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Developing Pre-algebraic Thinking in Generalizing Repeating Pattern Using SOLO Model*

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In this paper, researchers discussed the application of the generalization perspective in helping the primary school pupils to develop their pre-algebraic thinking in generalizing repeating pattern. There are two main stages of the generalization perspective had been adapted, namely investigating and generalizing the pattern. Since the Biggs and Collis' (1982) cognitive development model, SOLO model (structure of the observing learning outcome model), is more thoroughly worked out for describing and analyzing students' conceptualizations and understandings in the algebra learning, researchers claimed that the proposed framework enables primary school pupils' pre-algebraic thinking to be described systematically. Researchers hypothesized that primary school pupils could exhibit three levels of pre-algebraic thinking in solving repeating pattern task based on SOLO model, namely, uni-structural, multi-structural and relational. This paper drew an adaptation of the SOLO model that provides teachers and their pupils with a pedagogically sound template which can be used to develop pre-algebraic thinking.

Keywords: pre-algebraic thinking, generalization perspective, repeating pattern, SOLO model (structure of the observing learning outcome model)

Introduction

The technological future of a modern society depends in a large part on the mathematical literary, as mathematics is the most powerful technique for the understanding, generalizing of patterns and analysis of the relationship of the patterns. There is currently a general agreement among mathematics researchers that algebra is fundamentally the study of pattern and relationship. It is the most important topic to make generalization and interpretation of patterns and relationships. They have challenged the conventional view of algebra as a series of abstract rules regarding "*x*'s" and "*y*'s", formal structure, manipulation of symbols and rote skills, arguing that algebra as a tool for problem-solving and a method of expressing relationship (Day & Jones, 1997; Fernandez & Anhalt, 2001; NCTM (National Council of Teachers of Mathematics), 1989). Thus, algebraic thinking has become a catch-all phrase for the recent research. In the interest of building a strong foundation in algebra, preparation towards algebra at the primary school years deemed important, since algebra is a compulsory topics in secondary school and it is a prerequisite in going further to the more complex topics at the upper level of secondary school, such as straight line, gradient and area under graph, index and logarithm, matrix, variation and graph of function and quadratic equation (Teng, 2002). This seemed to suggest that the

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basic of algebraic thinking may be developed from arithmetic thinking and transitioned into algebraic thinking. Warren (2000) called this transition of thinking from arithmetic thinking to algebraic thinking as pre-algebraic thinking.

In spite of the difficult transition from arithmetic to algebraic thinking, research evidences indicated that the young children to sixth grade pupils are able to exhibit their pre-algebraic thinking and algebraic thinking (Cai, 1998; Lubinski & Otto, 1997; Slavit, 1999; Blanton & Kaput, 2004; Garrick, Threlfall, & Orton, 2005). In consequence, a number of mathematics researchers and educators have begun to focus on the investigation into the development of pre-algebraic thinking which can be viewed from different perspectives, such as generalization, problem-solving, modeling and functional (Fernandez & Anhalt, 2001; Friedlander & Hershkowitz, 1997; Herbert & Brown, 1997; Femiano, 2003; Ferrucci, Yeap, & Carter, 2003; Thornton, 2001). Apparently, all these perspectives have challenged the old world view of algebra, arguing that algebra is treated as a tool for expressing relationship, describing, analyzing and representing patterns and exploring mathematical properties in a variety of problem situation (Day & Jones, 1997; Fernandez & Anhalt, 2001; NCTM, 1989).

A deeper appreciation and knowledge of pupils' approaches to the pictorial pattern, especially that repeating pattern is essential, if we improve the quality of pupils' learning in generalizing of pattern in algebra. However, the most common type of algebra-related task found in the primary school mathematics textbook is where pupils are asked to continue or complete the number pattern. Thus, the search or construction of problem situation that promote a meaningful approach to pre-algebra is predominantly challenging task. In this paper, we discuss how the generalization perspective can be used to help the primary school pupils to develop their pre-algebraic thinking in generalizing repeating pattern. Since the Biggs and Collis' (1982) cognitive development model, which is called SOLO model (structure of the observing learning outcome model), is more thoroughly worked out for describing and analyzing student's conceptualizations in the algebra learning, we claim that the proposed framework enable primary school pupils' pre-algebraic thinking to be described systematically across three levels of this model, namely, uni-structural, multi-structural and relational.

What Is Repeating Pattern?

The repeating pattern contains a segment or core that continuously recurs. Richards and Jones (1990) suggested some examples of repeating pattern to help the primary school pupils to generate pattern, such as:

(1) Building bricks or beads;

(2) Using sound (clapping pattern);

(3) Using musical instruments (playing nursery rhymes);

(4) Movement (jump, hop and step);

(5) Arranging objects (shells and shapes);

(6) Making pictures (painting and drawing);

(7) Iconic and symbolic representations (color, letters and number).

Repeating pattern can be within one attribute (for example, color, size, shape, number, letters, etc.) or more than one (for example, using shapes of different size). The segment can vary in size and level of complexity, but the simplest includes just two elements. Figure 1 represents varies types of repeating pattern.

Exploration repeating pattern is an interesting activity that should occur in primary years' classroom. Typically pupils are asked to copy and continue the pattern, identify the segment of the pattern and find missing elements. In order to investigate their capabilities in generalizing of repeating pattern, several attempts were made to investigate the stages in the development of the primary school pupils' pattern abilities (Frobisher & Threlfall, 2005). Friedlander and Hershkowitz (1997), Herbert and Brown (1997) and Swafford and Langrall (2000) maintained that the ability of making generalization for the pattern involves a number of process or stages, namely investigating the pattern, representing and generalizing the pattern, interpreting and applying the equation. Since the primary school pupils are the early learner of investigating pattern, only the first two stages are discussed and adapted.

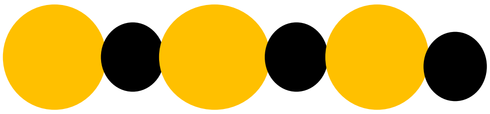
Examples of repeating pattern	The core of the pattern	The attribute of pattern
	• yellow-red (AB) • two elements	color
	• yellow-black(AB) • big-small (AB) • two elements	color and size
A B C B A B C B A B	• ABCB • four elements	letters
4 5 7 8 8 4 5 7 8 8 4 5 7	• 45788 (ABCDD) • five elements	numbers

Figure 1. Various types of repeating pattern.

The role of the initial process is to represent the pupils with the numerical examples that may turn out to be instances of an emerging pattern which is to be discovered. Pupils will notice and understand the pattern while working with the numerical examples which include the ability to extend to the next case and near cases. Data relating to pupils' responses and understandings of the pattern in the sequence can be collected in both oral form and written form.

The main contribution of pattern work is developing algebraic principle of generalization. Generalization is a process when the commonalties in the specific cases of the pattern are identified. More specifically, to generalize the pattern is to identify the sequence of operations that are common among the specific cases and extend them to the general case. Thus, making generalization through some specific cases is one of important perception to express generality in a lesson. Pupils are probably seeing the pattern through the particular number of specific cases and aware of generality. Friedlander and Hershkowitz (1997) and Mason (1996) noted that when students are confronted with "unfriendly" or large number of specific examples, it will push and force them to make a generalization for the pattern and they prompt to give the responses without having to see or draw them all. With respect to representation, researchers wanted to determine if pupils can refine their understanding of the pattern. In a repeating pattern context, identifying the core of repeat is the most frequent form of generalization.

In this paper, the two investigative processes identified by Friedlander and Hershkowitz (1997), Herbert and Brown (1997) and Swafford and Langrall (2000) were adopted to develop the pupils' pre-algebraic thinking across repeating pattern pictorial as shown in Figure 2.

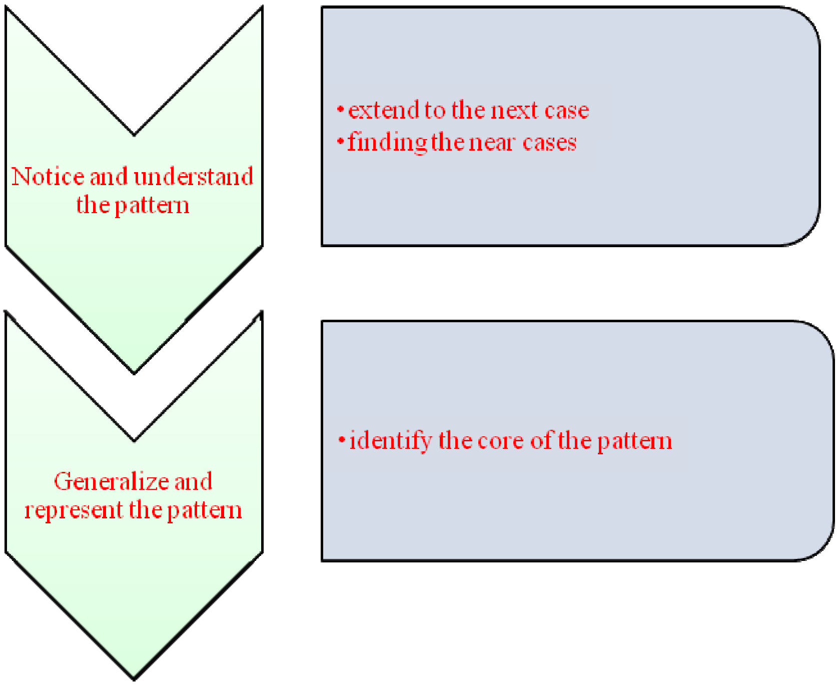


*Figure 2.*Algebraic process in investigating pattern.

Researchers hypothesized that young children could exhibit three levels of pre-algebraic thinking in investigating the repeating pattern based on SOLO model, namely uni-structural, multi-structural and relational. The description of this model and its application has been discussed in some detail as follows.

What Is SOLO Model?

SOLO model was developed by Biggs and Collis (1982). It was designed mainly as a mean to measure students' cognitive ability in academic learning context (Biggs & Collis, 1982; Collis & Romberg, 1991; Vallecillos & Moreno, 2002). The SOLO model has been used to analyze the structure of students' mathematical thinking, understanding of mathematical concepts and problem-solving ability over a wide educational span from primary to tertiary levels (Callingham, Pegg, & Wright, 2009; Reading, 1999; Chick, 1988; Collis, Romberg, & Jurdak, 1986; Lam & Foong, 1998; Panizzon, Callingham, Wright, & Pegg, 2007; Vallecillos & Moreno, 2002; Watson, Chick, & Collis, 1988; Wilson & Iventosch, 1988).

SOLO model suggested that when students answer the tasks given, their responses display a similar sequence across the task. This leads to the identification the stages at which a student is currently operating (Biggs & Collis, 1982). In these consistent sequences, the following stages occur:

(1) Pre-structural—response consists only of irrelevant information to the task. In other words, the task is

not attacked appropriately and the students have not really understood the task given;

(2) Uni-structural—response includes one obvious piece of information and it is treated independently. The information is obtainable from either the stem or from the diagram given. Thus, it may display premature response, because all available information has not been utilized;

(3) Multi-structural—response includes two or more pieces of relevant information without relating them to each other. They are seen as discrete and unrelated elements;

(4) Relational—response integrates all relevant pieces of information. It includes several conclusions from the available information given. However, the explanation is still relatively reality based on the context;

(5) Extended abstract—response not only includes all relevant pieces of information given, but also extends the response to integrate relevant pieces of information which are not given. It also includes hypothetical situation from a generalization.

The initial and current use of the levels of this model has been primarily diagnostic, focusing on the analysis of responses or learners' work from a particular content area. It suggests a basis way for progressing pupils up the four levels. There are clear implications for how educators can develop their lessons that enable pupils to enhance the depth and strength of their learning. Thus, it has the potential to be used as a teaching tool to make a more systematic and clearer expectations and mechanisms for interpreting and analyzing relevant repeating pattern and providing opportunities for developing this skill. This paper outlines an adaptation of the SOLO model that provides teachers and their pupils with a pedagogically sound template which can be used to develop pre-algebra thinking in generalizing of repeating pattern. The following discussion develops one repeating pattern tasks that a teacher can use to develop pre-algebraic thinking using SOLO-based technique. It provides a developmental pattern to help pupils to acquire a better understanding in the way described in Figure 3.

Level	Question	Descriptor
Uni-structural	Which would bethe 12th shape?	The question requires the response based on referring the given term of the pattern (rectangle). It requires the understanding of the pattern sequence by referring directly the diagram given in the stem.
Multi-structural	Which would bethe 13th, 14th and 15thshapes?	The question requires the given information are handled serially. That is, extend the pattern by finding the shapes. The information given in the stem is still used as sequence.
Relational	What would be the 96th shape?	The question "force" the pupils to integrate all given information to identify the "core" of the pattern. That is, making an informal generalization or generalization verbally.

Figure 3. Repeating pattern task and SOLO-based descriptors.

Conclusions

There are two main strands to the development in repeating pattern. First, the complexity of the pattern involves the number of attribute and the core of the pattern. The second strand is the way of "seeing" the pattern, concerning whether the pupils are aware of the unit of repeat. Threlfall (2005) found that the repeating pattern formed by concrete materials or media (e.g., musical pattern) are more helpful to lead early years pupils to attempting to operate on pattern and could be seen as an interesting form of assessment in searching the core of a repeating pattern. Besides, the study also revealed that the simplest alternating elements type of pattern can be seen to be more common among younger pupils. Hence, these two strands should be considered in developing repeating task.

The template structure allows teachers to formulate questions for almost any type of repeating pattern, and focuses the pupils' attentions on to a specific set of features at each stage of development. Besides, it also provides pupils with a useful tool to develop their underlying pre-algebraic thinking in generalizing pattern.

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