A Thank You to Our Corporate Sponsors

The Actuarial Foundation would like to thank the following sponsors for their generosity in funding the production of the Math Academy booklet.

By providing financial support for this project they are assisting The Actuarial Foundation in its pursuit of one of its many aspirations—to provide students with enriched learning opportunities that can help them succeed in their educational and professional endeavors.

The Actuarial Foundation and the companies listed below are collectively ensuring math education remains strong and enjoys a prosperous future.

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A Letter from the Author

Congratulations! You’ve just found your answer to the question, “What can I do to create real enthusiasm for mathematics in all my students?”

Created by teachers for teachers, the Math Academy tools and activities included in this booklet were designed to create hands-on activities and a fun learning environment for the teaching of mathematics to our students. On Math Academy days I often found that I couldn’t make it from my car to my classroom without being stopped by enthusiastic students wanting to know every detail of the upcoming day’s events. Math Academy days contributed to a positive school-wide attitude towards mathematics on our campus.

This booklet contains the Math Academy – Can You See It in Nature? Explorations in Patterns & Functions, which you can use to enhance your math instruction while staying true to the academic rigor required by the state standards framework.

This effort would not have been possible without support from The Actuarial Foundation. With the help of the Foundation’s Advancing Student Achievement (ASA) grant program, which is committed to funding projects that enhance the teaching of mathematics, this Math Academy program came into being.

I sincerely hope that you enjoy implementing the Can You See It in Nature? Math Academy with your students. When you do, you will find that your students engage with mathematics on a whole new level. The Actuarial Foundation is truly a great partner in furthering mathematics education!

For the kids!

Kimberly Rimbey, M.Ed., NBCT (Math)
Mathematics Specialist
Phoenix, AZ

If you wish to find out more about my experiences with this and other Math Academies, feel free to contact me via e-mail at mathacademy@actfnd.org.
What is a Math Academy?

When the Math Academy concept was first developed, it was designed as a half-day or full-day event which allowed students to deepen their understanding of math while interacting with volunteers from the community (see page 24 for ideas on working with community volunteers). The activities we selected for these events were hands-on, standards-based lessons which applied mathematical principles in real-world scenarios. Each student experienced three to five activities during the course of the event.

Each Math Academy began with a brief school assembly featuring a guest speaker who represented that day’s particular theme. Themes included math related to restaurants, sports, nature, shopping, fine arts and other topics, as well as focused on math-related careers. After the assembly, students rotated to different classrooms where they engaged in various activities related to mathematics and the day’s coordinating theme.

Included in this booklet is the *Math Academy – Can You See It in Nature? Explorations in Patterns & Functions*, which has all the activities we used for the patterns and functions Math Academy. This Math Academy is designed to help students understand the connection between geometric and numerical patterns (younger students) as well as between patterns and functions (older students). You may choose to implement a grade-level or school-wide Math Academy as we originally designed it, or you may prefer to implement these activities in your own classroom. Whichever format you use, keep in mind that the goal is to help your students see the relevance of mathematics in real-life contexts. If you would like more information on the set-up for a school-wide or grade-level Math Academy event, visit The Actuarial Foundation’s Web site at www.actuarialfoundation.org/grant/mathacademy.html
Getting Started

Math Academy Format

Announce to the students that today they will be working on projects dealing with patterns and functions in nature. Discuss the fact that patterns can be seen all around us and that looking for patterns and functions in nature help scientists make some of their greatest discoveries. Some people say that mathematics is the science of patterns, and this may not be far from the truth. Patterns of all kinds (numerical, spatial, sequential, etc.) help mathematicians solve problems and make new discoveries. In school, detecting and creating patterns produces better problem solvers and efficient students of mathematics. The natural themes selected include fish tails, baby snakes, picnic tables, flower petals and crystals. Although the students will not be working with real plants and animals, remind them that they will be focusing on the math knowledge that is needed to learn more about our world.

Math Academy Schedule

Schedule and times may vary depending on format being used.

- Opening assembly (optional) — 15 minutes
- Directions and Math Journals — 15 minutes
- Activity Rotations — 30–45 minutes per activity
- Assessment and Closure — 15 minutes

Opening Assembly/Directions

To build enthusiasm and to focus attention, have everyone participate in the Math Academy Chant (for younger students):

“You and me, we all agree —
Math is fun as you will see!
It makes us think, it makes us strong,
It helps us learn even when we’re wrong.
You and me, we all agree —
Math is fun at the Math Academy!”

Introduction

Announce to the students that today they will be working on projects dealing with patterns and functions in nature. Discuss the fact that patterns can be seen all around us and that looking for patterns and functions in nature help scientists make some of their greatest discoveries. Some people say that mathematics is the science of patterns, and this may not be far from the truth. Patterns of all kinds (numerical, spatial, sequential, etc.) help mathematicians solve problems and make new discoveries. In school, detecting and creating patterns produces better problem solvers and efficient students of mathematics. The natural themes include fish tails, baby snakes, picnic tables, flower petals and crystals. Although the students will not be working with real plants and animals, remind them that they will be focusing on the processes in which scientists engage to learn more about our world.

Guest Speaker Presentation (optional)

Beforehand, arrange for someone who works with plants or animals to talk to the students about the mathematics in nature. Possible guest speakers may include someone from the game and fish department, a local zoo, or a botanical

Theme

Patterns and Functions — Using Themes Seen in Nature to Examine Algebraic Relationships

Objective

Students will build, extend, describe, predict, graph, and generate rules for patterns and functions.
Getting Started (continued)

TEACHER TIPS

• We asked our students to use graphic organizers to record their thoughts and findings in their math journals. This creates a great connection to our language arts curriculum.

• The best results are achieved when 45 minutes are allowed in each classroom – about 30 minutes for the activity and 15 minutes for clean-up and reflection. The students can use their math journals to record their findings during the activities and write reflections on the journal’s blank pages.

• Following the pattern activities in this booklet, you will find a student quiz as well as sample surveys for students and teachers so that you can evaluate the success of your Math Academy. The Actuarial Foundation would love to hear all about your success and will send a $50 Scholastic Gift Certificate to the first 100 teachers who implement these activities and submit the results. For more information and to download the evaluation form, go to www.actuarialfoundation.org/grant/mathacademy.html.

Use of Math Journals

Students record their findings in their “Math Journals” during each activity. These journals should contain all recording sheets for the activities as well as blank paper for the extension activities. Before beginning the Activities Rotation, students should spend about 10 minutes writing in their math journals, including their reflections from the assembly as well as briefly describing what they already know about patterns & functions.

Activities Rotation

If multiple classes are participating in the Math Academy, each classroom should host a different activity so students will rotate from classroom to classroom in order to complete each activity. If only one class is participating, the students may rotate from one activity to the next around the room, or they may do each activity as a whole class, one game after the other. Activities begin on page 7. For best results, plan on three to five activities for your Math Academy.

Closure and Assessment

Once all activities are completed, the students may return to their homeroom classes for final reflections and assessment. See pages 21 and 22 for sample quiz and survey.

Key Vocabulary

• Pattern
• Geometric Pattern
• Number Pattern
• Growth Pattern
• Repeating Pattern
• Stage
• Recursive Rule
• Explicit Rule
• Function
• Function Table
• Horizontal
• Vertical
• Predict
• Coordinate Grid
• Coordinate Point
• Graph
• Discrete
• X-axis
• Y-axis
• Origin
• Equilateral Triangle
• Square
• Rhombus
• Trapezoid
• Hexagon

teacher tips: customizing these activities

• Use the “Procedures” section in each activity as a skeleton for the lesson.

• Look through the “Suggestions for Customizing This Activity” section.

• Select the suggestion(s) which fits best with the pattern/function concepts on which you want your students to focus.
The next three pages include activity work pages for your students to use as they record their thoughts during each activity in this booklet. Page 8 displays a format for younger students primarily focused on patterns while pages 9 and 10 display a two-sided format for older students to use when working on the pattern-function connection. The second side of this sheet provides space for the students to graph the function on a coordinate grid. Use whichever work-page best fits your instructional objectives (or feel free to reformat to fit your goals).
**Activity Work Page: Patterns**

<table>
<thead>
<tr>
<th>Task</th>
<th>Instructions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Draw and extend the geometric pattern.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Write and label the number pattern(s) to describe an attribute of the geometric pattern above.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Use words to describe the pattern.</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Draw and describe the _______ stage of this pattern.</strong></td>
<td></td>
</tr>
</tbody>
</table>
## Activity Work Page: Patterns & Functions

1. **Draw and extend the geometric pattern.**

2. **Create a function table** to describe an attribute of the geometric pattern above.
   - **Use words** to describe the pattern.

3. **Draw and describe the ________ stage.**

4. **What is the specific rule for this function?**

*continued on other side*
Activity Work Page: Patterns & Functions

Use the numbers in the function table below to create a graph of this function.

Coordinate Points:

Plot the coordinate points on the graph below. If the data is continuous, connect the points with a line. If the data is discrete, connect the points with a dotted line. Be sure to include a title and labels for the x- and y-axis (both numbers and words).

Title: 

0
Math Academy Activity 1: Fish Tails

Objective
Students will extend a toothpick pattern and describe the growth pattern associated with it. In addition, older students will describe the relationships between attributes of the pattern on a function table, with an equation, and on a coordinate graph.

Materials
- Toothpicks
- Pencils
- Work Page (or blank paper)

Procedures
1. Demonstrate the first four stages of this pattern using toothpicks to create the shapes.
2. Ask students to extend the pattern three more stages by adding one more square to the fish’s tail for each stage.
3. Ask students to record and describe their “fish” using picture, words, and numbers.
4. Ask students to predict and draw what stage 12 will look like without building the pattern any further. They should include a description of how they figured this out.

Ideas for Customizing This Activity
1. Distinguish between a pattern and a function (a pattern displays recursion within one set of numbers, usually demonstrated with a string of numbers, while a function displays the explicit relationship between sets of numbers).
2. Translate each geometric relationship into an algebraic expression or equation.
3. Have students create a function table demonstrating the number of toothpicks used in each stage of the pattern.
4. After creating the function table, ask students to generate the explicit rule (or function) which indicates the relationship between the two sets of numbers.
5. Use the numbers in the function table(s) to create coordinate points and graph the function on a coordinate grid.

Teachers’ Answer Key

Extend the Pattern:

Prediction:
Stage 12 will have 36 toothpicks.

Number Patterns:
- Toothpicks: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30 (recursive rule: add 3 to find the next number in the pattern – start with 3)

Function Table Sample:
- Function tables may be set up horizontally (as shown here) or vertically.

Number of Toothpicks at Each Stage

<table>
<thead>
<tr>
<th>Stage Number (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toothpicks (t)</td>
<td>3</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td>30</td>
</tr>
<tr>
<td>Explicit Rule</td>
<td>t  = 3n</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>f(n) = 3n</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Math Academy Activity 2: Baby Snakes

## Objective
Younger students will extend a snap cube pattern and describe the number pattern(s) associated with it. Older students will record the patterns related to a snap cube pattern on function tables, derive the functions (explicit formulas) for those patterns, and/or graph the function on a coordinate grid.

## Materials
- Snap cubes in 2 colors (the ends are one color and the middle are another color)
- Paper
- Pencils

## Procedures
1. Start a growing snake pattern using the snap cubes. The two end cubes should be one color, and all cubes in between should be the other color.
2. Snake Patterns:
   - Day 1 – red, 2 blues, red
   - Day 2 – red, 4 blues, red
   - Day 3 – red, 6 blues, red
   - Day 4 – red, 8 blues, red
3. Ask students to create the next three stages of this pattern using snap cubes.
4. Ask students to record and describe their blocks using pictures, words, and numbers.
5. Ask students to predict and draw what stage 12 will look like. They should include a description of how they figured this out.

## Ideas for Customizing This Activity
1. Distinguish between a pattern and a function (a pattern displays recursion within one set of numbers, usually demonstrated with a string of numbers, while a function displays the explicit relationship between sets of numbers).
2. Translate each geometric relationship into an algebraic expression or equation.
3. Have students create a function table demonstrating one (or more) of the following relationships:
   - Stage to blue
   - Stage to total snap cubes
4. After creating the function table, ask students to generate the function (or expression) which indicates the relationship between the two sets of numbers.
5. Use the numbers in the function table(s) to create coordinate points and graph the function on a coordinate grid.
Extend the Pattern:

![Pattern Diagram]

Prediction:
Stage 12 will have 2 reds and 24 blues for a total of 26 snap cubes.

Number Patterns:
- Red: 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2
  - (recursive rule: add 0 to find the next number in the pattern – start with 2)
- Blue: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20
  - (recursive rule: add 2 to find the next number in the pattern – start with 2)
- All Blocks: 4, 6, 8, 10, 12, 14, 16, 18, 20, 22
  - (recursive rule: add 2 to find the next number in the pattern – start with 4)

Function Table Sample:
- Function tables may be set up horizontally (as shown here) or vertically.
- Choose one of the following relationship on which to focus and use the others as extensions.

### Number of Red Cubes in Each Stage

<table>
<thead>
<tr>
<th>Stage Number (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red (r)</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Explicit Rule</td>
<td>( r = 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>( f(n) = 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Number of Blue Cubes in Each Stage

<table>
<thead>
<tr>
<th>Stage Number (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue (b)</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>Explicit Rule</td>
<td>( b = 2n )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>( f(n) = 2n )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Number of Cubes in Each Stage

<table>
<thead>
<tr>
<th>Stage Number (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Blocks (a)</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>Explicit Rule</td>
<td>( a = 2n + 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>( f(n) = 2n + 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Math Academy Activity 3: *Picnic Tables*

**Objective**  
Younger students will extend pattern block patterns and describe the number pattern(s) associated with the perimeter. Older students will record the patterns related to pattern block perimeters on function tables, derive the functions (explicit formulas) for those patterns, and/or graph the function on a coordinate grid.

**Materials**  
- Pattern blocks (triangles, squares, blue rhombi, trapezoids, hexagons)  
- Work-pages from this booklet or blank paper  
- Pencil

**Procedures**

1. Preparation: Put pattern blocks into baggies with only one shape in each bag – a minimum of 6 blocks per bag.
2. This activity encourages students to examine what happens to the perimeter of each “table” as the blocks are pushed together to create longer picnic tables.
3. Ask students to take one shape out of the bag and record how many people could sit around that table if one person can sit at each side.
4. Next, have students take out one more block and push it up against the first block so that they touch along one full side (see diagram to the left). Record how many people can sit around the perimeter of this table with this new configuration.
5. Have students continue to add one block at a time to their “picnic tables”, always pushing one full side against the full side at the end. Record the perimeter for each new “table”.

**Ideas for Customizing This Activity**

1. You may choose to put the students into groups of 2-4 students and have each group use a different shape. The students may create posters to display the geometric patterns, numbers patterns, descriptions, function tables, and/or graphs for their respective shapes.
2. Distinguish between a pattern and a function (a pattern displays recursion within one set of numbers, usually demonstrated with a string of numbers, while a function displays the explicit relationship between sets of numbers).
3. Translate each geometric relationship into an algebraic expression or equation.
4. Have students create a function table demonstrating one (or more) of the following relationships:
   - Triangles to seats(perimeter)
   - Squares to seats(perimeter)
   - Rhombi to seats(perimeter)
   - Trapezoids to seats(perimeter)
   - Hexagons to seats(perimeter)
5. After creating the function table, ask students to generate the function (or expression) which indicates the relationship between the two sets of numbers.

6. Use the numbers in the function table(s) to create coordinate points and graph the function on a coordinate grid.

Teachers’ Answer Key

Prediction:
Stage 12 for each pattern will have the following perimeters:
- Triangles: 14
- Squares: 26
- Rhombi: 26
- Hexagons: 50
- Trapezoids: 26

Number Patterns:
- Triangles: 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 (explicit rule: add 1 to find the next number in the pattern – start with 3)
- Squares: 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 (explicit rule: add 2 to find the next number in the pattern – start with 4)
- Rhombi: 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 (explicit rule: add 2 to find the next number in the pattern – start with 4)
- Trapezoids: 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 (explicit rule: add 2 to find the next number in the pattern – start with 4)
- Hexagons: 6, 10, 14, 18, 22,, 26, 30, 34, 38, 42, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 (explicit rule: add 4 to find the next number in the pattern – start with 6)

Function Table Samples:
- Function tables may be set up horizontally (as shown here) or vertically.
- Choose one of the following relationships on which to focus and use the others as extensions.

Number of Seats at Each Stage - Triangles

<table>
<thead>
<tr>
<th>Number of Triangles (t)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of “Seats” (p) (perimeter)</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Explicit Rule</td>
<td>( p = t + 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>( f(t) = t + 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of Seats at Each Stage – Squares* (same numbers for rhombi and trapezoids – just change the variable – NOTE: allow students to discover this similarity on their own)

<table>
<thead>
<tr>
<th>Number of Squares (s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of “Seats” (p) (perimeter)</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
</tr>
<tr>
<td>Explicit Rule</td>
<td>( p = 2s + 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>( f(s) = 2s + 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of Seats at Each Stage - Hexagons

<table>
<thead>
<tr>
<th>Number of Hexagons (h)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of “Seats” (p) (perimeter)</td>
<td>6</td>
<td>10</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>26</td>
<td>30</td>
<td>34</td>
</tr>
<tr>
<td>Explicit Rule</td>
<td>( p = 4h + 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>( f(h) = 4h + 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Math Academy Activity 4: Flower Petals

Objective
Younger students will extend a geometric pattern and describe it using numbers and words. Older students will extend a geometric pattern, describe it with numbers using function tables, define the function as an explicit rule, and graph the function on a coordinate grid.

Materials
- Pattern blocks (hexagons and squares)
- Work-page (or blank paper)
- Pencils

Procedures
1. Demonstrate the first three stages of this pattern.
2. Ask students to extend and record the next three stages using pattern blocks.
3. Ask students to record the number pattern associated with the number of squares at each stage. You may define how far they extend the number pattern (10 stages works well).
4. Ask students to predict, record, and describe what stage 10 will look like.

Ideas for Customizing This Activity
1. Distinguish between a pattern and a function (a pattern displays recursion within one set of numbers, usually demonstrated with a string of numbers, while a function displays the explicit relationship between sets of numbers).
2. Translate each geometric relationship into algebraic expression or equation.
3. Have students create a function table demonstrating the number of squares used in each stage of the pattern.
4. After creating the function table, ask students to generate the explicit rule (or function) which indicates the relationship between the two sets of numbers.
5. Use the numbers in the function table(s) to create coordinate points and graph the function on a coordinate grid.
Teachers’ Answer Key

Extend the Pattern:

```
Stage 10 will have 54 squares (9 squares lined up on each side of the hexagon)
```

Number Patterns:
- Squares: 0, 6, 12, 18, 24, 30, 36, 42, 48, 54 (recursive rule: add 6 to find the next number in the pattern – start with 0)

Function Table Sample:
- Function tables may be set up horizontally (as shown here) or vertically.

Number of Squares at Each Stage

<table>
<thead>
<tr>
<th>Stage Number ( (n) )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squares ( (s) )</td>
<td>0</td>
<td>6</td>
<td>12</td>
<td>18</td>
<td>24</td>
<td>30</td>
<td>36</td>
<td>42</td>
<td>48</td>
<td>54</td>
</tr>
<tr>
<td>Explicit Rule</td>
<td>( s = 6(n - 1) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>( f(n) = 6(n - 1) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Math Academy Activity 5: Growing Crystals

Objective
Younger students will extend the patterns using square tiles and describe the number pattern(s) associated with it. Older students will record the pattern related to square tile pattern on a function table, derive the function (explicit formula) for that pattern, and graph the function on a coordinate grid.

Materials
- 1-inch or 1-cm squares in two colors
- Work-page (or blank paper)
- Pencil
- Colored pencils (optional)

Procedures
1. Demonstrate the first 3 stages of the “growing crystals” pattern using squares.
2. Ask students to continue the next three stages of the pattern using squares.
3. Ask students to draw their results and describe the pattern using numbers and words.
4. Ask the students to draw and describe what the pattern would look like at stage 10.
5. Repeat with “Pattern 2”.

Ideas for Customizing This Activity
1. Distinguish between a pattern and a function (a pattern displays recursion within one set of numbers, usually demonstrated with a string of numbers, while a function displays the explicit relationship between sets of numbers).
2. Translate each geometric relationship into an algebraic expression or equation.
3. Have students create a function table demonstrating one (or more) of the following relationships:
   - Number of dark squares at each stage
   - Number of light squares at each stage
   - Number of total squares at each stage
4. After creating the function table, ask students to generate the explicit rule (or function) which indicates the relationship between the two sets of numbers.
5. Use the numbers in the function table(s) to create coordinate points and graph the function on a coordinate grid.
Math Academy Activity 5: *Growing Crystals*

**Teachers’ Answer Key**

**Pattern 1:**

**Extend the Pattern:**

![Pattern Example]

**Prediction:**  
Stage ten will have 10 dark squares and 26 light squares, for a total of 36 squares

**Number Patterns:**
- Dark Squares: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 (recursive rule: add 1 to find the next number in the pattern beginning with 1)
- Light Squares: 8, 10, 12, 14, 16, 18, 20, 22, 24, 26 (recursive rule: add 2 to find the next number in the pattern beginning with 8)
- Total Squares: 9, 12, 15, 18, 21, 24, 27, 30, 33, 36 (recursive rule: add 3 to find the next number in the pattern beginning with 9)

**Function Table Samples**

- Function tables may be set up horizontally (as shown here) or vertically.
- Choose one of the following relationships on which to focus and use the others as extensions.

**Number of Dark Squares in Each Stage**

<table>
<thead>
<tr>
<th>Stage Number (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dark Squares (d)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Explicit Rule (d = n)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function (f(n) = n)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Number of Light Squares in Each Stage**

<table>
<thead>
<tr>
<th>Stage Number (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light Squares (l)</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
<td>22</td>
<td>24</td>
<td>26</td>
</tr>
<tr>
<td>Explicit Rule (l = 2n + 6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function (f(n) = 2n + 6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Number of Blocks in Each Stage**

<table>
<thead>
<tr>
<th>Stage Number (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Squares (t)</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td>30</td>
<td>33</td>
<td>36</td>
</tr>
<tr>
<td>Explicit Rule (t = 3n + 6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function (f(n) = 3n + 6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
Math Academy Activity 5: Growing Crystals

Teachers’ Answer Key (continued)

Pattern 2:
Extend the Pattern:

Prediction:
Stage ten will have 100 dark squares and 44 light squares for a total of 144 squares.

Number Patterns:
- Dark Squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 (recursive rule: add consecutive odd numbers (beginning with 3) to the previous number to find the next number in the pattern – start with 1)
- Light Squares: 8, 12, 16, 20, 24, 28, 32, 36, 40, 44 (recursive rule: add 4 to find the next number in the pattern – start with 8)
- Total Squares: 9, 16, 25, 36, 49, 64, 81, 100, 121, 144 (recursive rule: add consecutive odd numbers (beginning with 7) to the previous number to find the next number in the pattern – start with 9)

Function Table Samples
- Function tables may be set up horizontally (as shown here) or vertically.
- Choose one of the following relationships on which to focus and use the others as extensions.

Number of Dark Squares in Each Stage

<table>
<thead>
<tr>
<th>Stage Number (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dark Squares (d)</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>49</td>
<td>64</td>
<td>81</td>
<td>100</td>
</tr>
<tr>
<td>Explicit Rule</td>
<td>(d = n^2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>(f(n) = n^2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of Light Squares in Each Stage

<table>
<thead>
<tr>
<th>Stage Number (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Light Squares (l)</td>
<td>8</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>24</td>
<td>28</td>
<td>32</td>
<td>36</td>
<td>40</td>
<td>44</td>
</tr>
<tr>
<td>Explicit Rule</td>
<td>(l = 4n + 4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>(f(n) = 4n + 4)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of Blocks in Each Stage

<table>
<thead>
<tr>
<th>Stage Number (n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Squares (t)</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>49</td>
<td>64</td>
<td>81</td>
<td>100</td>
<td>121</td>
<td>144</td>
</tr>
<tr>
<td>Explicit Rule</td>
<td>(T = (n + 2)^2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Function</td>
<td>(f(n) = (n + 2)^2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Sample Quiz

1. What are the next three stages in the following pattern? 5, 9, 13, 17, ______ , ______ , ______
2. What is the rule for extending the pattern in problem 1? __________________________
3. What are the next three stages in the following pattern? (Draw the next 3 stages.)
   
4. What is the number pattern that describes the pattern in problem 3?
   ______ , ______ , ______ , ______ , ______ , ______ , ______ , ______ , ______ , ______
5. What is the rule for extending the pattern in problem 3? __________________________
6. What are the next three stages in the following pattern? 3, 6, 12, 24, ______ , ______ , ______
7. What is the rule for extending the pattern? __________________________

For Older Students:

8. Create a function table for the pattern in problem 1 above.
   What is the explicit rule (or function) for this pattern?

   | Stage Number (n) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
   | Pattern (p)      | 5 | 9 | 13| 17|    |   |   |   |   |    |
   | Function         |   |   |   |   |    |   |   |   |   |    |

9. Create a function table for the pattern in problem 3 above.
   What is the explicit rule (or function) for this pattern?

   | Stage Number (n) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
   | X’s (x)          |   |   |   |   |    |   |   |   |   |    |
   | Function         |   |   |   |   |    |   |   |   |   |    |

10. Create a function table for the pattern in problem 6 above.
    What is the explicit rule (or function) for this pattern?

   | Stage Number (n) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
   | Pattern (p)      | 3 | 6 | 12| 24|    |   |   |   |   |    |
   | Function         |   |   |   |   |    |   |   |   |   |    |
Sample Student Survey

1. What job would you like to have when you grow up?

__________________________________________________________

2. What kind of math skills do you think you will need to do that job?

__________________________________________________________

3. Rank each of these statements on a scale of 1 to 5 (1 being lowest rank):

- I like math in school. 1 2 3 4 5
- I use math outside of school. 1 2 3 4 5
- The math I learn at school is helpful. 1 2 3 4 5
- I am good at math. 1 2 3 4 5
- I liked participating in the *Can You See It in Nature?* 1 2 3 4 5
- I learned a lot about math during the Math Academy. 1 2 3 4 5
- I would like to participate in Math Academies again in the future. 1 2 3 4 5

4. The best thing about the Math Academy was:

__________________________________________________________

__________________________________________________________

__________________________________________________________

__________________________________________________________

5. The worst thing about the Math Academy was:

__________________________________________________________

__________________________________________________________

__________________________________________________________

__________________________________________________________

6. Do you know what an actuary is?

__________________________________________________________
Sample Teacher Survey

1. What format did you use for this Math Academy?
   - ☐ classroom
   - ☐ all classrooms in grade level
   - ☐ school-wide to all grade levels

2. What grade-level(s) used this Math Academy lesson? _____________________________

3. Which Math Academy activities did you utilize with your students? 1 2 3 4 5 6

4. For the following two questions, please use a ranking scale of 1 through 5
   (5 = great; 3 = mediocre; 1 = poor).

   _____ Overall, how would you rank this Math Academy?
   _____ How would you rank the activities you presented/taught?

5. Would you recommend these activities be used again?
   - ☐ Yes
   - ☐ No
   Comments: ________________________________________________________________

6. Do you think your students now have a better understanding of patterns and functions?
   - ☐ Yes
   - ☐ No
   Comments: ________________________________________________________________

7. Would you like to participate in another Math Academy?
   - ☐ Yes
   - ☐ No
   Comments: ________________________________________________________________

8. Please let us know how well your Math Academy went.

   ________________________________________________________________

9. Comments, ideas, suggestions:

   ________________________________________________________________
   ________________________________________________________________
One of the most beneficial aspects of our Math Academy program is the actuarial mentors/volunteers who interact with our students during each Math Academy activity. Our actuarial mentors take time away from their usual work responsibilities on the days of our events so they can help out in the classrooms. Please take the time to contact The Actuarial Foundation at mathacademy@actfnd.org to find out if actuaries are available in your area.

If you do not have actuaries available in your community, you may want to consider requesting the assistance of parents, community members, or business partners who would be willing to work with the students during the activities.

After securing your mentors/volunteers, it is critical to identify a Lead Mentor who will serve as the liaison between you and the other mentors. The Lead should e-mail all communications from the teachers to the mentors, set up schedules, send reminders before each event, etc.

Mentor Training Session

- Distribute the Math Academy schedule.
- Distribute copies of the Math Academy activities.
- Pedagogy – Discuss some simple teaching techniques.
- Management – Assure mentors that the teachers will handle all discipline. Discuss preventative management techniques such as proximity and having activities well-prepared to avoid student down-time.
- Brainstorm – Allow some time during your training to take any ideas or suggestions from your mentors. Allow time for questions and answers.
- Assigning mentors – Assigning mentors to the same classroom throughout the year will help build stronger relationships with the students and teachers.
- E-mail exchange – Collect everyone’s e-mail addresses for easy communication between mentors and teachers.
- School tour – End your training with a school tour. Be sure your mentors know the key locations of your school including the sign-in book (and procedures), restrooms, principal’s office, and classrooms. If possible, include a map in their take-home materials so they can find their teachers’ classrooms once they receive their assignments.

What Is an Actuary?

An actuary is an expert who deals with numbers and percentages, also known as statistics. Actuaries provide advice to businesses, governments, and organizations to help answer questions about what to expect in planning for the future.

To find out more about the actuarial profession visit BeAnActuary.org.

Teacher Tips

- The number of volunteer mentors you have will depend upon the format and number of students involved in the program. Although you don’t need volunteer mentors at the classroom level, students find these volunteer mentors to be fun.
Alignment with Standards

The unit included in this Can You See It in Nature? booklet takes into account all of the process standards outlined in NCTM’s Principles and Standards for School Mathematics (NCTM, 2000), including communication, connections, problem solving, reasoning and proof, and representation. References to those types of processes are made throughout the activities. As for the content standards, the primary concentration is on Algebra (specifically, Patterns and Functions). The performance objectives for each of these areas include the following:

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

In grades 3-5, students should be able to—

- Describe, extend, and make generalizations about geometric and numeric patterns.
- Represent and analyze patterns and functions, using words, tables, and graphs.

In grades 6-8, students should be able to—

- Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules.
- Relate and compare different forms of representation for a relationship.
- Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

You will want to check with your own state framework to select performance objectives which are specific to your students.
Can You See It in Nature? Math Academy Checklist

☐ Determine the date, time, and schedule for your Math Academy

☐ Identify the objectives to be reinforced through this Math Academy

☐ Plan the opening assembly, if applicable

☐ Confirm the schedule and content with guest speaker, if applicable

☐ Customize the activities enclosed in this booklet

☐ Make copies of the activities, quiz and surveys

☐ Purchase and/or gather materials
  ☐ Pattern Blocks
  ☐ Toothpicks
  ☐ Snap cubes
  ☐ 1-inch Squares
  ☐ Pencils
  ☐ Colored pencils (optional)

☐ Make math journals for all students

☐ Distribute materials to other participating teachers, if applicable
More Resources from The Actuarial Foundation

Advancing Student Achievement Grants

Advancing Student Achievement helps support your efforts in the classroom by integrating hands-on, practical mathematics skills brought to life by practicing professionals into your everyday curriculum.

As part of that program, The Actuarial Foundation offers funding of mentoring programs that involve actuaries in supporting your school’s teaching of mathematics.

For information on this program and to find out how you can apply for an ASA grant, visit www.actuarialfoundation.org/grant/.

Expect the Unexpected with Math

*Shake, Rattle, & Roll* and *Bars, Lines, & Pies*, educational programs funded by the Foundation and developed and distributed by Scholastic, the global children’s publishing education and media company, are designed to provide teachers and students with math literacy-based materials that meet national standards and are in alignment with core school curriculum. These skill-building programs provide lesson plans, activities and other teaching resources while incorporating and applying actuaries’ natural mathematics expertise in real world situations.

To learn more about the program, or to download a copy, visit http://www.actuarialfoundation.org/grant/index.html.

Best Practices Guide

This guide features a compilation of research on the value of mentoring, combined with 15 case histories of programs funded by the Foundation, each of which includes information on program design and results.

To request a hard copy of the Best Practices Guide, send an e-mail to asa@actfnd.org or to download a copy, visit www.actuarialfoundation.org/grant/bestpractices.html.