Abstract

Critics of testing for admission purposes cite the moderate correlations of admissions test scores with success in college. In response, this study applies formulas from classical measurement theory to observed correlations to correct for restricted variances in predictor and success variables. Estimates of the correlations in the population of high school graduates are derived from two of the several formulas in the literature. This article describes limitations and encourages additional investigation into the use of the formulas for estimating correlations in unrestricted populations.

Critics of the use of test scores in college admissions cite the relatively low correlations between admission test scores and success in college, for example, first-year grade point average (GPA). They point out that a typical correlation of .40 between the admission test score and end-of-first-year GPA indicates that the test score predicts only 16% of the variance in the first-year grades. Critics assert that the percentage of variance is so low that colleges and universities place too much emphasis on test scores as predictors of college success (Kohn, 2001; Sternberg, Wagner, Williams, & Horvath, 1995; Vasquez & Jones, 2006; among others).

On the other hand, defenders of using test scores in admission decisions note that some of the criticism may be unwarranted. For example, Sackett, Borneman, and Connelly (2008) outlined several reasons why test scores might be more effective predictors of college success than what is typically reported. First, "variance explained" may not be the best metric for interpreting the predictive ability of test scores. That is, if you convert "variance explained" to "differences in odds of success," test scores predict the likelihood of a subject being successful quite well. Second, first-year GPA is not perfectly reliable,
and unreliability in this GPA depresses its correlation with the test score. Consequently, the observed correlation between test score and GPA understates the true relationship between the two variables. Third, admission tests do a reasonably good job predicting grades beyond the first year of college in addition to predicting the performance on other dependent variables such as GRE scores, LSAT scores, doctoral degree achievement, and getting tenure (Kuncel & Hezlett, 2007; Lubinski, Benbow, Webb, & Bleske-Rechek, 2006; Vey et al., 2003).

Fourth, concerns over using a restricted range or only examining the relationship between those students who take admission tests and students who actually go to college was cited as a major factor in limiting the predictability of test scores (Sackett et al., 2008). This final reason—restricted range—is the focus of the current study.

What may be overlooked by the critics is that the correlations in question are frequently calculated using a population of entering freshmen who completed a full year of college (Linn, 1990; Young, 2004; Zwick & Sklar, 2005; among others). In judging the value of the test in predicting college success, a more relevant population is that of all potential college students, specifically, that of all high school graduates. Clearly, this is a more heterogeneous population than the one used in calculating the correlations. The implication of this situation is that the calculated correlations understate the more meaningful correlations for the unrestricted population.

The population of high school graduates is initially restricted by the elimination of those who do not apply to college and next by those who apply but are not accepted and then by those who are accepted but do not enroll. Then, there is the group that enrolls on a part-time basis and may not be used in calculating the correlations of interest. Similarly, there is the further restriction from those who enroll but do not complete the first year with a GPA.

Indicators of success in high school are also used as predictors of success in college. As a matter of fact, the usual finding is that success in high school is a better predictor of success in college than is an admission test score (Hoffman, 2002; Munro, 1981; Zheng, Saunders, Shelley, & Whalen, 2002; among others). Further, the combination of test scores with measures of success in high school provides a better prediction of success in college than either predictor used individually (Eimers & Pike, 1997; Fleming, 2002; Geiser & Santelices, 2007; Kim, 2002; Linn, 1990; Mathiasen, 1984; Noble & Sawyer, 1997; Wolfe & Johnson, 1995; Zwick & Sklar, 2005).

### Purpose

The purpose of this study is to illustrate techniques for correcting a correlation between a predictor of success in college (admission test score or indicator of high school performance) with a measure of success in college (one-year retention or first-year GPA), given the restricted variances in the population used to calculate the correlations. In other words, this study demonstrates procedures for estimating correlations in the unrestricted population (students who attend college and students who do not attend college) based upon correlations calculated for the restricted population (students who attend college). A secondary purpose is to set the foundation for and stimulate additional studies designed to estimate these correlations in other unrestricted higher education and college student populations. This study focuses on correlations involving admission test scores, indicators of success in high school, and first-year college GPA.

### Formulas for Correcting Correlations

Classical measurement theory provides the means to “correct” correlations for restrictions in the variances of the correlated variables. The restriction in the variance of a predictor variable is due to selection on that variable or on a related variable or to selection on more than one variable. In the case of college admissions, students may be selected on the basis of an admission test, an indicator of high school success, or some combination of the two. In correcting correlations for restriction of variance, a distinction between explicit selection and implicit or incidental election is made. In order to correct a correlation, the variance of a relevant variable in the unrestricted population must be known. Selection on the basis of the predictor variable of interest for
which the variance in the unrestricted population is known as explicit selection, and that variable is referred to as the explicitly selected variable. Selection on the basis of some other variable that is related to the predictor variable of interest and for which the unrestricted population variance is known is considered to be implicit or incidental selection, and that predictor variable is the implicitly selected one.

This study describes three cases and associated formulas for estimating correlations in unrestricted populations. These three formulas are the ones derived or discussed in the seminal literature and are the formulas that have the most application in the current research. In the end, two of the formulas are actually employed in correcting correlations in this study. Assumptions underlying the three formulas include the conventional ones of linearity and homoscedasticity. Also assumed is that the selection of the restricted population from the unrestricted population is as described for the formulas being used.

The three situations and formulas for each are noted below. In these formulas, \( s \) (standard deviation), \( s^2 \) (variance), and \( r \) (correlation) are statistics from the restricted population, and \( S, S^2, \) and \( R \) are the corresponding statistics from the unrestricted population; \( X_1 \) is the predictor variable, \( X_2 \) is the variable being predicted, and \( X_3 \) is a third variable related to \( X_1 \).

Case 1. In this case, subjects are selected on the basis of \( X_1 \) and the values of \( s_{12}, s_{22}, S_{22}, \) and \( r_{12} \) are known. Gulliksen (1950) describes this as the case of incidental selection, because it is the variance of the criterion variable that is known, and derives the following formula:

\[
R_{12} = \frac{S_{1}r_{12}}{\sqrt{S_{1}^2 r_{12}^2 + S_{1}^2 - s_{12}^2 r_{12}^2}}
\]

Slightly different but equivalent formulas are given by Guilford (1965) and Cureton (1951). Gulliksen (1950) indicates that this formula was first derived by Pearson (1903) and that slightly different versions of it have been given by Kelley (1923), Holzinger (1928), Thurstone (1931), Thorndike (1947), Crawford and Burnham (1946), and others.

For example, if it is assumed that students are selected for admission on the basis of a predictor of college success, \( X_1 \), that the variance of the predictor variable in the unrestricted population of high school graduates, \( S_{12} \), and the variances of the predictor, \( s_{12}^2 \), and of the college success variable, \( s_{22}^2 \), as well as the correlation between the two variables, \( r_{12} \), in the restricted range population are known, then the correlation between the predictor variable and the college success variable, \( R_{12} \), for the population of high school graduates can be estimated using this formula.

A situation in which the Case 2 formula might be used is the one in which a test is given to a population of subjects and only those who score in the top, say, 40% on the test are selected for a program of instruction. At the end of the program, the selected subjects are given an achievement test on the subject matter of the instruction. The correlation, \( r_{12} \), of the selection test, \( X_1 \), and the achievement test, \( X_2 \), and the variances, \( s_{12}^2 \) and \( s_{22}^2 \), of the two variables are known for the selected subjects, those in the restricted population. Also known is the variance, \( S_{12}^2 \), of all subjects who took the selection test, those in the unrestricted population. The Case 2 formula then would be used to estimate the correlation, \( R_{12} \), between the two
tests for the population of subjects who took the selection test. This situation is illustrated by Berry and Sackett (2008) in a study involving students at 41 colleges. They found a correlation of .35 between SAT (Verbal + Math) score and first-year GPA. The corrected correlation for the population of students who took the SAT was .47. These situations are similar to the one for which the Case 2 formula is used in this study.

Case 3. In this case, subjects are explicitly selected on the basis of \(X_3\), and \(X_1\) is a third variable, related to \(X_3\), for which values are available for subjects in the restricted population. It is desired to estimate the correlation between \(X_1\), the incidental selection variable and the success variable, \(X_2\), for the unrestricted population. In this case, the values of \(s_{12}^2\), \(s_{22}^2\), \(s_{32}^2\), \(S_{32}^2\), \(r_{12}\), \(r_{13}\), and \(r_{23}\) are known. The formula for this case, given by Gulliksen (1950), follows:

\[
R_{12} = \frac{r_{12} - r_{13}r_{23} + r_{13}r_{23} \left( \frac{s_{32}^2}{s_{33}^2} \right)}{\sqrt{\left(1 - r_{23}^2 + r_{23}^2 \left( \frac{s_{32}^2}{s_{33}^2} \right) \right) \left(1 - r_{12}^2 + r_{12}^2 \left( \frac{s_{32}^2}{s_{33}^2} \right) \right)}}
\]

Gulliksen (1950) indicates that variants of this formula appear in Pearson (1903) and Thorndike (1947). The formula or its equivalent also appears in Cureton (1951) and Guilford (1965).

For example, if it is assumed students are selected on the basis of one predictor of success in college, \(X_3\), that the variance of that variable in the unrestricted population is known, and that values of a second predictor variable, \(X_1\), are available for the selected students, then the correlation of the second predictor variable and the success variable, \(X_2\), in the unrestricted population can be calculated from this formula.

Gulliksen (1950) provides two additional three-variable formulas for estimating correlations in unrestricted populations if the variance of the incidental selection in the unrestricted populations is known. These formulas are not given or discussed here. Sackett and Yang (2000) provide descriptions of 11 cases in which estimates of correlations in unrestricted populations might be estimated from data for restricted populations.

The three cases discussed above are included among the 11. This paper (Sackett & Yang, 2000) includes a comprehensive review of the literature on contributions to the matter of estimating population correlations from samples that have been restricted due to any of several types of selection. It is recommended to anyone seeking to investigate further the topic of the present study. The presentation by Thorndike (1949) is frequently cited in the literature on the topic and also is recommended to those interested in pursuing it further.

The Data and Their Application

The data for this study come from a population of first-time freshmen who entered a major research university with moderately selective admission standards in the fall 2008 semester, whose high school class percentile rank was 50 or greater, who entered the fall semester as full-time, degree-seeking students, and who completed both semesters with complete data for the study variables. There are 3,668 students in this population. The variables collected for these students are:

- ACT-C – ACT Composite Score
- HSCPR – High School Class Percentile Rank
- NHSCPR – Normalized High School Class Percentile Rank
- CCGPA – High School Core Course Grade Point Average
- FYGPA – Freshman Year Grade Point Average

The HSCPRs are transformed into normalized values (NHSCPRs) by converting cumulative percentile values from a normal curve tables into z-values, transformed to values with a mean of 50 and a standard deviation of 10. The NHSCPR variable is used in the data analysis because it has more desirable statistical properties than the HSCPR one. The CCGPA is an average on a 4-point scale calculated from the high school academic core courses in English, mathematics, science, social science, and fine arts. The other variables are as traditionally defined.
Some statistics for this population are shown in Table 1.

Table 1
Means, Standard Deviations, and Variances for Study Variables for Students in the Study Population, N = 3,668

<table>
<thead>
<tr>
<th>Statistic</th>
<th>ACT-C</th>
<th>NHSCPR</th>
<th>CCGPA</th>
<th>FYGPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>25.65</td>
<td>60.13</td>
<td>3.50</td>
<td>3.05</td>
</tr>
<tr>
<td>s</td>
<td>3.71</td>
<td>6.32</td>
<td>0.38</td>
<td>0.67</td>
</tr>
<tr>
<td>s²</td>
<td>13.76</td>
<td>39.94</td>
<td>0.14</td>
<td>0.45</td>
</tr>
</tbody>
</table>

For the study, the correlations between each of the three predictor variables, ACT-C, NHSCPR, and CCGPA and the college success variable FYGPA are calculated. These statistics are descriptive of the restricted population for which the data are collected. The applicable formulas described in the preceding section are then used to estimate the values of the correlations for all high school graduates, those in the unrestricted population.

The admission requirements for the subject university include minimum numbers of completed units of core subject areas of high school courses and standards based upon ACT-C and HSCPR. Specifically, students with an ACT-C of 24 or higher are admissible, and those with scores between 18 and 23 are deemed admissible on the basis of a sliding-scale combination of their ACT-C and HSCPR.

In calculating the desired estimates of correlations for all high school graduates, it is assumed that enrolled students are selected on the basis of HSCPR or NHSCPR. In order that the data to be analyzed conform as much as possible to this assumption, the study population is restricted to students whose HSCPR or NHSCPR is 50 or greater. In other words, this restricted population is treated as having been selected on the basis of HSCPR or NHSCPR. Even with this restriction, the assumption is not completely correct. First, high school graduates “self-select” in deciding whether or not to apply to the subject university. Second, students are also selected on the basis of core course requirements and ACT-C. Further, the population of admitted students is also restricted to those who enroll full-time and complete the freshman year with complete data on the study variables. Consequently, the estimated correlations will be in error to some undetermined degree. However, the derived values are certainly more accurate estimates of the correlations for all high school graduates than are the values calculated for the enrolled student population. Gulliksen (1950) suggests that “In many cases, however, it is clear that a given selection test was one of the major items in the selection procedure so that the results found by assuming that selection was solely on the basis of the test will not be far from the correct estimate” (p. 129).

The formulas for correcting correlations for restricted variances require that a standard deviation or variance of a relevant variable in the unrestricted population be known. For this study, the unrestricted population is that of all high school graduates. The variance of NHSCPR, the normalized values of HSCPR, for this population is 102 or 100. This is the value used to correct the prediction-of-success correlations.

If the population of “college bound” high school graduates were of interest, the correlations for this population could be estimated using ACT-C scores and the variance of these scores in the population of ACT test-takers. For this study, however, the population of all high school graduates is of more interest than the population of high school graduates who have taken the ACT. Thus, this application of the formula is not explored.

The degree of restriction for the study population is evident in the following: In recent years, the rate of college-going for the state in which the subject university is located and for the nation as a whole is around 60%. Thus, many students from the unrestricted population are clearly excluded from the study population. Next, approximately 14,500 prospective first-time freshmen applied to the subject university for the fall 2008, around 12,300 were admitted, and 5,800 enrolled. The study population of 3,668 includes those who enrolled full-time, completed both semesters, and had complete data on the study
variables. This is a noteworthy reduction in size of the population and clearly suggests a reduction in the variances of study variables.

Results

Correlations among the several study variables for the study population are contained in Table 2. These are the restricted population correlations.

Table 2

<table>
<thead>
<tr>
<th>Variable</th>
<th>NHSCPR</th>
<th>CCGPA</th>
<th>FYGPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACT-C</td>
<td>0.43</td>
<td>0.36</td>
<td>0.43</td>
</tr>
<tr>
<td>NHSCPR</td>
<td>0.78</td>
<td>0.49</td>
<td></td>
</tr>
<tr>
<td>CCGPA</td>
<td></td>
<td>0.56</td>
<td></td>
</tr>
</tbody>
</table>

Correlation of ACT-C with FYGPA

To correct the restricted population correlation, .43, the Case 3 formula is used. It is assumed that students are selected on the basis of NHSCPR, X3. The predictor variable is ACT-C, X1, and the variable being predicted is FYGPA, X2. The following values, from Table 1 and Table 2 (with four significant digits) are used in the formula:

\[ r_{12} = 0.4257, \quad r_{13} = 0.4281, \quad r_{23} = 0.4891, \]
\[ s_1^2 = 13.77, \quad s_2^2 = 0.4484, \quad s_3^2 = 39.94, \quad S_3^2 = 100. \]

The result is: \( R_{12} = 0.5630. \)

Correlation of NHSCPR with FYGPA

To correct the restricted population correlation, .49, the Case 2 formula is used. For this correlation, it is assumed that students are selected based on the predictor variable, NHSCPR, X1. The variable being predicted is FYGPA, X2. The following values, from Table 1 and Table 2 are used:

\[ r_{12} = 0.5582, \quad s_1^2 = 39.94, \quad s_2^2 = 0.4484, \quad S_1^2 = 100. \]

The result is: \( R_{12} = 0.7575. \)

Correlation of CCGPA with FYGPA

To correct this restricted population correlation, .56, the Case 3 formula is used. It is assumed students are selected on the basis of NHSCPR. The predictor variable is CCGPA, X1, and the variable being predicted is FYGPA, X2. NHSCPR, X3, is a third variable related to CCGPA. The following values, from Table 1 and Table 2 are used:

\[ r_{12} = 0.5582, \quad r_{13} = 0.7829, \quad r_{23} = 0.4891, \]
\[ s_1^2 = 1.1445, \quad s_2^2 = 0.4484, \quad s_3^2 = 39.94, \quad S_3^2 = 100. \]

The result is: \( R_{12} = 0.8025. \)

Discussion

Table 3 contains the correlation of each predictor variable with FYGPA for the study population, the selected population of enrolled students, and the corresponding estimate of the correlation for the unrestricted population of high school graduates. The table also includes percentages of the variance of FYGPA that is estimated by the predictor variable for each correlation. It is important to emphasize that the unrestricted population correlations are only estimates of the values that would be found were it possible to calculate them directly. To an unknown degree, the accuracy of the estimates is affected by the fact that the students in the study population were not selected purely on the basis of their HSCPR as assumed for the formulas used.

Table 3

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Restricted Population</th>
<th>Unrestricted Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACT-C</td>
<td>0.43</td>
<td>0.56</td>
</tr>
<tr>
<td>NHSCPR</td>
<td>0.49</td>
<td>0.76</td>
</tr>
<tr>
<td>CCGPA</td>
<td>0.56</td>
<td>0.80</td>
</tr>
</tbody>
</table>

There are four major findings that can be gleaned from this study. First, by applying the formulas and correcting the correlations, the relationship between the predictor variables and the measure of success in college increased significantly. From a formulaic perspective, if the variance for NHSCPR is 39.94 for the study...
population (see Table 1) and the variance for NHSCPR for all high school graduates is 100.0, similar restrictions should be realized in the variances of other study variables that are related to NHSCPR. These restrictions in variances indicate that the correlations in the population of high school graduates should be higher than those for the population of enrolled students. Accordingly then, the results are that the correlations increased from .43 to .56 for the ACT-C, from .49 to .76 for NHSCPR, and from .56 to .80 for CCGPA. By and large, these increases in correlations are quite significant, especially for NHSCPR and CCGPA. This finding also reinforces the argument that the predictive ability of these college admission indicators is relatively robust particularly when the broader population of all high school graduates is considered.

Second, the correlation of ACT-C with FYGPA in the study population is relatively modest at .43. For the unrestricted population, the estimated correlation of ACT-C with FYGPA is .56. This may not overwhelmingly justify the use of ACT-C as the sole criterion for college admission. At the same time, however, it is important to acknowledge that the percentage of variance in FYGPA explained by the ACT-C did increase by 78% (from 18% to 32%). This increase is not trivial.

Third, and as expected, the indicators of success in high school have higher correlations with the college success variable than does the admission test score. The differences between the correlation coefficients for these two types of predictor variables, .43 for ACT-C in contrast to .49 for NHSCPR and .56 for CCGPA, are fairly substantial. As noted earlier, this finding has been shared in several other studies using a wide variety of subjects and institutions (Hoffman, 2002; Munro, 1981; Zheng et al., 2002; among others).

Fourth, the differences between the corrected and uncorrected correlations are greater for the high school success variables than for the admission test variable. In comparison to the restricted population, the unrestricted percentage of variance explained increases 138% for NHSCPR, 106% for CCGPA, and 78% for the ACT-C. It is not clear why the restrictions in range have a larger impact on the correlations of the high school success variables with FYGPA than the corresponding correlation involving ACT-C. Perhaps there is something about predicting college performance from high school performance that underlies the difference. The estimated correlations for the high school success predictors are significant and certainly reinforce the importance of these variables in college admission decisions. Furthermore, these findings provide a counter argument for those colleges and universities that have minimized the importance of or even dismissed these predictor tools in admission decisions.

Further Research

Clearly, additional research on the effect of restriction of range on the correlations of predictor variables with indicators of college success is needed. The single institution study cannot be considered to be definitive. Results for colleges or universities with different degrees of selectivity may differ. Further, it would be desirable if a situation could be found where the assumptions of the correction formula are met more closely than in this study.

Conclusion

Restriction of range is clearly one reason that correlations calculated from enrolled student populations understate the true relationships between predictor variables and college success measures. The values of such correlations should not be the single criteria for decisions concerning the use of the predictors in college admissions. Other variables may also depress these correlations. For example, Pike and Saupe (1992) found that high school attended was a factor in the prediction of success in college. In that study, it was found that when high school attended was controlled, the correlation increased. Unreliability in the predictor and in the college success measures also can depress the correlations (Sackett et al., 2008).

In sum, the true relationships between predictor variables and college success measures can be masked by restricted range as well as other extraneous variables. The present study
demonstrates the influence that restricted range can have on this relationship and suggests that these predictor variables are probably more accurate than what is generally shared in the literature and in practice. This study will have been successful if it stimulates others to explore the use of the correction formulas to estimate correlations between predictor variables and indicators of success in college for unselected populations.

**Editor’s Note:**

The article by Saupe and Eimers is a delight. It might well be titled “Back to Basics” as it addresses one of the foundational issues in parametric statistics, the impact of restricting the range of variables. In classical test theory, also known as Weak True Score Theory because of its limited assumptions, the error variance is assumed to be consistent, so when the range of raw scores is restricted—which reduces the raw score variance—the true score variance is reduced. Hence, the scores become less reliable. The reduction of score range also reduces correlations, and this is the article’s focus. The other part of the assumptions includes the linearity of the relationship that is the same for the included scores and the excluded segment. In cases where the restriction is at the extremes, like excluding the bottom 10% of a distribution or selecting the top 25% of a distribution, one might want to empirically test the assumption, where feasible.

The first question the article raises, however, is about the use of test scores. It suggests the use of odds ratios, which are common for the independent variables in logistic regression but do not seem to be used that much as a dependent variable. This would seem to be an interesting alternative for some of our research, especially where the predicted result is a probability.

The article presents a discussion of how ranges of measures are restricted using the admissions decision. It would seem that there are other situations, like selecting a subgroup to engage in an intervention based on some criterion or selecting a group of students to be surveyed based on some criterion. This also raises the issue of the relationship of the selection criterion to the outcome(s) of interest. If we do select groups based on criterion, should we look at the correlations between the criterion and the key variables in the study?

There are a couple of points to make in this part of the discussion. First, while the earlier references include the classical derivations, the article by Sackett and Yang is, as the authors note, very good and includes a number of situations including cases when the restriction comes by deleting the center of the score distribution. They also show some of the restricted situations graphically. A second point is that the authors use a set of normalized scores for the high school rank scores. This is helpful since the standard deviation of the normalized scores is known by definition. It also is more appropriate since the percentile rank of high school scores is not an interval scale.

The authors clearly demonstrate that the correlations between attenuated distributions can significantly understate the correlation that would be obtained if the full range of scores were used. It also seems to indicate that the increase is proportional to the correlation of the attenuated measure and the measure(s) in the correlation. The largest increase seems to occur when the restriction is explicit on one of the measures in the correlation. In their suggestion for further research, they do note the need to replicate their results.

Another use of corrections, which would be a bit of an approximation, as was their case, would be to use the correction in support of a Meta analysis. For example, even though they mention that the ACT takers do not represent the population of high school graduates, it would still seem interesting to use a known ACT score deviation to correct reported correlations across studies based on the standard deviation of ACT in the individual study.

As in all good research, Saupe and Eimers raise more questions from their work and, in the process, remind us of some of the basic work in our field.
References


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