A 'knowledge quartet' Used to Identify a Second-Year Pre-service Teacher's Primary Mathematical Content Knowledge.

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This paper draws on observation of a primary mathematics lesson prepared and taught by a second-year pre-service teacher who lacked mathematical content knowledge. A 'knowledge quartet' (Rowland, Turner, Thwaites, & Huckstep, 2009) was used to investigate when and how a pre-service teacher drew on their knowledge of mathematics during primary teaching. Data were collected from field notes, audio recording of part of a lesson, and an interview with the pre-service teacher after the lesson. Discussion focuses on the four characteristics of the 'knowledge quartet': foundation, connection, transformation and contingency. Conclusions suggested that pre-service teachers need to continue developing their mathematical content knowledge to assist with future planning and teaching of primary mathematics lessons.

The following study was part of four-year longitudinal investigation of 17 pre-service teachers' primary mathematical content knowledge, demonstrated during their Bachelor of Education (Prep-12). The relationship between practice, mathematical content knowledge, (MCK) and pedagogical content knowledge (Chick, Baker, Pham, & Cheng, 2006; Schulman, 1987) is the focus of this paper. Lisa's case, a second-year pre-service teacher is presented. A four-part framework, the 'knowledge quartet' (Rowland et al., 2009) is used to map the relationship between her mathematical content knowledge with classroom practice. Lisa, a pre-service teacher with gaps in her MCK, was chosen to find out how the experience of planning and delivering a primary mathematics lesson demonstrated and/or enhanced her MCK and contributed to her awareness of pedagogical content knowledge.

During 2008, only 50% of the cohort (N = 283) of second-year pre-service teachers passed a Mathematical Competency, Skills and Knowledge Test (MCSKT). These pre-service teachers also taught in primary schools, on average once a week (30 days each year), in first and second-year for their practicum, known as their project partnership. Therefore half of the pre-service teachers were not necessarily prepared to engage with the mathematical content and the children in their primary practicum classrooms. This paper focuses on the interaction between Lisa's practice and her MCK during part of a grade three primary mathematics lesson.

Mathematical Content Knowledge Needed for Primary Teaching

The literature suggests MCK needed for teaching is very complex consisting of many features. Knowing and using mathematics for teaching entails making sense of methods and solutions different from one's own (Ball, Bass, & Hill, 2004). A teacher requires specialised content knowledge, more, not less mathematical knowledge than the average adult and this is unique to teaching (Ball, Thames, & Phelps, 2008).

Reynold's (1992) review of the literature concluded that effective teachers connect what they know to new information and rely on their subject matter knowledge to create good lessons and explanations for their students. Ma (1999) describes a teacher's deep knowledge of content as Profound Understanding of Fundamental Mathematics (PUFM), demonstrating breadth, depth, connectedness and thoroughness used for expressing

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mathematical solutions. MCK is important and used widely by the teacher within the classroom. A teacher draws on content knowledge to promote students' mathematical reasoning (Ball et al., 2009). The Carpenter, Fennema, Peterson, Chiang and Loef (1989) study investigated teachers' use of knowledge when teaching. They believed students will construct knowledge when the teacher builds on their students' existing knowledge, by adapting instruction to suit students' needs.

Schoenfeld and Kilpatrick (2008) described proficient teachers of mathematics as having many characteristics. One was the ability to use their knowledge of maths in ways to provide the tools to instil understanding or help students with misunderstandings. Teachers implement mathematical knowledge, drawing on: procedural knowledge, procedural fluency, conceptual knowledge and mathematical connections (Ball, 2003; National Curriculum Board, 2009). They demonstrate a broad understanding of the mathematical horizon linking their content knowledge with curriculum content and making connections between them (Ball et al., 2009). It is hoped that primary teachers develop their knowledge of content so they possess knowledge of the mathematical horizon and are aware of the range of strategies students will bring to mathematical tasks (Sullivan, Clarke, & Clarke, 2009).

The 'knowledge quartet'

Rowland and colleagues (2009) developed the 'knowledge quartet' framework to support beginning teachers. During a mathematics lesson, the teachers' actions were identified and recorded by an observer to provide feedback relating to the use of their MCK. There were many ways of looking at describing how the teachers used their MCK. The 'knowledge quartet' (Rowland et al., 2009) classified these into four 'big ideas' or dimensions: foundation, transformation, connections and contingency (Figure 1).

Foundation Is the knowledge a teacher brings to teaching, their content knowledge and beliefs about mathematics.	Transformation Is the choice of examples and representations the teacher uses, focusing on the teachers' knowledge in action.
Connection Identifies coherence and knowledge of the sequence of the topics from lesson to lesson and within the lesson.	Contingency How a teacher responds to a student's unexpected method or comment.

Figure 1. The codes of the 'knowledge quartet' and description for each category (Rowland et al., 2009).

Methodology

Lisa, who was not successful with passing a MCSKT, was selected from a larger longitudinal study because she was similar to half of her second-year cohort. At the time of the study Lisa was 20 years of age, she was enjoying her project partnership experiences and wanted to teach in a primary school on graduation. Prior to the observation reported in this paper she had taught ten mathematics lessons under the supervision of her mentor teacher. Lisa planned the activities presented to the students for the lesson this study is based on.

The study used a qualitative method to analyse the introduction of one primary mathematics lesson during her second-year. The researcher observed the lesson taking field notes. A digital voice recorder was used to record the lesson, which was later transcribed

for analysis. The researcher interviewed Lisa following the lesson, reflecting on her teaching. The interview was also digitally voice recorded and later transcribed.

Lisa's lesson was observed at her project partnership placement, towards the end of second-year, with twenty grade three students from a Catholic primary school. While Lisa taught the lesson, the students were seated at their tables, towards the front of the classroom. The researcher sat at the back of the classroom and did not interact with Lisa or the students during the lesson. The mentor teacher sat at her desk (to the side) and observed the lesson, occasionally interacting with the discussion.

The lesson took 60 minutes to complete. The introduction (i.e., the first 20 minutes) focused on subtraction of two-digit numbers and was used for analysis and discussion for this paper. The interview, after the lesson with Lisa and the researcher, took 40 minutes. The interview questions related to the lesson and other components of the longitudinal study, some interview data was used for this study.

Field notes, the transcription of the lesson introduction and interview were colour coded and matched with Rowland's et al., (2009) four categories from the 'knowledge quartet' (Figure 1). The four categories; foundation, connection, transformation and contingency were discussed with reference to Lisa's lesson to investigate when and what MCK had been demonstrated during the mathematics lesson.

Lesson Synopsis

Lisa's Grade 3 Subtraction Lesson

Lisa settled the students while they were seated at their tables. She stood at the front of the classroom introducing the lesson to the class demonstrating on the whiteboard. She commenced with a bingo game followed by a discussion that focused on drawing and using a subtraction ladder; this section lasted 20 minutes.

The students were asked to select their own numbers from zero to 20 for a three-by-three bingo grid they drew into their workbooks. Detailed instructions were not provided and it was assumed students knew the rules.

> Lisa: So we all know how to play bingo? As soon as you get three in a row, you can yell out bingo if you like. We are going to do subtraction problems. How you work it out it is up to you. If you want to use some scrap piece of paper and write it out you can do that or if you want to do it mentally, you might count by twos or fives. If you need to draw a number line you can do that.

Lisa tried to suggest to the students some strategies they could use for their subtraction facts. She did not explain what she meant by "count by twos or fives… draw a number line…" Lisa proceeded to ask subtraction basic facts between zero and twenty while the students located the answers on their bingo grid. After Lisa said one problem she asked a student for the answer, checking as the class completed the game.

> Lisa: "Twenty-eight take away eight, Isabel?"
>
> Isabel: "Twenty."
>
> Lisa: "Beautiful!"

For each question, a different student was asked to answer, and Lisa recorded correct responses next to the problem on the whiteboard. Lisa chose the following questions for the students to answer and find on their bingo board: $20 - 8 =$, $14 - 2 =$, $14 - 4 =$, $28 - 8 =$, $8 - 3 =$.

For the main activity, Lisa drew a subtraction ladder onto the whiteboard. (She adapted an idea from a maths book her mentor had shown her.)

> Lisa said, “Here is a subtraction ladder, you have probably never seen this before. We are going to make this work downwards… The first number is going to be ten and I want to put 5 here (the numbers were placed in the first and third spaces). I want you to tell me, what subtraction problem I can make to put a number in between? What is a number between ten and five?”
>
> Olivia: “Eight.”
>
> Lisa: “What do I do to get from ten to eight, what subtraction problem?”
>
> Darcy: “Ten take away two.”
>
> Lisa: “Who knows how to work that out?”

Lisa continued to work through this method placing a number into the ladder making it the difference. Then she used the number above it as the minuend to work out the subtrahend. Finally, she recorded the subtraction problem to the side. Figure 2 provides a copy of the subtraction ladder at the beginning of the discussion as well as the completed ladder at the conclusion of the discussion.

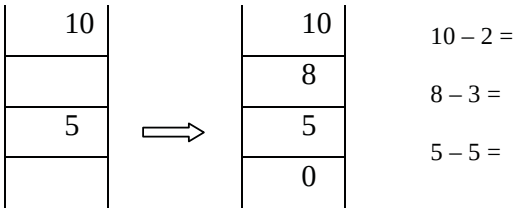

Figure 2. A copy of subtraction ladder at the beginning of the discussion and the same subtraction ladder as illustrated on the whiteboard after the discussion with the students.

Once the first ladder was completed Lisa suggested that these were pretty easy (Figure 2) and proceeded with a new example with larger numbers (Figure 3). For the second example she recorded the digits 50 and 35 into the ladder and asked the students to draw a ladder and use strategies to solve the problem.

> Lisa said, “You need 50 at the top and 25 half way down the ladder … What number might I put here?” (In the second space)
>
> Darcy: “25.” (The student may have been thinking the difference between 50 and 25 is 25 but Lisa prompted a different number)
>
> Lisa: “Maybe count by tens, 25.”
>
> Darcy: “35.”
>
> Lisa: “Yes. Do you maybe want to put 35 in here (she records 35 on whiteboard). OK, 50 take away 35. Write down or draw how you could solve that … You can draw maybe apples. You might have 50 apples. That’s a bit hard. So you might group them… You might want to use a number line.”

Lisa then asked the students to think of a number to record in the last place on the ladder.

> She said, “Could you put 40 in this box?”
>
> Ben replied, “No.”
>
> Lisa said, “Can we do 25 take away 40. No not really. Not properly. We need a number less than 25.”

The students were asked to copy the ladder (Figure 3) and write their own number into the last box of the ladder, before sharing their responses, for example 25 take away 10. The students then moved into three groups to complete further subtraction tasks to conclude the lesson.

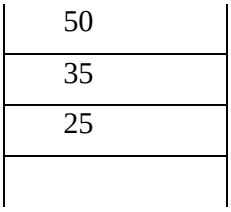

Figure 3. A copy of the second subtraction ladder used for providing subtraction questions for the students.

Results and Discussion

Identifying the Principles that Lisa Knew

The lesson introduction and a selection of reflections from Lisa's interview were further analysed, providing enough data to use with reference to the 'knowledge quartet' and the four dimensions; foundation knowledge, transforming knowledge, connection and contingency (Rowland et al., 2009). Discussion of the four categories of the knowledge quartet follows, focusing on Lisa's MCK implemented during her teaching and reflections from part of an interview after the lesson.

Foundation: For delivery of the introduction of the subtraction lesson, Lisa provided evidence that she could solve 2-digit subtraction problems. The most difficult example was 50 take away 35, during the subtraction ladder task. It is likely Lisa learnt how to subtract 2-digit numbers, as a primary student herself. Lisa was able to listen to the students' responses and knew the answers without needing to demonstrate her working out. Lisa most likely used a mental subtraction strategy in her head. Because Lisa answered the questions quickly, it could be assumed she was able to use known subtraction facts to work out her answers. This segment of the lesson shows that Lisa had the mathematical knowledge needed for solving a grade three level subtraction problem. It is hoped that she could solve subtraction questions needed for teaching higher grade levels, but this was not demonstrated during this lesson.

During her interview, Lisa thought she was "a bit rough" with her teaching of primary mathematics but felt that by the end of fourth-year she would be much more confident than she was now. She also explained that she felt her MCK was about average. Lisa provided the following reflection when asked to describe her own understanding of primary MCK:

> My content is just passing. I think there is a long way to go. After today I know now I needed to do this and I needed to do that and then I will go home and then I will read about it or learn in different ways. Until you are thrown in and experience it, I don't have an incentive to just read numbers … I think I am just over average.

This reflection indicates that Lisa is aware she needs to improve her content knowledge but does not clearly articulate what she plans to learn or needs to know. Later in the interview, Lisa said she had a grade five text book she was going to use for revision. Needing to revise a grade five textbook suggests Lisa currently lacks the content knowledge she would be expected to teach in the upper primary grades.

Connections: ***Lisa attempted to make connections during the lesson but without depth. She made reference to a previous lesson reminding students they had used a number line.*** During the interview Lisa vaguely suggested different subtraction strategies the students could use, for bingo, "write it out, do it mentally, count by twos or fives, or draw a number line." She also mentioned these during her lesson when using the subtraction ladder, but did not elaborate, demonstrate examples or provide materials.

To facilitate student learning teachers need to promote learning by making explicit connections of mathematical topics (Ma, 1999). Booker, et al (2004) says that children will construct meaning by the experiences provided with materials, reflecting and talking about their ideas to promote mathematical discussion of various interpretations. Lisa was aware that students can select a range of strategies when completing subtraction problems. However, she did not demonstrate strategies in depth within her lesson to promote learning. Lisa needs to further extend students' understanding of the different methods for finding solutions and incorporate these into her lesson to connect students' knowledge of 2- digit problems as the students explore harder questions.

Lisa made up the questions as she took the lesson. There was no evidence of planning the problems to use with the students, or use of notes from a lesson plan. The questions Lisa selected were more suitable for younger students. The content presented may have not engaged all students in the class. Lisa needs to ensure that she prepares her lessons well, and caters for all learning needs of all students. Further teaching experiences will assist to build coherence of the sequence of topics from lesson to lesson or within the lesson. A teacher catering for all learners would target questions to assist weaker learners and provide harder items to challenge the fluent learners. This would also demonstrate if Lisa had the knowledge to scaffold the level of difficulty of the questions to cater for the range of abilities within her grade three class. She should prepare her questions before the lesson by referring to a sequence for developing the subtraction concepts (Booker et al., 2004, p. 226).

Transformation: There were issues with the use of the subtraction ladder. When Lisa presented her second examples of subtraction problems (Figure 3) she changed the recording structure when using the subtraction ladder. The lesson began with students using the digits in the ladder to record the minuend and difference of the problem (10 – ? = 8). This method was then switched using the digits to record the minuend and subtrahend (50 – 35 = ?). Lisa was not aware that the subtraction ladder may have been confusing by swapping a change unknown structure to a result unknown structure (Carpenter, Fennema, Franke, Levi, & Empson, 1999).

While Lisa focused on her invented procedures she provided no connections or steps that scaffolded the students' understanding. The ladder was not an appropriate representation for modelling subtraction or difference concepts and was not useful for demonstrating procedures or mathematical thinking. During Lisa's interview she explained, "The ladder was a strength of my lesson, which was better than writing up equations." Lisa was more concerned about engaging the students by using a "game" and a different way of recording equations rather than focusing on mathematical connections.

During the lesson Lisa did not elaborate on the different student responses. Rowland et al., (2009) suggested in their study this could be because of time restraints or because it had been covered in a previous lesson or because the teacher did not have the confidence to do so. Lisa may not have developed the confidence or skills needed for explaining different strategies elaborating on the students' methods, she could have demonstrated her skill in transforming her MCK. Listening to different students' strategies improves students'

mathematical understanding and increases teachers' mathematical knowledge (Empson & Jacobs, 2008).

An inappropriate suggestion was made for solving, "50 take away 35… You can draw maybe apples". Story problems are used to introduce the subtraction concept. Students solving 2-digit problems should model, demonstrating their thinking in tens and extending their basic facts to ten, first without renaming, then with renaming (Booker et al., 2004). For example, the use of popsticks, bundles of tens and ones are used to develop 2-digit subtraction understanding before moving onto base ten materials.

Contingency: Within the lesson one misconception that arose was when Lisa spoke with Ben about whether 40 could be subtracted from 25 and her response was, "Not really". Her response was inappropriate, as 25 subtract 40 equals ¯15 and is a true mathematical statement. Maybe her response was affected by the awareness that this subtraction concept would not be introduced at this level and therefore would be too difficult to explain. Nevertheless, Ma (1999, p. 3) discussed a similar example believing that young students' future learning should not be confused by emphasizing a misconception. A better response from Lisa would have been to say, "Yes, it is possible and we can work on that later". Individually, Lisa could then chat with the student, providing an opportunity to explore the concept on a number line.

During the lesson there were no extending questions to challenge students. Lisa was able to respond to the students' comments and responses by asking closed questions requiring a one-word response. Open-ended questions with an open-ended answer would provide an opportunity for Lisa to further demonstrate her skills and knowledge of the topic. The question and answer approach during the lesson controlled the lesson format, ensuring the students were not diverted from her agenda and easy subtraction examples. Harder subtraction questions may have been difficult for Lisa to solve confidently while in front of the whole class.

Conclusion

Ma (1999) says that teachers can demonstrate a wide range of content knowledge, even with basic teaching of subtraction without regrouping. The use of the 'knowledge quartet' framework was able to identify that Lisa was not able to demonstrate a wide range of content knowledge during her lesson.

Findings suggested that a second-year pre-service teacher with gaps in MCK lacked the ability to implement a grade three subtraction lesson that promoted students' mathematical understanding. Lisa used procedures and closed questions, rather than articulate and demonstrate multi-solutions with materials. She did not discuss a range of thinking strategies that promoted learning. Focusing on improving her own mathematical knowledge might assist her with making the connections needed for mathematical explanations and develop the skills needed to acknowledge and use the range of strategies students will bring to each lesson.

Moreover during the follow up interview Lisa did not reflect on the way representations, materials and questions may promote or impede mathematical thinking or on her lack of teaching strategies to scaffold students' learning. While we need to be careful about drawing generalisations from one case, this study draws on a need for further study on how pre-service teachers' MCK can be enhanced through practice.

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