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LEGO-Method—New Strategy for Chemistry Calculation

József Molnár

Daniel Berzsenyi Lutheran Lyceum, Sopron, Hungary

Lívia Molnár-Hamvas University of West Hungary, Sopron, Hungary

The presented strategy of chemistry calculation is based on mole-concept, but it uses only one fundamental relationship of the amounts of substance as a basic panel. The name of LEGO-method comes from the famous toy of LEGO[®], because solving equations by grouping formulas is similar to that. The relations of mole and the molar amounts, as small perspicuous units (building blocks), are applied on the fundamental relationship (basic panel) and the problem can be solved as an algebraic operation. LEGO-method is much more simple than the other strategies (e.g., dimensional analysis, mole method or rule of three), because the students can understand and learn this procedure step by step at the very beginning of their science study. The measurements have demonstrated that those students, who learned the LEGO-method in the school, have used this strategy more frequently than other strategies for solving a stoichiometric problem. The success of these students indicates that the LEGO-method is a useful alternative strategy for teaching chemistry calculations and more complete understanding of problem-solving.

Keywords: chemistry calculation, problem-solving, LEGO-method, building blocks, basic panel

Introduction

Teaching of chemistry calculation seems to be not an easy or simple work for the teachers in the secondary school, because solving a chemistry problem requires both precise, analytical thinking and mathematical skill of the young students. That is the reason why the stoichiometric calculation belongs to the less attractive but more difficult areas in the secondary school chemistry. A fair proportion of school chemistry students find difficulty with mole calculations. One cause may be weakness in arithmetic, especially in handling ratio and proportion. Another reason of difficulty is that great numbers of different factors have to be correctly assembled in a mole calculation (Flood, 2004).

On the other hand, the skills of chemistry problem-solving are necessary to engineering or scientific fundamentals of university and college students, even if the chemistry course is not their professional subject. At the beginning, chemistry students find stoichiometric problems, which is one of the most difficulty aspects of the introductory chemistry course. Lots of them are unable to match analogs with the chemistry problems even after practice in using analogs (Gabel, 2010).

Problem-Solving Strategies

There are some classical and effective algorithmic strategies used in chemistry education for problem-solving. The main three types are the dimensional analysis, the mole method and the proportion method. All of them are based on the idea of the mole as the unit of the amount of substance, which is necessary to stoichiometric calculations.

József Molnár, Ph.D., Daniel Berzsenyi Lutheran Lyceum.

Lívia Molnár-Hamvas, Ph.D., associate professor, Institute for Chemistry, University of West Hungary.

Dimensional analysis (also called factor-label method or unit factor method) originally was developed for conversion of a given result from one system of units to another (e.g., convert the units of the English system to the metric system) (S. S. Zumdahl & S. A. Zumdahl, 2000). The strategy of dimensional analysis uses conversion factors in the stoichiometric calculations and sets up a joined relationship for solution (these conversion factors are provided by mole concept for problem-solving situation) (Petrucci, Hardwood, & Herring, 2002). The equalities (i.e., conversion factors) are set up in fraction form, then those are lined up sequentially, and the units used on the top and bottom of neighbouring fractions are alternated so that units cancel.

Most of the American and Canadian textbooks use this method steadily in the stoichiometric calculations, but these books considerably emphasize that the factor label method based on mole method (Petrucci et al., 2002; Masterton & Hurley, 2009). Both dimensional analysis and mole method get the result from the given quantity through the moles of the given and the wanted substances (Chang, 2002) (see Figure 1).

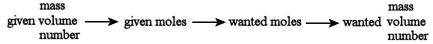


Figure 1. Schema of calculation strategy of dimensional analysis and mole method.

On the other hand, the mole method calculates the amounts, from the given quantity through the moles to the wanted amount, step by step (Chang, 2002). Chemical equations are written in terms of moles, but not in terms of masses, so to calculate the mass of product produced from a known mass of reactant, the mass must be first convert to moles. After it, the given number of moles has to be compared to the number of moles of product. And the last step is the converting of the moles to the mass of product (Chang, 2002; S. S. Zumdahl & S. A. Zumdahl, 2000).

The third one of the algorithmic methods, the proportion method, compares the given amounts to the wanted ones and sets up a relation between these amounts (Mascetta, 2003). It is to be regretted that this method is also widely used nowadays in the European textbooks and manuals (Ernst, Puhlfürst, & Schönherr, 2005; Knausz, Mörtl, & Szakács, 1997). The great drawback is that most of the Hungarian schoolbooks use classical "if...then..." reasoning in the cross-proportion, without specification and detailed explanation and does not use any units of molar quantities (Veszprémi, 2008). The proportion method emphasizes basic scientific principles through application during the process of solving numerical problems, which in turn promotes students' understanding of these principles by constantly reinforcing basic concepts (E. Cook & L. Cook, 2005).

Students' Problem-Solving Methods

Stoichiometry is fundamental to all aspects of chemistry and requires students' deep problem-solving skills. But, learning to solve stoichiometric problems demands not only good mastery of stoichiometry concepts, but also ability to construct and balance reaction equations and using them in calculation of the quantity of chemical substances. Solving a simple stoichiometric example might be a routine exercise for a practicing chemist, but it is a novel problem for students who encounter chemistry for the first time.

Researches of the students' problem-solving strategies established that the students use very different ways of stoichiometric calculation. Since stoichiometry has become an important topic in curricula and chemistry textbooks, many investigations have been carried out to understand students' problems in this field.

An extensive study of stoichiometry problem-solving strategies of German students by Schmidt (1994) established that senior high school students used well mathematical strategies, for example, a method suggested by textbooks and/or teachers. But 50%-60% of successful students also used other strategies, which were not illustrated by their teachers during instruction, or were not found in German textbooks. Comparing his studies with others, Schmidt (1997) concluded that students are more likely to use algorithmic strategies when solving more difficult problems, but tended to use reasoning strategies with easier problems. A decade later, Fach, de Boer, and Parchmann (2007) also found that German students are especially fond of the algorithmic methods, since seven of the 17 students used the equation to solve a stoichiometric problem, while five used the rule of proportion.

Schmidt and Jignéus (2003) reported that the high school students in Sweden successfully used their own strategies in solving simple stoichiometric problems on composition of binary compounds, but tended to use algorithmic methods thought at school in case of difficult problems.

Niaz and Robinson (1993) investigated the students' problem-solving strategies in the US (at Purdue University), where the factor-label approach is the most popular, almost the only one for solving stoichiometric problems. Sixty-two students with weak math skills, low high school math scores and no high school chemistry experience were selected for the study. Their paper concluded that the use of algorithmic solution strategies could require formal operational reasoning to a certain degree.

The study of BouJaoude and Barakat (2000) described and classified the strategies high school students use when solving stoichiometry problems. The students of a highly selective private school in Lebanon resorted mostly to algorithmic problem-solving, which may be viewed as a safe and sure way to the correct answer, and those were often similar to the strategies found in textbooks or used in class.

The Hungarian research of Tóth and Kiss (2005) found that the Hungarian secondary school students applied the strategies learned at school even in case of simple stoichiometric problems. The students used the mole method or the proportion method, but almost never used logical method. Kiss (2008) investigated what kind of strategy is preferred by the Hungarian students. It was found that they prefer the strategies taught at school, especially the mole method. The more qualified the students are in chemistry, the more they apply this method.

Tóth and Sebestyén (2009) studied the Hungarian secondary school students' problem-solving methods in stoichiometry based on the chemical equation. Three types of problem-solving strategies were discoverable in the calculations: mole method, rule of three and their mixed variation. The authors noted that the Hungarian schoolbooks generally discuss the first two solving methods. They found that only 40% of the Hungarian secondary students use any identifiable strategy in solving the complex stoichiometric problem. Students mainly used only two methods thought at school: mole method and proportion method (almost to the same extent). Only a few students used the mixed method and nobody tried to calculate by dimensional analysis. The results have shown that the two applied strategies are equivalent to each other both in their frequency and success rate.

The way, as the teacher actually teaches stoichiometry, seems to have a great impact on how young students solve stoichiometric problems. But, there are only few worked out examples in the European chemistry textbooks, and most of them discuss only one, special type of the strategies of chemistry calculations. That is the reason why the mentality and attitude of chemistry teachers play a significant role in the students' efficiency.

Teachers' Preferred Methods for Stoichiometry

The preferred strategy of teaching of stoichiometric problem-solving is very different in the American and the European countries. Dimensional analysis is the main, almost the only one, proposed problem-solving strategy in the US and Canada, in spite of the fact that the strategies of dimensional analysis and mole method are based on the same way of the stoichiometric calculation. DeMeo (2008) in his research asked near a thousand teachers through the US to identify which of the method is currently used by them in their chemistry classes. Dimensional analysis was found the dominant method, which was preferred by overwhelming 90.3% of the teachers. Using of this strategy was accounted for "issues of content", "easy to use and understand" or "specific cognitive issues" by more than the half of these teachers.

The traditional, algorithmic "rule of three" (proportion method) is widely used for solving numerical stoichiometric problems in the European countries, and it has much greater importance than other strategies. Fach et al. (2007) found that the "rule of proportion" method caused more troubles during the calculation, because students often mixed up numerators and denominators, which led to false results. There were made some unsuccessful attempts at spreading of dimensional analysis in European countries, too. A detailed booklet has been written by Flood (2004) about this strategy for the teachers, because the students are often not able to handle ratio and proportion well, and this method is relatively unknown in Scotland. Similarly, Tóth (2000) has also taken steps to teach the next generation of the Hungarian chemistry teachers for dimensional analysis, but this method has not been converted into the students' solving methods.

It is worthy of note that nowadays there are some endeavours in US to introduce again the proportion method, which was once widely used into the teaching of chemistry calculations. E. Cook and R. L. Cook (2005) proposed that this strategy regains currency as an alternative to the dimensional analysis method, particularly in lower-level chemistry courses. They established that dimensional analysis has emerged as the only problem-solving mechanism offered to high-school and general chemistry students in contemporary textbooks, replacing more conceptual methods, the cross-proportion included.

Strategy of LEGO-Method

It has been shown previously that many students solve chemistry problems using only algorithmic strategies and do not understand the chemical concepts on which the problems are based. Since the role of teachers is crucial in promoting students' conceptual understanding, it is important to explore how teachers teach stoichiometry. It also has been demonstrated that the chapters of stoichiometry in the Hungarian chemistry textbooks are unsuitable for the students' self-training and practice in calculations. Having recognized the students' difficulty with problem-solving, fundamental turning, i.e., an alternative strategy for teaching of calculations was proposed (Molnár & Molnárné, 2004).

The name of this problem-solving strategy was given by the students, because this method has similarity with the toy of LEGO® consisting small plastic bricks and other pieces, which can be joined together to make models of many different objects. Similarly, the LEGO-method is a strategy for chemistry calculation consisting a fundamental relationship and some small formulas, which can be joined together to solve many different calculation.

LEGO-method has been developed for enriching the algorithmic solution strategies and helping students working on stoichiometric problems. It was first published in Hungarian chemistry journals (Molnár & Molnárné, 2005; 2006). This new strategy of chemistry calculation was also expounded on posters (Molnár &

Molnár-Hamvas, 2005; 2006) and on workshops (Molnár-Hamvas & Molnár, 2010) of international conferences.

The name of this problem-solving strategy was given by the students, because this method has similarity with the toy of LEGO consisting small plastic bricks and other pieces, which can be joined together to make models of many different objects. Similarly, the LEGO®-method is a strategy for chemistry calculation consisting a fundamental relationship and some small formulas, which can be joined together to solve many different calculation.

LEGO-method is based on mole-concept, but it uses only one fundamental relationship as a basic panel. The fundamental principle is that "only unbroken particles are able to react with each other". Therefore, the moles of the given and wanted substances are in a fixed proportion.

$$n_{\text{wanted}} = \frac{\mathbf{u}_{\text{wanted}}}{\mathbf{u}_{\text{given}}} \cdot n_{\text{given}}$$

As shown in the above equation, the wanted amount of substance is equal with the product of moles of given substance and the ratio of their coefficients. The "u" coefficients of the given and the unknown amounts are whole numbers, which can be derived, e.g., from balanced chemical equation or from composition. This relationship of basic panel is suitable to solve almost any type of simple or complicated chemical problems.

The amounts of substance, the moles, can be expressed by some simple relations, by the building blocks. Most of these relations can be derived from the definitions of molar amounts (molar mass, molar volume, Avogadro's number, etc.). The needed building blocks can be put on either side of the basic panel and produce a great number of variations. The examples are as follows:

$$n = \frac{m}{M}; n = \frac{N}{N_{A}}; n = \frac{V_{gas}}{V_{m}}; n = \frac{p \cdot V}{R \cdot T}; n = \frac{\Delta H}{\Delta H^{\circ}_{r}}; n_{e} = \frac{I \cdot t}{F}; n = c \cdot V_{sol}; n = \frac{\pi \cdot V}{R \cdot T}$$

In the complex problems, the LEGO-method requires some other simple physico-chemical relationships, too, such as density, mass percent and mass concentration. The examples are as follows:

$$\rho = \frac{m}{V}, w_i = 100 \cdot \frac{m_i}{m_{total}}, c_i = \frac{m_i}{V_{solution}}, \text{ and so on.}$$

The strategy of LEGO-method is performed in four steps:

- (1) Find out what substance and property is wanted and what is given and find out what kinds of building blocks are required in the relationship;
 - (2) Build up the blocks to the panel and solve the formula for unknown;
 - (3) Fix the coefficients and find out what constants are needed;
 - (4) Replace the quantities by number values and perform the math on both the units and the numbers.

While some other strategies of problem-solving obtain the results by lots of calculative steps (e.g., mole method) or particular equations for different types of problems, LEGO-method needs to know only one relationship. Irrespective of the conditions, the building block of the wanted component (reagent or product, particle, atom or molecule and compound or solute) has to be on the left side of the basic panel. Due to building up the blocks to the basic panel, this strategy makes possible to solve stoichiometric problems much simpler. This strategy might be termed an edited version of mole method, which is founded on the equalities of the amounts of substances. But, this method might be also termed an alternative proportion method, because it uses a fixed relationship called basic panel.

Didactics

The most important object is that the principle of gradation has to be getting on from the beginning of teaching this strategy. Teaching the building blocks gradually, parallel with the definitions, allows enough time for the students to practice in using this method. The 13 years old students learn only two types of building blocks of mole (mass and number) and the relation of mass percent during the first chemistry year in the secondary school (the 7th grade). Knowledge and application of these relations enable the students to solve the simple types of exercises (see Sample exercises 1-2).

The first, fundamental step of problem-solving "find out what substance and property is wanted and what is given" calls for expends care in reading the text and the students have to understand well the meaning of words.

Another help of learning this method is that students might draw these relations on cards, which are allowed to use both on the lessons and on written tests from the beginning. The basic panel is sketched on a half of A4 size sheet of paper. The cards for building blocks of mole are smaller (one fourth of A4), and the two sides of the equation are sectioned out on the two sides of the sheet.

e. g., on the one side of the sheet is
$$n_{\text{wanted}}$$
 and $\frac{m_{\text{wanted}}}{M_{\text{wanted}}}$ is on the other.

Similarly, the additional relations of concentration and density are drawn even smaller cards (eighth of A4).

The students of the 7th grade find pleasure in building up the blocks to the panel by these cards and usually, a fair rivalry takes form for winning. Students can meet with success and they enjoy the chemistry lessons. These cards help them not only to solve the problems quickly, but also to learn definitions, accurately. That is the reason why these students have much less fault in correlation between particles, atoms and molecules. Students, who became skilled in LEGO-method, stop setting out the cards bit by bit and write down only the relationship, and some automatism can be turned up.

A periodic table containing molar amounts (mass, volume) and constants (N_A , R, F) is also a useful helper for the students' attainment of problem-solving practice. But, they must have proficiency in calculating of molar mass of the compounds as well as in using SI units (international system of units).

Further types of building blocks are taken in year by year. The students of the 9th grade meet with a great deal of studying in general chemistry. The main topics are ideal gas law, thermo- and electro- chemistry, solutions and concentrations, stoichiometric calculations, amounts of reactants and products, acids and bases, titration, etc.. During that school year, besides lots of new factual information, the students gain experience in balancing the chemical equations, in setting a value upon the significant figure in the calculation and in using the correct units of measurement. At that time, further building blocks of mole are introduced and some new cards are used again.

It is very important that the teacher must not permit the students to memorize lots of complicate final equations of the calculation, but require them to use consequently the basic panel and building blocks.

Sample Exercises

Samples for solution by strategy of LEGO-method, from the simple problems to the integrative and advanced problems, will be demonstrated in this paragraph. All of the solutions are built up by LEGO-method in four steps, which can express the perfectly learnable pathway of this strategy.

Sample exercises 1: Simple calculations (mass, volume and number of particles). These problems need only building up the simple relations of amount of substances to the basic panel and expression of the unknown amount. Since the unknown value is always on the left side of the equation, most often in the nominator, the calculation needs only very simple mathematical operation.

The basic panel is used in all cases:
$$n_{\text{wanted}} = \frac{u_{\text{wanted}}}{u_{\text{given}}} \cdot n_{\text{given}}$$

(a) A silicon chip contains 2.0×10^{20} atoms of silicon. How many grams of Si are there in this chip?

Step 1. The mass of Si is wanted, so the $n = \frac{m}{M}$ building block is needed on the left side, and the number of the silicon atoms is given, so $n = \frac{N}{N}$ is necessary on the right side of basic panel;

Step 2. Set up the panel
$$\frac{m_{\text{Si}}}{M_{\text{Si}}} = \frac{u_{\text{Si}}}{u_{\text{Si}}} \cdot \frac{N_{\text{Si}}}{N_{\text{A}}}$$
 and solve for unknown: $m_{\text{Si}} = \frac{u_{\text{Si}}}{u_{\text{Si}}} \cdot \frac{N_{\text{Si}}}{N_{\text{A}}} \cdot M_{\text{Si}}$

Step 3. Silicon is the wanted as well as the given substance, so $u_{\rm Si} = 1$. The Avogadro's number is needed $(N_{\rm A} = 6.022 \times 10^{23} \text{ L/mol})$, and the molar mass of silicon from the periodic table: $M_{\rm Si} = 28.09 \text{ g/mol}$;

Step 4. Calculate
$$m_{\text{Si}} = \frac{1}{1} \times \frac{2.0 \cdot 10^{20}}{6.022 \cdot 10^{23} \text{ L/mol}} \times 28.09 \text{ g/mol} = 9.3 \cdot 10^{-3} \text{ g} = 9.3 \text{ mg}$$

Answer: This chip contains 9.3 mg of silicon.

(b) Chalcanthite is a natural form of $CuSO_4.5H_2O$. Calculate the number of water molecules in a 500.0 mg weigh mineral.

Step 1. Number of molecules of water is wanted, so the $n = \frac{N}{N_A}$ building block is needed on the left side of basic panel. The mass of the mineral is given that is the reason why $n = \frac{m}{M}$ is necessary to the right side;

Step 2. Set up the panel
$$\frac{N_{\rm H_2O}}{N_{\rm A}} = \frac{u_{\rm H_2O}}{u_{\rm min}} \cdot \frac{m_{\rm min}}{M_{\rm min}}$$
 and solve for the unknown $N_{\rm H_2O} = \frac{u_{\rm H_2O}}{u_{\rm min}} \cdot \frac{m_{\rm min}}{M_{\rm min}} \cdot N_{\rm A}$

Step 3. Coefficients of water ($u_{\rm H_2O} = 5$) and the mineral ($u_{\rm min} = 1$) are needed, as well as the Avogadro's number ($N_{\rm A} = 6.022 \times 10^{23}$ L/mol), and have to calculate the molar mass of chalcanthite: $M_{\rm CuSO_4.5H_2O} = 249.7$ g/mol;

Step 4. Calculate
$$N_{\text{H}_2\text{O}} = \frac{5}{1} \times \frac{0.5000 \text{ g}}{249.7 \text{ g/mol}} \times 6.022 \cdot 10^{23} \text{ L/mol} = 6.029 \times 10^{21} \text{ L}$$

Answer: 500.0 mg sample of chalcanthite contains 6.029×10^{21} L water molecules.

(c) Molecule of cyanocobalamin, Vitamin B_{12} (M=1,357.4 g/mol), contains one atom of cobalt. What mass of Vitamin B_{12} would contain 1.00 μ g of cobalt?

Step 1. Mass of the cyanocobalamin is wanted and the mass of cobalt is given, so two $n = \frac{m}{M}$ building blocks are needed on the both sides of basic panel;

Step 2. Set up the panel
$$\frac{m_{\rm B12}}{M_{\rm B12}} = \frac{\rm u_{\rm B12}}{\rm u_{\rm Co}} \cdot \frac{m_{\rm Co}}{M_{\rm Co}}$$
 and solve for the unknown $m_{\rm B12} = \frac{\rm u_{\rm B12}}{\rm u_{\rm Co}} \cdot \frac{m_{\rm Co}}{M_{\rm Co}} \cdot M_{\rm B12}$

Step 3. The coefficients of cyanocobalamin ($u_{B12}=1$) and cobalt ($u_{Co}=1$) are used. The molar mass of vitamin is given ($M_{B12}=1,357.4$ g/mol), but molar mass of cobalt can be read from the periodic table ($M_{Co}=58.93$ g/mol). Convert the micrograms to grams;

Step 4. Calculate
$$m_{\rm B12} = \frac{1}{1} \times \frac{1.00 \cdot 10^{-6} \text{ g}}{58.93 \text{ g/mol}} \times 1357.4 \text{ g/mol} = 2.30 \times 10^{-5} \text{ g} = 23.0 \ \mu\text{g}$$

Answer: 23.0 µg cyanocobalamin contains 1.00 µg cobalt.

(d) A soda cartridge contains 8.00 g CO₂. What volume of CO₂ gas can be reached at 25 °C and 100.0 kPa? $(V_m = 24.79 \text{ dm}^3/\text{mol})$.

Step 1. The volume of carbon dioxide gas is wanted, so the $n = \frac{V_{gas}}{V_m}$ building block is needed on the left side of basic panel. The mass of the CO₂ is given; that is the reason why the $n = \frac{m}{M}$ building block is needed on the right side of basic panel;

Step 2. Set up the panel
$$\frac{V_{\text{gas}}}{V_m} = \frac{u_{\text{CO}_2}}{u_{\text{CO}_2}} \cdot \frac{m_{\text{CO}_2}}{M_{\text{CO}_2}}$$
 and solve for the unknown $V_{\text{gas}} = \frac{u_{\text{CO}_2}}{u_{\text{CO}_2}} \cdot \frac{m_{\text{CO}_2}}{M_{\text{CO}_2}} \cdot V_m$

Step 3. CO₂ is the wanted as well as the given substance, so $u_{\text{CO}_2} = 1$; the molar volume is given; the molar mass of CO₂ can be calculated $M_{\text{CO}_2} = 44.02 \text{ g/mol}$;

Step 4. Calculate
$$V_{\text{CO}_2} = \frac{1}{1} \times \frac{8.00 \text{ g}}{44.02 \text{ g/mol}} \times 24.79 \text{ dm}^3/\text{mol} = 4.51 \text{ dm}^3$$

Answer: The volume of 8.00 g carbon dioxide gas will be 4.51 dm³ litres.

(e) Nitrate content of a mineral water can be expressed as 3.25×10^{-5} mol/dm³ concentration of $Ca(NO_3)_2$ solution. How many nitrate ions are there in a 1.50 litre bottle of this mineral water?

Step 1. Number of nitrate ions is wanted, so the $n = \frac{N}{N_A}$ building block is needed on the left side of

basic panel. Molarity and the volume of $Ca(NO_3)_2$ solution are given, that is the reason why the $n = c \cdot V_{sol}$ is used on the right side of the basic panel;

Step 2. Set up the panel
$$\frac{N_{\text{NO}_3}}{N_{\text{A}}} = \frac{u_{\text{NO}_3}}{u_{\text{Ca(NO}_3)_2}} \cdot c \cdot V_{\text{sol}}$$
 solve for the unknown

$$N_{\text{NO}_3} = \frac{u_{\text{NO}_3}}{u_{\text{Ca(NO}_3)_2}} \cdot c \cdot V_{\text{sol}} \cdot N_{\text{A}}$$

Step 3. One unit of Ca(NO₃)₂ ($u_{\text{Ca(NO_3)}_2} = 1$) contains two nitrate ions ($u_{\text{NO}_3} = 2$); the Avogadro's number is also needed ($N_{\text{A}} = 6.022 \times 10^{23} \text{ L/mol}$);

Step 4. Calculate
$$N_{\text{NO}_3} = \frac{2}{1} \times 3.25 \times 10^{-5} \text{ mol/dm}^3 \times 1.50 \text{ dm}^3 \times 6.022 \cdot 10^{23} \text{ L/mol} = 5.87 \times 10^{19} \text{ L}$$

Answer: 1.50 litre of this mineral water contains 5.87×10^{19} L nitrate ions.

Sample exercises 2: Calculation by reaction equation. This type of problem-solving needs a reaction equation and the values of "u" coefficients are established from the balanced stoichiometric equation.

The basic panel is used in all cases:
$$n_{\text{wanted}} = \frac{u_{\text{wanted}}}{u_{\text{given}}} \cdot n_{\text{given}}$$

(a) What mass of CO_2 is formed in the reaction of 1.00 kg octane, with an excess of oxygen? The combustion reaction is:

$$2C_8H_{18}(l) + 25 O_2(g) \rightarrow 16 CO_2(g) + 18 H_2O(l)$$
.

Step 1. Mass of the carbon dioxide is wanted and the mass of octane is given, so the $n = \frac{m}{M}$ building blocks are needed on both sides of basic panel;

Step 2. Set up the panel
$$\frac{m_{\text{CO}_2}}{M_{\text{CO}_2}} = \frac{u_{\text{CO}_2}}{u_{\text{C}_8\text{H}_{18}}} \cdot \frac{m_{\text{C}_8\text{H}_{18}}}{M_{\text{C}_8\text{H}_{18}}}$$
 and solve for the unknown

$$m_{\text{CO}_2} = \frac{\mathrm{u}_{\text{CO}_2}}{\mathrm{u}_{\text{C}_8 \text{H}_{18}}} \cdot \frac{m_{\text{C}_8 \text{H}_{18}}}{M_{\text{C}_8 \text{H}_{18}}} \cdot M_{\text{CO}_2};$$

Step 3. From the reaction equation, the coefficients are $u_{\text{CO}_2} = 16$ and $u_{\text{C}_8\text{H}_{18}} = 2$. The molar masses can be determined. $M_{\text{CO}_2} = 44.02$ g/mol and $M_{\text{C}_8\text{H}_{18}} = 114.22$ g/mol;

Step 4. Calculate
$$m_{\text{CO}_2} = \frac{16}{2} \times \frac{1.00 \text{ kg}}{114.22 \text{ g/mol}} \times 44.02 \text{ g/mol} = 3.08 \text{ kg}$$

Note: The kilogram need not change to grams.

Answer: 3.08 kg of carbon dioxide is formed by the combustion of 1.00 kg octane.

- (b) How many O_2 molecules are reacted with 12.5 g aluminium to produce Al_2O_3 ?
- Step 1. Number of oxygen molecules is wanted, so the $n = \frac{N}{N_A}$ building block is needed on the left side

of basic panel. The mass of aluminium is given and that is the reason why $n = \frac{m}{M}$ is necessary to the right side;

Step 2. Set up the panel
$$\frac{N_{\text{O}_2}}{N_{\text{A}}} = \frac{u_{\text{O}_2}}{u_{\text{Al}}} \cdot \frac{m_{\text{Al}}}{M_{\text{Al}}}$$
 and solve for the unknown $N_{\text{O}_2} = \frac{u_{\text{O}_2}}{u_{\text{Al}}} \cdot \frac{m_{\text{Al}}}{M_{\text{Al}}} \cdot N_{\text{A}}$

Step 3. A balanced equation is needed for the coefficients: $4 \text{ Al}(s) + 3 \text{ O}_2(g) \rightarrow 2 \text{ Al}_2\text{O}_3(s)$.

The coefficients of oxygen ($u_{O_2} = 3$) and the aluminium ($u_{Al} = 4$) are from balancing, the Avogadro's number is needed ($N_A = 6.022 \times 10^{23}$ L/mol), and the molar mass of Al is from the periodic table: $M_{Al} = 26.98$ g/mol;

Step 4. Calculate
$$N_{O_2} = \frac{3}{4} \times \frac{12.5 \text{ g}}{26.98 \text{ g/mol}} \times 6.022 \cdot 10^{23} \text{ L/mol} = 2.09 \cdot 10^{23} \text{ L}$$

Answer: 2.09×10^{23} L oxygen molecules are reacted with 12.5 g aluminium.

(c) Air bags are activated when sodium azide (NaN_3) is decomposed explosively, according to the following reaction:

$$2 \text{ NaN}_3(s) \rightarrow 2 \text{ Na}(s) + 3 \text{ N}_2(g)$$

What mass of NaN₃(s) must be reacted in order to inflate an air bag to 75.0 litres at STP?

Step 1. The mass of sodium azide is wanted, so the $n = \frac{m}{M}$ building block is needed on the left side, and the volume of nitrogen is given, so $n = \frac{V_{gas}}{V_m}$ is necessary on the right side of basic panel;

Step 2. Set up the panel
$$\frac{m_{\text{NaN}_3}}{M_{\text{NaN}_3}} = \frac{u_{\text{NaN}_3}}{u_{\text{N}_2}} \cdot \frac{V_{\text{gas}}}{V_m}$$
 and solve for unknown: $m_{\text{NaN}_3} = \frac{u_{\text{NaN}_3}}{u_{\text{N}_2}} \cdot \frac{V_{\text{N}_2}}{V_m} \cdot M_{\text{NaN}_3}$

Step 3. The coefficients are $(u_{\text{NaN}_3} = 2 \text{ and } u_{\text{N}_2} = 3)$ from the reaction equation; the molar volume $(V_m = 22.71 \text{ dm}^3/\text{mol})$ and the molar mass of NaN₃ $(M_{\text{NaN}_3} = 65.02 \text{ g/mol})$ are needed;

Step 4. Calculate
$$m_{\text{NaN}_3} = \frac{2}{3} \times \frac{75.0 \text{ dm}^3}{22.71 \text{ dm}^3/\text{mol}} \times 65.02 \text{ g/mol} = 143 \text{ g}$$

Answer: 143 g of NaN₃ will produce 75.0 litres of nitrogen at 0 °C and 100 kPa.

(d) What mass of AlCl₃ can be produced by the reaction of aluminium oxide and 250.0 cm³ of a 0.1450 mol/dm³ solution of hydrochloric acid?

Step 1. The mass of aluminium chloride is wanted, so the $n = \frac{m}{M}$ building block is needed on the left side; molarity and volume of HCl solution are given, and that is the reason why the $n = c \cdot V_{sol}$ is used on the right side of the basic panel;

Step 2. Set up the panel
$$\frac{m_{\text{AlCl}_3}}{M_{\text{AlCl}_3}} = \frac{u_{\text{AlCl}_3}}{u_{\text{HCl}}} \cdot c \cdot V_{\text{sol}}$$
 and solve for the unknown

$$m_{\text{AlCl}_3} = \frac{\mathbf{u}_{\text{AlCl}_3}}{\mathbf{u}_{\text{HCl}}} \cdot c \cdot V_{\text{sol}} \cdot M_{\text{AlCl}_3}$$

Step 3. A reaction equation is needed for the coefficients: $Al_2O_3(s) + 6 HCl(aq) \rightarrow 2 AlCl_3(aq) + 3 H_2O(l)$. The coefficients are $u_{AlCl_3} = 2$ and $u_{HCl} = 6$ from the reaction equation; and the molar mass of AlCl₃ is needed ($M_{AlCl_3} = 133.33 \text{ g/mol}$). Change the volume (250.0 cm³ = 0.2500 dm³);

Step 4. Calculate
$$m_{\text{AlCl}_3} = \frac{2}{6} \times 0.1450 \text{ mol/dm}^3 \times 0.2500 \text{ dm}^3 \times 133.33 \text{ g/mol} = 1.611 \text{ g}$$

Answer: 1.611 g aluminium chloride can be produced.

(e) How much heat, in kilojoules, is associated with the production of 285 kg of slaked lime, $Ca(OH)_2$? $\Delta H + CaO(s) + H_2O(l) \rightarrow Ca(OH)_2(s)\Delta H_r^\circ = -65.2 \text{ kJ/mol}$ Step 1. The heat of the production of slaked lime is wanted, so the $n = \frac{\Delta H}{\Delta H^{\circ}_{r}}$ building block is needed on the left and $n = \frac{m}{M}$ on the right side of basic panel, because the mass of slaked lime is given;

Step 2. Set up the panel
$$\frac{\Delta H}{\Delta H^{\circ}_{r}} = \frac{u_{\text{heat}}}{u_{\text{Ca(OH)}_{2}}} \cdot \frac{m_{\text{Ca(OH)}_{2}}}{M_{\text{Ca(OH)}_{2}}}$$
 and solve for the unknown

$$\Delta H = \frac{\mathrm{u_{heat}}}{\mathrm{u_{Ca(OH)_2}}} \cdot \frac{m_{\mathrm{Ca(OH)_2}}}{M_{\mathrm{Ca(OH)_2}}} \cdot \Delta H^{\circ}_{r}$$

Step 3. The coefficients are $u_{\text{heat}} = 1$ and $u_{\text{Ca(OH)}_2} = 1$ from the reaction equation and the molar mass of Ca(OH)_2 is needed $(M_{\text{Ca(OH)}_2} = 74.10 \text{ g/mol})$. Change the mass $(285 \text{ kg} = 2.85 \times 10^5 \text{ g})$;

Step 4. Calculate
$$\Delta H = \frac{1}{1} \times \frac{2.85 \cdot 10^{5} \text{ g}}{74.10 \text{ g/mol}} \times (-65.2 \text{ kJ/mol}) = -2.51 \times 10^{5} \text{ kJ}$$

Answer: 251 MJ is associated in this reaction with the production of 285 kg of slaked lime.

(f) The combustion of methane gas is represented by the reaction:

$$\Delta H + \text{CH}_4(g) + 2 \text{ O}_2(g) \rightarrow \text{CO}_2(g) + 2 \text{ H}_2\text{O}(l); \Delta H^\circ = -890.3 \text{ kJ/mol}.$$

What volume of methane, at 18.6 °C and 102.4 kPa, must be burned to liberate 2.80×10^7 kJ of heat?

Step 1. Volume of methane gas is wanted at 18.6 °C and 102.4 kPa, so the $n = \frac{p \cdot V}{R \cdot T}$ building block is needed on the left side of the basic panel and $n = \frac{\Delta H}{\Delta H^{\circ}_{r}}$ on the right side due to given heat;

Step 2. Set up the panel
$$\frac{p \cdot V}{R \cdot T} = \frac{\mathbf{u}_{\text{CH}_4}}{\mathbf{u}_{\text{heat}}} \cdot \frac{\Delta H}{\Delta H^{\circ}_r}$$
 and solve for the unknown $V = \frac{\mathbf{u}_{\text{CH}_4}}{\mathbf{u}_{\text{heat}}} \cdot \frac{\Delta H}{\Delta H^{\circ}_r} \cdot \frac{R \cdot T}{p}$.

Step 3. The coefficients are $u_{\text{CH}_4} = 1$ and $u_{\text{heat}} = 1$ from the reaction equation and the Clapeyron-Mendelejev equation contains pressure in unit of Pa (= kg/m·s²) and $R = 8.314 \text{ m}^2 \cdot \text{kg/s}^2 \cdot \text{K·mol}$. Change pressure (102.4 kPa = 1.024×10⁵ Pa) and temperature (18.6 + 273.1 K);

Step 4. Calculate
$$V = \frac{1}{1} \times \frac{(-2.80 \times 10^7) \text{ kJ}}{(-890.3) \text{ kJ/mol}} \times \frac{8.314 \text{ m}^2 \cdot \text{kg/s}^2 \cdot \text{K} \cdot \text{mol} \cdot (18.6 + 273.1) \text{ K}}{1.024 \times 10^5 \text{ kg/m} \cdot \text{s}^2} = 745 \text{ m}^3$$

Answer: The given quantity of liberated heat needs burning of 745 m³ methane at these conditions.

(g) What volume of Cl_2 gas is produced, at 25 °C and 100.0 kPa, when molten NaCl is electrolyzed by a current of 10.0 A for 2.00 hours? ($V_m = 24.79 \text{ dm}^3/\text{mol}$).

Step 1. Volume of chlorine gas is wanted (the molar volume is known), so the $n = \frac{V_{gas}}{V_m}$ building block is needed on the left side of the basic panel. The parameters of electrolysis are given, therefore, $n_e = \frac{I \cdot t}{T}$ is

needed on the right side for moles of electrons. $V_{\text{even}} = U_{\text{electron}}$

Step 2. Set up the panel
$$\frac{V_{\text{gas}}}{V_m} = \frac{\mathbf{u}_{\text{Cl}_2}}{\mathbf{u}_{\text{e}}} \cdot \frac{I \cdot t}{F}$$
 and solve for the unknown $V_{\text{gas}} = \frac{\mathbf{u}_{\text{Cl}_2}}{\mathbf{u}_{\text{e}}} \cdot \frac{I \cdot t}{F} \cdot V_m$

Step 3. A reaction equation is needed for the coefficients: $2 \text{ Cl} \rightarrow \text{Cl}_2 + 2 \text{ e}^-$.

The coefficients are u_{Cl_2} = 1 and u_{e} = 2 from the reaction equation and the molar volume is given and the Faraday-constant is needed (F = 96485 As/mol). Change the time (2.00 h = 7,200 sec);

Step 4. Calculate
$$V_{\text{gas}} = \frac{1}{2} \times \frac{10.0 \text{ A} \times 7200 \text{ s}}{96485 \text{ As/mol}} \times 24.79 \text{ dm}^3/\text{mol} = 9.25 \text{ dm}^3$$
.

Answer: 9.25 dm³ litres of chlorine gas can be produced by these conditions.

(h) Citric acid has the molecular formula $C_6H_8O_7$. A 0.250g sample of citric acid dissolved in 25.0 cm³ of water requires 37.2 cm³ of 0.105 mol/dm³ NaOH for complete neutralization. How many acidic hydrogens per molecule does citric acid have?

Step 1. From the equation of neutralization $C_6H_8O_7 + x$ NaOH \rightarrow Na_xC₆H_{8-x}O₇ + x H₂O can be stated that $x = u_{\text{NaOH}}$ is wanted, and the $n = c \cdot V_{sol}$ building block is needed on the left side of basic panel. The mass of citric acid is given, so $n = \frac{m}{M}$ building block gets to the right side, and the volume of water is not needed for solving the problem.

Step 2. Set up the panel
$$c \cdot V_{\text{sol}} = \frac{\mathbf{u}_{\text{NaOH}}}{\mathbf{u}_{\text{C}_6\text{H}_8\text{O}_7}} \cdot \frac{m_{\text{C}_6\text{H}_8\text{O}_7}}{M_{\text{C}_6\text{H}_8\text{O}_7}}$$
 and solve for the unknown

$$c \cdot V_{\rm sol} \cdot {\bf u}_{{\rm C}_6{\rm H}_8{\rm O}_7} \cdot \frac{M_{{\rm C}_6{\rm H}_8{\rm O}_7}}{m_{{\rm C}_6{\rm H}_8{\rm O}_7}} = {\bf u}_{\rm NaOH}$$

Step 3. $u_{C_6H_8O_7}=1$; the molar mass of citric acid can be determined $M_{C_6H_8O_7}=192.12$ g/mol; change volume of NaOH (37.2 cm³ = 0.0372 dm³);

Step 4. Calculate
$$u_{NaOH} = 0.105 \text{ mol/dm}^3 \times 0.0372 \text{ dm}^3 \times 1 \times \frac{192.12 \text{ g/mol}}{0.250 \text{ g}} = 3.00$$

Answer: Citric acid has three acidic hydrogens.

Sample exercises 3: Advanced exercises—Also use other building blocks. Other simple building blocks are also used in these examples of problem-solving, e.g., mass percent, volume percent, density, but the calculation is not so difficult.

The basic panel is used in all cases:
$$n_{\text{wanted}} = \frac{u_{\text{wanted}}}{u_{\text{given}}} \cdot n_{\text{given}}$$

- (a) What volume of a cubic mineral Fluorite (CaF_2) contains 1.55×10^{21} L fluoride ions? Density of the mineral is 3.18 g/cm³.
- Step 1. Volume of the solid compound (CaF₂) is wanted. The mass can be expressed by the volume from the relation of density $m = \rho V$, that is the reason why the $n = \frac{m}{M}$ is necessary to the left side of the panel.

Numbers of fluoride ions are given, so the $n = \frac{N}{N_A}$ building block is needed on the right side;

Step 2. Set up the panel
$$\frac{m_{\text{CaF}_2}}{M_{\text{CaF}_2}} = \frac{u_{\text{CaF}_2}}{u_{\text{F}}} \cdot \frac{N_{\text{F}}}{N_{\text{A}}}$$
 import the relation of volume $\frac{\rho \cdot V_{\text{CaF}_2}}{M_{\text{CaF}_2}} = \frac{u_{\text{CaF}_2}}{u_{\text{F}}} \cdot \frac{N_{\text{F}}}{N_{\text{A}}}$ and solve for unknown: $V_{\text{CaF}_2} = \frac{u_{\text{CaF}_2}}{u_{\text{F}}} \cdot \frac{N_{\text{F}}}{N_{\text{A}}} \cdot \frac{M_{\text{CaF}_2}}{\rho}$.

Step 3. The coefficients are $u_{\text{CaF}_2} = 1$ and $u_{\text{F}} = 2$ from the formula; the Avogadro's number ($N_{\text{A}} = 6.022 \times 10^{23} \text{ L/mol}$) and the molar mass of CaF₂ ($M_{\text{CaF}_2} = 78.08 \text{ g/mol}$) are needed;

Step 4. Calculate
$$V_{\text{CaF}_2} = \frac{1}{2} \times \frac{1.55 \times 10^{21}}{6.022 \times 10^{23} \text{ L/mol}} \times \frac{78.08 \text{ g/mol}}{3.18 \text{ g/cm}^3} = 3.16 \times 10^{-2} \text{ cm}^3$$

Answer: The volume of the mineral is 31.6 mm³.

(b) What mass of 65.0% phosphoric acid solution, by mass, can be produced from 150 tons of white phosphorous? Use a symbolized reaction equation:

$$P_4(s) \rightarrow 4 H_3PO_4(aq)$$
.

Step 1. The total mass of phosphoric acid solution is wanted, so the $w\% = 100 \cdot \frac{m_{\rm H_3PO_4}}{m_{total}}$ relation has to use for expressing the total mass of H₃PO₄ solution, and $n = \frac{m}{M}$ building block for $m_{\rm H_3PO_4}$ is needed on the left side of basic panel. The mass of the reactant phosphorous is given and that is the reason why $n = \frac{m}{M}$ is necessary to the right side, too;

Step 2. Set up the panel $\frac{m_{\rm H_3PO_4}}{M_{\rm H_3PO_4}} = \frac{u_{\rm H_3PO_4}}{u_{\rm P_4}} \cdot \frac{m_{\rm P_4}}{M_{\rm P_4}}$, solve for unknown $m_{\rm H_3PO_4} = \frac{u_{\rm H_3PO_4}}{u_{\rm P_4}} \cdot \frac{m_{\rm P_4}}{M_{\rm P_4}} \cdot M_{\rm H_3PO_4}$ and express the total mass of solution $m_{total} = \frac{100}{w_{\rm H_3PO_4}} \cdot m_{\rm H_3PO_4} = \frac{100}{w_{\rm H_3PO_4}} \cdot \frac{u_{\rm H_3PO_4}}{u_{\rm P_4}} \cdot \frac{m_{\rm P_4}}{M_{\rm P_4}} \cdot M_{\rm H_3PO_4}$.

Step 3. The coefficients are $u_{\rm H_3PO_4}$ = 4 and $u_{\rm P_4}$ = 1 from the equation and the molar masses are also needed $(M_{\rm P_4}$ = 123.88 g/mol, $M_{\rm H_3PO_4}$ = 97.99 g/mol);

Step 4. Calculate
$$m_{total} = \frac{100}{65.0} \times \frac{4}{1} \times \frac{150 \text{ t}}{123.88 \text{ g/mol}} \times 97.99 \text{ g/mol} = 730 \text{ t}$$

Note: The tone need not change to grams.

Answer: The total mass of produced phosphorous acid solution is 730 tones.

(c) On a hot summer day (35.5 °C, 1020 hPa), how many O_2 molecules can an adult breathe in once by 0.500 litre of air? Air contains 29.05% O_2 by volume.

Step 1. Number of molecules of oxygen is wanted, so the $n = \frac{N}{N_A}$ building block is needed on the left side of basic panel. Volume of the air and its oxygen content are given at 35.5°C and 1020 hPa, so the $n = \frac{p \cdot V}{R \cdot T}$ building block is needed on the right side of the basic panel and the relation of percent by volume

also have to use $\varphi\% = 100 \cdot \frac{V_{\rm O_2}}{V_{air}}$;

Step 2. Set up the panel $\frac{N_{\mathrm{O_2}}}{N_{\mathrm{A}}} = \frac{\mathrm{u_{\mathrm{O_2}}}}{\mathrm{u_{\mathrm{O_2}}}} \cdot \frac{p \cdot V_{\mathrm{O_2}}}{R \cdot T}$, import the relation of $V_{\mathrm{O_2}}$: $V_{\mathrm{O_2}} = \frac{\varphi\% \cdot V_{air}}{100}$

$$\frac{N_{\mathrm{O_2}}}{N_{\mathrm{A}}} = \frac{\mathrm{u_{\mathrm{O_2}}}}{\mathrm{u_{\mathrm{O_2}}}} \cdot \frac{p}{R \cdot T} \cdot \frac{V_{\mathrm{air}} \cdot \varphi\%}{100} \quad \text{and solve for unknown:} \quad N_{\mathrm{O_2}} = \frac{\mathrm{u_{\mathrm{O_2}}}}{\mathrm{u_{\mathrm{O_2}}}} \cdot \frac{p}{R \cdot T} \cdot \frac{V_{\mathrm{air}} \cdot \varphi\%}{100} \cdot N_{\mathrm{A}}$$

Step 3. Oxygen is the wanted as well as the given substance, so u_{O_2} = 1; the Avogadro's number (N_A = 6.022×10^{23} L/mol) is needed; the Clapeyron-Mendelejev equation contains pressure in unit of Pa (= kg/m·s²), volume in unit of m³ and R =8.314 m²·kg/s²·K·mol (change volume, pressure and temperature!)

Step 4. Calculate

$$N_{\rm O_2} = \frac{1}{1} \times \frac{1.020 \times 10^5 \text{ kg/m} \cdot \text{s}^2}{8.314 \text{ m}^2 \cdot \text{kg/s}^2 \cdot \text{K} \cdot \text{mol} \cdot (35.5 + 273.1 \text{ K})} \times \frac{5.00 \times 10^4 \text{ m}^3 \times 29.05}{100} \times 6.022 \cdot 10^{23} \text{ L/mol} = 3.48 \times 10^{21} \text{L} \cdot \frac{1.020 \times 10^5 \text{ kg/m} \cdot \text{s}^2}{100} \times 6.022 \cdot 10^{23} \text{ L/mol} = 3.48 \times 10^{21} \text{L} \cdot \frac{1.020 \times 10^5 \text{ kg/m} \cdot \text{s}^2}{100} \times 6.022 \cdot 10^{23} \text{ L/mol} = 3.48 \times 10^{21} \text{L} \cdot \frac{1.020 \times 10^5 \text{ kg/m} \cdot \text{s}^2}{100} \times 6.022 \cdot 10^{23} \text{ L/mol} = 3.48 \times 10^{21} \text{L} \cdot \frac{1.020 \times 10^5 \text{ kg/m} \cdot \text{s}^2}{100} \times 6.022 \cdot 10^{23} \text{ L/mol} = 3.48 \times 10^{21} \text{L/mol} = 3.48 \times 1$$

Answer: 3.48×10^{21} oxygen molecules are breathed by 0.500 litre of air.

(d) What volume of 2.430 mol/dm³ sodium hydroxide is required to neutralize 25.00 g of sulphuric acid solution 12.45% by mass?

Step 1. The volume of NaOH solution is wanted, so the $n = c \cdot V_{sol}$ building block is needed on the left side of basic panel. Percent by mass of sulphuric acid solution and its mass are given; that is the reason why the $n = \frac{m}{M}$ building block has to be used on the other side. The relation of $w\% = 100 \cdot \frac{m_{\rm H_2SO_4}}{m_{total}}$ also is used for calculation;

Step 2. Set up the panel
$$c \cdot V_{\text{sol}} = \frac{\mathbf{u}_{\text{NaOH}}}{\mathbf{u}_{\text{H}_2\text{SO}_4}} \cdot \frac{m_{\text{H}_2\text{SO}_4}}{M_{\text{H}_2\text{SO}_4}}$$
, import the relation of $m_{\text{H}_2\text{SO}_4}$: $m_{\text{H}_2\text{SO}_4} = \frac{w^0 / \cdot m_{\text{total}}}{100}$

$$c \cdot V_{\rm sol} = \frac{\mathrm{u_{\,NaOH}}}{\mathrm{u_{\,H_2SO_4}}} \cdot \frac{w\% \cdot m_{\rm total}}{100 \cdot M_{\,H_2SO_4}} \quad \text{and solve for unknown} \quad V_{\rm sol} = \frac{\mathrm{u_{\,NaOH}}}{\mathrm{u_{\,H_2SO_4}}} \cdot \frac{w\% \cdot m_{\rm total}}{100 \cdot M_{\,H_2SO_4} \cdot c}$$

Step 3. Write the balanced equation of neutralization: $H_2SO_4(aq) + 2 \text{ NaOH}(aq) \rightarrow \text{Na}_2SO_4(aq) + 2 H_2O(l)$. The coefficients are: $u_{\text{NaOH}} = 2$ and $u_{\text{H}_2SO}_4 = 1$; the molar mass is $M_{\text{H}_2SO}_4 = 98.09 \text{ g/mol}$;

Step 4. Calculate
$$V_{\text{sol}} = \frac{2}{1} \times \frac{12.45 \cdot 25.00 \text{ g}}{100 \cdot 98.09 \text{ g/mol} \cdot 1.215 \text{ mol/dm}^3} = 0.05223 \text{ dm}^3$$

Answer: Neutralization needs 52.23 cm³ of NaOH solution.

(e) How many grams of gaseous F_2 are needed to produce 124.0 g of PF_3 if the reaction has a 78.5% yield?

$$P_4(s) + 6 F_2(g) \rightarrow 4 PF_3(g)$$
.

Step 1. The percent yield is meaning a ratio between the actual and the theoretical amount of the component $\eta\% = 100 \cdot \frac{m_{\text{actual}}}{m_{\text{theoretical}}}$. The theoretical mass of fluorine is wanted, but the actual mass of PF₃ is

given. On both sides of basic panel $n = \frac{m}{M}$, building blocks are needed and the actual mass of fluorine can be determined at first;

Step 2. Set up the panel for actual (given) masses $\frac{m_{\rm F_2\,(act)}}{M_{\rm F_2}} = \frac{{\rm u}_{\rm F_2}}{{\rm u}_{\rm PF_3}} \cdot \frac{m_{\rm PF_3}}{M_{\rm PF_3}}$ solve for unknown

$$m_{\mathrm{F}_{2}(act)} = \frac{\mathrm{u}_{\mathrm{F}_{2}}}{\mathrm{u}_{\mathrm{PF}_{3}}} \cdot \frac{m_{\mathrm{PF}_{3}}}{M_{\mathrm{PF}_{3}}} \cdot M_{\mathrm{F}_{2}}$$
 and express the relation for $m_{\mathrm{F}_{2}(theor)}$

$$m_{\rm F_2(\it{theor})} = \frac{100}{\eta\%} \cdot m_{\rm F_2(\it{act})} = \frac{100}{\eta\%} \cdot \frac{{\rm u_{\rm F_2}}}{{\rm u_{\rm PF_3}}} \cdot \frac{m_{\rm PF_3}}{M_{\rm PF_3}} \cdot M_{\rm F_2}$$

Step 3. The coefficients are u_{F_2} = 6 and u_{PF_3} = 4 from the equation; the molar masses are also needed (M_{F_2} = 38.00 g/mol, M_{PF_3} = 87.97 g/mol);

Step 4. Calculate
$$m_{\text{F}_2(theor)} = \frac{100}{78.5} \times \frac{6}{4} \times \frac{124.0 \text{ g}}{87.97 \text{ g/mol}} \times 38.00 \text{ g/mol} = 102.4 \text{ g}$$

Answer: At this yields of the reaction 102.4 g, F₂ is needed for 124.0 g of PF₃.

Sample exercises 4: Challenge problems. These difficult types of problems need not only mechanical

use of LEGO-method, but also logical thinking and systematic building up of problem-solving. Sometimes, identifying the wanted value demands hard thinking or the given data are hidden in the exercise.

The basic panel is used in all cases:
$$n_{\text{wanted}} = \frac{\mathbf{u}_{\text{wanted}}}{\mathbf{u}_{\text{given}}} \cdot n_{\text{given}}$$

(a) Fungal laccase, a blue protein found in wood-rotting fungi. It contains 0.390% Cu by mass. If the fungal laccase molecule contains four copper atoms, what is the molar mass of fungal laccase?

Step 1. The molar mass of fungal laccase is questioned, so the $n=\frac{m}{M}$, building block is needed on the left side of basic panel. The copper content is given in percent by mass and that is the reason why the $n=\frac{m}{M}$ is needed on the right side and the relation of $w\%=100\cdot\frac{m_{\text{Cu}}}{m_{laccase}}$ also has to use;

Step 2. Set up the panel
$$\frac{m_{\text{laccase}}}{M_{\text{laccase}}} = \frac{u_{\text{laccase}}}{u_{\text{Cu}}} \cdot \frac{m_{\text{Cu}}}{M_{\text{Cu}}}$$
, solve for the unknown $M_{\text{laccase}} = \frac{u_{\text{Cu}}}{u_{\text{laccase}}} \cdot \frac{m_{\text{Cu}}}{M_{\text{Cu}}}$ and replace the relation of percent $M_{\text{laccase}} = \frac{u_{\text{Cu}}}{u_{\text{laccase}}} \cdot \frac{100}{w^{0/6}} \cdot M_{\text{Cu}}$

Step 3. One fungal laccase molecule contains 4 copper atoms, so $u_{\text{laccase}} = 1$ and $u_{\text{Cu}} = 4$; the molar mass of Cu is from the periodic table: $M_{\text{Cu}} = 63.55 \text{ g/mol}$;

Step 4. Calculate
$$M_{\text{laccase}} = \frac{4}{1} \times \frac{100}{0.390} \times 63.55 \text{ g/mol} = 6.52 \cdot 10^4 \text{ g/mol}$$

Answer: The molar mass of fungal laccase is 6.52×10^4 g/mol.

(b) A 0.755 g sample of hydrated copper (II) sulphate (CuSO₄ · xH_2O) was heated carefully, until it had changed completely to anhydrous copper (II) sulphate (CuSO₄) with a mass 0.483 g. Determine the value of x.

Step 1. Value of x can be determined by the help of the chemical equation of decomposition of hydrated copper (II) sulphate. From the equation of $CuSO_4.xH_2O \rightarrow CuSO_4 + x H_2O$ can be stated that $x = u_{H_2O}$ is wanted. The mass of the anhydrous copper (II) sulphate is given, as well as the mass of water can be calculated by subtraction, so the $n = \frac{m}{M}$ building blocks are needed on both sides of basic panel;

Step 2. Set up the panel
$$\frac{m_{\rm H_2O}}{M_{\rm H_2O}} = \frac{u_{\rm H_2O}}{u_{\rm CuSO_4}} \cdot \frac{m_{\rm CuSO_4}}{M_{\rm CuSO_4}} \quad \text{and solve for the unknown}$$

$$\frac{m_{\rm H_2O}}{M_{\rm H_2O}} \cdot \frac{M_{\rm CuSO_4}}{m_{\rm CuSO_4}} \cdot u_{\rm CuSO_4} = u_{\rm H_2O}$$

Step 3. The u_{CuSO_4} coefficient is equal with one; the mass of water is $m_{\text{H}_2\text{O}} = m_{\text{sample}} - m_{\text{CuSO}_4}$; and the molar masses also can be determined $M_{\text{H}_2\text{O}} = 18.02$ g/mol, $M_{\text{CuSO}_4} = 159.62$ g/mol;

Step 4. Calculate
$$u_{H_2O} = \frac{(0.755 \text{ g} - 0.483 \text{ g})}{18.02 \text{ g/mol}} \times \frac{159.62 \text{ g/mol}}{0.483 \text{ g}} \times 1 = 4.99$$

Answer: The formula of hydrated copper (II) sulphate is CuSO₄.5H₂O.

(c) The density of phosphorus vapour at 310 °C and 103325 Pa is 2.64 g/dm³. What is the molecular formula of the phosphorus under these conditions?

Step 1. Molecular formula of the phosphorus can be determined by the help of calculation its molar mass, so the $n = \frac{m}{M}$ building block is needed on the left side of basic panel. Parameters of the phosphorus vapour are given. Therefore, $n = \frac{p \cdot V}{R \cdot T}$ building block has to be used on the other side. Moreover, the value of density can be used instead of the mass and volume ratio of phosphorus: $\rho = \frac{m}{V}$;

Step 2. Set up
$$\frac{m_{P_x}}{M_{P_x}} = \frac{u_{P_x}}{u_{P_x}} \cdot \frac{p \cdot V_{P_x}}{R \cdot T}$$
 and regroup the panel $\frac{m_{P_x}}{V_{P_x}} = \rho = \frac{u_{P_x}}{u_{P_x}} \cdot \frac{p}{R \cdot T} \cdot M_{P_x}$ and solve for the unknown $M_{P_x} = \frac{u_{P_x}}{u_{P_x}} \cdot \frac{\rho \cdot R \cdot T}{p}$ and finally determine x : $x = \frac{M_{P_x}}{M_{P_x}}$.

Step 3. Phosphorus is the wanted as well as the given substance, so $u_{P_x} = 1$; the Clapeyron-mendelejev equation contains pressure in unit of Pa (= kg/m·s²), $R = 8.314 \text{ m}^2 \cdot \text{kg/s}^2 \cdot \text{K·mol}$ and volume in unit of m³ (change density and temperature!), and the molar mass of P is from the periodic table: $M_P = 30.97 \text{ g/mol}$;

Step 4. Calculate
$$M_{P_x} = \frac{1}{1} \times \frac{2.64 \cdot 10^3 \text{ g/m}^3 \cdot 8.314 \text{ m}^2 \cdot \text{kg/s}^2 \cdot \text{K} \cdot \text{mol} \cdot (310 + 273) \text{ K}}{103325 \text{ kg/m} \cdot \text{s}^2} = 123.8 \text{ g/mol}$$

$$x = \frac{123.8 \text{ g/mol}}{30.97 \text{ g/mol}} = 4.00$$

Answer: The molecule consist of four phosphorus atoms, so the formula of the phosphorus vapour is P₄.

(d) 6.00 mol/dm³ sulphuric acid, $H_2SO_4(aq)$, has a density of 1.338 g/cm³. What is the percent by mass of sulphuric acid in this solution?

Step 1. Percent by mass of sulphuric acid is questioned, so the relation of $w\% = 100 \cdot \frac{m_{\text{H}_2\text{SO}_4}}{m_{sol}}$ has to be

used. The $n=\frac{m}{M}$ building block is needed on the left side of basic panel. The molarity of the solution is given, which is contained in the $n=c\cdot V_{sol}$, so it comes to the right side of the basic panel. Neither mass nor volume of the solution is known, but the value of density can be used for express the volume of solution: $V_{sol}=\frac{m_{sol}}{\rho}$;

Step 2. Set up the panel
$$\frac{m_{\mathrm{H_2SO_4}}}{M_{\mathrm{H_2SO_4}}} = \frac{\mathrm{u_{\mathrm{H_2SO_4}}}}{\mathrm{u_{\mathrm{H_2SO_4}}}} \cdot c \cdot V_{\mathrm{sol}}$$
 and replace the volume $\frac{m_{\mathrm{H_2SO_4}}}{M_{\mathrm{H_2SO_4}}} = \frac{\mathrm{u_{\mathrm{H_2SO_4}}}}{\mathrm{u_{\mathrm{H_2SO_4}}}} \cdot c \cdot \frac{m_{\mathrm{sol}}}{\rho}$ and express the wanted percent by mass $w\% = 100 \cdot \frac{m_{\mathrm{H_2SO_4}}}{m_{\mathrm{sol}}} = 100 \cdot \frac{\mathrm{u_{\mathrm{H_2SO_4}}}}{\mathrm{u_{\mathrm{H_2SO_4}}}} \cdot c \cdot \frac{M_{\mathrm{H_2SO_4}}}{\rho}$.

Step 3. $u_{\text{H}_2\text{SO}_4}$ = 1; the molar mass can be determined $M_{\text{H}_2\text{SO}_4}$ = 98.09 g/mol; change density!;

Step 4. Calculate
$$w\% = 100 \times \frac{1}{1} \times 6.00 \text{ mol/dm}^3 \times \frac{98.09 \text{ g/mol}}{1338 \text{ g/dm}^3} = 44.0$$

Answer: The 6.00 mol/dm³ sulphuric acid solution has 44.0 percent by mass of H₂SO₄.

Conclusions

LEGO-method does not require students to memorize lots of final formulas, but only to use consistently

the panel and building blocks. Both understanding of the problem and correct use of definitions as well as units have a significant role in application of this method. Therefore, the first requirement for successful problem-solving—that the problem solver has to understand the meaning of the problem—is realized, because finding out the wanted and given amounts is the first step of LEGO-method.

First of all, a great difference can be disclosed between the American and the European (first of all Hungarian and German) printed books, namely, the numbers of worked out examples. Ever so many sample exercises are there in the US or Canadian university textbooks, but not any or only few solutions can be found in the European ones (Schröter, Lautenschläger, Bibrack, & Schnabel, 1990; Gergely, Erdődi, & Vereb, 2001).

General character of the American and Canadian textbooks that those apply thoroughly integrated, step-by-step approach to problem-solving and build a lasting conceptual understanding of key chemical concepts, contains hundreds of examples, solved problems and practice exercises.

Our non-published measurements have demonstrated that eighth to 12th graders (N = 255), who learned the LEGO-method in the school, have used this strategy more frequently (ca. 60%) than mole or proportional methods for solving a stoichiometric problem.

Students, who have learned this method of problem-solving from the beginning, do not hate chemistry calculations and they can solve problems more effective than others, because they understand well the concepts on which the problems are based (first of all the meaning of mass, volume, mole, concentration, etc.), and have less "misconceptions". The success of these students (ca. 77 %) indicates that the LEGO-method is a useful alternative strategy for teaching calculations and a more complete understanding of problem-solving.

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