US-China Education Review B 1 (2012) 17-30
Earlier title: US-China Education Review, ISSN 1548-6613

Elementary School Teachers' Understanding Towards the Related Knowledge of Number Sense

Yea-Ling Tsao
Minnesota State University, Mankato, USA

Yi-Chung Lin
Taipei Municipal Jingxing Elementary School, Taipei, Taiwan

The goal of this study was to investigate understanding of in-service elementary school teachers in Taiwan about number sense, teaching strategies of number sense and the development of number sense of students. Data were gathered through interviews of nine elementary mathematics teachers, regarding their understanding about number sense. The data indicated the followings: (1) The intuitive of number sense of teachers described as good intuition or sensitive about number sense. They stress on understanding number meanings and ignore these relationships about quantity and operation; (2) The teachers have similar views about teaching strategies of number sense; (3) The teachers believe that children with good number sense have well-understood number meanings and relationships, awareness that multiple strategies exist, and recognize reasonableness of data and calculation; (4) The teachers lacked for knowledge of number sense, so that they cannot address the complete teaching strategies of number sense; and (5) The background variables of teachers are related to the understanding of number sense.

Keywords: number sense, in-service teacher, teaching

Rationale and Purpose

Number sense is highlighted in current mathematics education reform documents, because it typifies the theme of learning mathematics as a sense-making activity (NCTM (National Council of Teachers of Mathematics), 2000). The development of number sense is important in mathematics education. The NCTM (2000, p. 32), in their "Principles and Standards for School Mathematics", noted that number sense is one of the foundational ideas in mathematics in that students: (1) understand number, ways of representing numbers, relationships among numbers and number system; (2) understand meanings of operations and how they related to one another; and (3) compute fluently and make reasonable estimates.

Over the past decades, considerable research has dealt with the mathematical performance of elementary school students, but far less research has dealt with what their teachers understand and teaching practice about number sense. The few studies that have investigated the mathematical understanding of elementary teachers and pre-service elementary teachers indicate that many exhibit weakness in mathematics and their, may misapply mathematical rules, do not understand true meanings of mathematical concepts, and that they are, generally, not prepared to teach the mathematical subject matter entrusted to them (Cuff, 1993; Tsao, 2004, 2005; Yang, 2007).

Yea-Ling Tsao, Ph.D., associate professor, Department of Mathematics and Statistic, Minnesota State University.
Yi-Chung Lin, Taipei Municipal Jingxing Elementary School.

In that elementary, teachers provide the first formal mathematical training children receive, and it is reasonable that the educators responsible for preparing them to teach should know what skills they possess and what skills they lack in order to design their curricula. Although considerable attention to number sense is occurring in Taiwan, the term "number sense" is rarely heard in teacher practice research. Believing that understanding the level of number sense should play an important role in pre-service teaching programs and professional development, the motive for conducting this study rises from a deep concern for the development of number sense for all teachers.

Research Question

The research question is: What is the understanding of Taiwan elementary school teachers with respect to the significance and importance of number sense, the strategies to involve number sense in the instruction and the children's number sense development?

Literature Review

The "Commission on Standards for School Mathematics" of NCTM in 1987 described children with number sense as those children who understand number meaning, develop multiple relationships among numbers, know relative sizes of numbers and comprehend how arithmetic operations affect results (Howden, 1989). It seems intuitive that students who have more opportunities to learn and explore mathematics would develop greater number sense. The NCTM "Curriculum and Evaluation Standards" (1989, p. 38) defined that,

> Children with good number sense: (1) have well-understood number meaning; (2) have developed multiple relationships among numbers; (3) recognize the relative magnitude of numbers; (4) know the relative effect of operating on number; and (5) develop a referent for measures of common objects and situations in their environment.

Three of these components—developing number meaning, understanding relative size of numbers and developing familiar referents—may be considered understanding that is related to number sense. Exploring number relationships with manipulatives is a teaching method used to promote the student's development of number sense and other ideas. The last component, developing referents for measures of common objects and situations, utilizing operation sense, as well as number sense.

Several researchers cited earlier made suggestions about how to facilitate students' development of number sense (Gurganus, 2004; Hope & Small, 1994; Reys, 1994; Sowder, 1992). These researches provide the following list of factors as an overview of general suggestions to facilitate number sense development: (1) working with concrete materials and familiar ideas; (2) discussing and sharing their discoveries and solutions; (3) composing and recomposing different arrangements and representations of numbers; (4) investigating the realistic uses of numbers in their everyday world; (5) exploring number patterns and number relationships; (6) creating alternative methods of calculation and estimation; (7) solving realistic problems using a variety of approaches; (8) calculating for the purpose rather than for the sake of calculating; (9) gathering, organizing, displaying and interpreting quantitative information; (10) understanding what characterizes number sense and need to prepare activities that present students with opportunities to explore within that framework; (11) measuring and then making measurement estimates and planning powerful estimation experiences; and (12) exploring very large number and representations and providing experience with number line.

Reys (1994) pointed out that teaching for the development of number sense requires conscious and coordinated effort to build connections and meaning on the part of the teacher. Teachers play an important role

in building number sense in the type of classroom environment they create, they employ in the teaching practices and they select in the activities. Some strategies teachers might consider when teaching number sense are to: (1) use process questions; (2) use writing assignments; (3) encourage invented methods; (4) use appropriate calculation tools. Number sense can be promoted by ensuring that students learn to calculate in various ways including written, mental, approximate and electronic methods; (5) help students establish benchmarks. Approximate computation or estimation is another important tool for encouraging students to use what they already know about numbers to make sense of new numerical situations; and (6) promote internal questioning. An important role for teachers in the development of number sense is helping students learn to ask themselves key questions before, during and after the solution process.

Over the past two decades, studies have highlighted that teachers play a key role in helping children develop number sense through well-designed number sense activities and by creating a good learning environment and promoting class discussion (Markovits & Sowder, 1994; Reys, 1994; Tsao & Rung, 2006, 2007; Cheng & Tsao, 2008). Ma (1999, p. 144) also pointed out that, "While we want to improve students' mathematics education, we also need to improve their teachers' knowledge of school mathematics". While there is no doubt that the goal is to improve number sense in children, we also need to examine ways of improving the number sense of teachers.

At present, the term "number sense" is not prevailing in Taiwan and most of the teachers have never reached number sense, not to mention the attention on number sense instruction. However, the studies related to the teachers' cognition of number sense and instructions are still insufficient. Yang (1998; 2000; 2002) has managed some studies upon the students' capacities used when answering the questions of number sense and we still need further research on the teachers' roles in number sense instruction. At the time, when grade one to nine curriculum guidelines proposed by Ministry of Education in Taiwan value number sense instruction, the exploration on teachers' recognition on number sense and teaching practice becomes more significant. Therefore, this research used qualitative research method to explore elementary school teachers' understanding toward the related knowledge of number sense. The result of this research will provide important data, allow the teachers to value the students' number sense development and improve the students' mathematics and problem-solving performances as the base for teacher educator to improve and strengthen on-the-job teachers' mathematical knowledge or for teacher education institutions to design mathematics curriculum.

Methodology

Research Design

This research adopted qualitative research method and since qualitative research valued in-depth and detailed exploration, the number of samples was usually limited (Bogdan & Biklen, 1982). Upon this consideration, this research used small number of samples to proceed with in-depth interview and expected to acquire profound and abundant data. The research subjects were the mathematics teachers of the grade six in elementary school. At the beginning of the interview, the researcher initially accessed to the nine teachers' backgrounds, history of mathematics learning, mathematics learning methods and the teachers' perception of the purpose of mathematics education. Subsequently, the researcher used the research tool of "the significance of number sense, general situation of number sense combined in teaching and children's number sense ability development" to interview the teachers in order to understand the participant teachers' number sense, the related knowledge of number sense combined in teaching and their knowledge of children's number sense

ability development. Each teacher received the interviews for three times and the interview time was about 90 minutes.

Instruments

The main interview contents included three sections. The first section was the teachers' recognition towards number sense. For example, have they ever heard of this term "number sense"? How would they explain it? What is its importance?

The second section included the general situation of number sense integrated in teaching. The design of interview content focused on the general situation of how the teachers combined number sense in teaching and the teachers' views of problem-solving teaching. In addition, as to the interview of problem-solving teaching, the researcher designed three questions to understand how the teachers develop problem-solving process. For example, one sample of question, "How do you allow the students to understand the perspective that there are infinite fractions between $\frac{4}{9}$ and $\frac{5}{9}$?".

The third section referred to the teachers' cognition towards the children's number sense development. The main interview content included: for the teachers, what kinds of mathematical abilities should the students possess? What kinds of conditions do the students with number sense capacity (characteristic) have? The fraction addition question, such as $\frac{2}{3}+\frac{1}{2}=\frac{3}{5}$, reveals a wrong answer. Please do not calculate and describe the reason. The researcher used above to explore if the teachers understand the purpose of this kind of question.

Research Subjects

The research subjects were the mathematics teachers of the grade six in elementary school. The researcher chose nine teachers in Taipei City who were willing to cooperate and proceeded with semi-structural in-depth interview. The participant teachers were the volunteer teachers for this research. For respecting the participants, the teachers' name mentioned in the article would be represented by code names. The researcher respectively used A, B, C, D … as the code names of nine teachers. The following is the descriptions of the participants' backgrounds.

Teacher A. After graduating from Physical Education Department of Teachers College, in order to upgrade the professional teaching knowledge, Teacher A attended department of language and literature education of teachers college to study the related courses. Currently, Teacher A was studying in graduate school of physical education. During his 16-year teaching experience, he has been physical education teacher for five years and nature teacher for five years. Until recent six years, he started to be the guide teacher of senior grade and began to teach mathematics. Teacher A thought that during the whole mathematics teaching process, a teacher should understand the students' thoughts, needs and the capacities lacked at any time and then gradually lead the students to have some interests for mathematics and increase the students' mathematics capacity in lives instead of resisting mathematics.

Teacher B. After graduating from Physics Department of University, because of the enthusiasm for education, Teacher B decided to become a teacher. Thus, he attended teacher education course of teachers college to study elementary education curriculum. After practicing for one year, being the guide teacher of medium grade for three years and high-grade guide teachers for four years, Teacher B is currently the guide teacher of the grade six students. Teacher B's mathematics performance was outstanding, since he was young

and he had great interest in mathematics. For the mathematics concept, he did not understand and he would try the best to understand it instead of memorizing the formulas and rules. After being a teacher, he also actively participated in the related studies of mathematics in order to increase his mathematics teaching capacity.

Teacher C. Teacher C graduated from Department of Elementary Education of Teachers College and practiced in medium grade for one year. After becoming the formal teacher, he has been the fourth grade guide teacher for one year and senior grade guide teacher for six years. Teacher C liked to accept challenges. She who was interested in mathematics would ponder on the questions she has never met and solve the problems with her own solution. Thus, Teacher C, mathematics was not a difficult subject.

Teacher D. After graduating from Department of Oceanography of University, Teacher D has worked in a private institute and been the substitute teacher in junior high school. He then attended teacher education course. He had 12-year guide teacher experience in senior grade. Teacher D's teaching was interesting and he was patient, kind and expected to cultivate the children's active learning and independent thinking habit and allowed the children to cooperate with the group, learn to communicate and have the abilities of negotiation and problem-solving to create a safe, harmonious and pleasant learning environment.

Teacher D gained the sense of achievement from mathematics learning. Thus, he thought that mathematics was not a difficult subject and had great interest in mathematics. Teacher D expected himself to guide the students by encouragement during the mathematics teaching process to make the students like mathematics and feel that mathematics was not a difficult subject.

Teacher E. Teacher E had the teaching experience for only less than three years and was considered as a young teacher. After graduating from Department of Elementary Education of Teachers College, Teacher E was the grade two practice teacher for one year, the grade three guide teacher for one year and the grade four guide teacher for half year. Currently, she was the grade six guide teachers. Thus, she only had one-year grade six mathematics teaching experience. In mathematics teaching experience, Teacher E emphasized the construction of the basic concept of mathematics. With solid mathematical basic capacity, the students could have good mathematics ability. Teacher E also believed that the grade six students had already had higher level of mathematics ability. Thus, she also provided the mathematical questions related to mathematics curriculum in junior high school to allow the students brainstorm and upgrade their mathematical thinking.

Teacher F. Teacher F graduated from Chinese Department of University. After finishing the teacher education courses, he/she practiced the training, taught in senior grade and had the teaching experience for six years. Teacher F was a serious teacher, valuing the students' individual difference and interacting well with the parents and students. Since she has learned mathematics by memorizing formulas since children, Teacher F did not really understand the concept of mathematics. During the process of mathematical teaching, the teacher was more likely to understand the thoughts of the students with unsatisfying mathematics performance. She guided the students to learn mathematics through the method she could understand.

Teacher G. Teacher G graduated from Art Education Department of Teachers College. In her 11-year working experience, she had two-year administration experience and has been the art teacher for three years. For the recent four years, she has been the senior grade guide teacher. In recent years, she worked hard to cultivate herself and has been graduated from institute of visual art education. The teacher, thus, had special talents in art and human aspects. Teacher G was devoted to the students and expected the students to have diverse skills. Since she contacted mathematics teaching in recent years, she had close connection with mathematics. Therefore, mathematics for her who studied art was not as important.

Teacher H. Teacher H had abundant teaching experience. After graduating from Language Department of Teachers College, Teacher H became a teacher. In order to increase her teaching effects, she attended the Chinese department of University to study professional teaching capacity. In 26-year teaching lives, she has been senior grade guide teacher for 11 years. At elementary school and junior high school stages, besides the teachers' instruction, Teacher H made efforts practicing calculation every day and establish good mathematical base. Thus, in present mathematical teaching, she could provide the students clear mathematical concept. Teacher H has also been the practical training guidance teacher for years, and she worried that the system of present teacher education act was not complete and it could not cultivate the fresh teachers with correct academic knowledge. The education for future hopes of the country was actually a worry.

Teacher I. Teacher I graduated from Mathematics and Physics Education Department, and had been the substitute teacher of the grade four students for one year and senior grade guide teacher for four years. She was studying in the graduate program of mathematics and computer science education, mastered mathematics education and had unique opinions about mathematics. In mathematics teaching, he emphasized the in-depth understanding of mathematics concepts and cultivated the students' ability of independent thinking. Therefore, Teacher I indicated that for the students' understanding of mathematics, the teachers played an extremely important role. She held the teaching idea of "happy learning and happy growing" and developed the students' active and useful capacities.

Results and Discussions

Teachers' Related Knowledge of Number Sense

This section was divided into four parts. The first part was to explore the participant teachers' understanding towards the significance and importance of number sense. The second part analyzed the participant teachers' strategies of number sense combined in teaching mentioned in the interview. The third part explored the participant teachers' understanding towards the children's number sense development. The last part analyzed the number sense cognition of the teachers with different learning and teaching backgrounds.

Teachers' understanding toward the significance of number sense and importance. The following were the discussion and analysis of the teachers' understanding towards the significance and importance of number sense.

At present, not many elementary school teachers have heard of the term "number sense". Among nine participants, six of them heard "number sense" for the firs time. Among others, Teachers A and B, the author have heard "number sense". However, only Teacher A had deeper study on number sense. Teachers B and I have heard of or seen this term once.

> Teacher A: … the children with better number sense ability might manage deeper questions, such as the jumping pattern of 2, 4, 6, 8 and 10 I mentioned. He can even control the amount and order of numbers. For example, he could reverse 1, 2, 3, 4, 5 and 6 into 6, 5, 4, 3, 2 and 1. In other words, he can grasp the amount of the numbers. In fact, the number sense capacity is connected to his calculations of numbers in the future. The children's number sense ability is not established in his childhood and not even in low grade which will affect some of his numbers checking calculation. … he cannot think in other direction. He can only think by one and single direction…. my wife works in children education circle and finds out that the culture at home stimulates the children's concepts of numbers. It is very important… actually, since his childhood, he has some initial concept with what he has seen and heard. He has this concept already….

Teacher A indicated that number sense was "a kind of instinct toward numbers and it was developed

gradually". Number sense was built from the children's daily life experience since their childhood. By the cultural stimulation of number concepts at home, the children would develop their own understanding toward number pattern and could further judge the amount of numbers and understand the influence of arithmetic on numbers. After entering the school, the conceptual knowledge of numbers would develop with the related process knowledge, such as number counting skills, the identification and writing of numbers. The children could master the understanding and application of the number concept, think flexibly and have logic reasoning. Thus, their number sense would be more complete.

Eight out of nine participant teachers described the direct perception that number sense was "the feelings toward numbers" and a kind of sensitivity toward numbers. Teacher A indicated that number sense should be established since childhood and be constructed by gradual development. If the children's number sense was not established well in childhood, it would affect his/her mathematics learning in the future. Only Teacher E thought that number sense was a kind of connection of number concept and high level thinking of logic reasoning.

Teacher A pointed out that in childhood, number sense has started to be developed. In daily lives, parents counted the fingers and toys for their children. The pre-schoolers would gradually have some ideas of numbers and counting numbers. The establishment of the initial number sense would further affect their arithmetic learning in the future. The logic thinking of numbers, arrangement of numbers and the model of problem-solving strategies would make the children to have more feelings towards numbers with their growing. They would have some basic concepts instinctively for the numbers they saw and heard.

> Teacher E: At the beginning, I would think about music sense. In music, it means the rhythm. I grasp the tempo and easy… if I transform it here, I think, in terms of mathematics, it becomes the relation addition, subtraction, multiplication and division and numbers. With regard to the teaching of speed, when I apply the relation among distance, speed and time, it is smooth to some degree. I think number sense should be like this.

Teacher E defined number sense as the connection of number concept to understand the relations among numbers, the basic arithmetic of addition, subtraction, multiplication and division, logic reasoning and thinking and being able to construct their own problem-solving strategies and successful problem-solving. They gained the sense of achievement from problem-solving, because they liked mathematics.

Teachers C, D, F and G defined the number sense in narrow sense that it was the understanding toward the significance of numbers and dealing with numbers as quantities instead of treating numbers as the instinct and perception of abstract and formal things.

> Teacher F: It might be the students… more sensitive to numbers. He knows the meanings and for him, during the process of arithmetic, he would not manage addition when he sees the numbers or only do some simple addition and subtraction and would not.

> Teacher G: … actually number sense is the sensitivity to numbers. He might acquire it through constant practice and generalize in his mind. Sometimes, the teachers have to remind them of this, such as ratio of the circumference, 3.14…

According to the above data, the researcher found out that the participant teachers neglected the communication of conceptual network of the situations generated from arithmetic, numbers and arithmetic and the capacity of meta-cognition.

Only Teachers E and F indicated the importance of number sense in daily lives. Teacher G thought that in daily lives, the application of number sense or mathematics was not relatively important.

> Teacher E: … as to the children with stronger number sense ability, for example, when we just finished a charity bazaar, it was easy for him to judge, if they had profits in this business or he could be sure that he would earn the money. Thus, in the price-fixing or preparation in advance, he would not buy the things with high unit prices in case that he would not be able to sell the things or lose money. Then, I observed the children with worse number sense ability and I found that their costs were higher and they sold the same amount. Thus, he lost some money…
>
> Teacher F: I think he can understand it logically and then he will be able to solve the problems in daily lives. I think it is important.

Based on the views of two teachers, we realize the influence of arithmetic on numbers and have efficient. In logic thinking, the children will be more able to understand the significance of mathematics and find out efficient and rapid problem-solving strategies and they can also apply mathematical knowledge to solve the problems in daily lives.

Teacher B proposed the importance of number sense in integer scale. However, in terms of decimal and fraction, the characteristic of number sense was not easy to reveal. In senior high school, in particular, and the mathematics learning in university, the position of number sense was even less significant.

> Teacher B: … For example, when your numbers are more and more difficult, such as fraction, decimal fractions or even unknown numbers. It will become more and more difficult. You see, what they learn in senior high school and university does not include so many numbers! I think when you talk about number sense at last, it becomes less important!

Generally speaking, Teacher B thought that number sense was extremely important for mathematical "integer" learning. Number sense meant the sensibility to numbers. When people are highly sensitive to numbers, they are more likely to understand the influence of arithmetic on numbers and access to the significance of arithmetic to quantities and the change of size. During the process of problem-solving, they could judge and reflect the rationality of the strategies. However, as to non-integer mathematics, such as fraction and unknown numbers or even the mathematical learning in senior high school and university. Teacher B thought that number sense became less important.

In current teaching, the teachers tended to value the precise answers and mathematical teaching at school also spent most of the time to make the students have the mechanical practice of calculation which resulted in the situation in which the children's inherent instinct and flexible thinking of mathematics aspect were gradually limited to rapid and effective arithmetic rules and formulas. Thus, the teachers' in-depth understanding toward number sense, spending more time to maintain and increase the children's number sense capacity and flexibly using proper, creative and efficient strategies to solve the problems will increase the students' interests of mathematical learning.

Analyzing the teachers' understanding toward number sense combined in teaching strategy in the interview. The instruction of number sense could allow the students to actively perceive, discover and explore, they could recognize that mathematics was around us during the process of learning mathematics. Using mathematical knowledge could explain the mathematical phenomenon in lives and solve the problems in lives. How to combine number sense in mathematics teaching? The teachers should have the basic knowledge of mathematics and understand the students' development of mathematics concept. Besides, many experts and scholars pointed out that interesting and live curriculum design could construct the students' clear mathematical concept which emphasized meaningful learning instead of traditional arithmetic and formula. It could often use concrete objects and lines for instruction, and propose open questions which could frequently be combined with daily life experience. Using process questions and mathematical writing, providing irregular problems,

encouraging the students to discuss and present various problem-solving strategies, frequently examining the rationality of answers, repetitive practice after class, encouraging the children to read the related books of mathematics, frequently establishing students' reference point in teaching and having classroom decoration were all the methods which could increase number sense.

According to the above views, the researchers explored the participant teachers' understanding towards number sense combined in teaching strategies. Eight out of nine participant teachers thought that in mathematics teaching, constructing the students' clear basic mathematics concept was the most essential law to increase number sense. Teachers A and D pointed out that the deeper and broader concept was one of the strategies to develop number sense.

Teacher A indicated that the grade six children were going to enter the curriculum of junior high school. Thus, when having the instruction, they deepened and broadened the questions with respect to the mathematical ability they must learn after attending junior high school and asked the students to discuss and manage the problem-solving of their current mathematical knowledge and abilities. The teachers properly participated in to strengthen the students' concepts and help the students to understand the meaning implied in the questions to construct efficient problem-solving strategies. However, after the introduction of concepts and being familiar, the mathematical concepts and the mastering of calculation could not be neglected. Teacher D thought that the mathematical instruction of daily lives could skillfully increase the students' number sense. For example, as to the sensitivity to numbers, besides encouraging the students to observe the relations among numbers, the teachers could also present by different expressions with deeper and broader methods.

Six participant teachers, Teachers A, C, F, G, H and I, thought that practice was necessary for the upgrading of number sense. After constructing the mathematical concepts, too complicated calculation or process would reduce the students' learning motivation and they would be lack of confidence when operating the numbers. Only the master of practice could make the students control the amount of numbers, the influence of arithmetic on numbers and the relations among numbers which allowed the students to precisely, rapidly and successfully solve the problems. Their number sense ability was also increased. However, noticeably, as to this kind of practice of traditional calculation process, the students tended to "to know the hows but not the whys" (Yang, 2000; 2002).

> Teacher H: … for the children with good quality, approaching him more, I think that more practice makes better mathematics! His pattern practice… I saw the news last time and it is two and three years ago! The contestant of International Mathematics Olympic should join the game in the world. I saw the champion from Europe or something; he said in one month, he was trained with 1,000 mathematical questions.

This kind of mechanical operation of arithmetic rules or formulas was meaningless for the students. Only the students mastered the rules and formulas of arithmetic, increased their understanding toward the relations among numbers and arithmetic, and internalized them as their own knowledge and reflecting instinctively could the number sense ability be developed.

Five teachers (Teachers A, C, D, E and G) proposed that classroom discussion could increase the students' number sense. In the teaching activities, the teachers focused on the arrangement of the questions and created the situations for the students to explore and learn. During the discussion between the teachers and students, mathematical essence was controlled and the students' learning keys were examined. Through debate and clarification, the students could really understand the content of the courses and finally had feedbacks and

reflection.

> Teacher E: I think the discussion in class is very important. …although I use discussion to enhance my teaching, it is useless for some kids with inferior level! Sometimes, I would increase the students' interests of mathematics learning though games and encourage them, so that they would be more interested in the learning.

Thus, five participant teachers thought that with the influence of class discussion culture, when the speakers described the process to others, they would certainly review the concept in their minds or the problem-solving process. The reflection process allowed the learners to acquire the shared characteristics of the similar questions to form the concepts or problem-solving strategies. The discussants would compare their own solutions with the speakers and reach the purpose of reflection. In addition, from the process of skepticism and debate could also accelerate the construction of mathematics concept and problem-solving strategies and further increase the number sense.

Five participant teachers (Teachers A, B, C, D and I) thought that the operation of concrete objects and semi-concrete objects could help the students to "discover" the relationship between concrete object world and formal mathematics and further gradually cultivate the basic concepts and arithmetic. During the simulation situation, the students understood the meaning of mathematics which could help the students construct solid connection between mathematical symbols and significance and internalize number sense.

> Teacher D: For example, for this kind of comparison among numbers, I would draw the lines to make them judge and tell them, see! You do not have to count. You can find out by drawing directly. We can use lines like this. In fact, I think now some of the courses allow the kids to understand the relations among numbers. For example, they can use simpler numbers to count the numbers! There are many related questions in appendix which makes them observe.

At present, the whole Taiwan mathematics educational reform focuses on life application and problem-solving and mathematics in elementary school targets on the solutions for daily life issues. However, only three (Teachers A, E and I) out of nine participant teachers emphasized the combination of mathematics and daily life experience during the teaching process.

> Teacher A: … A watermelon is divided for eight people and another one is for four people. Which one is more? How much does each person have when a watermelon is divided for eight people? They would know it is 1/8. Then, I asked them which is more? 1/8 or 1/4? They would tell me, sir, the more people have the watermelon, the less a person would have. The less people have the watermelon, the more a person could eat. They would gradually internalize the concept. Thus, when denominator is more, numerator is the same. When denominator is more, it will become less. Thus, they will gradually understand. There are many examples in lives for application.

However, the application targets of the grade five and six mathematics were gradually not limited to direct life situations. The reason was that the level of mathematical application was deepened. Thus, the students must have more complete and abstract control on the mathematical materials in order to efficiently and accurately solve and apply the questions. However, when we opened the textbooks of the elementary school, we find out that most of the questions were "life situation questions". Although it seemed that the questions were interesting, the subjects were the characters, space, time and incidents in daily lives. It was like that this kind of questions could solve the problems in lives. However, the narrow training of problem-solving did not develop the students' smooth number sense. Thus, how to truly and actually combine mathematics in daily lives is a question for in-depth reflection.

Teachers B, C, D and I thought that in the mathematics learning process, the children were the individuals

participating in mathematics learning activities. It was the process of creating, modifying or practicing the activities through complete interaction and share and it was a "reinvention" process. Ordinary students could not understand the essence and characteristics of mathematics at once and they must experience several reflections, in-depth studies and constant adjustment in order to acquire real mathematics knowledge and skills. It required good reflection. Otherwise, it was the accumulation of knowledge learned and could not form good number sense. Therefore, during the process of teaching, valuing the reflection of learning process and the rationality of judgment results could efficiently increase the students' number sense abilities.

Three participant teachers (Teachers E, F and H) proposed intellectual and challenging questions. This kind of irregular question tended to stimulate the students' learning interests, allow them to have pleasant mathematics experience and facilitate the students to connect old knowledge and experience of mathematics, actively construct effective problem-solving strategies and further develop number sense ability. However, due to the limitation of current teaching hours, only Teacher E would frequently provide students with the opportunities, and Teachers F and H were unable to do what they wanted very much to do.

Teachers C and D pointed out that in the teaching process, encouraging and praising the students would enhance the students' learning confidence and interests. Thus, students would like mathematics and be willing to count mathematics, approach and seriously understand mathematics. Thus, when the students had in-depth understanding of number concept, they were able to efficiently apply arithmetic and constructed effective problem-solving strategies, and their number sense was also increased.

Teacher E thought that games were one of the teaching strategies to facilitate number sense. Games were the children's favorite. Combining mathematics learning through games strengthened the playfulness of mathematics courses and the students would think learning mathematics interesting. Thus, lively curriculum design could increase the students' learning interests and further develop their number sense ability. Teacher H thought that each person had some inherent talents for certain things. Likewise, number sense, the instinct to numbers, was also a kind of talent which could not be efficiently developed by teaching.

Teachers' Cognition Towards the Students' Number Sense Abilities

Teachers D, E, F and H believed that the children with number sense had complete understanding towards the significance and concept of the numbers. Thus, they could construct efficient problem-solving strategies. Teachers B, D and H pointed out that the children with number sense reflected during the problem-solving process and judge the rationality of the answers. According to above, Teacher D's number sense meant that the children could think and observe the numbers, have feelings, further understand the significance of the numbers and be highly sensitive to numbers with stronger judgmental capacity in the situations of frequent practice and contact with numbers. Thus, during the process of problem-solving, they could use more efficient strategies to solve the problems successfully. Thus, they could be interested in numbers and feel that mathematics learning was interesting.

Teachers C, D, E and H pointed out that the children with number sense were confident of mathematics and had sense of achievements which further increased their interests of mathematics learning.

> Teacher H: Firstly, it can help their arithmetic ability, since he is very familiar with and sensitive of it. Secondly, if he has enough sensibility, he will love mathematics instead of rejecting it. Since he loves mathematics, it will increase his confidence of the related subjects or knowledge which will expand his learning field.

According to the above descriptions, we can find out that the possession of number sense would influence

of the children's interests in mathematics. The children with number sense ability will apply the mathematics knowledge learned in daily lives, such as the mathematical problems of payment and return when buying things, and how to make money when doing business. When solving the problems, they could apply their understanding toward numbers, deeply understand the meaning of questions, use drawing or the related supporting tools to analyze the problems and construct their own problem-solving strategies. Thus, the children with number sense capacity could have confidence of mathematics learning and further increase their interests in mathematics learning.

Five teachers (Teachers A, B, F, G and H) proposed that the children with number sense could think more flexibly and have logic reasoning.

> Teacher G: … For example, recently, we are solving a question: there is a circle and a diameter. How many sectors are there? There are two diameters. How many sectors are there? There are three diameters. How many sectors are there? …some children realized it immediately! Two times. It becomes two after one cut and four after two cuts. Thus, it is $2x$. Some children are quick and I think it is number sense!

Based on the descriptions of the above teachers, we thought that the people with stronger number sense abilities will show their mathematical knowledge accumulated from the past or because they have met more mathematical question patterns, they could be familiar with the mathematical logic thinking. Thus, when a number appears, he could immediately understand the meaning of the number and have the mathematical thinking model matching logic reasoning. Thus, number sense was the sensibility to numbers.

Six out of nine participant teachers (Teachers A, C, D, E, F and H) thought that the students who could understand the significance of numbers and their relations, develop different problem-solving strategies and have reasonable judgment possessed good number sense ability.

> Teacher E: … you wanted him to add from 1 to 100 and he might forget the trapezoid formula… He would try to grasp by himself. He might use the concept of trapezoid formula… I think he did use the strategies, only he had no idea of its relation with the formula…

Teachers A, B and D pointed out that the children with number sense ability could distinguish the amount of numbers. Teachers A and B indicated that the children's identification of the influence of arithmetic on numbers was the indicator of number sense performance.

> Teacher B: Some kids might keep on counting and after having the answers, he would not think. Some kids would not count at all. I mean some kids could think without counting. Some questions did not require counting which was what you called number sense ability.

The neglect of reference point was worthy of further attention. Among nine teachers, only Teacher E mentioned that the construction of reference point could enhance the students' number sense development. Five teachers (Teachers C, F, G, H and I) all agreed that the children with good arithmetic ability had better development of number sense. Teachers E, G, H and I thought that the children who could manage the abstract thinking of mathematics had better number sense ability.

> Teacher G: First, he was more interested in mathematics and did not resist numbers. Secondly, he might have stronger abstract thinking and more imagination. Thirdly, he might have stronger ability to memorize, or even he had stronger calculation ability so that he could influence these numbers immediately.

In addition, only Teacher H proposed that the children with number sense ability had clear thinking and

were clearer about the significance and concept of numbers. They could have bright verbal expression and learning mathematics and doing mathematics questions in organized way and possess the estimation ability.

> Teacher H: When managing division, you had to estimate, you know the multiple of dividend and divisor. You can estimate so that you can control the speed. I think it is quite important.

Most of the teachers thought that good basic arithmetic ability and abstract thinking ability could develop the children's number sense abilities. Having good arithmetic ability, the degree of sensitivity to numbers would be high. The children could understand the relation between arithmetic and numbers, and thus, they could significantly increase number sense. The children with abstract thinking ability, first could clearly understand the meanings of numbers and arithmetic, and could use the relationship among numbers, quantities and shapes, use proper mathematical language and further solve mathematics or daily life issues. Thus, number sense was developed.

Number Sense Cognition of Teachers With Different Backgrounds

Teachers A, B, D and I had mathematics and physics backgrounds, whereas Teachers C, E, F and G, H had abundant language backgrounds. Teacher A had the longest teaching years and Teacher E had the shortest teaching years. Teachers F and G were not interested in mathematics and Teachers C and E were extremely interested in mathematics.

In the interview, all of the teachers said that during their teacher education, they did not receive the related knowledge of number sense. What they have learned was the traditional narration. Only Teacher I started to understand the related knowledge of number sense in the study of graduate school and knew the complete influence of number sense on students learning.

After analyzing the teaches' understanding towards number sense and their backgrounds, the data shown that the teachers with mathematics and physics backgrounds had more complete interpretation towards the related knowledge of number sense. The teachers who did not have mathematics and physics backgrounds but were extremely interested in mathematics had initial understanding towards the related knowledge of number sense. The teachers without mathematics and physics were not interested in mathematics and had less knowledge of number sense.

Conclusions

With regard to the findings of the study, this research proposed the following suggestions. "Number sense" has been valued by many countries. However, most of the teachers have never encountered number sense, not to mention their values for number sense combined in teaching. The teachers must first develop their number sense and related knowledge of number sense. Raising the awareness of the importance of number sense in the mathematical development of in-service teachers is essential for mathematics education. The learning of mathematics emphasized meaningful learning. When the teachers were having mathematics, they still could not break through the limitation of traditional calculation instruction. The classroom teachers should participate in advanced mathematics courses more frequently in order to understand the importance and significance of number sense. Teachers should also understand how to integrate number sense into mathematics teaching. Therefore, the educational administration institutions should develop a professional training program with number sense for in-service teachers to enhance their understanding towards number sense, so that number sense could be efficiently combined into mathematics teaching for proceeding with meaningful mathematics

teaching. At meantime, mathematics education programs should design more specific number sense activities and integrate them into the pre-service teacher training courses. When teachers understand number sense and know how to teach it, they will be able to help their children develop number sense. The researcher hopes the findings could encourage more teacher education programs to direct their instructions towards facilitating the understanding and meaningful learning of number sense.

References

Bogdan, R. C., & Biklen, S. K. (1982). *Qualitative research for education: An introduction to theory and methods.* Boston: Allyn and Bacon.

Cheng, C. Y., & Tsao, Y. L. (2008). The design of number sense with fraction teaching. *Journal of Elementary Education, 48*(6), 79-85.

Chen, P. C., & Yang, D. C. (2006). Exploring the importance of number sense from problem of solving. *Journal of the National Institute for Compilation and Translation, 34*(2), 83-94.

Cuff, C. (1993). Beyond the formula-mathematics education for prospective elementary school teachers. *Education, 114*(4), 221-223.

Gurganus, S. (2004). Promote number sense. *Intervention in School and Clinic, 40*(1), 55-58.

Hope, J., & Small, M. (1994). Number sense in interactions. *Program Information, 4,* 18-19. Toronto, Ontario: Ginn Publishing Canada Inc..

Howden, H. (1989). Teaching number sense. *Arithmetic Teacher, 36*(6), 6-11.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, N. J.: Lawrence Erlbaum.

Markovits, Z., & Sowder, J. T. (1994). Developing number sense: An intervention study in grade 7. *Journal for Research in Mathematics Education, 25*(1), 4-29.

Ministry of Education (2001). *Nine-year-integrated mathematics curriculum standards for schools from grade 1 to 9 in Taiwan.* Taiwan, Ministry of Education.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, V. A.: NCTM.

National Council of Teachers of Mathematics. (2000). *The principles and standards for school mathematics.* Reston, V. A.: NCTM.

Reys, B. (1994). Promoting number sense in the middle grades. *Mathematics Teaching in the Middle School, 1*(3), 114-120.

Sowder, J. (1992). Making sense of numbers in school mathematics. In G. Leinhardt, & R. Hattrup (Eds.), *Analysis of arithmetic for mathematics teaching* (pp. 1-51). Hillsdale, N. J.: Erlbaum.

Tsao, Y. L. (2004). Effects of a problem-solving-based mathematics course on number sense of preservice teachers. *Journal of College Teaching & Learning, 1*(2), 33-49.

Tsao, Y. L. (2005). The number sense of preservice elementary teachers. *College Student Journal, 39*(4), 647-679.

Tsao, Y. L., & Rung, L. J. (2007). The study of developing the fifth grade students' number sense through teaching activities. *Research and Development in Science Educational Quarterly, 1,* 78-119.

Tsao, Y. L., & Rung, L. J. (2006). Promoting number sense in class activity. In C. H. Yang (Ed.), *Educational consulting book: Enthusiasm remarkable and new model* (pp. 209-232). Taiwan, Taipei: National Taipei University of Education.

Yang, D. C. (2000). The methods used by sixth grade students in Taiwan when responded number sense questions. *Chinese Journal of Science Education, 8*(4), 379-394.

Yang, D. C. (2002). A study of sixth grade students' development of number sense through well-designed number sense activities. *Chinese Journal of Science Education, 10*(3), 233-259.

Yang, D. C. (2003). Teaching and learning number sense—An intervention study of fifth grade students in Taiwan. *International Journal of Science and Mathematics Education, 1*(1), 115-134.

Yang, D. C. (2006). Developing number sense through real-life situations in school of Taiwan, *Teaching Children Mathematics, 13,* 104-110.

Yang, D. C. (2007). Investigating the strategies used by pre-service teachers in Taiwan when responding to number sense questions. *School Science and Mathematics, 107*(7), 293-301.