US-China Education Review B 2 (2011), 203-212
Earlier title: US-China Education Review, ISSN 1548-6613

Comparison of the Mathematics Expectations Related to Number and Quantity for Student Learning*

Jung-chih CHEN
National Chiayi University, Chiayi, Taiwan

Chia-huang CHEN
Kun Shan University, Tainan, Taiwan

In the study reported here, the authors examined the LEs (learning expectations) related to Grades 1 to 8 number and quantity in mathematics across several US states and high performing TIMSS (Third International Mathematics and Science Study) Asian countries, including Singapore, Taiwan and Japan. The general strategy used is based on the topic tracing method. Several approaches and lenses have been used to analyze the LEs in each document. In order to simplify the procedures, only one topic within the number and quantity strand is reported here. Results of this study indicated that the mathematics contents, grade placement and cognitive level of LEs related to selected topic vary markedly across documents. Thus, these differences may have impact on students' opportunities to learn.

Keywords: learning expectation, number and quantity strand, opportunity to learn, TIMSS (Third International Mathematics and Science Study)

Introduction

In recent years, international studies of mathematics and science achievement have consistently reported that students in Asian countries usually demonstrate higher levels of mathematics achievement than those in the US (Mullis, Martin, & Foy, 2008; Mullis, Martin, Gonzalez, & Chrostowski, 2004; Wilson & Blank, 1999). Although the reasons are complex, educators generally agree that OTL (opportunity to learn) is a contributing factor. Floden (2002) has noted that "If OTL is not taken into account; its effect may be mistakenly attributed to some other attributes of the educational system" (p. 239).

The NRC (National Research Council of the National Academies) (2002) provided a framework for examining various channels of influence on the educational system (see Figure 1). We see that curriculum is one of channels which have influence on student learning, it includes state and district policy decisions, particularly those Les (learning expectations). Also, the TIMSS (Third International Mathematics and Science Study) used a model called Potential Educational Experience (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997) to capture different aspects of how educational opportunities were shaped and how they were potentially related. In the TIMSS model, national/regional curriculum goals at the system level represent the intended curriculum which contained what students were expected to learn. However, little was known about how the curricula described in the official documents differed from state or Asian country curriculum frameworks.

---

* The work reported in this paper was partly supported by the National Science Council in Taiwan (NSC) (Grant number: NSC 96-2521-S-415-003). However, the opinions reflected in this paper are solely those of the authors and do not necessarily reflect the policy or position of the NSC.
Jung-chih CHEN, Ph.D., associate professor, Department of Applied Math, National Chiayi University.
Chia-huang CHEN, Ph.D., assistant professor, Center for General Education, Kun Shan University.

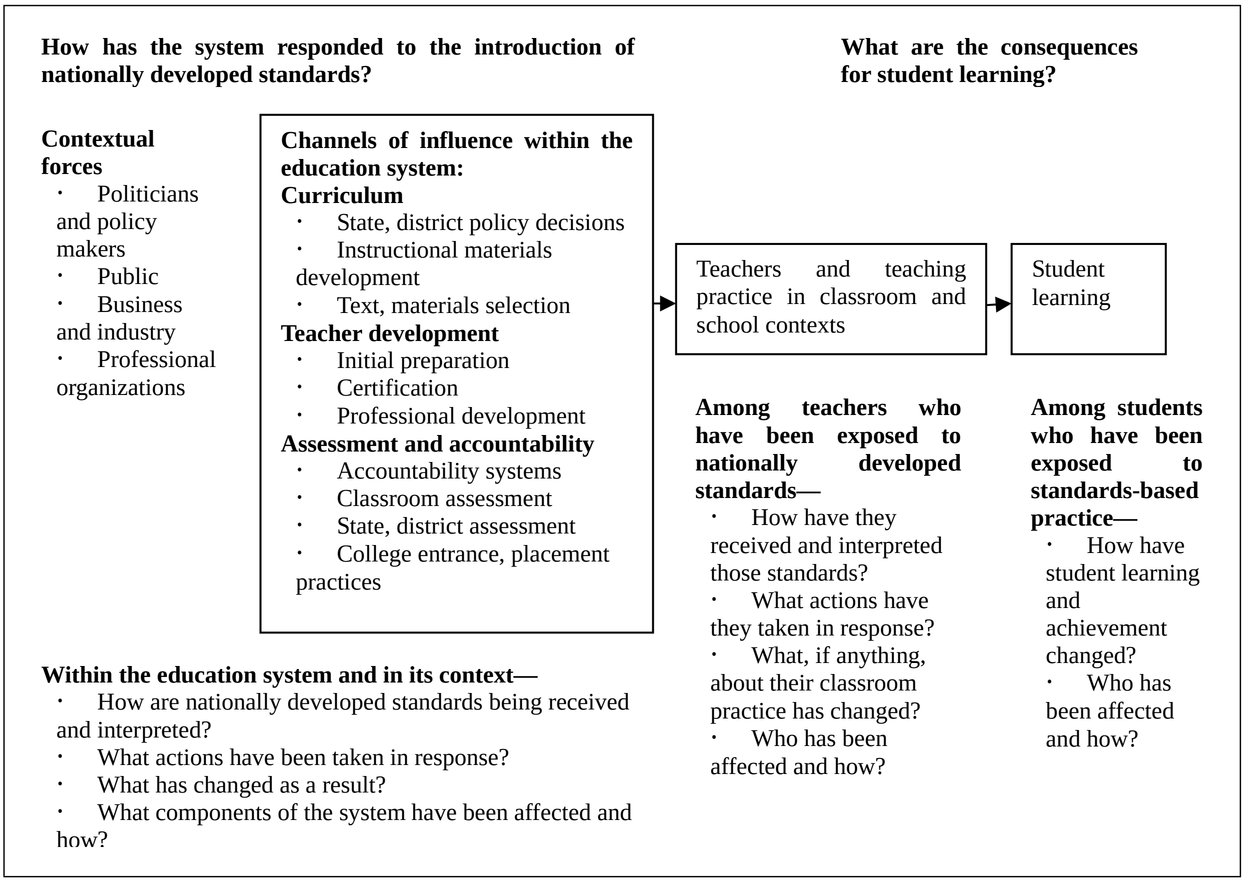

Figure 1. Framework for examining various channels of influence on the educational system. Source: NRC, 2002.

Research Question

In this study, the authors focus on one topic within the number and quantity strand in the official documents. More specifically, this study examines the following research question:

To what extent and in what ways are LEs associated with "decimal" in the number and quantity strand similar or different in emphasis and grade placement in some Asian countries and US states as described in their official mathematics curriculum documents?

Certainly, this analysis may partially explain differences in performance among students in several countries and states, particularly if the intended curriculum is an important contributor to what students have an opportunity to learn.

Source Documents

In this study, the primary data sources under examination are the state or country official mathematics curriculum frameworks. They include the following documents provided by the state or country government:

(1) SP (Singapore): Primary Mathematics Syllabus and Lower Secondary Mathematics Syllabus (2001);

(2) TW (Taiwan): Mathematics Curriculum Guidelines for Grade 1 to Grade 9 (2003);

(3) JP (Japan): Mathematics Program in Japan (including Elementary, Lower Secondary and Upper Secondary Schools) (2000);

(4) MN (Minnesota): Minnesota Academic Standards in Mathematics K-12 (2003);

(5) MO (Missouri): Mathematics Grade-Level Expectations (2004);

(6) CA (California): The California Mathematics Content Standards (2000);

Moreover, k-8 mathematics expectations developed by Achieve (2004) were also reviewed because they represent a new proposal for curricular emphases by an independent national organization which was created by governors and corporate leaders in 1996 to help raise states' standards and student performance;

(7) AC (Achieve): MAP (Mathematics Achievement Partnership) K-8 Mathematics Expectations (2004).

National documents, such as the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000), identify some content strands, such as number, algebra, geometry, measurement and data analysis and probability. Table 1 illustrates how these strands are reflected in the various documents that were analyzed in this study. This table briefly summarizes the organization of curriculum frameworks.

Table 1

Summary of Strand Organization in Curriculum Standards

Country/State	Year	Grades	Strands
SD*	2001	Grades 1-5	Whole number
		Grades 1-8	Measurement, statistics, geometry
		Grades 2-5	Fractions
		Grades 3-5	Decimals
		Grades 6-8	Algebra
TW	2003	Grades 1, 3, 4, 5, 6	Number and quantity; geometry; algebra; statistics and probability
		Grades 2, 8	Number and quantity; geometry; algebra
		Grade 7	Number and quantity; algebra
JP	2000	Grades 1-6	Number and calculations; quantities and measurement; geometrical figures; math relation
		Lower Secondary 1 and 2	Numbers and algebraic expressions; geometrical figures; math relations
MN	2003	Grades 1-8	Mathematical reasoning; number sense and computation and operations; patterns and functions, and algebra; data analysis and statistics, and probability; spatial sense and geometry, and measurement
MO	2004	Grades 1-8	Number and operations; algebraic relation; measurement; data and probability.
CA	2000	Grades 1-7, Grades 8-12**	Number sense; algebra and functions; measurement and geometry; statistics and data analysis, and probability; math reasoning
AC, Inc.	2004	Grade K-8	Algebra; data and measurement; geometry; number and operations

Notes. *Other strands are emphasized in the Singapore framework on one or two grades levels which include: ratio/proportion, percentage, problem solving and trigonometry; ** A Geometry course is also provided in Grades 8-12.

In addition to differing by strand organization, the "grain size" of the level of specificity of the statements of LEs also differs. It can be illustrated, in part, by examining the number of LEs at each grade in the documents. Based on the collected documents, Table 2 provides the number of LEs within the number and quantity strand by grade.

Furthermore, comparison can be made about number and percent distribution of LEs in number and quantity strand versus others strands (see Table 3).

Methodology

The selection of countries for this study was based on the performance on the TIMSS assessment. The selection of US states was based on student performance on the NAEP (national assessment of educational progress)-2000 (Kloosterman & Lester, 2004) assessment and the evaluation of official state curriculum documents by the Fordham Foundation. Basically, number and quantity strand was only one of studies; other strands included measurement, geometry, and algebra and statistics/data analysis.

Table 2

Number of LEs Within the Number and Quantity Strand of Each Document by Grade

Grade	1	2	3	4	5	6	7	8	Total
SD	28	20	17	27	26	5	22	2	147
TW	10	17	18	17	19	13	19	8	120
JP	4	7	10	12	10	8	6	0	57
MN	8	9	11	11	11	13	11	11	85
MO	5	8	10	9	10	8	10	10	70
CA	13	15	17	17	10	8	12	0	92
AC	23	29	36	32	28	30	33	6	217

Table 3

Number and Percent Distribution of Learning Expectations

Attribute county or state	Total LEs, Grades 1-8	LEs in number and quantity strand	LEs in other strands
SP	287	147 (51%)	140 (49%)
TW	188	120 (64%)	68 (36%)
JP	97	57 (59%)	40 (41%)
MN	147	85 (58%)	62 (42%)
MO	159	70 (44%)	89 (56%)
CA	178	92 (52%)	86 (48%)
AC	330	217 (66%)	113 (34%)

A coding system was developed which consisted of the general categories: object, action, tools and cognitive domain. For each LE in the selected topic of the curriculum documents, the following information was coded:

(1) Object—the main noun(s) in the learning expectation;

(2) Action—the main verb(s) in the learning expectation;

(3) Tools—equipment specified for use within the learning expectation;

(4) Cognitive domain—identification of cognitive level of learning expectation based on the Survey of Enacted Curriculum protocol (CCSSO (Council of Chief State School Officers), 1999) (see Table 4).

A sample of how learning expectations were coded is provided in Table 5.

Analysis of the LEs Related to the Selected Topic

The general strategy for analysis was based on the "topic tracing" method developed by TIMSS researchers. That is, for each topic, all LEs related to that topic within each curriculum document (SP, TW, JP, MN, MO, CA, AC) were identified and the following information was compiled:

(1) A description of the focus of the topic by grade level and document;

(2) The grade where the topic is intended to be first introduced to students;

(3) The range of grades during which instruction was intended to take place on the topic;

(4) Any grade for which the topic was to be a special emphasis.

Table 4

Cognitive Level of LE

Level No.	Main goals of learning
Level 1	Memorize facts/definitions/formulas: Recite basic mathematics; Recall mathematics terms and definitions; Recall formulas and procedures.
Level 2	Perform procedures: Use numbers to count/order/denote; Do computational procedures or algorithms; Follow procedures/instructions; Solve equations/formulas/routine word problems; Organize or display data; Read or produce graphs and tables; Execute geometric constructions.
Level 3	Demonstrate understanding of mathematical ideas: Communicate mathematical ideas; Use representations to model mathematical ideas; Explain findings and results from data analysis strategies; Develop or explain relationships among concepts; Show or explain relationships among models, diagrams and other representations.
Level 4	Conjecture/generalize/prove: Determine the truth of a mathematical pattern or proposition; Write formal or informal proofs; Recognize/generate or create patterns; Find a mathematical rule to generate a pattern or number sequence; Find and investigate mathematical conjectures; Identify faulty arguments or misrepresentations of data; Reason inductively or deductively.
Level 5	Solve problems/make connections: Apply and adapt a variety of appropriate strategies to solve problems; Apply mathematics in contexts outside of mathematics; Analyze data/recognize patterns; Synthesize content and ideas from several sources.

Table 5

Sample of Coded LEs

LE	Grade	Action	Object	Cognitive demand	Tools
Pupils can understand the meaning that two triangles are congruent through construction with straightedge and compass (TW).	8	Understand	Congruent triangles	Level 3	Straightedge/ compass
Student will know and use the decimal notation and the dollar and cent symbols for money (CA).	2	Know, use	Notation and symbol	Level 1	—
Use fractions, decimals and percents to solve problems (MO).	8	Solve problems	Fraction, decimals and percent	Levels 2, 5	—
The decimal expansion of an irrational number never ends and never repeats (AC).	8	Expand	Irrational number	Level 4	

Summary of LEs Related to "Decimal"

The concept of "decimal" was one of the topics analyzed within the number and quantity strand. Based on the analysis, a summary of the content emphasis and grade placement for this topic was provided.

All 74 LEs related to decimal were identified across the seven documents. The earliest LE appeared in Grade 2 of the California document and states: Know and use the decimal notation and the dollar and cent symbols for money. (CA, 2)

Other early grade decimal LEs include:

(1) Pupils should be able to add and subtract money in compound units using the decimal notation (SP, 3);

(2) Pupils can realize one-digit number decimal to make comparisons and add and subtract calculating (TW, 3);

(3) To know the units 100 million and one trillion, and to have an overall understanding of the decimal system (JP, 4).

A sample of Grade 7 and 8 LEs related to decimal includes:

(1) Student will represent rational numbers as fractions, mixed numbers, decimals or percents and convert among various from as appropriate (MN, 7);

(2) Using fractions, decimals and percents to solve problems (MO, 8);

(3) The decimal expansion of an irrational number never ends, and never repeats (If it did, then the number would be rational.) (AC, 8).

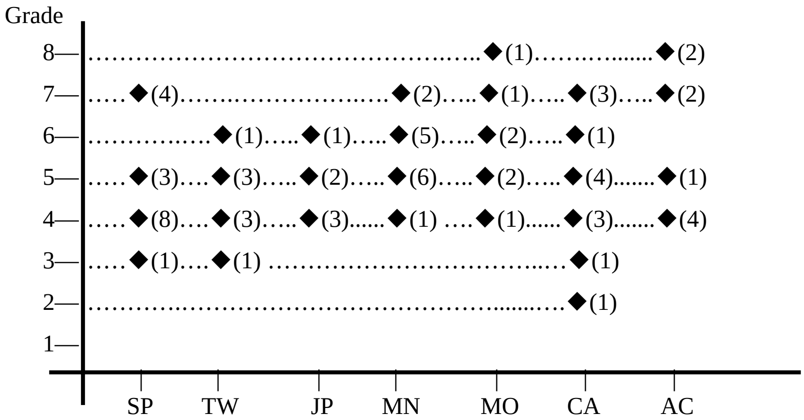

Figure 2. Number and grade placement of LEs related to "decimal" within number and quantity strand (Remark: The number inside parentheses indicates the number of LEs).

Common LEs related to decimal include: using decimal to solve problems (e.g., word problem, mathematical problem and solve problems in daily life), adding and subtracting with decimals (e.g., adding and subtracting money, addition and subtraction without calculator), multiplying and dividing with decimals (e.g., multiplication without calculator), etc..

Three common learning goals were noted within the set of LEs (see Table 6). For example, in seven of the documents, students were expected to identify decimal, including addition, subtraction, multiplication and division with decimal across these seven documents.

Table 6

Common LEs Related to the "Decimal" Topic

Common LE	SP	TW	JP	MN	MO	CA	AC
Use decimal to solve problems	G4	G4	G5	G4	G7	G3	G5
Add and subtract with decimals	G3	G3	G4	G5	G6	G5	G4
Multiply and divide with decimals	G4	G5	G5	G5	-	G5	-

Notes. G1 means the LE is provided for Grade 1; "-"indicates no specific statement in the LEs.

Among the 74 LEs, some were noted only within one or two documents. For example, JP, MN, CA and

AC documents included LEs not found in other documents. They included:

(1) To use decimals to express the size of remainders, and know how to express decimals, and the one-tenths position (JP, 4);

(2) Student will order and compare integers, fractions, decimals and mixed numbers with >, <, and =. Locate and compare positive and negative rational numbers on a numbers on a number line (MN, 6);

(3) Student will represent rational numbers as fractions, mixed numbers, decimals or percents and convert among various from as appropriate (MN, 7);

(4) Knowing and using the decimal notation and the dollar and cent symbols for money (CA, 2);

(5) Knowing that every rational number is either a terminating or repeating decimal and be able to convert terminating decimals into reduced fractions (CA, 7);

(6) The decimal expansion of an irrational number never ends and never repeats (If it did, then the number would be rational.) (AC, 8).

Based on the analyses of the collected documents, Table 7 summarizes the grade at which the topic of the decimal receives special emphasis. “Special emphasis” indicates that common LEs of this topic are addressed and that a substantial amount of time (in proportion to other topics from the strand) is devoted to the decimal. In general, attention to this topic is concentrated on Grades 4-8.

Table 7

Grades for Special Emphasis on “Decimal” Topic

	Grade 1	Grade 2	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8
SP				√	√		√	
TW				√	√			
JP				√	√			
MN					√	√		
MO					√	√		
CA				√	√		√	
AC				√			√	√

Summary of Emphasis on Decimal by Country/State

As noted in Figure 2, the SP document includes the largest number of LEs related to decimal (16 in all) with the greatest emphasis in Grade 4. Students in Grade 3 and Grade 4 should be able to add, subtract, multiply and divide decimals up to two decimal places. Students in Grade 5 should be able to solve word problem involving decimals. Students in Grade 7 should be able to compare and order fractions and decimals. They also should be able to use the four operations for calculations with fractions and decimals.

In contrast, JP has fewer LEs related to decimal. Most of LEs emphasize understanding the meaning of multiplication and division when the multipliers and divisors are decimals. In other words, these LEs related to basic terms are mostly categorized into the mid-levels.

TW has a very condensed set of LEs related to decimal (eight LEs spanning Grades 3 to 6). In Grade 3 students can realize one-digit decimal to make comparisons and add/subtract calculating. In Grade 4 and Grade 5, students can deal with the calculation and multiplier with the vertical form and solve problems in daily life. In Grade 6, students can deal with addition, subtraction, multiplication and division of the decimal with the vertical form and solve the problems in daily life.

In the MN document, LEs related to decimal appear in Grade 4 through Grade 7. In Grade 4, students will use decimal to solve mathematical problems. In Grade 5 and Grade 6, students will add, subtract, multiply and divide, without calculator, a multi-digit decimal.

In MO, only seven LEs appear in Grade 4 through Grade 8. In Grade 4 and Grade 5, students read, write and compare decimals. In Grade 6, students recognize, generate equivalent forms of decimals and describe the effects of addition and subtraction on decimals. In Grade 7 and Grade 8, students use decimals to solve problems.

In CA, the LEs appear in Grade 2 through Grade 7, the earliest LEs appear in Grade 2, students know and use the decimal notation and the dollar and cent symbols for money. In Grade 3, students solve problems involving addition, subtraction, multiplication and division of decimals. In Grade 4 and Grade 5, students identify and represent on a number of line decimals. In Grade 6 and Grade 7, students know that every rational number is either a terminating decimal and be able to convert terminating decimals into reduced fractions.

In AC, the main emphasis for this topic is in Grade 4, Grades7 and Grade 8. Students in Grade 4 should understand the values of the digits in a decimal and express them in alternative notations, they also should be able to add and subtract two-decimal numbers, notably currency values, in vertical form. In Grade 7, students should transform numbers from one form to another with efficient calculation. In Grade 8, students will use the first few digits in the decimal expansion of an irrational number to locate it in the number line.

Weight to Topic Within Number and Quantity Strand

In order to gauge the relative emphasis (weight) of the decimal within the number and quantity strand, Table 8 provides a summary of the number of LE associated with the decimal and the percent with respect to the total number of LEs within the number and quantity strand.

Table 8

The Weight of Topic-Decimal

	SP	TW	JP	MN	MO	CA	AC
Number of LEs	16	8	7	14	7	13	9
Percent of total num. Les (%)	10.9	6.7	12.3	16.5	10.0	14.1	4.1

Table 8 indicates that MN has the highest percentage related to this topic within the number and quantity strand. By contrast, AC has relatively low weight.

Cognitive Level of LEs Related to Decimal

Recall that the cognitive level for each LE was coded using the SEC (Survey of Enacted Curriculum) protocol (CCSSO, 1999). Table 9 provides a summary of the distributions of levels in cognitive demand, noting that each LE may be coded into double levels. It has also provided evidence that most LEs under this topic are categorized into the second and third levels of cognitive domain, noting that each LE might be double coded.

Conclusions

Great efforts to reform school mathematics education have been made in recent years. Mathematics curriculum frameworks typically contain statements that specify the subject contents for particular grades. These statements are intended to be a set of expectations for mathematics curriculum development and assessment. They indicate the scope of contents and highlight the specific topics at all levels for students to learn (Reys, Robinson, Sconiers, & Mark, 1999).

Table 9

Number and Distribution of Level in Cognitive Domain for LEs Related to Decimal

SEC country/state	*N*	Memorize fact, definition/formula (%)	Perform procedures (%)	Demonstrate understanding (%)	Conjecture, generalize and prove (%)	Solve problems, connect (%)
SP	16	-	81	19	-	-
TW	8	13	50	13	-	50
JP	7	14	57	71	-	-
MN	14	-	79	29	7	-
MO	7	-	29	29	29	29
CA	13	-	54	54	8	8
AC	8	-	78	22	-	-

In this study, we learn that some content similarities and differences are evident across the different documents. More specifically, this examination reveals that the MN document has exceptional high weights about "decimal" topic than TW and AC documents (see Table 8). Also, special emphasis to this topic is mainly placed at Grades 4 through Grade 8. Meanwhile, most LEs are categorized into the "perform procedures" level. It is also clear that many LEs related to this topic in particular documents, such as Missouri include multiple cognitive domains. Generally, each document might have its strength and weakness depending on the topic chosen. Understanding the attention focused on the topic in the intended curriculum may help clarify the context for differences in students' opportunities to learn.

References

Achieve, Inc. (2004). *Mathematics achievement partnership, K-8 mathematics expectations* (draft of December 2004). Retrieved from http://www. achieve.org/

Bakeman, R. (2000). Behavioral observation and coding. In H. T. Reis, & C. M. Judge (Eds.), *Handbook of research methods in social and personality psychology*. New York: Cambridge University Press.

CCSSO (Council of Chief State School Officers). (1999). *Survey of instructional content Grade K-8 mathematics.* Washington, D. C.: Council of Chief State School Officers.

De Lange, J. (2007). Large scale assessment of mathematics education. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1111-1144). Charlotte, N. C.: Information Age Publishing.

Floden, R. E. (2002). The measurement of opportunity to learn. In A. C. Porter, & A. Gamoran (Eds.), *Methodological advances in cross-national surveys of educational achievement.* Washington, D. C.: National Academy Press.

Kloosterman, P., & Lester, F. K. (2004). *Results and interpretations of the 1990-2000 mathematics assessments of the national assessment of educational progress.* Reston, V. A.: National Council of Teachers of Mathematics.

Mullis, I. V. S., Martin, M. O., & Foy, P. (2008). *TIMSS 2007 international mathematics report: Findings from IEA's trends in international mathematics and science study at the fourth and eighth grades.* Chestnut Hill, M. A.: Boston College.

Mullis, I. V. S., Martin, M. O., Gonzalez, E. J., & Chrostowski, S. J. (2004). *TIMSS 2003 international mathematics report: Findings from IEA's trends in international mathematics and science study at the eighth and fourth grades.* Chestnut Hill, M. A.: Boston College.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematic*s. Reston, V. A.: Author.

National Research Council of the National Academies. (2002). *Investigating the influence of standards—A framework for research in mathematics, science, and the technology education*. Washington, D. C.: The National Academies Press.

Porter, A. C. (2002, October). Measuring the content of instruction: Uses in research and practice. *Educational Researcher, 31*(7), 3-14.

Reys, B. J. (Ed.). (2006). *The intended mathematics curriculum as represented in state-level curriculum standards*. Charlotte, N. C.: Information Age Publishing.

Reys, B. J., Robinson, E., Sconiers, S., & Mark, J. (1999). Mathematics curricula based on rigorous national standards: What, why and how? *Phi Delta Kappan, 80*(6), 454-456.

Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997). *Many visions, many aims: A cross-national investigation of curricular intentions in school mathematics* (Vol. 1). Dordrecht, the Netherlands: Kluwer.

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, H. C., Wiley, D. E., Cogan, L. S., & Wolfe, R. G. (2001). *Why schools matter: A cross-national comparison of curriculum and learning*. JOSSEY-BASS, A Wiley Company, San Francisco, C. A., USA.

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Kluwer Academic Publishers, the Netherlands.

Weiss, I., Pasley, J., Smith, P., Banilower, E., & Heck, D. (2003). *Looking inside the classroom: A study of k-12 mathematics and science education in the United States*. Chapel Hill, N. C.: Horizon Research.

Wilson, L. D., & Blank, R. K. (1999). *Improving mathematics education using results from NAEP and TIMSS*. Council of Chief State School Officers, State Education Assessment Center.