A Thank You to Our Corporate Sponsors

The Actuarial Foundation would like to thank the following sponsors for their generosity in funding the production of the Math Academy booklet.

By providing financial support for this project they are assisting The Actuarial Foundation in its pursuit of one of its many aspirations—to provide students with enriched learning opportunities that can help them succeed in their educational and professional endeavors.

The Actuarial Foundation and the companies listed below are collectively ensuring math education remains strong and enjoys a prosperous future.

Ernst & Young

New York Life Foundation

Swiss Re

The Actuarial Foundation’s mission: To develop, fund and execute education and research programs that serve the public by harnessing the talents of actuaries.

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Congratulations! You’ve just found your answer to the question, “What can I do to create real enthusiasm for mathematics in all my students?”

Created by teachers for teachers, the Math Academy tools and activities included in this booklet were designed to create hands-on activities and a fun learning environment for the teaching of mathematics to our students. On Math Academy days I often found that I couldn’t make it from my car to my classroom without being stopped by enthusiastic students wanting to know every detail of the upcoming day’s events. Math Academy days contributed to a positive school-wide attitude towards mathematics on our campus.

This booklet contains the themed program Are You Game? Math Academy – Explorations in Probability, which you can use to enhance your math instruction while staying true to the academic rigor required by the state standards framework.

This effort would not have been possible without support from The Actuarial Foundation. With the help of the Foundation’s Advancing Student Achievement (ASA) grant program, which is committed to funding projects that enhance the teaching of mathematics, this Math Academy program came into being.

I sincerely hope that you enjoy implementing the Are You Game? Math Academy – Explorations in Probability with your students. When you do, you will find that your students engage with mathematics on a whole new level. The Actuarial Foundation is truly a great partner in furthering mathematics education!

For the kids!

Kimberly Rimbey, M.Ed., NBCT (Math)
Mathematics Specialist
Phoenix, AZ

If you wish to find out more about my experiences with this and other Math Academies, feel free to contact me via e-mail at mathacademy@actfnd.org.
When the Math Academy concept was first developed, it was designed as a half-day or full-day event which allowed students to deepen their understanding of math while interacting with volunteers from the community (see page 24 for ideas on working with community volunteers). The activities we selected for these events were hands-on, standards-based lessons which applied mathematical principles in real-world scenarios. Each student experienced three to five activities during the course of the event.

Each Math Academy began with a brief school assembly featuring a guest speaker who represented that day’s particular theme. Themes included math related to restaurants, sports, nature, shopping, fine arts and other topics, as well as focused on math-related careers. After the assembly, students rotated to different classrooms where they engaged in various activities related to mathematics and the day’s coordinating theme.

Included in this booklet is the *Math Academy – Are You Game? Explorations in Probability*, which has all the activities we used for The Game Company Math Academy. This Math Academy is designed to help students understand probability through playing games. You may choose to implement a grade-level or school-wide Math Academy as we originally designed it, or you may prefer to implement these activities in your own classroom. Whichever format you use, keep in mind that the goal is to help your students see the relevance of mathematics in real-life contexts. If you would like more information on the set-up for a school-wide or grade-level Math Academy event, visit The Actuarial Foundation’s Web site at [www.actuarialfoundation.org/grant/mathacademy.html](http://www.actuarialfoundation.org/grant/mathacademy.html).
Getting Started

Math Academy Format
You may choose to conduct your Math Academy as a school-wide event, as a grade-level rotation, or as a single-classroom experience. If you will be holding your Math Academy for a single classroom, you may want to invite a guest speaker to speak with your class about how mathematics is used in his/her job (rather than putting on a school-wide or grade-level assembly as described below). You can still set up the day in a similar way by explaining the Game Company scenario, but you will simply do this in your classroom rather than creating a large-scale production.

Math Academy Schedule
Schedule and times may vary depending on format being used.

- Opening assembly (optional) — 15 minutes
- Directions and Math Journals — 15 minutes
- Activity Rotations — 30–45 minutes per activity
- Assessment and Closure — 15 minutes

Opening Assembly/Directions
To build enthusiasm and to focus attention, have everyone participate in the Math Academy Chant (for younger students):

“
You and me, we all agree —
Math is fun as you will see!
It makes us think, it makes us strong,
It helps us learn even when we’re wrong.
You and me, we all agree —
Math is fun at the Math Academy!”

Introduction
Announce to the students that today they will be working for a company that invents and sells games. Tell them you have reason to believe that the company’s lead game inventor has lost his touch and has stopped creating games that are fair. If games aren’t fair for all players, they aren’t fun, and as a result, people will stop buying them. Today the students will be playing several game prototypes to determine whether or not they are fair. The goal is to find out what’s wrong with the games and to fix them before it’s too late! They will be asked to play each game prototype several times and to record the data they collect to determine if a game is fair or not. They may also be asked for recommendations on how to fix the games to make them fair.

Guest Speaker Presentation (optional)
Beforehand, arrange for someone in the game industry to talk to the students about the mathematics and probability of games. Possible guest speakers may include a chess coach, an actuary, or a game store employee.

Theme
The Game Company —
Understanding Probability Through Playing Games

Objective
Understand and apply concepts of probability while playing games to determine fairness
Getting Started (continued)

Use of Math Journals
Students record their findings in their “Math Journals” during each activity. These journals should contain all recording sheets for the activities as well as blank paper for the extension activities. Before beginning the Activities Rotation, students should spend about 10 minutes writing in their math journals, including their reflections from the assembly as well as briefly describing what they already know about probability.

Activities Rotation
If multiple classes are participating in the Math Academy, each classroom should host a different activity so students will rotate from classroom to classroom in order to complete each activity. If only one class is participating, the students may rotate from one activity to the next around the room, or they may do each activity as a whole class, one game after the other. Activities begin on page 7. For best results, plan on three to five activities for your Math Academy.

Closure and Assessment
Once all activities are completed, the students may return to their homeroom classes for final reflections and assessment. See pages 21 and 22 for sample post test and survey.

Key Vocabulary
- Probability
- Chances
- Fair
- Unfair
- Certain
- Possible
- Impossible
- Likely
- Unlikely
- Equally likely
- More likely
- Less likely
- Outcomes
- Theoretical probability
- Experimental probability
- Complementary
- Mutually exclusive
- Experiments
- Organized list
- Tree diagram
- Area model (array)
- Fraction
- Decimal
- Percent

Teacher Tips: Customizing These Activities
- Use the “Procedures” section in each activity as a skeleton for the lesson.
- Look through the “Suggestions for Customizing This Activity” section.
- Select the suggestion(s) which fits best with the probability concepts on which you want your students to focus.
Math Academy Activity 1: Race to the Top

Objective
The students will examine the probability of rolling sums, 0-12, with two dice. Older students will list all possible outcomes and the probability of each.

Materials
- Two dice, each numbered 1–6, per team
- “Race to the Top” game sheet (1 per team)

Procedures
1. Students roll the dice and record an “x” above the sum of the two dice. Students continue rolling, adding, and marking the sums for the duration of the game.

2. The first number to reach the finish line is the winner. Play should continue until the first 3 places are determined (or time runs out).

3. The winning number should now be referred to as the “mode” (most frequently occurring number).

4. After all the game sheets are complete, ask students the following questions:
   - Was this game fair for all of the numbers? (Fair means that all sums had equally likely chances to be rolled.)
   - Which sums were impossible? (0 and 1)
   - Which sums were more likely to be rolled? (5, 6, 7, 8, 9)
   - Which sums were less likely to be rolled? (2, 3, 4, 10, 11, 12)

Suggestions for Customizing This Activity

All Students:
1. Ask students to work in groups to determine a way to make this game fair for two people playing the game (e.g., player 1 gets a point when 4, 5, 6, or 7 is rolled and player 2 gets a point when anything else is rolled – the goal here is for each player to have 18 of the 36 possible outcomes).

2. Ask students to explain orally or in writing why certain sums are more likely to be rolled than others. Sophistication of answers should increase through the grade-levels.

3. Create a new game board for multiplying the numbers on the two dice.

Older Students:
4. Ask students to make a chart showing all of the sums possible with two 6-sided dice (a 6x6 array works nicely; although you may also have students design a tree diagram or other type of representation).

5. Ask the students to state the probability for each sum as a fraction, decimal, and/or percent.

NOTE
This game is non-competitive in this format. However, students will be asked to predict the game’s fairness in a situation where they are playing against other students and in which each student can only move if their number is rolled.
Math Academy Activity 1: Race to the Top

2. Less Sophisticated Sample: The numbers near the ends (2, 3, 11, 12) have fewer possible combinations. For example, 1 + 1 is the only combination that creates the number 2. The numbers in the center have more chances.

More Sophisticated Sample: There are six possible outcomes for the sum of 7, five outcomes for a sum of 6 or 8, four possible outcomes for a sum of 5 or 9, three possible outcomes for a sum of 4 or 10, two possible outcomes for a sum of 3 or 11, and only one possible outcome for the sum of 2 or 12. 0 and 1 are impossible.

4. There are 36 possible outcomes when adding the numbers of two rolled dice as shown in the array below.

4. Possible Outcomes for Adding Two 6-Sided Dice (array model)

<table>
<thead>
<tr>
<th>Sum</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimal</td>
<td>0</td>
<td>0</td>
<td>.03</td>
<td>.06</td>
<td>.08</td>
<td>.11</td>
<td>.14</td>
<td>.17</td>
<td>.14</td>
<td>.11</td>
<td>.08</td>
<td>.06</td>
<td>.03</td>
</tr>
<tr>
<td>Percent</td>
<td>0</td>
<td>0</td>
<td>3%</td>
<td>6%</td>
<td>8%</td>
<td>11%</td>
<td>14%</td>
<td>17%</td>
<td>14%</td>
<td>11%</td>
<td>8%</td>
<td>6%</td>
<td>3%</td>
</tr>
</tbody>
</table>

5. The possible sums from adding the numbers on two rolled dice are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12. The chart below shows the probability for each sum.
Race to the Top Game Board

Sum Winners

1st Place

2nd Place

3rd Place
Math Academy Activity 2: *Fair Spinners*

**Objective**
The students will determine what makes a spinner fair or unfair.

**Materials**
- Spinners
- Game Board
- Counters (for game pieces)
- Paper clips
- Paper
- Pencils

**Procedures**
1. Put the students in groups of four. One player will spin the paper clip on the spinner while the other three players move their individual game pieces based on the outcome of each spin. (*Note:* students should use the paper clip as an arrow – see diagram below.)
2. Each student should choose a game piece before spinners are passed out.
3. Give the students spinner #1 to play the first round.
4. Each player gets to move when the paper clip points to his/her color on the spinner.
5. Have students discuss why the spinner causes this game to be unfair.
6. Have students design a new spinner that IS fair.
7. Play the game again with the new spinner. (*For older students:* require that they design a spinner with more than three sections.)
8. As a class, discuss exactly what makes a spinner fair or unfair. Also discuss different ways to make fair spinners.

**Suggestions for Customizing This Activity**

**All Students:**
1. Ask students to list all of the possible outcomes for each spinner.
2. Ask students to design spinners that are more complicated but still remain fair (include several spaces, mix the colors, etc.).
3. Collect class data for which color won the first game. Ask students to create a graph to display the results.
4. Ask the students to design spinners for a FAIR game with four players.

**Older Students:**
5. Ask the students to state the probability for each spinner as a fraction, decimal, and/or percent.

**Teachers’ Answer Key**
1. Possible Outcomes for Spinner 1: yellow, red, blue

5. **Theoretical Probability for the Outcomes on Spinner 1**

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>$\frac{1}{2}$</td>
<td>0.5</td>
<td>50%</td>
</tr>
<tr>
<td>Blue</td>
<td>$\frac{1}{4}$</td>
<td>0.25</td>
<td>25%</td>
</tr>
<tr>
<td>Yellow</td>
<td>$\frac{1}{4}$</td>
<td>0.25</td>
<td>25%</td>
</tr>
</tbody>
</table>
### Fair Spinners Game Board

<table>
<thead>
<tr>
<th>Finish</th>
<th>28</th>
<th>27</th>
<th>26</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td>19</td>
<td>18</td>
<td>17</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
</tr>
<tr>
<td>Start</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

**Game 1 Spinner**

- Yellow
- Red
- Blue

**Game 2 Spinner**

- Blank
Math Academy Activity 3: *It’s in the Bag*

**Objective**  
Students will explore probability by predicting how many tiles of each color are in the “mystery bags”.

<table>
<thead>
<tr>
<th>Materials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three paper bags (label bags “1–3”; place ten colored tiles inside each bag):</td>
</tr>
<tr>
<td>– Bag 1: 8 yellow and 2 blue</td>
</tr>
<tr>
<td>– Bag 2: 5 yellow and 5 blue</td>
</tr>
<tr>
<td>– Bag 3: 2 yellow and 8 blue</td>
</tr>
<tr>
<td>1 mystery bag for each pair of students</td>
</tr>
<tr>
<td>Red and green colored tiles (10 tiles in each bag – different ratios in each)</td>
</tr>
<tr>
<td>Tally charts</td>
</tr>
</tbody>
</table>

**Procedures**

1. **Class discussion** –
   - Beginning with Bag 1, remove a tile, record its color on a tally chart, and put back in the bag. Repeat 10 times.
   - Ask the students, “Do you think there are more yellow or blue tiles in the bag? Why?”
   - Repeat with bags 2 and 3.
   - Show them the actual contents of each bag & ask if they were correct. Then say, “So…let’s say you could win $1,000,000 for drawing out a blue tile with only one try, which bag would you want to draw from?”

2. **Partner activity** –
   - Choose a mystery bag.
   - Ask the partners to pull out one tile at a time, record each tile’s color on a tally chart, and put it back in the bag. Repeat 10 times.
   - Predict if there are more reds, more greens, or an equal number of each.
   - Take out the tiles and record the number of tiles in each color.
   - Compare the prediction to the actual number of colored tiles.
   - Trade bags with another group and try again. Repeat at least three times.

3. **Class discussion** –
   - “Which mystery bags are the most difficult to predict?” (the ones in which there are almost the same number of each color)
   - “What makes a mystery bag easier to predict?” (when there’s many more of one color than the other).
   - If you were playing a game with these tiles and player A gets to move every time yellow is pulled out, and player B gets to move every time blue is pulled out, which bag would make a FAIR game for everyone?

**NOTE**

These directions mention using 1-inch plastic tiles for this game. You may use colored candies, colored disks, or any type of colored manipulative in place of the colored tiles.
Objective
Students will explore probability by predicting how many tiles of each color are in the “mystery bags”.

Materials
- Three paper bags (label bags “1–3”; place ten colored tiles inside each bag):
  - Bag 1: 8 yellow and 2 blue
  - Bag 2: 5 yellow and 5 blue
  - Bag 3: 2 yellow and 8 blue
- 1 mystery bag for each pair of students
- Red and green colored tiles (10 tiles in each bag; different ratios in each)
- Tally charts

Procedures
1. Class discussion –
   - Beginning with Bag 1, remove a tile, record its color on a tally chart, and put back in the bag. Repeat 10 times. Ask the students, “Do you think there are more yellow or blue tiles in the bag? Why?” Repeat with bags 2 and 3. Show them the actual contents of each bag & ask if they were correct. Then say, “So…let’s say you could win $1,000,000 for drawing out a blue tile with only one try, which bag would you want to draw from?”

2. Partner activity –
   - Choose a mystery bag. Ask the partners to pull out one tile at a time, record each tile’s color on a tally chart, and put it back in the bag. Repeat 10 times. Predict if there are more reds, more greens, or an equal number of each. Take out the tiles and record the number of tiles in each color. Compare the prediction to the actual number of colored tiles. Trade bags with another group and try again. Repeat at least three times.

3. Class discussion –
   - “Which mystery bags are the most difficult to predict?” (the ones in which there are almost the same number of each color)
   - “What makes a mystery bag easier to predict?” (when there’s many more of one color than the other).

   If you were playing a game with these tiles and player A gets to move every time yellow is pulled out, and player B gets to move every time blue is pulled out, which bag would make a FAIR game for everyone?

Suggestions
for Customizing This Activity

All Students:
1. Ask students to explain orally or in writing why some mystery bags are easier to predict than others. Answers will vary in sophistication.
2. As a class or in small groups, create a two-player game with each tile draw as the determining factor in which player gets to move. Students may create game boards, choose the themes, etc. You may want to connect this activity with another content area (e.g., science or social studies). Emphasize that the game must provide equally likely chances for each player to win.

Older Students:
3. Ask students to make a chart showing all of the possible combinations for ten tiles.
4. Ask the students to state the probability for each of the above outcomes as a fraction, decimal, and/or percent.
5. Ask students to create a double-bar graph representing their results from 10 draws with one bar and the actual number of tiles with the other bar (e.g., graph how many red/green drawn vs. how many red/green were actually in the bag). This helps them visualize the difference between experimental probability and theoretical probability.
6. Have the students place 20 tiles in the bag and draw 20 times.

Teachers’ Answer Key

3. Possible Combinations for Ten Tiles and the Probability for Drawing a Red Tile*

<table>
<thead>
<tr>
<th>Red</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Fraction</td>
<td>0/10</td>
<td>1/10</td>
<td>1/5</td>
<td>3/10</td>
<td>2/5</td>
<td>1/2</td>
<td>3/5</td>
<td>7/10</td>
<td>4/5</td>
<td>9/10</td>
<td>1</td>
</tr>
<tr>
<td>Decimal</td>
<td>0.0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
<td>0.6</td>
<td>0.7</td>
<td>0.8</td>
<td>0.9</td>
<td>1.0</td>
</tr>
<tr>
<td>Percent</td>
<td>0%</td>
<td>10%</td>
<td>20%</td>
<td>30%</td>
<td>40%</td>
<td>50%</td>
<td>60%</td>
<td>70%</td>
<td>80%</td>
<td>90%</td>
<td>100%</td>
</tr>
</tbody>
</table>

*designate the color for which you want students to fill out this chart
## It’s in the Bag Game Board

<table>
<thead>
<tr>
<th>Bag #</th>
<th>Circle one: 1 2 3 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tallies</td>
<td>Red</td>
</tr>
<tr>
<td></td>
<td>Green</td>
</tr>
<tr>
<td>Prediction</td>
<td>Circle one: More Red More Green Equal</td>
</tr>
<tr>
<td>Actual Number</td>
<td>_______ Red _______ Green</td>
</tr>
<tr>
<td>Did your prediction match the actual number?</td>
<td>Yes No</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bag #</th>
<th>Circle one: 1 2 3 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tallies</td>
<td>Red</td>
</tr>
<tr>
<td></td>
<td>Green</td>
</tr>
<tr>
<td>Prediction</td>
<td>Circle one: More Red More Green Equal</td>
</tr>
<tr>
<td>Actual Number</td>
<td>_______ Red _______ Green</td>
</tr>
<tr>
<td>Did your prediction match the actual number?</td>
<td>Yes No</td>
</tr>
</tbody>
</table>
Math Academy Activity 4: Odd Man Out

Objective
The students will determine the probability of getting an even or an odd product using the numbers on a regular deck of cards. In the follow-up activities, the students will prove by example why the even products are more likely to occur than the odd products.

Materials
- One deck of cards per pair of students
  (Note: Remove all face cards; Aces count as ones)
- Game board (one per pair of students)
- Game pieces (one per student)
- OPTIONAL: one multiplication chart per student

Procedures
1. Students should work in pairs.
2. Players determine who will be player 1 and player 2.
3. Players take turns flipping over two cards and finding the product. Regardless of who flipped the cards, player 1 always moves one space if the product is even, and player 2 always moves one space if the product is odd.
4. Continue playing the game until someone reaches the finish line.
5. Players switch roles and repeat the same procedure (player 1 now becomes player 2 and moves on odd products, and player 2 now becomes player 1 and moves on even products).
6. Each pair should play the game a minimum of three times.
7. As a class, discuss the fairness of this game and how to alter it to make it fair.

Suggestions for Customizing This Activity

All Students:
1. Ask the students to explain orally or in writing why even products are more likely to be created than odd products. You may want to ask the students to examine a multiplication chart showing products through 10x10 to complete this activity.
2. Collect class data on which player “won” the game. Ask students to create a graph representing the number of times each outcome “won.” Responses will vary in sophistication.
Suggestions for Customizing This Activity

Older Students:

3. Ask students to make a chart showing all of the products possible when the numbers on two cards are multiplied (a multiplication chart can be used to help them with this task).

4. Ask the students to state the probability for each product as a fraction, decimal, and/or percent. Then ask the students to state the probability for creating an even product and for creating an odd product.

5. Use the probability information from the step above to create new rules for the game wherein the even and odd players both have equally likely chances of winning. For example, an “even” product results in the “even” player moving one space and an “odd product results in the “odd” player moving 3 spaces.

Teachers’ Answer Key

3. There are 100 possible outcomes for multiplying the numbers on two cards, 1-10. Seventy-five of the possible outcomes are “even” products, and twenty-five of the possible outcomes are “odd” products.

Possible Outcomes for the Products of Two Cards

<table>
<thead>
<tr>
<th>×</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
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<td>2</td>
<td>2</td>
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<td>6</td>
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</tr>
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<td>3</td>
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<td>6</td>
<td>9</td>
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<td>24</td>
<td>28</td>
<td>32</td>
<td>36</td>
<td>40</td>
</tr>
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</tr>
</tbody>
</table>
Odd Man Out Game Board

Start

Finish
Math Academy Activity 5: *Tossing Coins*

**Objective**

The students will determine the probability for the outcomes of tossing two coins simultaneously. In the follow-up activities, the students will prove by example why there are four possible outcomes, and they all have an equally likely chance of occurring (see note).

**Materials**

- Two U.S. coins per trio of students (do not use plastic coins)
- Game board (one per trio of students)
- Game pieces (one per student)

**Procedures**

1. Students should work in groups of three.
2. Players decide who will move for each coin combination (player 1 moves if the coins land on two heads, player 2 moves if the coins land on two tails, and player 3 moves if the coins land on one head and one tail. (Note: player 3 has the advantage.)
3. Players take turns tossing coins. The player whose coin combination comes up moves one space. (Note: It does not matter who tossed the coins — players move their game pieces whenever their coin combination comes up.)
4. The first player to reach “finish” wins the game.
5. Students should play the game at least three times.
6. Collect class data to show how many times each coin combination won the game. (Note: player 3 should have won the game far more often than players 1 or 2, since the outcome of one head and one tail actually has two possibilities to occur while each of the other coin combinations has only one possibility.)

**Suggestions for Customizing This Activity**

**All Students:**

1. Ask the students to explain orally or in writing why there are four possible outcomes and why they have equally likely chances of occurring.
Math Academy Activities: Tossing Coins

Suggestions for Customizing This Activity

Older Students:

2. Ask students to make an outcome chart that shows all of the possible outcomes for tossing two coins. (Note: there are four possible outcomes with equally likely chances — 2 heads, 1 head and 1 tail, 1 tail and 1 head, and 2 tails; 1 head then 1 tail is a different outcome than 1 tail then 1 head. This is easier to see if you use two different coins, like a penny and a nickel, to demonstrate.)

3. Collect class data and create graphs to represent the number of times each outcome won. Compare these graphs to the theoretical probability.

4. Ask the students to state the theoretical probability for each outcome as a fraction, ratio, decimal, and/or percent.

5. Ask the students to toss three coins rather than just two (adapt game rules accordingly). Ask them to determine all possible outcomes (a total of eight).

6. Use the probability information from the above step to create new game rules wherein each player has an equally likely chance to win.

Teachers’ Answer Key

2. Possible Outcomes for Tossing Two Coins

<table>
<thead>
<tr>
<th>Coin 1</th>
<th>Coin 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>H</td>
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<td>T</td>
<td>H</td>
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<td>T</td>
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</tbody>
</table>

5. Possible Outcomes for Tossing Three Coins

<table>
<thead>
<tr>
<th>Coins</th>
<th>Outcomes</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>H H H</td>
</tr>
<tr>
<td>2</td>
<td>H H T</td>
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<tr>
<td>3</td>
<td>H T H</td>
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<td>H T T</td>
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<td>T H H</td>
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</table>

Theoretical Probability for Two Coins

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Fraction</th>
<th>Decimal</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Heads</td>
<td>½</td>
<td>0.25</td>
<td>25%</td>
</tr>
<tr>
<td>Two Tails</td>
<td>¼</td>
<td>0.25</td>
<td>25%</td>
</tr>
<tr>
<td>One Head, One Tail</td>
<td>½</td>
<td>0.5</td>
<td>50%</td>
</tr>
</tbody>
</table>

Theoretical Probability for Three Coins

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Fraction</th>
<th>Decimal</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Three Heads</td>
<td>⅙</td>
<td>0.125</td>
<td>12.5%</td>
</tr>
<tr>
<td>Three Tails</td>
<td>⅛</td>
<td>0.125</td>
<td>12.5%</td>
</tr>
<tr>
<td>Mixed</td>
<td>⅓</td>
<td>0.75</td>
<td>75%</td>
</tr>
</tbody>
</table>
Tossing Coins Game Board
Sample Quiz

1. You and your two friends have been asked to try out a new game. These are the rules:
   - Players take turns rolling two dice and adding the numbers together.
   - Player A gets one point if the sum is 1, 2, 3, or 4.
   - Player B gets one point if the sum is 5, 6, 7, or 8.
   - Player C gets one point if the sum is 9, 10, 11, or 12.
   - Roll the dice 15 times. The player with the most points wins.

   Is this a fair game?  ○ Yes  ○ No

   If not, which player is most likely to win the game? Explain your answer.

2. There are 5 slices of pizza in a box – 2 pepperoni, 1 mushroom, and 2 sausage. If you reach into the box without looking and take one slice, what is the probability you will get a pepperoni slice?

   [Extension: Create a diagram showing the probability of each possible outcome as a fraction, decimal and percent.]

3. Design a spinner in which 3 players would have an equally likely chance of winning. You may use words to represent colors, numbers, etc. Your spinner MUST have more than 3 spaces.

   Explain why your spinner is fair:

   [Extension: Include a diagram showing all of the possible outcomes.]

4. Rachel put 3 red marbles, 2 blue marbles, 1 yellow marble, and 4 green marbles into a bag. All the marbles were the same shape and size. Without looking, Rachel pulled 2 marbles out of the bag without putting the first back in. Which of the following would be an impossible outcome for this event?
   A. red, then red
   B. blue, then green
   C. yellow, then yellow
   D. green, then red
1. What job would you like to have when you grow up?

2. What kind of math skills do you think you will need to do that job?

3. Rank each of these statements on a scale of 1 to 5 (1 being lowest rank).

   - I like math in school.  1 2 3 4 5
   - I use math outside of school.  1 2 3 4 5
   - The math I learn at school is helpful.  1 2 3 4 5
   - I am good at math.  1 2 3 4 5
   - I liked participating in the *Are You Game* Math Academy.  1 2 3 4 5
   - I learned a lot about math during the Math Academy.  1 2 3 4 5
   - I would like to participate in Math Academies again in the future.  1 2 3 4 5

4. The best thing about the Math Academy was:

5. The worst thing about the Math Academy was:

6. Do you know what an actuary is?
Sample Teacher Survey

1. What format did you use for this Math Academy?
   ○ classroom ○ all classrooms in grade level ○ school-wide to all grade levels

2. What grade-level(s) used this Math Academy lesson? ___________________________________

3. Which Math Academy activities did you utilize with your students? 1 2 3 4 5

4. For the following two questions, please use a ranking scale of 1 through 5
   (5 = great; 3 = mediocre; 1 = terrible).
   ______ Overall, how would you rank this Math Academy?
   ______ How would you rank the activities you presented/taught?

5. Would you recommend these activities be used again?
   ○ Yes ○ No
   Comments: _____________________________________________________________

6. Do you think your students now have a better understanding of probability?
   ○ Yes ○ No
   Comments: _____________________________________________________________

7. Would you like to participate in another Math Academy?
   ○ Yes ○ No
   Comments: _____________________________________________________________

8. Please let us know how well your Math Academy went.
   _____________________________________________________________
   _____________________________________________________________
   _____________________________________________________________

9. Comments, ideas, suggestions:
   _____________________________________________________________
   _____________________________________________________________
   _____________________________________________________________

* The Actuarial Foundation is proud to offer a $50 Scholastic Classroom gift certificate to the first 100 teachers who implement this Math Academy using these materials and complete and return the teacher survey. Go to www.actuarialfoundation.org/grant/mathacademy.html to download a copy of the official survey.
One of the most beneficial aspects of our Math Academy program is the actuarial mentors/volunteers who interact with our students during each Math Academy activity. Our actuary mentors are released from their usual work responsibilities on the days of our events so they can help out in the classrooms. Please take the time to contact The Actuarial Foundation at mathacademy@actfnd.org to find out if actuaries are available in your area.

If you do not have actuaries available in your community, you may want to consider requesting the assistance of parents, community members, or business partners who would be willing to work with the students during the activities.

After securing your mentors/volunteers, it is critical to identify a Lead Mentor who will serve as the liaison between you and the other mentors. The Lead should e-mail all communications from the teachers to the mentors, set up schedules, send reminders before each event, etc.

Mentor Training Session

- Distribute the Math Academy schedule.
- Distribute copies of the Math Academy activities.
- Pedagogy – Discuss some simple teaching techniques.
- Management – Assure mentors that the teachers will handle all discipline. We also discussed preventative management techniques such as proximity and having activities well-prepared to avoid student down-time.
- Brainstorm – Allow some time during your training to take any ideas or suggestions from your mentors. We have always received great ideas for future events and speakers during these meetings. Also, allow time for questions and answers.
- Assign mentors to classrooms for consistency & relationship building – Assigning mentors to the same classroom throughout the year really helps them interact with the students and the teachers on a deeper level.
- E-mail exchange – Collect everyone’s e-mail addresses for easy communication. Also, once mentors and classroom teachers have been matched, be sure they have each other’s e-mail address so that they can communicate prior to each event.
- School tour – End your training with a school tour. Be sure your mentors know the key locations of your school including the sign-in book (and procedures), restrooms, principal’s office, and classrooms. If possible, include a map in their take-home materials so they can find their teachers’ classrooms once they receive their assignments.

WHAT IS AN ACTUARY?

An actuary is a statistical expert who determines the financial impact of randomly occurring events like earthquakes, injuries from an accident, even death. Actuaries work primarily in the insurance industry and for state and federal government agencies. Actuaries use math and statistics to determine the probability of major events when evaluating and managing risk.

To find out more go to BeAnActuary.org.

TEACHER TIPS

- The number of volunteer mentors you have will depend upon the format and number of students involved in the program. Although you don’t need volunteer mentors at the classroom level, students find these volunteer mentors to be fun.
Alignment with Standards

The unit included in this Are You Game? booklet takes into account all of the process standards outlined in NCTM’s Principles and Standards document, including communication, connections, problem solving, reasoning and proof, and representation. References to those types of processes are made throughout the activities. As for the content standards, the primary concentration is on Probability and Data Analysis. The performance objectives for each of these areas include the following:

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- Formulate questions that can be addressed with data, as well as collect, organize, and display relevant data to answer them;
- Select and use appropriate statistical methods to analyze data;
- Develop and evaluate inferences and predictions that are based on data;
- Understand and apply basic concepts of probability.

In grades 3-5, students should be able to—

- Describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible;
- Predict the probability of outcomes of simple experiments and test the predictions;
- Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.

In grades 6-8, students should be able to—

- Understand and use appropriate terminology to describe complementary and mutually exclusive events;
- Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations;
- Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.

You will want to check with your own state framework to select performance objectives which are specific to your students.
Are You Game?  
Math Academy Checklist

- Determine the date, time, and schedule for your Math Academy
- Identify the objectives to be reinforced through this Math Academy
- Plan the opening assembly, if applicable
- Confirm the schedule and content with guest speaker, if applicable
- Customize the activities enclosed in this booklet
- Make copies of the activities, quiz and surveys
- Purchase and/or gather materials
  - Dice
  - Paper clips
  - Coins
  - Cards
  - Pencils
  - Student/class gifts (optional)
- Make math journals for all students
- Distribute materials to other participating teachers, if applicable
More Resources from The Actuarial Foundation

Advancing Student Achievement Grants

Advancing Student Achievement helps support your efforts in the classroom by integrating hands-on, practical mathematics skills brought to life by practicing professionals into your everyday curriculum.

As part of that program, The Actuarial Foundation offers funding of mentoring programs that involve actuaries in supporting your school’s teaching of mathematics.

For information on this program and to find out how you can apply for an ASA grant, visit www.actuarialfoundation.org/grant/.

Shake, Rattle, & Roll

Shake, Rattle, & Roll, a sponsored education program funded by the Foundation and developed and distributed by Scholastic, the global children’s publishing education and media company, is designed to provide teachers and students with math literacy-based materials that meet national standards and are in alignment with core middle school curriculum. This skill-building program provides lesson plans, activities and other teaching resources while incorporating and applying actuaries’ natural mathematics expertise in real world situations, namely disasters.

To learn more about the program, or to download a copy, visit www.actuarialfoundation.org/youth/Shake-Rattle-Roll.html

Best Practices Guide

This guide features a compilation of research on the value of mentoring, combined with 15 case histories of programs funded by the Foundation, each of which includes information on program design and results.

To request a hard copy of the Best Practices Guide, send an e-mail to asa@actfnd.org or to download a copy, visit www.actuarialfoundation.org/grant/bestpractices.html