Mathematics Remediation for Indigenous Students With Learning Difficulties: Does It Work?*

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Over-reliance on prescriptive pedagogies, such as explicit instruction, could hamper students with learning difficulties from sense-making and thus limit their acquisition of conceptual understanding. To help them in constructing mathematical knowledge, manipulative and drawing could be used to solve problems in a meaningful context. Using a case study design, teaching and learning process of a native teacher and her six indigenous students in a mathematics remediation classroom at an elementary school located in the interior area was investigated. Qualitative data were collected using observation, interview, and students’ work. Research findings showed that participating teacher tended to use manipulative through explicit instruction to explain meaning of number operations. Students were taught drawing to get answers for computational problems. Problem-solving process was teacher-directed but students were capable to perform simple reasoning. However, students were weak in communicating mathematical ideas. Teachers should provide hands-on activities involving manipulative or drawing that stimulate sense-making among students with learning difficulties. They should be engaged in mathematical processes rather than merely involved in re-enacting procedures that were demonstrated by their teacher.

**Keywords:** conceptual understanding, drawings, manipulative, procedural knowledge, mathematics remediation

**Introduction**

Development of mathematical knowledge is fundamental in mathematics learning. Most of the work related to mathematics learning of students with learning difficulties in mathematics focuses on the use of instructional approach which is based on a behaviorist framework of learning (Cawley & Parmar, 1992; Mercer & Miller, 1992; L. S. Fuchs & D. Fuchs, 2001; Tournaki, 2003; D. P. Bryant, B. R. Bryant, Gersten, Scammacca, & Chavez, 2008). A systematic and explicit instruction that involves CRA (concrete-representation-abstract) sequence is commonly used in providing intervention for these students. Several researches (Mercer & Miller, 1992; L. S. Fuchs & D. Fuchs, 2001; Tournaki, 2003; Bryant et al., 2008; Flores, 2009) supported the use of this strategy in facilitating a student’s understanding of mathematics ideas.

The above approach appears to be moderate for supporting students with learning difficulties in learning...
mathematics but students can only benefit from their learning if they are encouraged to think and reason (Cawley & Parmar, 1992; Ketterlin-Geller, Chard, & Fien, 2008; Moscardini, 2009). Merely performing steps in solving problems by following what is demonstrated does not help children to internalize the concepts and thus might not understand those steps. This view is supported by Ma (1999) that students’ misconceptions in mathematics are likely to be a result of being taught rules and algorithms which are demonstrated by their teacher in early mathematics. Teachers with traditional disposition might use materials to demonstrate procedures for their students to re-enact. As a result, students might be involved in learning activities that foster over-reliance on prescriptive pedagogies that prevent them from active thinking and sense-making process. Although students are able to apply certain concepts and perform procedures during initial instruction, they might not maintain their knowledge and skills over time (Ketterlin-Geller et al., 2008). Hence, the diagnostic and remedial approach which is based on traditional learning theories might limit the acquisition of conceptual understanding of students with learning difficulties. To help them in mastering both conceptual understanding and procedural knowledge, they must engage in sense-making and reasoning.

**Instructional Approach for Mathematics Remediation**

Instructional activities in this research project were planned in order to help native students at the Ulu Baram area in mastering arithmetic skill of addition and subtraction. The authors designed instructional activities that could motivate these students to gain conceptual and procedural knowledge through modelling manipulative and drawing. The design would be different from the CRA sequence used in the systematic and explicit instruction (Mercer & Miller, 1992; L. S. Fuchs & D. Fuchs, 2001; Tournaki, 2003; Bryant et al., 2008; Flores, 2009) which emphasizes the direct involvement of teachers and thus separates it from constructivist approaches. Participating teacher of this research was encouraged to provide learning environment which facilitates the use of constructivist approaches rather than explicit instruction during the teaching and learning process.

**Conceptual and Procedural Knowledge**

Students should construct their knowledge through active participation in learning activities. Instructional activities which are meant to develop students’ conceptual understanding tend to downplay the development of skill proficiency (Evans, 2007). If traditional algorithms are introduced only until students have gained a strong understanding of basic concept, students with learning difficulties might only learn traditional algorithms when they are in their fourth year of schooling (Evans, 2007). Therefore, Rittle-Johnson, Siegler, and Alibali (2001) suggested that both should be emphasized during instruction to enable students to develop their conceptual understanding optimally and become procedurally proficient as they have more cognitive resources to apply their knowledge and skills. Hence, teachers should make links between procedural knowledge and conceptual understanding explicit during instruction (Evans, 2007).

In developing students’ computational fluency with addition and subtraction, they are required to learn the procedures for particular algorithms which are supported by sufficient conceptual understanding (Reys, Lindquist, Lembdin, & Smith, 2007). Concept of place value supports the computation of whole numbers and might help students work efficiently with the algorithms. As students explore algorithms for addition and subtraction, they could participate in trading activities which are accompanied by renaming activities (Reys et al., 2007). Students could learn to quantify sets of object by grouping by 10 and use the structure of the written notation to record the information about grouping.
Modelling Manipulative

To ensure that students learn algorithms with understanding, not by rote, Reys et al. (2007) suggested the use of manipulative materials. Materials function as a link between a real-life problem situation and the abstract algorithm. It also helps students to recognize that what is written down represents real objects and actions. Through experiences in counting objects, students should be guided to build a part-part-whole schema for numbers (Resnick, 1989; Van de Walle, 2001). In the process, they need to understand the key principles of additive composition by which parts are combined to form a whole. As they develop a concrete understanding of addition, they may use a more efficient counting strategy. Fluency with basic addition and subtraction requires a great deal of experiences in counting objects (Reys et al., 2007; Byrnes, 2008). To improve students’ thinking strategies for these operations, they need to learn counting on, counting back, and compensation (Cathcart, Pothier, Vance, & Bezuk, 2011). However, according to Mayer and Wittrock (2006), when students are not used to hands-on activities, cognitive load may actually increase. Hence, students must be given sufficient time in this process (Slavin, 2009; Thompson, 1991). Besides, teachers must be aware that materials themselves carry no actual mathematical information (Moscardini, 2009; Reys et al., 2007; Thompson, 1994).

In the process of manipulating concrete materials, teachers must consciously encourage students to develop their understanding of the relationships within the number system and establish connections between concepts and processes (Moscardini, 2009).

Drawings

After sufficient manipulation with objects, students should be encouraged to make its transition to pictures and symbols (Thompson, 1991). When students could possess an image of object that no longer depends on action (N. A. Sprinthall, R. C. Sprinthall, & Oja, 1994), they should proceed to the representational level. This level of instruction provides a transition between the concrete and abstract levels. Students are expected to create drawings and use them to illustrate a mathematical procedure. Flores (2009) suggested the use of mnemonic device as a scaffold between the representation and abstract levels. It is used to help students in remembering how to solve a particular problem and provide a cue for how to proceed if they could not remember a particular fact or procedure.

Research Purpose

This research investigated the instructional practice in a mathematics remediation classroom at a primary school located at the interior area of Ulu Baram. With the understanding gathered at the site, instructional activities were planned to help students with mathematics learning difficulties. Hence, this research was carried out to investigate the teaching and learning process of students with learning difficulties during instruction based on the use of manipulative and drawings to improve their mathematical knowledge.

Method

Research Design

This research was carried out to investigate instructional practice in a mathematics remediation classroom. Hence, a case study research design (Creswell, 2008; Merriam, 1998) was used to understand the process of teaching and learning. Besides obtaining an in-depth understanding of the usual practice and later the effect of instruction planned by us, it also enabled us to reflect on that process (Merriam, 1998).
Setting and Participants

Participants of this research were a remediation teacher and her students from a primary school in the interior area at Ulu Baram. Most of the students in this school were native from nearby villages. The teacher, Miss Bong, was a native who was posted to the school and had six years of experience in teaching students with learning difficulties in mathematics. Six students were selected for this research project after administration of a screening test. All of them could count whole numbers up to 50 and could get basic addition facts by counting on from the second addend, but they still needed remedial intervention in concept of place value and arithmetic skills of addition and subtraction of whole number. All the six students are Penan children. Max and Diane were students with 4-year and 3-year school experience. Tom and Rex were with 2-year school experience. Sandy and Esther were also with 2-year school experience, but they only started their schooling two months before the research was started. In spite of such short schooling experience, these two girls were able to master skill in counting.

Collection and Analysis of Data

To understand the teaching and learning process of the usual practice and the remedial intervention prepared for this research, the authors used classroom observations, interviews with teacher, and students’ work and tests. Data collected with qualitative approach from classroom observation were recorded in the form of video clips and analyzed using a qualitative approach recommended by Creswell (2008) which involved transcribing, segmenting, coding, creating themes, and inter-relating themes. To obtain a holistic understanding of the students’ work, such as drawings and answer sheets, the authors compared it with the related video clips of classroom observation. It also enabled the authors to understand the conditions under which the students produced their work.

Validity and Reliability

Understanding is the core of this investigation. Hence, the criteria for trusting this research would be definitely different from that of the experimental study. Triangulation was used to improve the accuracy of the research findings. Evidences from different individuals, types of data, and methods of data collection could be used to support each other (Creswell, 2008). Hence, observational field notes, interviews, and students’ work were collected to enable us to examine each information and find evidence to support a theme. As suggested by Merriam (1998), it is aimed at providing “enough detailed description of the study’s context to enable readers to compare the fit with their situations”. A qualitative research aims to describe and explain human behaviours instead of confirming laws of those behaviours. Hence, to assess the reliability of documents and personal accounts, we applied techniques recommended by Merriam (1998), such as using triangulation and describing the process of data collection, analysis, and interpretation.

Instructional Activities

Conceptual understanding and procedural knowledge of addition and subtraction were emphasized and taught simultaneously. Students participated in classroom activities which were carried out in three stages.

At the first stage, the teacher explained a problem context related to a favourite leisure time activity of the students. As the boys liked to go hunting birds and fishing with their father, the problem involved addition to find the total number of fish caught by them. Another problem based on subtraction was related to the number of fish left after several fish were cooked. The students were taught to use concrete materials such as straws to illustrate a problem situation which was explained by their teacher. Subsequently, they needed to solve an
arithmetic problem based on their understanding obtained from the manipulation of the concrete materials.

Next, a problem related to the number of insects lived on two leaves was explained. The students were required to represent a problem situation using drawings after a problem situation was explained and illustrated using concrete materials. Afterwards, they were required to solve arithmetic problems using drawing.

Finally, students were asked to solve arithmetic problems using their preferred strategy. They were given individual practices with support from their teacher.

Results

Pre-intervention: Current Practice of Teaching and Learning

The children who participated in this research project preferred quiet and relaxing environment for teaching and learning. They became panic easily if their teacher talked loudly as they liked people to talk gently and slowly. They were found not used to talking in class. According to the headmaster and their teachers, these children were naturally shy and sensitive. During instructional activities, it is found that although they were slow in doing their work, they were careful and always tried to do it correctly. The students’ attendance record indicated that Rex tended to escape from school. He explained that he was not interested in school and mathematics learning was boring.

All the students could count the correct number of straws to represent a number shown to them. However, when they were asked to explain the meaning of the numeral in the tens and ones, they could not answer or show with the straws. For instance, if Miss Bong wrote “16”, the students would count 16 straws correctly and show them to the class. If she pointed to the “6” in “16”, all the students would be able to show six straws. However, if she pointed to the “1” in “16”, they showed one straw and said it is “one”.

All the students were very confident in using finger to count on from the second addend for arithmetic combinations of addition. For example, when they wanted to compute “3 + 4”, they would point to their head and say “three in head, four on fingers”. Then they would put up four fingers and say “four, five, six, seven” before writing the answer on paper. Although this strategy is considered immature, these students could perform it correctly, quickly, and confidently. For basic subtraction facts, they would put up their fingers according to the minuend in the combination. Then they would put down the fingers one by one by counting up to the number of the subtrahend. However, they did not know any strategy to get basic subtraction facts if the minuend was more than 10.

It is found that these students did not know when they should apply addition and subtraction. When a problem situation was explained to them, they did not know which number operation to use in solving the problem. For a given mathematical sentence of addition, they also could not tell a story or situation which could represent it. The concept of part-part-whole of addition was never taught. Students did not know when they should use the skill of addition. They also had difficulties in learning addition and subtraction with regrouping. Their main problem was in the procedure of regrouping due to lack of understanding in place value and the procedure.

Miss Bong usually used explicit instruction and drill-and-practice approach in the mathematics remediation classroom. First, she would explain and demonstrate the steps in solving an arithmetic problem. Students observed and listened to her explanation. This was followed by guided practice. Students were instructed to copy mathematical sentences from the blackboard and change that to standard written form. However, all of them made mistakes as shown in Figure 1. Hence, Miss Bong showed them the standard
written form for every question on the blackboard. She did not explain and students simply copied that into their exercise book.

![Figure 1. Mistakes of a student.](image)

According to Miss Bong, it is very hard to teach her students using constructivist approaches, such as guided exploration and authentic problem-solving as these students were too weak in their mathematics. She believed that they needed explicit instruction and plenty of exercises to master the basic skills and maintain these skills over time. Although she believed in behaviourist framework of learning, she admitted that her instruction was not effective for some of her students. In her opinion, these students were “just too weak to learn mathematics”.

**Explicit Instruction in Using Manipulative for Addition**

After a problem was explained, Miss Bong instructed two students to arrange two groups of straws on the table and wrote “□ + □ = □” on the blackboard. Max, Diane, Tom, and Sandy filled in the addends and total correctly. Rex and Esther did not response. Thus, Miss Bong explained very explicitly that the first two squares should be filled with the number of objects in two groups that were to be combined. Then, she instructed Max to combine the two groups of straws and counted them while Rex tied every 10 straws into a bundle. When she asked them how many bundles and how many straws they had altogether, Max counted the bundles while Rex counted the unit straws left and they told her the correct answer. Yet, when they were asked to explain the meaning of the mathematical sentence, they kept quiet.

**Explicit Instruction in Using Manipulative for Subtraction**

Miss Bong showed five straws and told the students that she had five fish. Then she asked the students to find the fish left after two fish were cooked. At first, all of them were very quiet. Suddenly, Diane suggested that three fish were left. Prompted by her teacher, she counted five straws to represent the number of fish they initially had. Then she removed two straws and counted the remaining straws to answer the question. To help students connect the ideas to its abstract representation, the teacher wrote “□ - □ = □” on the blackboard and asked them what should be filled in each square. Students managed to write a correct mathematical sentence.
Simple Reasoning Performed by Students

As first lesson was ended here, the authors asked Rex what he understood of Diane’s action and the math sentence. Surprisingly, the authors found that he was able to connect each number in the math sentence with Diane’s action. He told the authors that the sentence meant “what we have initially minus ‘take-away’ equals to what is left”.

Next, the authors asked the students to compare “+” with “−” and tell the difference between these two operations, students looked puzzled. Therefore, the authors asked them to revise the examples given by their teacher earlier: “6 + 10 = 16” and “5 − 2 = 3”. After thinking for a while, Max told us that the number became bigger in the case of “6 + 10 = 16” but the number became less in the case of “5 − 2 = 3”. Diane suggested that they “get more” in the case of addition but they “lose something” in the case of subtraction.

Transition From Manipulative to Drawing

Students were asked to perform a task individually but allowed to discuss with their peers. The teacher drew two leaves and asked students what living things would most probably live on the leaves. Diane suggested insects. Hence, the teacher told the story of two groups of insects that lived on two different leaves. This problem should be solved using addition without regrouping. Tom manipulated straws to illustrate meaning of that problem. Each student was given a piece of paper with two leaves drawn on it. First, they were encouraged to draw insects on the paper but soon they found it very hard to draw so many insects. Hence, their teacher guided them to draw simple lines to represent the number of insects in tens and in ones as shown in Figure 2. They also wrote on the paper the number of insects on each leaf. Students wrote a math sentence and found the total by counting the drawing of the number of tens and of ones. It seemed very easy for them to write the correct math sentence but they made mistakes when changing the math sentence to the standard written form. Apart from that, in case of addition with regrouping, some students tended to make mistakes after computing the numerals in the ones and writing the sum wrongly. They “carried” the wrong numeral to the tens and put the other numeral at the ones. They failed to identify the error when they checked their answers. Obviously, they were lack of experiences in connecting place value with symbols and procedures of number operations.

Figure 2

Place Value Box and Standard Written Form

Miss Bong asked the students to draw a table with two columns and three rows which we call it as “place value box”. At the top row, “pu” was written to represent tens and “sa” was written to represent ones. Then
they were guided to fill in the two addends into the middle row. All the students, except Esther, were able to fill the answer into the bottom row correctly. Obviously, Esther still could not understand the idea of place value. Hence, the teacher put two boxes in front of Esther. The left box was labelled “pu” while the right box was labelled “sa”. She asked Esther to draw a “place value box” on a piece of paper. First, she told Esther that there were six insects living on a leaf and put six straws into the “sa” box. Then she asked Esther to write “6” at the “sa” column. Next, she asked Esther to fill the second row when she put a bundle of ten straws into the “pu” box and said that another 10 insects lived at another leaf. She guided Esther to write the numerals and explained the meaning of each numeral in each place by referring to the manipulation of straws. Finally, Esther was guided to find the total number of insects and thus fill the answer in the “place value box” as shown in Figure 3.

Guided Participation in Using Drawing for Subtraction

Students were encouraged to manipulate straws and draw insects according to the problem situation mentioned by their teacher for the next two problems which involved addition with regrouping and subtraction without regrouping. The teacher prompted them to understand the connection between the manipulation of straws, drawings, mathematical sentence, and the standard written form. However, for subtraction with regrouping, the teacher thought it would be very difficult for the students and thus she decided to use explicit explanation, demonstration, and discussion with students. She explained and demonstrated a way to break a ten to ten ones by drawing. For example, to solve the problem “13 – 5”, she guided them to draw a row of ten segments and another three segments below. She explained to the students that these segments represented the 13 insects and asked students to find the number of insects left if five insects flew away. Hence, students slashed five segments to get the answer.

Behaviour of Students During Individual Practice

During individual practices, Esther had showed that she was not confident of herself in doing mathematics. She approached the teacher very often to check every step of her work and asked the teacher what she should do next. Obviously, she preferred a one-to-one instruction which is teacher-centred. Her peer, Sandy, who also started schooling only two months before this research project, was more independent. Sandy was used to sitting alone when she was doing her work. According to her, doing work together with peers would disturb her,
and thus, she could not concentrate on her work. Sandy and Esther liked to do their work at the table. Max, Rex, and Diane were flexible. They could work alone and also work with peers. They liked to work together and often discussed about the algorithm. Tom always worked alone but if any peer approached him to have discussion, he would help them willingly. All these students were allowed to choose their own learning style in order to learn mathematics in a relaxing and favourable environment (Gan & Poon, 2008) or the appropriate climate as suggested by Slavin (2009).

We found that all the students were drawing segments and lines which represent ones and tens to help them solve the arithmetic problems which involve regrouping. For other problems, they used finger-counting and count-all technique to get a basic addition or subtraction fact. None of them used straws in solving the problems. They told us that they preferred using fingers or drawings because this strategy was “easy to use”. The movement and actions during manipulation of objects were confusing.

Discussion

Tool of Demonstration and Computation

Application of explicit instruction in the mathematics remediation classroom has resulted in the role of manipulative as a tool for explaining mathematical ideas and demonstrating mathematical procedures in arithmetic. The teaching and learning process was teacher-directed. Although the curriculum for mathematics remediation is set by the administration, teachers could plan the instruction according to the learning needs and styles of their students. The instructional approaches and methods used in the classroom depend on the belief and perception of the teachers towards construction of knowledge (Westwood, 2003). As the participating teacher believed in the behaviourist framework of learning, she used manipulative to explain and demonstrate mathematical ideas to her students. For example, when she realized that her students did not understand the concept of place value, straws in units and bundles of 10 were used to help her students understand the meaning of tens and ones in a whole number. The manipulation of straws was often either carried out by herself or by her students under her instruction step by step.

Through explicit instruction, the teacher could maximize students’ learning time as suggested by Joyce, Weil, and Calhoun (2009). Although her explanation was explicit and clear, her students were not involved in decision-making and sense-making processes. They only needed to act following her instructions instead of reflecting on their actions. Using manipulative as a tool of explanation and demonstration was also found on the teachers in the research of Moscardini (2009). Those teachers also used manipulative as an apparatus for doing computation. However, our participating teacher thought that drawing should be used as a tool for doing computation as manipulative would not be provided during examinations.

Obviously, explicit instruction in using manipulative and drawing to learn mathematics might result in students’ difficulties to acquire conceptual understanding and procedural knowledge. The students were not given opportunity to solve the problems through their own initiation as the process was directed by their teacher. Thus, they were not required to justify and explain their solution. However, they were capable of representing their answer as a number sentence which is an abstract representation. The passive mode of learning might result in their weakness in conceptual understanding of place value and number operations. For example, they could not apply the concept of place value when solving arithmetic problems in the standard written form. Some students still could not understand the concept while others could not identify the connection between the mathematical ideas. This finding is supported by Ma (1999) that students’ misconception in mathematics might
be a result of rote learning in early mathematics. They might be unable to maintain their conceptual understanding over time (Ketterlin-Geller et al., 2008).

In view of the difficulties encountered by these students, teachers should reduce the use of explicit instruction in the mathematics remediation classroom. Students should be guided to participate in hands-on activities where they could use manipulative and produce drawing to gain conceptual understanding and thus master procedural knowledge and skills.

**Tool of Reasoning**

If students were provided support and prompt, they could perform simple reasoning. Diane had shown their ability to solve problems through imagination upon the objects provided. After observing manipulation done by a peer, Rex could connect the actions to the addends and sum of a number sentence. Max and Diane could make comparison between addition and subtraction after they solved problems using manipulation of objects. Apart from sense-making, all of them had used simple language to express their mathematical ideas. Hence, manipulative could function as a tool to cultivate active thinking and intrinsic motivation to mathematics learning.

Prompt and motivation provided by teacher during hands-on activities could help students to think and reflect on their learning. Besides, they could improve their language skill, particularly in speaking, as they were supported to communicate their ideas. In our research, objects enabled problem-solving without manipulating physically when Diane performed the “manipulation” mentally. Students were able to make connection between a real situation and its abstract representation if they were required to make sense. In short, teachers should reduce explicit instruction but instead always ask students to make sense and reflect on their learning.

As suggested by Slavin (2009) and Thompson (1991), time-pressure should be avoided as students must gain experiences in modelling manipulative and make sense of their actions. As contrast to explicit instruction which emphasizes teacher-directed instruction (Joyce et al., 2009), students should be allowed to perform the task individually (Reys et al., 2007) and this process might be time-consuming. The students need time to think and make the link between the actions and the mathematical ideas. Most students preferred using drawing in learning mathematics rather than manipulative as they found manipulation of objects confusing. According to Mayer and Wittrock (2006), when students are not used to hands-on activities, their cognitive load might be increased. Hence, it is important that the students are given sufficient time and experiences in manipulating objects and drawing. Another important factor in modelling manipulative and producing drawing is the use of proper language (Usiskin, 2007). Teachers should use simple but proper language in describing the manipulation or the meaning of the drawings. This would improve the language skills of students and make the link between the real situation or context and the representation explicit.

**Instructional Approach**

In this research, students were taught formal and abstract mathematical ideas throughout the process in which they had been working with. Instead of only teaching the abstract representation at the end of the mathematical process after students had gained firm conceptual understanding which was suggested by Slavin (2009), it is found that it was more efficient to learn both simultaneously during the mathematical process. Teaching both conceptual understanding and procedural knowledge simultaneously was appropriate as students could make the connection instantly. This strategy is also supported by Rittle-Johnson et al. (2001) who...
suggested an iterative model that asserted the interactive relationship between conceptual and procedural knowledge.

Conclusions

This research project focused on mathematics teaching approach which involved students in doing mathematics by manipulating concrete materials and drawing. Students were involved in instruction that emphasized extensive use of physical, pictorial, and symbolic representations of mathematical ideas. In the process, teacher tended to use that as a tool of demonstration but students with learning difficulties managed to use it in simple reasoning. To involve students in hands-on activities, teachers should avoid placing time pressure on their students. Manipulation of concrete materials and use of drawings should be based on a problem context as materials do not carry any meaning. Some students respond positively towards teaching and learning which is based on the constructivist approach and others might not, thus, teachers should be tactful and monitor the process of learning actively. For students who need explicit and individual instruction, teacher should be flexible in adjusting the instruction. The purpose is to create an appropriate climate during problem-solving process.

References


