Dilemma in Teaching Mathematics

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The challenge in mathematics education is finding the best way to teach mathematics. When students learn the reasoning and proving in mathematics, they will be proficient in mathematics. Students must know mathematics before they can apply it. Symbolism and logic is the key to both the learning of mathematics and its effective application to problem situations. Above all, the use of appropriate language is the key to making mathematics intelligible. In a very real sense, mathematics is a language. Proficiency in this language can be acquired only by long and carefully supervised experience in using it in situations involving argument and proof. Mathematics is essentially a structured hierarchy of proposition forged by logic on a postulation base. However, nowadays, the teaching of mathematics is more on focusing the mathematical procedures. Students learn variety of problem-solving in mathematics and then on its application. In addition, the teaching of mathematics is moving towards the use of mathematics in the real world. This method is said to motivate students to learn mathematics and enable them to transfer this knowledge in their daily live and future career. There are beliefs that due to the current over-emphasis on problem-solving and applications, students who enjoy working the problems in high school math, and hence decide that they like mathematics and want to major in it, find that the nature of the subject changes abruptly when they encounter the proof courses. They are bewildered and dismayed. Their previous problem-solving courses did not prepare them for this abrupt changed. Is argument and proof in the applications-problem centered domain of school mathematics being postponed, suppressed and downgraded? Are we sacrificing the essence of mathematics in order to motivate students to learn and appreciate mathematics? In this paper, the authors will discuss on this issue.

Keywords: mathematics education, real life, motivation

Introduction

The involvement of a society in mathematics is determined by cultural and functional factors. Mathematics has its own intrinsic beauty and aesthetic appeal, but its cultural role is determined mainly by its perceived educational qualities. The achievements and structures of mathematics are recognized as being among the greatest intellectual attainments of the human species, and therefore, are seen as being worthy of study in their own right, while the heavy reliance of mathematics on logical reasoning is seen to have educational merit in a world where rational thought and behaviors are highly valued. Furthermore, the potential for sharpening the wit and problem-solving abilities fostered by study of mathematics is also seen as contributing significantly to the general objectives of acquiring wisdom and intellectual capabilities. These
cultural aspects affect our society to some degree through our formal educational processes, reflecting the degree to which the society is committed to “liberal” or “humanist” education. In particular, it is a point of view adopted by many professional mathematicians in their teaching and research activities.

The “functional” aspect of mathematics stems from its importance as the language of science, engineering and technology, and its role in their development. This involvement is as old as mathematics itself and it can be argued that, without mathematics, there can be neither science nor engineering. In modern times, adoption of mathematical methods in the social, medical and physical sciences has expanded rapidly, confirming mathematics as an indispensable part of all school curricula and creating great demand for university-level mathematical training. Much of the demand stems directly from the need for mathematical and statistical modeling of phenomena. Such modeling is basic to all engineering, plays a vital role in all physical sciences and contributes significantly to the biological sciences, medicine, psychology, economics and commerce.

**Challenge in Mathematics Education**

The challenge in mathematics education is finding the best way to teach mathematics. When students learn the reasoning and proving in mathematics, they will be proficient in mathematics. Students must know mathematics before they can apply it. Symbolism and logic is the key to both the learning of mathematics and its effective application to problem situations. Above all, the use of appropriate language is the key to making mathematics intelligible. In a very real sense, mathematics is a language. Proficiency in this language can be acquired only by long and carefully supervised experience in using it in situations involving argument and proof. Mathematics is essentially a structured hierarchy of proposition forged by logic on a postulation base. However, nowadays, the teaching of mathematics is more focusing on the mathematical procedures. Students learn variety of problem-solving in mathematics and then on its application. In addition, the teaching of mathematics is moving towards the use of mathematics in the real world. According to Sargent (2000), in Md Kamaruddin and Wan Ahmad (2007b), the workplace/community learning focuses on the subject matter that connects to the workplace or the real world in America. In 1997, the Technical Education Department under the Ministry of Education, Malaysia, introduced the contextual approach where they got the concept from America, in the teaching and learning mathematics and additional mathematics in all Malaysian technical secondary schools (Md Kamaruddin & Wan Ahmad, 2007). Students were able to understand abstract concepts through concrete experiences. A good student commented that she would still understand the concept by either method but she understood faster with the contextual approach (Md Kamaruddin & Wan Ahmad, 2007). Students preferred this method, because usually they learned mathematics very mechanistic, which was memorizing the formula and solving problems using the formula (Md Kamaruddin & Md Amin, 2007a). Not only the students were able to learn faster by this method, but the lab activities helped students develop critical thinking skills (Md Kamaruddin & Wan Ahmad, 2007c). Besides using concrete experiences, the use of video clips helped students know how mathematics is used in the real world. The illustration with video attracted their attention during the lesson and this approach was able to motivate them to focus on the lessons (Md Kamaruddin & Abdul Hamid, 2005). This method of teaching mathematics is good for technical students (Md Kamaruddin & Wan Ahmad, 2006). In problem-based learning, the contextual concept is used as this method of learning is built around the delivery of the real-world learning experience (Md Kamaruddin & Wan Ahmad, 2007d). From the research done in Universiti Tun Hussein Onn Malaysia, students preferred to learn statistics using the contextual concept and they scored better than the group who did not use this concept (Md Kamaruddin & Wan
Ahmad, 2007d).

There are beliefs that due to the current over-emphasis on problem-solving and applications, students who enjoy working the problems in high school math, and hence decide that they like mathematics and want to major in it, find that the nature of the subject changes abruptly when they encounter the proof courses. They are bewildered and dismayed. Their previous problem-solving courses did not prepare them for this abrupt change. Is argument and proof in the applications-problem centered domain of school mathematics being postponed, suppressed and downgraded? Are we sacrificing the essence of mathematics in order to motivate students to learn and appreciate mathematics?

The Conflict

We think in terms of words. Yet today, anyone, who identifies lexical reasoning, based on gradually formalized natural language, as the key to the learning of mathematics, is filing a minority report. The tendency to downplay this important idea has been dominant in mathematics education circles for many years.

A somewhat parallel situation exists in English classes of secondary school. Nowadays, English teachers who seek to clarify meaning and facilitate intelligible communication by insisting on the enforcement of the accepted rules of grammar and syntax are fighting a losing battle, and worse, are patronized and ridiculed for being “uptight”. Those of us who insist on the crucial importance of lexical reasoning and proof in the teaching and learning of secondary school mathematics are in the same boat, even though this was once the dominant viewpoint. We live in an age of ambiguity where it is far more profitable to be fluent and verbose than it is to be articulate. At such a time, the learning of mathematics, which requires understanding of carefully worded expositions as well as appreciation of carefully drawn distinctions, is bound to suffer, and it is suffering. Judging by the performance levels, is the school mathematics in Malaysia in regression? Perhaps, we should reassess some of the highly controversial doctrines which have, in recent years, gained a large measure of uncritical acceptance.

At any given time, there are certain widely accepted viewpoints that are difficult to challenge. Unpopular ideas of that question, this prevailing orthodoxy are silenced or ignored, even though their logic is simple and their validity is clear.

In the present context, the widely accepted viewpoints are essentially these: If we emphasize the applications of school mathematics in a wide variety of problem-solving situations, two good things will happen: (1) The student will somehow learn the mathematics needed to solve these problems; and (2) Seeing that mathematics is useful, he/she will be motivated to learn more mathematics. This viewpoint recommends that the mathematics curriculum be organized, not around its own internal structure, but around problem-solving.

Does the current emphasis on the application and problem-solving aspects of school mathematics indicate that the utility criteria are being “strictly and shortsightedly applied”? We think it does. We think that utility is being overemphasized at the expense of certain intrinsic values of school mathematics which would serve the student better in the long run. We never hear anymore about the beauty of mathematics or about its structure and internal consistency or about mathematics as an ideal arena for the application of logic to the thinking process or indeed about any of the cultural values of mathematics that have been cherished by the race for generations.

All this is ignored by the sweeping recommendation that the mathematics curriculum should be organized around problem-solving. In fact, this recommendation almost denies that school mathematics exists as a
separate entity apart from its applications. It implies, moreover, that mathematics in and of itself is pretty dismal stuff which can interest no one, unless it is attached to some supposedly interesting situation in the “real world”. If we make this attachment, maybe some of the interest derived from these external situations will “rub off” on the mathematics, and thus, it will render more palatable to the student.

This brings us to our recommendation that natural language, gradually expanded to include symbolism and logic, is the key to both the learning of mathematics and its effective application to problem situations. And above all, the use of appropriate language is the key to making mathematics intelligible. Indeed, in a very real sense, mathematics is a language. Proficiency in this language can be acquired only by long and carefully supervised experience in using it in situations involving argument and proof.

Due to the current overemphasis on problem-solving and applications, the student of school mathematics does not get nearly enough experience with the various aspects of proof.

Can we suppress proof without distorting school mathematics and seriously impairing the students’ ability to understand it? Mathematics is essentially a structured hierarchy of propositions forged by logic on a postulation base. For how long, do we protect our students from the pedagogical consequences of this fact? And indeed, whom are we protecting? Those who advocate this language of logical discourse are seeking clarity of exposition rather than pretentious rigor in proof.

Also, due to our preoccupation with applications, there is not nearly enough time spent in deriving key propositions and theorems. When you and the author say that we understand the theorem “the determinant of a square matrix is zero if and only if its rows are linearly dependent”, we are saying that we understand how this theorem fits into a hierarchy of propositions, and could given time, derive it from first principles.

Why should our students be any different? To be sure, they are working at a more elementary level. But, this nagging question remains: What basis do they have for understanding anything without seeing how it fits into a structure based on something?.

Yet, when high school students say that they understand the formula for the cosine of a difference, they generally mean that they have memorized the formula and can apply it. They generally do not mean that they can derive it from first principles. Very often they do not realize that this essential dimension of understanding even exists, i.e., their understanding is deficient. So, they get by on memory and facility until the cumulative effects of these deficiencies ultimately overwhelm them, and they leave mathematics in frustration and despair.

We can prevent this by equipping the student with the essentials of the language needed to understand mathematical reasoning.

Conclusions

Due to our reluctance to use a handful of universally valid and easily understood logical principles, we are denying our students the opportunity to understand.

For too long, have we sheltered our students from explanations deemed to be too difficult for them? When explanations are inadequate or missing entirely, students attribute their resulting lack of understanding to the idea that there is something abstruse and forbidding about mathematics which they will never be able to understand. The result may be permanent alienation from both mathematics and the mathematical sciences. This alienation not only cripples thousands of students, it cripples our economy and our defense posture as well.

Problem-solving, important as it is, does not provide adequate preparation. Many students who enjoy
working the problems in high school math and even in calculus, and hence decide that they like mathematics and want to major in it, find that the nature of the subject changes abruptly, when they encounter the proof courses which follow calculus. They are bewildered and dismayed in courses, such as Linear Algebra, Advanced Calculus and Topology where proof is the name of the game. Their previous problem-solving courses did not prepare them for this abrupt change in emphasis. Their rude awakening often comes so late that it is difficult to change majors. This is tragic. As educators, we have become increasingly aware of the dilemma facing those teaching mathematics. How can we best provide an appropriate environment to motivate students whilst maintaining its essence?

References