Assessing the Number Knowledge of Children in the First and Second Grade of an Indonesian School

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An assessment approach from Mathematics Recovery was used to document the number knowledge of 20 first-graders and 20 second-graders in an Indonesian school. Sixteen first-graders were at the advanced-counting-by-ones stage and fourteen second-graders were facile. As well, fifteen first-graders and eleven second-graders were at the level of an intermediate concept of 10. Other findings were nine of the second-graders used the erroneous 'subtract smaller from larger bug' and five first-graders used Jarimatika (Chisanbop). Results are discussed in light of the literature.

An important part of teaching children well is first to understand if they already have some degree of number knowledge. Vygotsky's theory of learning, related to his notion of the Zone of Proximal Development (Vygotsky, 1978), proposes that students learn best if they are challenged within close proximity to, and slightly above, their current level of development. However, in Indonesia, teachers seldom begin teaching number by considering children's current number knowledge. They emphasize the teaching of procedures, such as the standard written algorithm, rather than developing children's strategies (Marsigit, 2004).

There are many studies in the English language literature that investigate children's number knowledge (e.g. Clarke, 2006; Gervasoni, 2007). However, research studies involving Indonesian children are rare. This paper reports a study that aimed to document the number knowledge of first- and second-graders in an Indonesian school including their strategies used to solve number problems. This paper also aims to identify possible ways of improving the teaching of early number in Indonesian schools.

Literature Review

The emphasis on detailed assessment as a basis for teaching has strongly influenced recent initiatives in early number learning by several school systems, especially in English speaking countries. In Australia, drawing on the work of Wright, the developer of Mathematics Recovery (MR) (Ewing-McMahon, 2000), several systemic initiatives have been implemented to change the approach to instruction in number in the early years (Perry & Dockett, 2007). These include *Count Me in To* in New South Wales, the *Early Numeracy Program* in Victoria and the *New Zealand Numeracy Development Project* (Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge & Gould, 2005). The assessment approach used in the MR program has been acknowledged as distinctive and enables detailed documentation of children's current number knowledge (Williams, 2008).

A model of stages in strategies in early arithmetical learning (Steffe & Cobb, 1988) and a model of levels in base-ten arithmetical strategies (Cobb & Wheatley, 1988) have been adapted by the Mathematics Recovery Program to document children's progress in arithmetical learning. This study used an assessment theory, technique and tools developed and used in this program (Wright, Martland & Stafford, 2006).

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Method

Study site

The study was conducted in an elementary school in Yogyakarta, Indonesia. The school is a co-educational Islamic-based private school catering for children from the first to the sixth grade. The data was collected in February and March 2009, around the middle of the second half of the Indonesian school year. There were 167 children in the first grade and 172 children in the second grade.

Participants

All children in the first and second grades were given a one-minute basic number facts test for addition and one for subtraction adapted from Westwood (2000). In each class, the children were listed from the highest to the lowest score on this test. Then the children were divided into five groups with approximately equal numbers in each group. The first group contained the highest listed and so on. One child from each group was selected by the teachers and asked whether they agreed to be interviewed. If a child refused, then another child from his/her group was selected, and so on until for each group, there was a child who agreed to be interviewed. In total there were 20 first (11 boys and 9 girls) and 20 (9 girls and 11 boys) second grade children who agreed to be interviewed. Their parents' consent was sought. The mean age of the first-graders was 7 years and of the second-grades was 7 years 11 months. All the interview sessions were videotaped. During the selection and data collection process, the Australian Human Research Ethics Committee guidelines about recruiting children for research were strictly followed.

Interview

The interview consists of two parts. The first part, strategies in early arithmetical learning was conducted to determined children's stages in early arithmetical learning, and the second part, base ten arithmetical strategies was conducted to determine children's level in base-ten arithmetical strategies.

Stages of Early Arithmetical Learning (SEAL). The SEAL model used in MR is adapted from Steffe and Cobb (1988). The SEAL sets out a progression of stages in early arithmetical learning. The child is asked to solve number problems involving collections of counters, which may be screened. An addition task for example, involves presenting 7 red counters and 5 blue counters, screening the collections and asking how many counters there are altogether. The interviewer observes how the child solves the problem. If the child is unsuccessful, the counters are unscreened. If the child is successful, their strategy is noted. Furthermore, the child's stage is determined using the model shown in Table 1.

Base Ten Arithmetical Strategies (BTS). The BTS model used in MR is adapted from Cobb and Wheatley (1988). Three types of tasks are presented; ten and ones tasks using ten-dot strips, uncovering board tasks and horizontal written number sentences. For the first two types, the interviewer observes whether the child increments by tens or counts on. For the horizontal written number sentences such as 42+23 (no-carry addition), 38+24 (carry addition), 26─12 (no-borrow addition) and 41─24 (borrow addition), the interviewer observes the child's strategies. Furthermore, the child's level is determined using the model shown in Table 2.

Table 1
The model for stages of early arithmetical learning

Stage	Name of stage	characteristic
0	Emergent	Cannot count visible items
1	Perceptual	Can count visible items only.
2	Figurative	Can count invisible items, but starts from one.
3	Advanced-counting-by-ones	Can count invisible items, using a counting-on strategy to solve addition or missing addend tasks, and may use a counting-back strategy (counting back-from or counting-back-to) to solve missing subtrahend or removed items tasks.
4	Facile	Can use non-counting-by-one strategies, such as doubles, add through ten, compensation, etc.

Adapted from Wright, et al., 2006

Table 2
The model for the development of base-ten arithmetical strategies

Level	Level	characteristic
1	Initial concepts of ten	Not able to see ten as a unit composed of ten ones. The child solves tens and ones tasks using a counting-on or counting-back strategy.
2	Intermediate concepts of ten	Able to see ten as a unit composed of ten ones. The child uses incrementing and decrementing by tens, rather than counting on by ones to solve an uncovering board task. The child cannot solve addition and subtraction tasks involving tens and ones when presented as horizontal written number sentences.
3	Facile concepts of ten	Able to solve addition and subtraction tasks involve tens and ones when presented as horizontal written number sentences by adding and/or subtracting units of ten and ones.

Adapted from Wright, et al., 2006

Data Analysis

The videotapes were reviewed and transcribed. The transcriptions consisted of descriptions of the child's words and actions during the interview. The transcriptions were written in Indonesian language and then translated to English language. Based on the transcriptions, a description of each child's strategies was written and general insights about their strategies were noted. Furthermore, each child's stage and level were determined using the models in Tables 1 and 2. Notes were made about distinctive features of each child's strategies. Two phenomena related to children's number knowledge emerged during the data analysis. These are the use of an erroneous algorithm called the 'subtract smaller from larger bug' (Brown & Van Lehn, 1982) and the use of the Jarimatika (chisanbop) method (Wulandari, 2004). These phenomena are discussed in detail in the results and discussion section.

Results and Discussion

Table 3 shows the numbers of first- and second-graders at each stage. No children were found to be in the emergent, perceptual or figurative stages. Two explanations of this are: 1) The interview was conducted in February-March and the school started in July of the previous year. Thus first-graders were already in the elementary school for eight months and second-graders for one year and seven months. As well, children were in the kindergarten school for at least one year before starting elementary school. So, they were used to solve number problems without seeing real objects. 2) In the first semester of first grade, children were taught one-digit addition. In an informal discussion the teacher said she taught the children about counting-on. She taught that the easiest way to do one-digit addition such as 7+5 is by saving 7 in their head, making 5 with their fingers and then counting on, 8,9,10,11,12. Thus the children in the study had already been taught a count-on strategy. Sixteen of the 20 first-graders and six of the 20 second-graders were in the advanced-counting-by-ones stage while four first-graders and 14 second-graders were in the facile stage. Children in the facile stage solved most of the tasks quickly or used a non-count-by-ones strategy such as using doubles or an adding through ten strategy.

Table 3
The numbers of first- and second-graders at each stage of early arithmetical learning

Stage	Name of Stage	First grade	Second grade
0	Emergent	0	0
1	Perceptual	0	0
2	Figurative	0	0
3	Advanced-counting-by-ones	16	6
4	Facile	4	14

Table 4
The numbers of first- and second-graders at each level of base ten arithmetical strategies

Level	Name of level	First Grade	Second Grade
1	Initial concepts of ten	5	4
2	Intermediate concepts of ten	15	11
3	Facile concepts of ten	0	5

Table 4 shows the numbers of children at each level of base ten arithmetical strategies. Fifteen first-graders and 11 second-graders were at the intermediate concepts of ten level, that is they were able to see ten as a unit composed of ten ones but were not yet facile in solving written number tasks. Five second-graders were able to solve successfully all types of addition (no-carry and with-carry) and subtraction written tasks (no-borrow and with-borrow) mentally, and were judged to be at the facile concepts of ten level. No first-graders were judged to be at this level. This is to be expected because at the time of the interviews, the first-graders were just starting to learn two-digit addition and subtraction in class, while the second-graders had already learnt this topic.

Among the five second-graders, three children successfully imagined the standard column algorithms for addition and subtraction. One child used a jumping strategy (N10) and one child used a splitting strategy (1010) (Beishuizen, 1993). Data analysis also showed that 11 second-graders used an erroneous algorithm, and nine of these used 'the smaller from larger bug' in which, for example they say that 31−23=12 or 41−24=13. Three first-graders who had already learned the standard written algorithm at home also used this erroneous algorithm.

Further findings were that many children used fingers, and across the children there were several strategies. To solve 7+5 for example, some children directly opened 7 fingers and continued counting 8, 9, 10. When they ran out of fingers they closed all fingers and then opened two fingers sequentially. Other children simultaneously opened five fingers and closes them sequentially, saying 8, 9, 10, 11, 12. The first example shows that some children re-used their fingers to symbolise numbers above 10. This is the same as the method used by Korean children as reported by Fuson and Kwon (1992). For some first-graders use of fingers seems to be like a vestige. As well, five first-graders used the Jarimatika method. This involves using fingers to count as learned in an out-of-school course, along with other strategies. Some children were confused when their result using Jarimatika differed from their result using another strategy.

An erroneous algorithm 'the subtract smaller from larger bug'

'The subtract smaller from larger bug' has been identified by many researchers as one of the errors frequently made by children (e.g. Young & O'Shea, 1981; Brown & VanLehn, 1982; Ashlock, 1982). The teaching of the standard written algorithm has been identified as a factor which contributes to this error. This has led to the suggestion to defer the teaching of standard written algorithms until later years of schooling, and to wait until children have a strong conceptual understanding of tens and ones. In the early years it is better to support children to invent their own algorithms (Kamii, 1998).

Netral is a first-grader who has already learned the no-carry and carry addition standard written algorithms at home. He solved 42+23 by adding 2 and 3, and then 4 and 2, and solved 38+24, by adding 8+4=12 (counting with fingers), adding 1 to the result of 3+2 and then answering 62. He solved 26−12, by subtracting 2 from 6 and 1 from 2. When asked to solve 31−23, he tried to use the same strategy. However, he was confused when trying to subtract 3 from 1. He changed his answer 3 times, saying that 1−3=2, 1−3=3 and 1−3=0. Given that he had not learned a procedure for borrow-subtraction problems, his three answers seemed meaningful to him. However, it seems that he was not satisfied with these answers. His case indicates that he used the erroneous algorithm 'the subtract smaller from larger bug' because he had been taught the standard written algorithm for addition, and tried to use it to solve subtraction problems. Use of these algorithms may constrain a child's ability to reflect on the ten structure of the number system (Ebby, 2005). Working from the right (ones), Netral seems unaware of the tens value of the digits on the left side.

Limas, a second grader who had already been taught all of the standard written algorithms for addition and subtraction, used the erroneous algorithm 'the subtract smaller from larger bug' without hesitation because in her opinion this was the easiest way to find the answer. This finding accords with that of Hatano, Amiwa and Inagaki (1996) that even though children already know the 'correct' procedure, the buggy algorithm could be an attractive variant to save them from a long meaningless procedure. Furthermore, Limas also said that the easiest way to solve two-digit addition and subtraction is to work from right (the ones) to left as told by her teacher, instead of working from left to right which children who invent their own strategy usually do (Thompson, 1994). This finding suggests that Limas prefers to use the standard written method, which was taught by her teacher instead of using her own thinking. This is likely to have serious consequences for her overall mathematics achievement in the future.

Use of the Jarimatika method by the first-graders

Jarimatika or Chisanbop is an abacus-like method (Sun, 2008) of doing basic arithmetic attributed to a Korean tradition, in which the numbers 1 to 9 are symbolised by fingers on the right hand (Figure 1), and the numbers 10 to 90 are symbolised similarly using the fingers on the left hand. Using two hands, one can display any number from 1 to 99, and perform addition or subtraction by closing and opening the fingers.

Figure 1. Fingers representation for numbers 1 to 9 (adapted from Wulandari, 2004)

To carry out an addition such as 21+55 (Figure 2), first 21 is symbolised, then both thumbs are opened (55). The result can now be read on both hands as 76.

Figure 2. Addition 21+55 using the Jarimatika method (adapted from Wulandari, 2004)

When there are not enough fingers to complete addition or subtraction, the method involves the use of complementary numbers of 5 (called little friends) and 10 (called big friends). Therefore children have to be facile with combinations of 5 and 10. To carry out 9+4 for example (Figure 3), children recall that the big friend of 4 is 6, so they have to open the forefinger of the left hand which symbolises 10, and close the thumb and little finger of the left hand which symbolises 6. In this way the task 9+4 is changed to 9+10–6.

Figure 3. Addition using Jarimatika method (adapted from Wulandari, 2004)

Rahman, a first grader who has learned Jarimatika in an out-of-school course, consistently made mistakes for one-digit addition such as 7+6 (=14), 8+7 (=10) and 6+7 (=15). Only once did he solve an addition task correctly and quickly. This was 5+6. He said that he knows 5+5=10 and then add one more. He kept changing between the Jarimatika method and other strategies, and his answer did not indicate reflective thinking.

His case is similar to Nitas. As shown in the following excerpt, Nitas, a first-grader who had learned Jarimatika and also the standard written algorithm at home, seems confused about which of those methods to use.

Interviewer : So, how would you solve this (show card 31−23)?
Nitas : (She tries to use Jarimatika. She stops her attempt and suddenly say the answer) 12.
Interviewer : How did you get 12?
Nitas : I used the way mom taught me.
Interviewer : How?
Nitas : 3−2=1, 1−3=2
Interviewer : How about the way you are taught in the Jarimatika course?
Nitas : She tries to use Jarimatika again but fails and is back to her previous answer) 12
Interviewer : Please take time, and try again by using the Jarimatika method.
Nitas : (She uses Jarimatika, and she looks hesitant) 8.
Interviewer : So, according to Jarimatika the answer is 8, and according to mom's method the answer is…
Nitas : 12.
Interviewer : Which one is the correct answer?
Nitas : 12

The cases of Rahman and Nitas suggest that the Jarimatika method and the standard algorithms can be confusing and inhibit children from using intuitive reasoning. Rahman who showed an ability to use his own reasoning to answer 5+6, relied on using his taught Jarimatika method, without considering whether it produces the correct answer. While in Nitas's case, it is clear that she doesn't use intuitive reasoning at all. Rather, she merely attempts to recall methods taught to her.

Conclusion

The results show that the first- and second-graders in this study already have some degree of arithmetical knowledge. Most first graders were in the advanced-counting-by-ones stage in which they are able to solve number problems using a counting-on or a counting-back strategy. Most second graders were in the facile stage in which they can solve number problems using a non-counting-by-ones strategy. Most of them are also able to see ten as a unit composed of ten ones but are not yet facile in two-digit addition and subtraction problems. Their strategies were influenced by the teaching approaches used in their school, home and out-of-school number lessons, which did not always positively enhance their reasoning and problem solving ability.

The first grade of Indonesian schools, includes teaching of the standard written algorithms to solve two-digit no-carry addition and no-borrow subtraction and in the second grade, two-digit carry-addition and borrow-subtraction are taught. Researchers have warned about the harmful effects of teaching algorithms in the early years when children have not developed adequate understanding about tens and ones. The findings of this research accord with this, and indicate that instead of teaching procedures or algorithms, teachers should facilitate children's development of their own strategies and reasoning (Beishuizen & Anghileri, 1998; Kamii, 1998).

Furthermore, the findings from this study do not support the teaching of the Jarimatika method to young children. Similar to teaching standard written algorithms in the first and second grade, teaching the Jarimatika method to children at those grade levels may confuse them and tend to inhibit their use of reflective thinking and their ability to develop strategies which are meaningful to them.

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