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Project DIVIDE Instrument Development

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Abstract

In this technical report, we describe the development of cognitive diagnostic test items that form the basis of the diagnostic system for Project DIVIDE (Dynamic Instruction Via Individually Designed Environments). The construct underlying the diagnostic test is division of fractions. We include a description of the process we used to identify the cognitive attributes for division of fractions. We present the hierarchical association between attributes that lead up to mastery of the domain. Finally, we describe item writing procedures and evidence for technical adequacy of the resulting item bank.

Introduction

The purpose of project DIVIDE (Dynamic Instruction Via Individually Designed Environments) in mathematics is to develop an integrated cognitive diagnostic assessment and instructional delivery system. Applying the design flexibility and analytic power of recent developments in computer technology, this system will provide individualized instruction in mathematical computation skills and support conceptual understanding to students studying division of fractions. This system is designed to support students with disabilities in the general education setting.

A component of this project is the development of a computer-based cognitive diagnostic assessment system. The results from administering this test are used to classify students into the instructional system that most closely aligns with their mastery of the prerequisite knowledge. The purpose of this technical report is to describe the development of the cognitive diagnostic test items. Included in this description is the process used to identify the cognitive attributes underlying division of fractions. We present the hierarchical association of skills and knowledge associated with mastery of division of fractions. Additionally, we describe item writing procedures and steps taken to evaluate the technical adequacy of the items.

Identification of Cognitive Attributes

In this project, we assigned students to instructional modules based on results from the cognitive diagnostic assessment. Accurate classification of students into instructional modules requires carefully designed assessments that align with the cognitive model underlying performance in the domain. The cognitive model is composed of attributes that are domain specific prerequisite skills and knowledge needed to demonstrate mastery in the targeted task (Chipman, Nichols, & Brennan, 1995; Leighton & Gierl, 2007; Tatsuoka & Tatsuoka, 1986).

Attributes are typically isolated through careful task analyses, expert review, verbal protocols, and other inquiry methods for analyzing student thinking processes (Gorin, 2006).

Task analysis. To begin this project, the research team conducted a task analysis of the skills and knowledge needed to divide fractions (see Appendix A for biographies of the research team). First, we identified the types of division of fractions problems. Using the general model of division of fractions, $a\frac{b}{c} \div d\frac{e}{f}$ (where a, b, c, d, e, f are whole numbers; $c \neq 0$ and $f \neq 0$), division of fractions problems can be classified into three general categories: a) a proper fraction is divided by a proper fraction (e.g., $\frac{2}{3} \div \frac{1}{2}$ where $a = 0$ and $d = 0$), b) a fraction is divided by a whole number (e.g., $\frac{2}{3} \div 2$ where $a = 0$ and $e = 0$), and c) problems involving dividing mixed numbers (e.g., $3\frac{1}{2} \div 4$, $5\frac{1}{2} \div 2\frac{1}{2}$ where $a \neq 0$ or $d \neq 0$).

Second, the research team consulted mathematics textbooks and discussed the process for dividing fractions with a mathematician to determine the precise steps involved in this procedure. The mathematical rationale can be summarized:

$$M \div N = X \quad (N \neq 0) \Leftrightarrow M = XN \quad (N \neq 0) \text{ ----- (1)}$$

Put $M = \frac{a}{b}$, $N = \frac{c}{d}$, and $X = \frac{x}{y}$ ($b \neq 0, d \neq 0,$ and $y \neq 0$).

$$\frac{a}{b} \div \frac{c}{d} = \frac{x}{y}$$

$$\xrightarrow{\text{by (1)}} \frac{a}{b} = \frac{x}{y} \times \frac{c}{d}$$

$$\xrightarrow{\text{multiply both sides by } \frac{d}{c}} \frac{a}{b} \times \frac{d}{c} = \frac{x}{y} \times \frac{c}{d} \times \frac{d}{c}$$

$$\xrightarrow{\frac{c}{d} \times \frac{d}{c} = 1} \frac{a}{b} \times \frac{d}{c} = \frac{x}{y}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

The attributes emerged by examining the mathematical rationale and isolating the specific steps needed for students to understand and execute each step of the procedure.

Expert review. To verify the attributes, members of the research team presented the mathematical rationale and proposed attributes to two mathematics education experts (for biographies of the Mathematics Education Expert and Mathematics Instructional Design Expert see Appendix A). The purpose of this review was to refine the attribute list by removing irrelevant attributes or adding necessary skills that were missed during the task analysis. The review included a description of the project, review of the vocabulary associated with cognitive diagnosis, and a discussion of the proposed attribute structure. We presented the mathematical rationale for division of fractions, discussed the conceptual and procedural knowledge needed to solve division of fractions problems, and solved several sample problems using various approaches students might try.

As a result of this review, the original set of attributes was modified. Four attributes were omitted because the skills and knowledge were identified as unnecessary to master the procedures or concepts of division of fractions. Four attributes were added to encompass the range of skills and knowledge students needed to master division of fractions. The revised list of attributes is provided in Appendix B.

We conducted verbal protocols with students to collect additional evidence about the cognitive model (see Ketterlin-Geller and Liu, in preparation).

Association Between Attributes

To determine the hierarchical structure of skills and knowledge needed to solve division of fractions problems, the research team evaluated the association among attributes. Articulating

these relations is an essential step for determining the necessary item types that must be sampled in the cognitive diagnostic test. From the administration of these items, it should be clear which attributes students have and have not mastered. As such, the relation across attributes can significantly influence the resulting instructional decisions.

To determine these relations, the research team carefully analyzed the sequence of skills and knowledge needed to divide fractions. Each attribute was evaluated to determine the knowledge and skills that were required to master the range of examples within that attribute. We considered sample problems for each attribute that included whole numbers as well as rational numbers.

To avoid confirmation bias, each member of the research team independently created a diagram specifying the association among attributes. Then, the research team met to discuss the diagrams. Because mastery of the subsequent attributes requires that students understand, recognize, and define fractions, each member of the research team identified *Definition of Fractions* as the prerequisite skill for division of fractions. The remaining associations are specified in Appendix C.

Item Writing

Instrument Development

The cognitive diagnostic test is designed for computer delivery in a group setting. Members of the research team along with a Mathematics Content Expert (see Appendix A for biography) wrote a total of 252 items. Multiple-choice items were created for efficiency in the computer delivery. Each item had three distractors and one correct answer. Distractors were carefully written to reflect potential errors in student thinking related to the attribute. Errors included computational errors, conceptual errors, procedural errors, and strategy errors. The computer

randomly assigned the location of the correct answer in the list of possible answers. Scoring was dichotomous.

The content of the test encompasses the attributes for division of fractions. Items included a range of problems from computational to situated within a context. Some items were designed to elicit conceptual understanding or required reasoning skills by the student. Graphics were used in instances where they explained the problem, provided a visual clue to clarify the context, or were integral to the stem or answer choices. Irrelevant graphics were not included. The reading level of the items was intentionally constrained to the 5th grade level, however, readability statistics were not calculated for each item. Whenever possible, plain language and simple, straight-forward statements were incorporated into the items.

Content-Related Evidence for Validity

Internal review. Members of the research team conducted an internal review of the items to verify the content and alignment with the attributes. Each item was reviewed for the appropriateness of the content, graphics, and language. Additionally, the reviewers assigned an attribute(s) to the item. These classifications were used to verify the attribute assignment for each item. At least two internal reviewers provided feedback for each item. According to the reviewers' comments, the original item writer modified the items.

Teacher review. Eight teachers from local public schools reviewed the items. All participating teachers taught mathematics at public schools from 2 years to 22 years, with an average of 12 years. Four teachers earned Masters degrees in mathematics or education. Three teachers had bachelor degrees in mathematics.

Teachers analyzed each item for grade-level appropriateness in terms of understandability of language and vocabulary, content or concepts, graphics, potential bias in language and/or

content, clarity of directions and answers, and effectiveness of distractors. The test items and distractors were rated on a scale of 1 to 4 for each criterion. A rating of 1 meant that the item/distractors were not at all appropriate based on the criterion (or very biased); a rating of 2 meant that the item/distractors were somewhat appropriate based on the criterion (or somewhat biased); rating of 3 meant that the item/distractors were appropriate based on the criterion (or not biased); and a rating of 4 meant that the item/distractors were extremely appropriate based on the criterion (or not biased *and* has multi-cultural components to it). In instances where the teachers provided a rating of 3 or lower, they were asked to provide additional suggestions and comments regarding the measures.

For 112 items, teachers pointed out possible problems regarding grade-level appropriateness. Twenty items were noted as having language or vocabulary that was too complex for 5th grade students; simplified text was suggested. Twenty-two items were identified as having complex concepts. The graphics for 6 items were considered inappropriate. Bias was identified for two items. Clarity and effectiveness of the distractors was questioned for 52 items. For these items, teachers provided alternate distractor suggestions. The research team reviewed all suggestions and made revisions based on teacher feedback.

Math expert review. Five graduate students in mathematics at the University of Oregon reviewed all items. All reviewers were enrolled in the Ph.D. program in mathematics and had earned Bachelors degrees in mathematics. Three students had completed a Masters degree in mathematics or physics. All participants had previous experience teaching mathematics to college students.

The math experts were asked to identify the attribute(s) measured by each item and to evaluate the adequacy of measurement and effectiveness of distractors for each item. The

specific considerations are as follows. Items and distractors were evaluated on a 4-point scale for each criterion. A rating of 1 meant that the item did not measure the attribute identified, and the distractors were effective; a rating of 2 meant that the item somewhat measured the attribute identified, and the distractors were somewhat effective; rating of 3 meant that the item mostly measured the attribute identified, and the distractors were mostly effective; and a rating of 4 meant the item effectively measured the attribute identified, and the distractors were extremely effective. If they give the item a score of 1 or 2 on a category, they were encouraged to provide a suggestion.

The attribute classifications by the math experts differed from those made by the research team for 86 items. In most of these items, the math experts suggested that the items were measuring additional attributes that had not previously been assigned to the item. For example, in most cases where the item measured the relationship between multiplication and division, the math experts also suggested that these items were measuring whether or not students could balance an equation. Based on reviewer feedback, attribute classifications were changed for 47 items when the suggestion was justifiable.

Additionally, the math experts suggested revising the language for 17 items to increase the mathematical precision of the concepts presented, clarity of the distractors, and/or appropriateness of the graphics. The research team reviewed the wording suggestions and modified the language according to suggestions.

Pilot Test

Subjects. Six hundred four students in grades 5-7 participated in the pilot study. Most participants lived in Oregon; however, students from Washington, Missouri, Idaho, and

Massachusetts also participated. Roughly equal numbers of female (52%) and male (48%) students participated.

Procedures. A convenient sample of teachers was sent an email inviting their students to participate in a computer-based test on division of fractions. Approximately 24 teachers agreed to participate. Once enrolled in the testing session, students were randomly assigned to take an 18-item subtest. Each subtest included 5 anchor items and 13 unique items that were purposefully selected to represent a range of attributes at varying difficulty levels. The computer program provided all relevant instructions. Teachers proctored the testing session using standardized administration procedures specified in the testing manual. Fidelity of implementation was not monitored due to the distributed nature of the participants.

Results. Results were analyzed using a 1-parameter item response theory (IRT) model. Of the 252 items, 13 items under-fit and 4 items over-fit the model based on mean square residual fit statistics. The average item fit was 1.00 (SD=0.32). The item calibrations are reliably estimated with a reliability of .85. Item difficulty ranged from -3.99 to 3.65 on the logit scale. The item/person map (see Appendix D) indicates that the calibration sample is well targeted with the items and the items provide a wide measure.

Conclusions

The purpose of this technical report was to describe the procedures used to develop the cognitive diagnostic test items. We described the process used to define the cognitive model underlying division of fractions. In addition, we discussed item writing efforts as well as procedures for evaluating content-related evidence for validity. Pilot test data were presented indicating that the items represent a range of difficulty levels and appropriately fit the IRT

model. The evidence presented in this technical report indicates that the cognitive model as well as the diagnostic test items are appropriate for the purposes of Project DIVIDE.

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Appendix A: Biographies of Researchers and Consultants

Principal Investigator

The Principal Investigator has been the Director of Research Projects with Behavioral Research and Teaching at the University of Oregon since 2003. Additionally she has served as an assistant professor with Educational Leadership since 2005. She holds a Ph. D. in Educational Leadership with an emphasis on applied measurement. She earned her teaching credentials for secondary science as well as K-12 administrative licensure. She is also the co-principal investigator for a research project examining the alignment of content standards in mathematics with IEP goals and how skills and perceptions influence goals. She has been working as a key researcher on projects investigating features of universally designed mathematics assessments and research on effects of accommodations on measurement of mathematics performance.

Research Team Member 1

The Research Team Member holds a Bachelor of Science and has been a research assistant and project coordinator for federally funded grants and state contracts for 5 years. He assisted in the creation of a web-delivered math assessment researching effectiveness of accommodations. Additionally, he has been involved in the development of the alternate assessment in mathematics for students with significant cognitive disabilities at both the elementary and high school levels for 4 years. The Research Team Member also was a team member of an item writing team for creating math items for math screening tests and assisted in development of accommodated versions of math items. He has also been a part of several research teams conducting multi-state research projects examining comparability of math performance on different alternate assessments. He is completing a Master's degree in Special Education and has taken coursework in mathematics instruction and assessment design.

Research Team Member 2

The second Research Team Member has a B. S. in Mathematics as well as a B.A. and a M.A in Special Education. She is currently pursuing a Ph. D. in Special Education at the University of Oregon. She has experiences in developing and validating Mathematics Curriculum-Based Measures (CBM) for grades 2 through 4 in her master’s program. In addition, she created math computation items for students in grades 3 to 8 for a project examining information sources for Individualized Education Program (IEP) decision making. She also revised alternate assessment items in mathematics for significant cognitive disabilities at both the elementary and secondary level. She has a secondary special educator license specialized in mathematics. She taught students with significant cognitive disabilities, hearing impaired, blindness, and autism at the elementary, middle, and high school levels as a teacher and a volunteer for several years in Korea.

Research Team Member 3

The third member of the research team completed his Ph.D. in Educational Psychology with an emphasis on psychometric theory, statistical decision analysis, and behavioral research methods. The Research Team Member was a special education teacher in New York State and completed a Masters degree in Special Education. He has remained active in the field as a senior research associate, associate professor, and consultant to government and research organizations. Current research interests include item response theory (IRT) modeling in the context of assessment for special populations, and the specification of cut scores for optimal decision making. The Research Team Member presents regularly at national conferences, addressing scale construction, scale performance standards, and bias in measurement systems designed for diverse populations.

Mathematics Education Expert

The Mathematics Education Expert was an associate professor of the department of Special Education and Clinical Sciences at the University of Oregon for 7 years and is currently serving as a Dean of the School of Education and Human Development at the Southern Methodist University. He has a teaching license of mathematics in California and Michigan. He is a member of National Council of Teachers of Mathematics and American Mathematical Association. He has diverse experiences in math teaching as a classroom teacher for several years in California and an associate training director at U. S. Peace Corps in Lesotho, Southern Africa. He worked as a principal investigator of Early Mathematics Project from 2004 to 2007.

Mathematics Instructional Design Expert

The Mathematics Instructional Design Expert, M. A., has been a Senior Instructor and a Research Assistant in the Department of Special Education at the University of Oregon for over 20 years. She taught for twelve years with students with learning disabilities, behavior disorders, deaf-blind, and autism at both elementary and middle school levels. She taught math to students in early math preskills through middle school math. Additionally, she has taught special education math instructional methods for about 15 years at the University of Oregon. She is a supervisor for practicum students and student teachers in implementing math instruction in special education settings. She also participated in writing a K math curriculum. She holds an Oregon teaching license in Elementary education (K-8) with endorsements in special education (K-12) and hearing impairment (K-12).

Mathematics Content Expert

The Mathematics Content Expert is working as a high school math teacher as well as a math coach at Willamette High School. She has been a high school math teacher (all grades 9 to 12)

for ten years. The Content Expert has a B. S. in Mathematics with minor in Economics. She also holds a M. A. in Economics and a MAT in Advanced Mathematics. She is a committee of 2007 Review Benchmark cut scores for Oregon State Assessment at Oregon Department of Education. Additionally, she will work for 2008 Mathematics State Adopted Instructional Material Review as a committee. The Content Expert is a member of NCTM (National Council of Teachers of Mathematics).

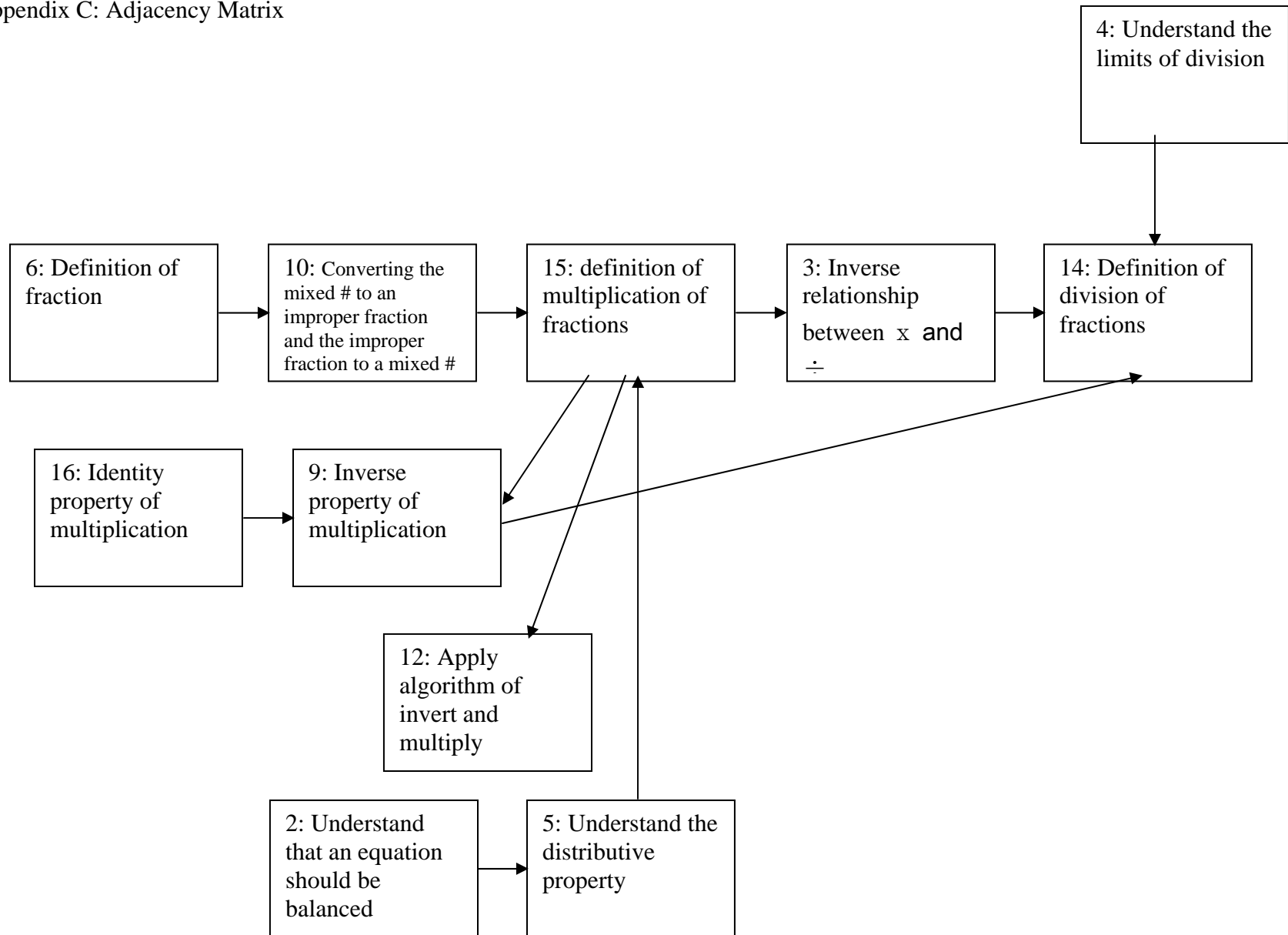
Appendix B: Attribute List

Understand that an equation should be balanced so that if you do something to one side, you have to do it to the other
Understand inverse relationship between multiplication and division
Understand the limits of division (cannot divide by a zero because you can't have equal parts of nothing)
Understand the distributive property
Understand the definition of a fraction <ul style="list-style-type: none"> A. Relationship between two numbers; ration/proportion B. Sometimes includes all elements in a set
Understand and apply inverse property of multiplication <ul style="list-style-type: none"> A. Any fraction multiplied by its inverse, equals 1 B. Example: $\frac{a}{b} \times \frac{b}{a}$ equals 1
Know and apply the procedures for converting the mixed number to an improper fraction (and converting an improper fraction to a mixed number) when $a \neq 0$ or $d \neq 0$.
Apply algorithm of invert and multiply <ul style="list-style-type: none"> A. Invert the second fraction B. Change the sign to a times sign C. Multiply
Understand definition of division of fractions <ul style="list-style-type: none"> A. “m” divided by “n” = how many copies or parts of “n” are there in “m” B. Any dividend (n) divided by a number between 0-1 results in a larger number

<p>than the dividend (n)</p>
<p>Understand definition of multiplication of fractions</p> <ul style="list-style-type: none"> A. Divide the whole into parts B. Take the parts of interest C. “m/n” multiplied by “1/k” = divide “1/k” into “n” equal parts and take “m” of them D. Any factor multiplied by a number between 0-1 results in a smaller number than the factor
<p>Understand and apply identity property of multiplication</p> <ul style="list-style-type: none"> A. Multiplying anything by 1, results in itself

* Note: attributes are not in order

Appendix C: Adjacency Matrix



Appendix D: Item/Person Map

