_1$ (se)						
$\phi_1$ (se)						
$\Theta_1$ (se)			.29 (.05)		.34 (.07)	
$\Theta_2$ (se)					.39 (.06)	
$\Phi_1$ (se)		.49 (.06)		.31 (.07)		.24 (.06)
$\Phi_2$ (se)				.37 (.07)		.48 (.06)
Oct 30 (t=38) (se)						-24.45 (5.51)
Feb 11 (t=107) (se)						-31.03 (5.5)
Mar 19 (t=128) (se)						-17.24 (5.49)
$\sigma^2$	78.86	59.62	68.14	51.21	55.78	38.59
LL	-659.3	-634.41	-646.15	-621.2	-628.61	-595.88
AIC	1322.61	1274.82	1298.29	1250.4	1265.23	1203.75

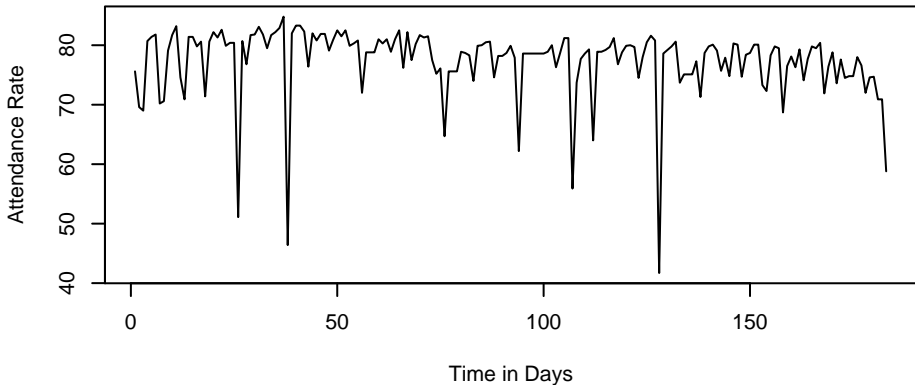
**Table 5. Conditional Variance Models: School 2**

Model	Order		Coefficients				Goodness of Fit		
	AR(P)	GARCH (q, p)	$\omega$	$\alpha_1$	$\beta_1$	$\beta_2$	LL	df/k	AIC
IVa	2	0,1	50.34 (3.76)	.02 (.04)			-616.57	2	1237.14
IVb	2	1,1	3.43 (6.03)	.02 (.03)	.91 (.14)		-615.93	3	1240.86
VIa	2 with outliers	0,1	38.3 (4.81)	.01 (.07)			-590.9	2	1185.8
VIb	2 with outliers	1,1	1.41 (1.64)	.04 (.04)	.92 (.07)		-589.22	3	1187.45
VIc	2 with outliers	2,1	32.98 (184.3)	.02 (.07)	.13 (2.77)	.00 (4.67)	-588.1	4	1192.21

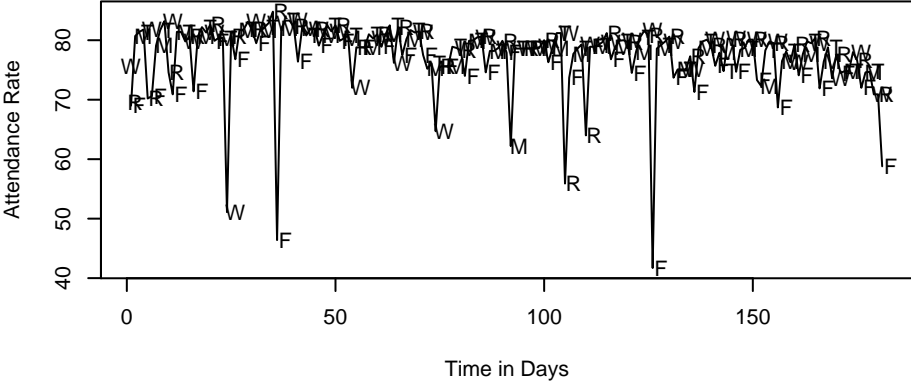
## FIGURES



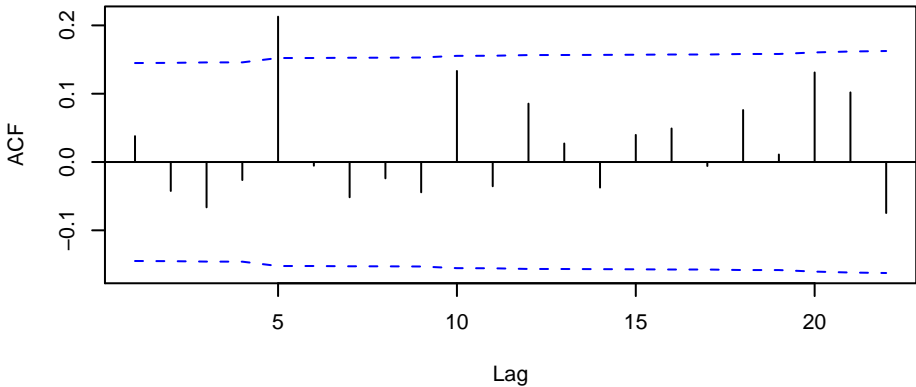
**Figure 1a. Daily Attendance 2009–2010: School 1**



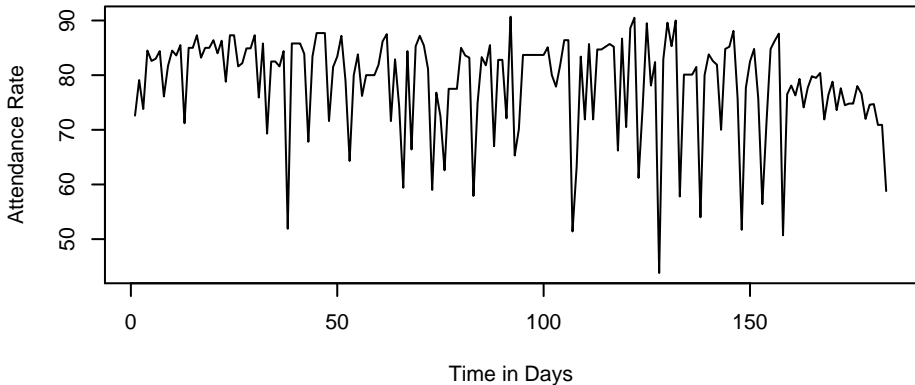
**Figure 1b. Daily Attendance 2009–10: School 1 (Days Marked)**



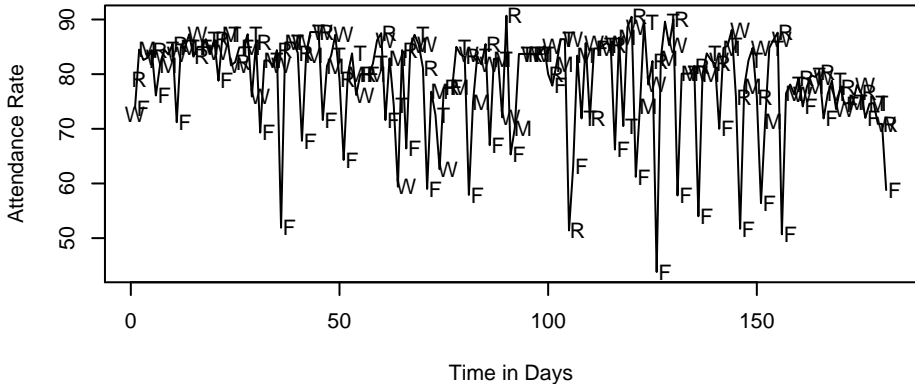
**Figure 2. Autocorrelation Function (ACF): Attendance Rates School 1**



**Figure 3a. Daily Attendance Rates 2009–2010: School 2**



**Figure 3b. Attendance Rates 2009–2010: School 2 (Days Marked)**



**Figure 4a. Autocorrelation Function (ACF): Attendance Rates School 2**

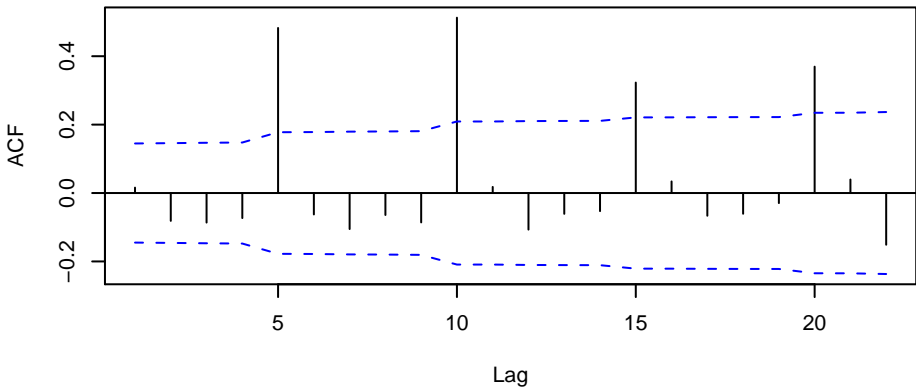
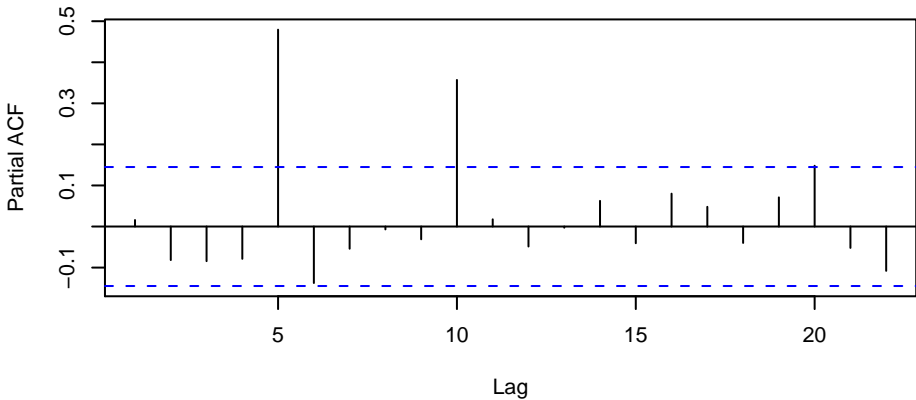


Figure 4b. Partial ACF: Attendance Rates School 2



**Figure 5. First Difference  $Y_t - Y_{t-1}$  Attendance Rates 2009–10 School 2**

