Images, Anxieties and Attitudes toward Mathematics

Shashidhar Belbase

Graduate Student of Mathematics Education
College of Education, University of Wyoming, Laramie, Wyoming

December 3, 2010

Images, Anxieties and Attitudes toward Mathematics

Shashidhar Belbase
Graduate Student of Mathematics Education
College of Education, University of Wyoming, Laramie, Wyoming
December 3, 2010

Abstract

Images, anxieties, and attitudes towards mathematics are common interest among mathematics teachers, teacher educators and researchers. The main purpose of this literature review based paper is to discuss and analyze images, anxieties, and attitudes towards mathematics in order to foster meaningful teaching and learning of mathematics. Images of mathematics are greatly shaped by epistemological and philosophical perspectives of one who views mathematics either as priori or posteriori, absolute or relative, and concrete or nominal. These images as perceived by an individual play a significant role in the development of attitudes towards mathematics in long run. Images of mathematics have possible negative and positive impacts in teaching and learning of mathematics with the subsequent development of attitude toward mathematics as positive or negative (Goolsby, 1988; Ma and Kishor, 1997; & Lakoff and Nunez, 2000). Mathematics anxiety is developed as a result of having a poor image of mathematics due to a general lack of comfort that someone might experience when required to perform mathematically(Wood, 1988), or the feeling of tension, helplessness, and mental disorganization one may have when required to manipulate numbers and shapes (Richardson & Suinn, 1972; Tobias, 1978). In the long run, mathematics anxiety can produce different feelings such as like/dislike (an attitudinal element), happiness/worrisome (a cognitive element), and motivation/fear (an emotional element) (Hart, 1989; Wigfield & Meece, 1988). A theoretical model with different combinations of images, anxieties, and attitudes toward mathematics can be a helpful tool to develop an understanding of the different relationships among them for significant pedagogical implications.

How do students perceive mathematics in schools? What are different images of mathematics that students perceive? How these images impact in their learning? What is math anxiety? What are the causes of math anxiety? What is relation of image of mathematics as perceived by students with math anxiety? What are different attitudes toward mathematics? How these attitudes impact in learning mathematics? How images, anxieties and attitudes related to each other or how they form personality of students in terms of mathematics? These are some questions that this paper is based on. There are large number of researchers and published papers in attitude towards mathematics but none of them clearly discuss the relationship among images, anxieties and attitudes toward mathematics. This paper tries to bring them together with a hypothetical model and seeks to understand the impact of different combinations in teaching and learning mathematics.

It seems that the number of dissertations and published articles dealing with attitude towards mathematics increased geometrically since Feierabend's (1960) report "Review of research on psychological problems in mathematics education" (Aiken, 1970). This indicates that there is growing interest in research about attitude toward mathematics. In this context, mathematics educators have regarded the relationship between attitude toward mathematics and achievement in mathematics as their major concern (Ma & Kishor, 1997). Ma and Kishor further stated that "the research literature, however, has failed to provide consistent findings regarding the relationship between attitude toward mathematics and achievement in mathematics" (p. 27). Some researchers (e.g., Deighan, 1971; Vachon, 1984) demonstrated that there is low correlation (below 0.5) between attitude toward mathematics and achievement in mathematics; however, other researchers (e.g. Kloosterman, 1991) demonstrated that the attitudinal variables are significant indicators of math achievement. This paper is an attempt to analyze the images of mathematics in relation to anxieties and attitudes toward mathematics and their impacts in teaching and learning mathematics in classroom from the literatures of mathematics education.

From a psychological perspective, there is a general myth that mathematics is a mysterious subject. Some people claim that they like mathematics while others claim that they dislike mathematics. Some people even are scared of mathematics while others enjoy problem solving in mathematics. The people who claim that they like mathematics often choose mathematics in their college study, while those who prefer to say they dislike mathematics view mathematics as a difficult subject (Sam, 1999) and most possibly they discontinue mathematics.

According to different perspectives, mathematics can be a battle, a mountain, or a bridge, and mathematics can be viewed differently in terms of inherent characteristics as perceived by teachers and students (Sterenberg, 2008). Metaphorical images of mathematics held by students and teachers play a significant role in developing beliefs and attitudes toward mathematics in terms of having a favorable or disfavorable opinion. These images reveal that relationships and meanings are produced metaphorically through a transfer between domains of mathematics and terms related to representing mathematics. Such a transfer forces us to make sense of mathematical objects (Game & Metcalfe, 1996). Many people tell stories of their childhood when they were frustrated in mathematics class and/or scared of problem solving in mathematics. The long thread of their struggle in learning mathematics in schools my create different images of mathematics; many of them unfortunately negative (Sterenberg, 2008).

In a study, university professors of mathematics and mathematics education in Canadian Universities have two images of mathematics: it is either a formal abstract system ruled by logic, or as a model of the real world (Mura, 1995). Some considered mathematics to be both an art and science, and while others see it as a language and a set of specific contents. These images of mathematics value inductive or deductive reasoning as a way to learn and teach mathematics. Researchers have claimed that those images of mathematics have possible negative and positive impacts in teaching and learning of mathematics with the subsequent development of attitude toward mathematics as being positive or negative (e.g. Goolsby, 1988; Ma & Kishor, 1997; Lakoff & Nunez, 2000). Negative attitude of mathematics means having a dislike of mathematics, which in turn may result in the avoidance of using mathematics in daily life. This may create a low self-esteem or less confidence in using mathematics in daily life and, ultimately, having a low interest towards learning mathematics (Sam, 1999). A negative image of mathematics discourages students from choosing mathematics as their major in schools and colleges and, finally, may keep them away from careers in science and technology.

There is an increasing recognition that affective factors play a critical role in the teaching and learning of mathematics (McLeod, 1992, 1994). Mathematics anxiety is often referred to as the general lack of comfort that someone might experience when required to perform mathematically (Wood, 1988), or the feeling of tension, helplessness, and mental disorganization one has when required to manipulate numbers and shapes (Richardson & Suinn, 1972; Tobias, 1978). Mathematics anxiety can take multidimensional forms including, for example, dislike (an attitudinal element), worry (a cognitive element), and fear (an emotional element) (Hart, 1989; Wigfield & Meece, 1988). The anxieties of dislike, worry, and fear are created due to poor image of mathematics creating negative perception towards mathematics leading to negative attitude in the long run. The relationship of images, anxieties and attitudes toward mathematics can be discussed within epistemological and philosophical foundations mathematics, teaching and learning of mathematics in schools.

Epistemological and Philosophical Foundation

What is the nature of mathematics? How this nature is perceived by an individual? What are personal epistemologies and philosophies? How these epistemologies and philosophies impact in our understanding of mathematics? There are some questions we need to think in order discuss the relationship among images, anxieties and attitude toward mathematics.

The nature of mathematics can be viewed differently from different epistemological and philosophical perspectives. For *Realists*, mathematics is viewed as the science of numbers, sets, functions, etc., just as physical science is the study of ordinary physical objects, astronomical bodies, subatomic particles, and so on (Maddy, 1990). According to the *Realist* school, images of mathematics relate to the nature of mathematics as being fixed, epistemologically priori, and it is infallible. This epistemological perspective believes that mathematical knowledge is fixed, and it is out there that we have to discover it. Many mathematicians have this dominant image of mathematics, and their teaching and learning in the classroom is affected by such image influenced by their epistemological perspective and personal philosophy. *Intuitionism* is based on the idea that mathematics is a creation of the mind. The truth of a mathematical statement can only be conceived via mental construction that proves it to be true and the communication between mathematicians only serves as a means to create the same mental process in different minds. The image of mathematics from this epistemological perspective and philosophical lens is that mathematics is a mental creation; mathematical objects are created by the intuition of mind, irrespective of language in which one thinks (Iemhoff, 2008).

The epistemology and philosophy of *Constructivism* conceives that the learners actively construct their own knowledge, rather than passively receive it. *Constructivists* argue that the term knowledge is problematic because it evokes a static, rather than dynamic image of learning, and they prefer to talk about learning or knowing, interpreting and making sense of experiences. A popular conception of *Constructivism* claims that learners can only construct meaningful understanding in relation to their prior knowledge. The image of mathematics is viewed from this epistemological and philosophical lens as a co-construction of mathematical ideas through social critical discourse of various mathematical phenomena (Confrey & Kazak, 2006).

There exists a common feature of all the views just described, that is, that they all take mathematics to deal with abstract objects, whether one takes these to have an independent existence in their own right, or to be abstracted from our experience (Avigad, 2007). An alternative, as suggested by Avigad, is simply to deny such object's ontological status in the first place, and think of mathematics, instead, as a science governing the use of (relatively concrete) signs. The challenge then is to give an account of mathematical knowledge that explains what it is that gives certain manipulations of signs normative force and also explains the applicability of mathematics to the sciences. Positions that adopt such an approach fall under the rubric of *Nominalism* (Avigad, 2007). According to *Nominalism*, mathematical objects do not exist or, at least, they need not be taken to exist for us to make sense of mathematics (Bueno & Zalta, 2005).

Strawderman (2010) has proposed three domains to study mathematics anxiety: *social/motivational* domain, *intellectual/educational* domain, and *psychological/emotional* domain. Strawderman clarified that the *social/motivational* domain includes those forces that act upon a person through the agencies of family, friends, and society as a whole. The *intellectual/educational* domain is comprised of those influences that are cognitive in nature. These cognitive influences include but are not limited to, the knowledge and skills an individual has and or is expected to acquire and his or her perception of success or failure in them. The *psychological/emotional* domain is formed by the faculties that are affective in nature. It is largely comprised of the individual's emotional history, reactions to stimuli, and arousal states. Hence, the continuum associated with this domain is feelings. At either end of the feelings continuum lie anxiety and confidence. These domains of anxiety are related to attitude towards mathematics in terms of emotions, expectations and values (Hannula, 2002). These discussions lead us to conclude that images of mathematics are greatly shaped by the epistemological and philosophical perspectives of one who views mathematics either as priori or posteriori, absolute or relative, and concrete or nominal. These images are further associated with perceptions, feelings and anxiety of mathematics leading to negative or positive attitude toward mathematics.

Images of Mathematics

When I talk or think about images of mathematics, two things come into my mind: images as objects or images as abstraction. I think images as objects in relation to mathematics are related to symbols (+, - , %, [ ], Δ, π, <, =, >, √, $\sum$, $\int$ etc.) and images as abstraction are related to operations that go in our mind. The image as an object is static and it visualizes mathematics as a subject matter. The image as an abstraction is dynamic and it visualizes mathematics as a process or operation that goes on in our mind.

Tall and Vinner (1981) define a concept image as all the cognitive structures, conscious or unconscious, associated with a concept, including mental images and words. A concept, such as an apple, must allow for variability. If we imagine an object shaped like an apple that is purple, we can still believe that it is an apple. We have the freedom to recombine familiar ideas in novel ways. But, since we have never seen a purple apple, it is unlikely that we would form an image of one, when hearing the word apple (Browne, 2009). McGinn (2004) asserts that images are part of our active nature, since they are subject to the will of the viewer. Percepts belong to the passive part of thinking and imagination. In other words, one must make an effort to form an image of something, while the same may not hold true for just looking. That is to say that something that we see may have different mental image than that appears to us. McGinn classifies images as a distinct mental category, separating them from percepts. In mathematics, images represent perception in terms of nature of mathematics as viewed by a person. Lakoff and Nunez (2000) have argued that the conceptual metaphor plays a fundamental role in mathematical understanding because it provides a means to map ideas in one conceptual domain to corresponding ideas in another conceptual domain. For example, it makes possible for us to understand difficult ideas such as infinity.

Based on our experience, we all may develop different images in relation to mathematics and its nature. Specifically, we all have developed images of and about mathematics. Ernest (2008) argues that there are many components of learner attitudes and beliefs about mathematics. These attitudes and beliefs play an important role in problem solving and in learner participation in advanced mathematical studies and careers. According to Ernest, developing a positive image of mathematics leads a learner toward advancement and to the benefit of society.

In *absolutist* perspective, images of mathematics are viewed as an objective, absolute, certain, and incorrigible body of knowledge, which rests on the firm foundations of deductive logic. Among twentieth century philosophies, *Logicism*, *Formalism,* and, to some extent, *Intuitionism* and *Platonism* may be said to be *Absolutist* in this way (Ernest, 1991). However, Ernest (2008) claimed that absolutist philosophies of mathematics are not concerned about describing mathematics or mathematical knowledge as they are practiced or applied in the world around us.

Rensaa (2006) asserts that in the past few decades a new wave of epistemology and philosophy of mathematics have been gaining ground and these propose a non-absolutist account of mathematics. Kitcher and Aspray (1988) described this as the ‘maverick’ tradition that emphasizes the practice of, and human side of mathematics, and characterizes mathematical knowledge as historical, changing, and corrigible. Image of mathematics is viewed as falsifiable, contextual, and relative.

According to Ernest (1994), one of the innovations associated with a fallibilist view of mathematics is a reconceptualized view of the nature of mathematics. It is no longer seen as a body of pure and abstract knowledge which exists in a superhuman, objective realm. The perfection of mathematics is ideal and, therefore, the false image of perfection of mathematics must be dropped (Davis, 1972).

Before discussing the relationship of images of mathematics with values and epistemology it is necessary to indicate what we mean by an image of mathematics in this context. Taking an image of mathematics is a representation of mathematics that is either *social* or *personal. Social images* of mathematics are public representations encompassing mass media representations including films, cartoons, pictures, popular music; presentations and displays in school mathematics classrooms and the related learning experiences relating to them; parent, peer, or other narratives about mathematics; and representations of mathematics utilizing any other semiotic education modes or means. These public images of mathematics may possibly have a significant impact in shaping children's attitudes toward mathematics (Ernest, 2008).

Ernest (2008) further claimed that *personal images* of mathematics are personal representations of mathematics utilizing some form of mental picture, visual, verbal, narrative or other personal representation, originating from past experiences of mathematics, or are from social talk or other representations of mathematics, which may potentially compromise cognitive affective and behavioral dimensions. To me, these *personal images* in relation to *social images* develop our perceptions, values, and attitudes towards mathematics. The conception of mathematics as it is represented in such images may vary across a range encompassing research mathematics and mathematicians, school mathematics, and mathematical applications in everyday or otherwise.

A widespread *public image* of mathematics in the West is that it is difficult, cold, abstract, theoretical, and ultra-rational, and, also important and largely masculine (Ernest, 2008). It also has the image of being *remote and inaccessible* to all but a few super-intelligent beings with 'mathematical minds' (Buerk, 1982; Buxton, 1981; Ernest, 1996; Lim & Ernest, 1998; Picker & Berry, 2000). For many people, this *negative image* of mathematics is also associated with anxiety and failure. When Brigid Sewell was gathering data on adult numeracy for the Cockcroft (1982) inquiry, she asked a sample of adults on the street if they would answer some questions. Half of them refused to answer further questions when they understood it was about mathematics, suggesting *negative attitudes*. Extremely *negative attitudes* such as '*mathephobia*' (Maxwell, 1989) probably only occur in a small minority in Western societies, and may not be significant at all in other countries. In fact, the world-wide consensus of mathematics educators is that school mathematics must counter that image, and offer instead something that is personally engaging, and useful, or motivating in some other way, if it is to fulfill its social functions (Howson & Wilson, 1986; NCTM, 1989; Skovsmose, 1994).

In a broad sense, images of mathematics as separated and connected to values that further lead us to formulation of a school mathematics either as *disconnected or connected images* of mathematics. Finally, teaching and learning of mathematics influenced by the *absolutist* epistemology and philosophy help students develop *separate (fragmented) image* of mathematics, while *fallibilist* epistemology and philosophy help students to develop a *connected (logically related) image*. However, for many pupils the image about mathematics become influenced by dichotomous thought as mathematics is *absolute* or *fallible*, they are gradually changing from *positive* to *negative* feelings through schools because of not being able to understand that they can create mathematical objects rather than imitate from others. Such negative feelings about mathematics, for instance, have been seen in reports from the United States, Austalia, or closer in Norway (Ernest, 2008). Renssa (2006) also claimed in the same line as Ernest that pupil's images of mathematics and mathematicians are derived as a result of social experiences, either through school, peers, parents, or mass media. In real life the picture is more complex as these influences interact each other.

Within the public society, adults and parents' images of mathematics are important when it comes to influence children's perceptions. As stated by Ernest (1996) there is no doubt about impact of adult's and parents' perception and attitude toward mathematics on children's attitude to the subject. It indicates that how important parental encouragement is to children's learning of mathematics. Ferry et al. (2000) found in their research on family background context variable, parental encouragement in mathematics and science significantly influence learning experiences. Learning experiences, in turn, were found to significantly influence self-efficacy and outcome expectations. These results support the role of family context in Lent et al.'s social cognitive career development model (Lent et al., 1994). Children not having this support may therefore have a drawback when it comes to continuing with *negative images* of mathematics.

Mathematics Anxiety

When I think or talk about mathematics anxiety, two things come to my mind: one is anxiety as progressive thinking and the other is anxiety as regressive thinking. To me all anxieties are not bad things. Anxieties can be both good and bad. If it promotes for progressive thinking (like when I am puzzling in a mathematics problem for a few days and I am trying to solve it in a variety of ways), then certainly it is good thing. But anxiety is mostly taken as regressive thinking in which a person having anxiety tries to go away or get rid of problem simply by avoiding it and taking it negatively.

Mathematics anxiety is an anxious state in response to mathematics-related situations that are perceived as threatening to self-esteem. Cemen (1987) proposed a model of mathematics anxiety reaction consisting of environmental antecedents (e.g., negative mathematics experiences, lack of parental encouragement), dispositional antecedents (e.g., negative attitudes, lack of confidence), and situational antecedents (e.g., classroom factors, instructional format) are seen to interact to produce an anxious reaction with its physiological manifestations (e.g., perspiring, increased heart beat). Many researchers (e.g., Ma, 1997; Richardson & Suinn, 1972; Tobias & Weissbrod, 1980) have reported the consequences of being anxious toward mathematics, including the inability to do mathematics, the decline in mathematics achievement, the avoidance of mathematics courses, the limitation in selecting college majors and future careers, and the negative feelings of guilt and shame. Ma (1997) claimed that mathematics anxiety is usually associated with mathematics achievement individually. A student's level of mathematics anxiety can significantly predict his or her mathematics performance (Fennema & Sherman, 1977; Wigfield & Meece, 1988).

In their study Miller and Bichsel (2004) claimed that math anxiety appears to primarily impact visual working memory, contradicting previous findings that anxiety is primarily processed in verbal working memory and supporting the hypothesis that math anxiety does not function similarly to other types of anxiety. They referred to past researches which investigated the underlying cognitive processes that contribute to individual differences in math ability, the most investigated of which appears to be working memory. Many researchers in the past have shown that the processes involved in working memory, namely, temporary retrieval, processing, and storage, explain much of the variance in math ability (e.g., Adams & Hitch, 1998; Ashcraft, 1995; Miller & Bichsel, 2004). The concluding remarks can be made based on these claims that individuals who are more efficient and adept in carrying out these processes are likely to perform better on tests of math ability.

Miller and Bichsel (2004) identified two general types of anxiety: trait and state. They clarified that individuals experiencing trait anxiety have a characteristic tendency to feel anxious across all types of situations. In contrast, individuals possessing state anxiety tend to experience it only in specific personally stressful or fearful situations. Trait anxiety is more related to wide range of situations to which one feels a kind of threat, unsecured, and challenge all the time. In mathematics, students under this anxiety have fear of mathematics class, homework, exam and any situation when comes with mathematics. According to Spielberger et al. (1970), state anxiety reflects a transitory emotional state or condition of the human organism that is characterized by subjective, consciously perceived feelings of tension and apprehension, and heightened autonomic nervous system activity. Several past studies demonstrated that both state and trait anxiety affect task performance (e.g., Leon & Revelle, 1985; MacLeod & Donnellan, 1993; Miller & Bichsel, 2004). Concluding the findings from these researches, Miller and Bichsel stated that individuals with high trait anxiety show poorer performance on various tasks than low trait anxiety individuals. This difference tends to be exacerbated in a high state anxiety condition. With reference to research on impact of gender on math anxiety, Hembree (1990) found math anxiety being more predictive of math performance in males than in females.

Attitude toward Mathematics

When we talk about attitude toward mathematics, we directly to jump on to dichotomies of positive and negative attitude. I think attitude toward mathematics cannot be completely understood in terms of just as positive or negative. Mathematics is a broad field and one cannot be completely positive or negative to the whole domain of mathematics rather we can see if one is positive or negative to different areas of mathematics such as geometry, arithmetic, algebra, statistics, analysis etc. There are many researches in attitude toward mathematics but rarely these studies analyzed whether the attitudes developed due to nature of mathematics or processes of doing mathematics.

Images of mathematics as perceived by a person develop his or her *positive* or *negative* attitude towards mathematics. These images, as perceived by students as social and personal in a positive or negative ways, have a significant impact in their choice of mathematics as major in higher education. In this context, many studies have been conducted on attitudes toward mathematics (e.g., Eleftherios & Theodosius, 2007; Fonseca, 2007; Hannula, 2002, 2004; Norton & Rennie, 1998; Swetz et al., 1983; Zan & Di Martino, 2007). Eleftherios and Theodosius (2007) used the term 'beliefs' in the meaning of personal judgments and views, which constitute one's subjective knowledge, does not need formal justification. They investigated the students' beliefs and attitudes, which mainly concerned studying and learning mathematics. Specifically, they explored their factorial structure. Also, they investigated whether there were differences in students' beliefs and attitudes regarding their social status and gender; they examined whether these factors correlated and influenced students' performance at school and their ability to understand mathematical proofs. In their study they found a significant statistical difference between female and male students concerning "mathematical understanding is achieved through procedures and studying mathematics with understanding" (p. 101). Love of mathematics was found to be correlated positively with studying of mathematics involving understanding and reflection, with high performance at school and with the ability to understand mathematical proofs. The results from this study identified the factors that lead to the development of students' positive and negative attitude towards mathematics with a significant impact in their learning of mathematics and achievement.

According to Zan and Di Martino (2007), the phenomenon of ‘negative attitude towards mathematics’ is related to the learning of the discipline. They further claimed that the negative attitude towards mathematics affects various aspects of the social context: the refusal of many students to enroll in scientific undergraduate courses due to the presence of exams in mathematics, a worrisome mathematical illiteracy, or an explicit and generalized refusal to apply rationality characterizing scientific thinking, or, vice versa, a tendency to uncritically accept models that are only apparently rational. Their results suggested that the attitude construct – as used by teachers – although rich enough to deal with the complexity of learning mathematics, does not seem to have the characteristics of a theoretical instrument capable of directing their work. They found from personal essays that the two dimensions - vision of mathematics and like/dislike - are mutually independent. They further noticed that this independence was strongly expressed in characterizing mathematics as useful/useless and easy/ difficult.

Hannula (2002, 2004) asserted on *everyday-notion-of-attitude* referring as some one’s basic liking and disliking of a familiar target. He discussed students’ attitude towards mathematics separating into four different evaluative processes: emotions the students experience during the mathematics-related activities, the emotions that students automatically associate with the concept mathematics, evaluations of situations that students expect to follow as a consequence of doing mathematics, and the value of mathematics related goals in the students’ global structure. Through an action research the researcher was successful to change attitudes, beliefs, and behaviors of a participating student. He also proposed a theoretical framework about emotions, associations, expectations, and values to study attitude towards mathematics. The most important conclusion from this study was that the proposed framework of emotions, associations, expectations, and values was useful in describing attitudes and their changes in detail. He further concluded that attitudes sometimes could change dramatically, in a relatively short time and the negative attitude towards mathematics could be a successful defense strategy of a positive self-concept.

Norton and Rennie (1998) reported a study concerning the continuing debate regarding the differences between boys and girls in their attitudes towards mathematics and their participation and performance in this subject at school. They asserted that it was difficult to attribute a lack of participation by girls compared to boys to suggest inferior performance. To them, the nature and extent of gender differences in mathematical performance remains a controversial topic. In this study, they measured attitudes using five affective variables defined by Fennema and Sherman (1976): mathematics anxiety, confidence in learning mathematics, effectance motivation, attitude towards success in mathematics, and mathematics as a male domain. Mathematics Attitude Scales (MAS) developed by Fennema and Sherman were adopted to measure those affective variables. They concluded that it was much more likely that the findings reflected some middle position.

Swetz et al. (1983) claimed that assessing mathematics performance and potential of students' attitudes toward mathematics and mathematics learning were frequently cited as factors contributing to success. They further referred to several studies (e.g. Aiken, 1976; Carey, 1958; Goolsby, 1988; Fennema, 1976; Head, 1981; Hungerman, 1967; Neale, 1969; & Poffenberger, 1959) showing that positive attitudes were conducive to good performance. However, an individual's attitude towards mathematics can be influenced by many factors. They identified two such factors as the focus of investigation: the individual's sex and socioeconomic standing. They also generalized that females exhibit less positive attitudes toward mathematics than males do.

Ifamuyiwa and Akinsola (2008) investigated the effects of self and cooperative instructional strategies on senior secondary school students' attitude towards mathematics. The results of their study revealed that self and cooperative instructional strategies through use of learning packages were effective for learning mathematics. The self-instructional strategy through the use of self-instructional package was found to be more effective in improving students' attitude towards mathematics than cooperative-instructional strategy. Olatunde (2009) reported conflicting results concerning the relationship between students' attitudes and academic achievement.

Image, Anxiety and Attitude

Images, anxieties, and attitude play a significant role in learning mathematics. These attributes are related to personal psychology, philosophy and epistemology. There are several researches and published papers on one or two of these attributes and rarely do they discuss explicitly about relationship among the three attributes at a time. The complexity of these attributes in relation to mathematics should be discussed with various possibilities and their consequences in teaching and learning mathematics.

Wigfield and Meece (1998) assessed relations between math anxiety and other key math attitudes, beliefs, values, and math performance measured in the larger study as one way of assessing the distinctiveness of math anxiety as a construct. Several researchers (e.g., Fennema, 1977; Fennema & Shermon, 1977; Fox, 1977; Richardson & Suinn, 1972; Richardson and Woolfolk, 1980; Suinn, Edie, Nicoletti, & Spinelli, 1972; Tobias & Weissbrod, 1980) reported negative correlation between math anxiety and low performance in mathematics, and then poor images associated with negative attitude towards mathematics. Although research studies have been undertaken to examine the affective domain, it has become important to describe a person's attitude towards mathematics using precise but connected terminology, e.g. beliefs, emotions, confidence, anxiety, self-concept or image (McLeod, 1992).

Norwood (1994) emphasized that math anxiety did not appear to have a single cause, but was, in fact, the result of many different factors such as truancy, poor self image, poor coping skills, teacher attitude, and emphasis on learning maths through drill without understanding. However, Greenwood (1984) further stated that the principal cause of mathematics anxiety has been in teaching methodologies. A review of current research suggests that low achievers in mathematics frequently accompany to the incidence of mathematics anxiety (Zakaria & Nordin, 2008). In order to reduce mathematics anxiety and increase achievement, Miller and Mitchell (1994) suggested that teacher should create a positive learning environment, free from tension and possible causes of embarrassment or humiliation.

There are some commonly held beliefs about mathematics which are still true today as they are associated with maths-anxiety and mathematics avoidance (Kogelman & Warren, 1978). Sam (1999) reported that these beliefs are: (a) inherited mathematical ability or some people have a mathematical mind and some don't., (b) you must always know how you got the answer., (c) there is one best way to do a mathematics problem., (d) mathematics requires a good memory., (e) men are better at mathematics than women., (f) it is always important to get the answer exactly right., (g) mathematicians solve problems quickly in their heads., and (h) it is bad to count on your fingers.

Based upon above discussions on image, anxiety and attitude towards mathematics, a model leading to success or failure of mathematics teaching and learning with regard to student achievement, and motivation can be suggested. Images of mathematics as infallible or fallible, mathematics anxiety as high or low self-esteem, and attitude towards mathematics as positive or negative can be modeled into a triangular relation leading to a perception about mathematics. This theoretical model can be represented in diagram as in figure 1. This hypothetical model can be considered as a basis to relate image, anxiety and attitude together with several possibilities of combinations.

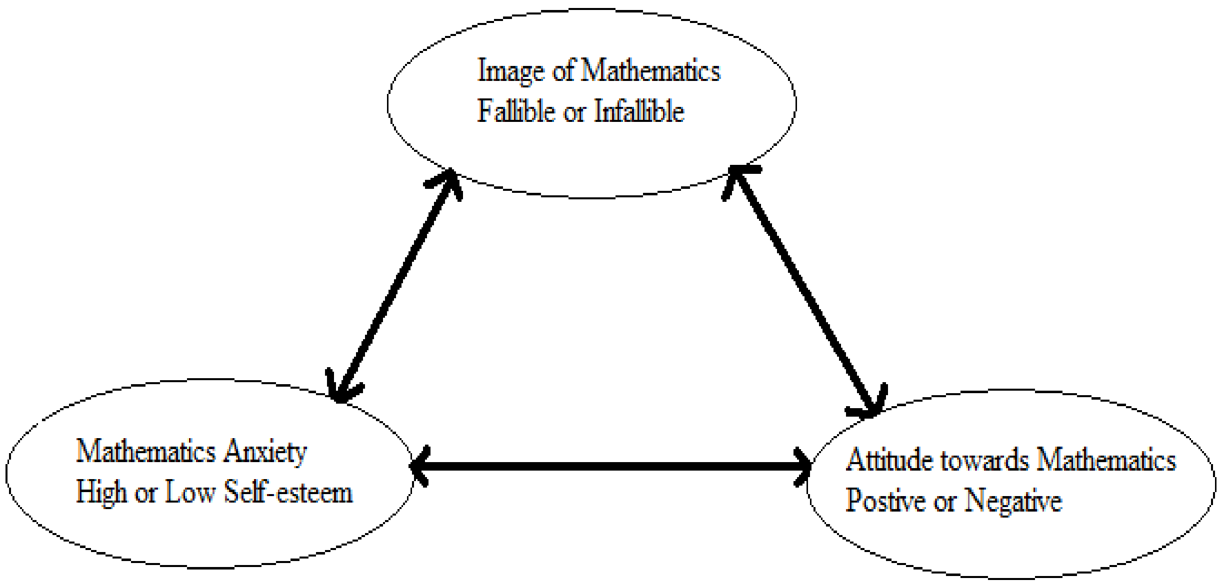

Figure 1: Model of triangular relation of image, anxiety and attitude towards mathematics

There are eight possible outcomes from the model representing different perceptions about mathematics: (1) infallible, high self-esteem, positive attitude; (2) infallible, high self-esteem, negative attitude; (3) infallible, low self-esteem, positive attitude; (4) infallible, low self-esteem, negative attitude; (5) fallible, high self-esteem, positive attitude; (6) fallible, high self-esteem, negative attitude; (7) fallible, low self-esteem, positive attitude; and (8) fallible, low self-esteem, negative attitude. Among these combinations, the combinations (1), (4), (5) and (8) are practically viable situations. Rests of the combinations are theoretically viable but they seem to be non-practical because high self-esteem and negative attitude, and low self-esteem and positive attitude towards mathematics seem contradicting. The contradiction in high self-esteem and negative attitude, and low self esteem and positive attitude is obvious as they represent opposite characters about perception towards mathematics. Among the four possibilities the first one is combination of infallible (image), high self-esteem (low degree of anxiety), and positive attitude. This combination is possible to develop a perception towards mathematics as absolute, infallible and incorrigible; however, the student has high self-esteem and positive attitude towards mathematics. The view of mathematics as absolute and infallible leads the student to develop a positivistic philosophy that can lead to development of his or her personality as an *Absolutist*. The student with this kind of personality enjoys routine problem solving, follows a rigid procedure to solve problems, and values high scores in test.

The fourth combination of infallible, low self-esteem and negative attitude is a problematic situation. Teacher centered teaching and learning that has fewer activities in the class for students, less emphasis to group or peer work, less questioning by students, and authoritative instruction may result into low self esteem and negative attitude towards mathematics. Teaching and learning mathematics guided by drill, practice and copy from board instead of construction of ideas by students may possibly leads to this situation impacting severely in students' achievement.

The fifth combination of fallible, high self-esteem and positive attitude leads to develop a perception that mathematical objects are socially constructed, fallible, and questionable, and student has a high self esteem towards mathematics leading to positive attitude. This combination develops personality of students to question mathematical objects, maintain high self-esteem about learning mathematics, and think positively about his or her ability to learn mathematics. These students value the process of learning mathematics and they try to understand the nature of mathematics from examples and practices. They enjoy non-routine type unstructured problem solving.

The eighth combination of fallible, low self-esteem and negative attitude leads to develop a perception that mathematical objects are socially constructed, fallible, and questionable; however, the student has low self-esteem due to some internal and external problems to cope with the situation in the classroom that ultimately leads to development of negative attitude. The teacher can help such students to develop high self-esteem by changing the pace of learning and helping him or her to learn from context with unstructured problem solving. The triangular relation of image, anxiety and attitude towards mathematics leads us to some pedagogical implications that have been discussed in the next section.

Implication of Model in Teaching and Learning

Choice of instructional methods and resources and their appropriate use in classroom teaching and learning mathematics largely depends upon images of mathematics as perceived by teacher and students. Images based on an absolutist view of mathematics as neutral and value-free regarding mathematics teaching as necessitating the adoption of humanistic, connected values have raised the issue of the relationship between epistemology and philosophy of mathematics, values and teaching (Ernest, 1995). Empirical research (e.g. Cooney, 1988) has confirmed claims that teachers' views, beliefs, and preferences about mathematics do influence their instructional practice (Thompson, 1984). “Thus it may be argued that any philosophy of mathematics (including personal philosophies) has many educational and pedagogical consequences when embodied in teachers' beliefs, curriculum developments, or examination systems” (Ernest, 1995, p. 457). The images of mathematics from the philosophical perspectives value inductive and deductive reasoning as a way to learn and teach mathematics. Those images of mathematics have possible negative and positive impacts in teaching and learning of mathematics with development of attitude toward mathematics as positive or negative (e.g. Goolsby, 1988; Ma & Kishor, 1997; Lakoff & Nunez, 2000).

Ernest (1995) developed a model for relationship between personal philosophies of mathematics, values, and classroom images of mathematics. This model compares the impacts of absolutist and fallibilist philosophies of mathematics that generate authoritarian and humanistic views of school mathematics, respectively. When the constraints and opportunities afforded by social context intervene in authoritarian and humanistic views of mathematics, they give rise to separated or connected images of mathematics realized in classroom.

Based upon the above discussion, we can say that effective teaching depends on one's image of mathematics based on personal epistemology and philosophy. It is up to a teacher to select a method of instruction in classroom to engage students in learning mathematics. If the teacher views that school mathematics is merely a collection of formulas, rules, and procedures that must be memorized and mastered, then he may apply traditional teaching techniques like drilling, individual worksheet practice, and flashcards could be considered effective. If the teacher believes that mathematics is an integrated whole, a study of structures and the relationships between things, and a way to study and understand the world around us, then the goal of teaching mathematics changes. Now the teacher helps students develop the skills they will use every day to solve mathematical and non-mathematical problems, which include the ability to reason, to explain and justify ideas, to use resources to find needed information, to work with other people on a problem, and to generalize to different situations, as well as the traditional ability to carry out computations and procedures (Zemelman et al., 1998).

Bishop et al. (1993) identified four groups of influences which appear to be crucial for learners of mathematics: influence from society, the socio-cultural context of mathematical thinking, influence of teaching materials, and role of teacher in children's learning of mathematics. Teacher's personal beliefs and attitudes play a vital role in dealing with these four groups of influences in order to teach mathematics in a way that shapes images of mathematics to the children's mind and their attitudes towards mathematics. Bishop et al. further claimed that there are the demands, constraints and influences from the society in which the mathematics learning is taking place. These demands, constraints and influences set the knowledge and emotional context within which the meaning and importance of teaching and learning mathematics are established through social and personal images that can develop positive attitude towards mathematics.

The second set of influences concern the knowledge, skills and understanding which the learners develop outside the school setting and which have significance for their learning inside the school. One of the great educational challenges of the present time concerns how school mathematics teaching should take learners' out-of-school knowledge into account. The third set of influences on children's learning of mathematics come from the teaching materials and aids to learning in the classroom. These have become more subtle and varied - from the textbook to the computer and have increased in number and importance considerably over the last decades. In the face of such development, the need for continuing research and analysis concerning the significance of these influences has become even more important.

The fourth and final influence is not a new one, indeed, it could be thought of as the oldest influence on mathematics learning, the teacher. Every learner can quote the memory of a particularly influential teacher, whether good or bad, and every teacher knows the feeling of influence which the position gives them. There has, in the last decade, been a growth of interest in research into the influence which teachers can and do have on the mathematics learners in their charge.

Several researchers (e.g., Aiken, 1970 & 1976; Eleftherios et al., 2007; Fonseca, 2007; Hannula, 2002; Hungerman, 1967) have shown that student's attitude towards mathematics is associated with his or achievement in the subject. In a study report, Eleftherios et al. (2007) reported that there was a significant statistical difference between female and male students concerning factors mathematical understanding through procedures and studying mathematics with understanding. They also found that female students have a stronger belief that mathematical understanding is achieved through procedures than male students do. It also emerged from their study that females study mathematics more carefully than males do. The result from the multivariate analysis showed that there was no significant statistical difference for all the factors concerning the social status of the students. Swetz et al. (1983) reported in another study that males were slightly more positive in their attitudes toward mathematics than were females, as were urban dwellers in comparison with rural dwellers.

Buxton (1981) and Ma (1999) came to a conclusion that a meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics can be used to derive a psychological framework in which the relationship between mathematics anxiety and mathematics achievement can be understood as a psychological function of emotional reaction. Buxton (1981) mentioned that panic, fear, anxiety, and embarrassment have been identified as the results of emotional reaction to mathematical tasks. Emotional reaction to academic situations involving mathematics seems more likely to trigger mathematics anxiety that relates to mathematics performance than do both beliefs and attitudes that are considered "cold" and "cool" (McLeod, 1992).

In summary, different studies (as discussed above) indicated a positive relationship among images, anxieties and attitudes towards mathematics, and these emotional factors had negative relationship with student's achievement. However, there is a lack of research that examines students' achievement in relation to different combinations of images, anxieties, and attitudes. The theoretical model presented in this paper with varying combinations of fallible or infallible images, high or low self esteem, and positive or negative attitudes can have significant pedagogical implications. Teacher's awareness to these combinations can help him or her to balance between the different approaches of teaching and learning mathematics as per need and context in the classroom. Such a balance of teaching and learning approaches, followed by a constructivist approach in combination with instructionist approach, can be helpful to teach mathematics lessons in a meaningful way where learning takes place. By engaging students in routine and non-routine problem solving includes structured and unstructured problem solving methods, empowering students' perception towards mathematics leading to success. Research based evidences should be developed in order to foster the positive learning environment combining images, anxieties and attitudes appropriately.

References

Adams, J. W., & Hitch, G. J. (1998). Children's mental arithmetic and working memory. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 153–173). East Sussex, UK: Psychology Press Ltd.

Aiken, L.R. (1970). Attitude towards mathematics. *Review of Educational Research,* 40 (4), 551-596.

Aiken, L.R. (1976). Update on attitude and other effective variables in learning mathematics. *Review of Educational Research*, 46, 293-311.

Ashcraft, M. H. (1995). Cognitive psychology and simple arithmetic: A review and summary of new directions. *Mathematical Cognition*, 1, 3–34.

Avigad, J. (2007). Philosophy of mathematics. In C.V. Baundas (Ed.), *Twentieth century philosophies* (pp.234-251). Edinburgh: Edinburgh University Press.

Bishop, A. J., Hart, K., Lerman, S., & Nunes, T. (1993). Scientific influences on children's learning of mathematics. *Science and technology education document series*, 47. Paris: UNESCO.

Browne, M.E. (2009). Learner's mental images of the mathematical symbol 'x'. *Philosophy of Mathematics Education Journal*, 24, 1-18.

Bueno, O., & Zalta, E.N. (2005). A nominalist's dilemma and its solution. *Philosophia Mathematica*,13 (3), 294-307.

Buerk, D. (1982). An experience with some able women who avoid mathematics. *For the Learning of Mathematics,* 3(2), 19-24.

Buxton, L. (1981). *Do you panic about maths?* London: Heinemann.

Carey, G. (1958). Sex differences in problem-solving performances as a function of attitude differences. *Journal of Abnormal and Social Psychology*, 56, 256-260.

Cemen, P. B. (1987). *The nature of mathematics anxiety*. ERIC Document No. ED 287 729. http://www.eric.ed.gov/ERICWebPortal/detail?accno=ED287729

Cockcroft, W. H. (1982). *Mathematics counts*. London: Her Majesty's Stationery Office.

Confrey, J., & Kazak, S. (2006). A thirty year reflection on constructivism in mathematics education in PME. In A. Gutierrez, & P. Boero (Eds.), *Handbook of research on the* *psychology of mathematics education: past, present and future* (pp.305-345). Rotterdam: Sense Publishers.

Cooney, T. J. (1988). The issue of reform. *Mathematics Teacher,* 80, 352-363.

Davis, P. J. (1972). Fidelity in mathematical discourse: Is one and one really two? *American Mathematical Monthly*, 79(3), 252-263.

Deighan, W. P. (1971). An examination of the relationship between teachers' attitudes toward arithmetic and the attitudes of their students toward arithmetic (Doctoral dissertation, Case Western Reserve University, 1970). *Dissertation Abstracts International*, *31*, 3333A.

Eleftherios, K., & Theodosius, Z. (2007). Students' beliefs and attitude about studying and learning mathematics. In J.H. Woo, H.C. Lew, K.S. Park, & D.Y. Seo (Eds.), *Proceedings of the 31st Conference on the International Group for the Psychology of Mathematics Education*, 3, 97-104. Seoul: Psychology of Mathematics Education.

Ernest, P. (1991). *The philosophy of mathematics education.* London: Falmer Press.

Ernest, P. (1995). Values, gender and images of mathematics: A philosophical perspective. *International Journal of Mathematics Education in Science and Technology*, 26(3), 449-462.

Ernest, P. (1996). Popularization: myths, mass media and modernism. In A.J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.). *International Handbook of Mathematics Education*. Dordrecht: Kluwer.

Ernest, P. (2008). Epistemology plus values equals classroom image of mathematics. *Philosophy of Mathematics Education Journal*, 23, 1-12.

Ernest, P. (Ed.) (1994). *Mathematics, education and philosophy: An international perspective.* London: Falmer Press.

Feierabend, R.L. (1960). Review of research on psychological problems in mathematics education. *Cooperative Research Monograph*, 3, 3-46. U.S. Office of Education.

Fennema, E. (1977). *Influence of selected cognitive, affective, and educational variables on sex-related differences in mathematics, learning, and studying.* (National Institute of Education Papers in Education and Work, No. 8, U.S. Department of Health, Education, and Welfare). Washington, DC: U.S. Government Printing Office.

Fennema, E., & Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization, and affective factors. *American Educational Research Journal, 14, 51-71.*

Ferry, T.R., Fouad, N.A., & Smith, P.L. (2000). The role of family context in a social cognitive model for career-related choice behaviour: A math and science perspective. *Journal for Vocational Behaviour,* 57, 348-364.

Fonseca, J.R.S. (2007). Can we reduce students' negative attitude towards math? *Proceedings of the 2007 Informing Science and IT Education Joint Conference*. Retrieved on April 6, 2010 from http://proceedings.informingscience.org/InSITE2007/InSITE07p057-064Fons315.pdf

Fox, L. H. (1977). *The effects of sex role socialization on mathematics participation and achievement* (National Institute of Education Papers in Education and Work, No. 8, U.S. Department of Health, Education, and Welfare). Washington, DC: U.S. Government Printing Office.

Game, A., & Metcalfe, A. (1996). *Passionate sociology*. London: SAGE Publications.

Goolsby, C.B. (1988). Factors affecting mathematics achievement in high-risk college students. *Research and Teaching in Developmental Education,* 4 (2), 18-27.

Greenwood, J. (1984). My anxieties about math anxiety. *Mathematics Teacher, 77*, 662-663.

Hannula, M.S. (2002). Attitude towards mathematics: emotions, expectations, and values. *Educational Studies in Mathematics*, 49, 25-46.

Hannula, M.S. (2004). *Affect in mathematical thinking and learning*. Doctoral dissertation, University of Turku, Finland.

Hart, L. E. (1989). Describing the affective domain: Saying what we mean. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp.37-45). New York: Springer.

Head, J. (1981). Personality and the learning mathematics. *Educational Studies in Mathematics*, 12, 339-350.

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education,* 21, 33–46.

Howson, A. G., & Wilson, B. (Eds.). (1986). *School Mathematics in the 1990.* Cambridge: Cambridge University Press.

Hungerman, A.D. (1967). Achievement and attitude of sixth-grade pupils in conventional and contemporary mathematics. *Arithmetic Teacher*, 14, 30-39.

Iemhoff, R. (2008). Intuitionism in the philosophy of mathematics. *Standford encyclopedia of philosophy*. CA: Standford University. Online Encyclopedia of Philosophy retrieved on March 2, 2010 http://plato.stanford.edu/entries/intuitionism/

Ifamuyiwa, S.A., & Akinsola, M.K. (2008). Improving senior secondary school students' attitude towards mathematics through self and cooperative-instructional strategies. *International Journal of Mathematics Education in Science and Technology*, 39(5), 569-585.

Kitcher, P., & Aspray, W. (1988). An opinionated introduction. In W. Aspray, & P. Kitcher (Eds.), *History and philosophy of modern mathematics.* Minneapolis: University of Minnesota Press.

Kloosterman, P. (1991). Beliefs and achievement in seventh-grade mathematics. *Focus on Learning Problems in Mathematics*, *13*(3), 3–15.

Kogelman, S., & Warren, J. (1978). *Mind over math*. New York: McGraw-Hill.

Lakoff, G., & Nunez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic Books.

Lent, R., Brown, S., & Hackett, G. (1994). Towards a unifying social cognitive theory for career and academic interest, choice and performance. *Journal of Vocational Behaviour*, 45, 79-122.

Leon, M. R., & Revelle, W. (1985). The effects of anxiety on analogical reasoning: A test of three theoretical models. *Journal of Personality and Social Psychology*, 49, 1302–1315.

Ma, X., & Kishor, N. (1997). Assessing the relationship between attitude towards mathematics and achievement in mathematics: A meta-analysis. *Journal for Research in Mathematics Education*, 28(1), 26-47.

MacLeod, C., & Donnellan, A. M. (1993). Individual differences in anxiety and the restriction of working memory capacity. *Personality and Individual Differences*, 15, 163–173.

Maddy, P. (1990). *Realism in mathematics*. New York: Oxford University Press.

Maxwell, J. (1989). Mathephobia. In P. Ernest, (Ed.). *Mathematics teaching: The state of the art*. London: Falmer Press.

McGinn, C. (2004). *Mindsight*. Cambridge, MA: Harvard University Press.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575-596). New York: Macmillan.

McLeod, D. B. (1994). Research on affect in mathematics learning in the journal for research in mathematics education: 1970 to the present. *Journal for Research in Mathematics Education*, 25, 637-647.

Miller, H., & Bichsel, J. (2004). Anxiety, working memory, gender, and math performance. *Personality and Individual Differences*, 37, 591-606.

Mura, R. (1995). Images of mathematics held by university teachers of mathematics education. *Educational Studies in Mathematics*, 28, 385-399.

NCTM. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, Virginia: National Council of Teachers of Mathematics.

Neale, D.C. (1969). The role of attitudes in learning mathematics. *Arithmetic teacher*, 16, 631-640.

Norton, S.J., & Rennie, L.J. (1998). Students' attitude towards mathematics in single sex and coeducational schools. *Mathematics Education Research Journal*, 10(1), 16-36.

Norwood, K.S. (1994). The effects of instructional approach on mathematics anxiety and achievement. *School Science and Mathematics, 94*, 248-254.

Olatunde, Y.P. (2009). Students' attitude towards mathematics and academic achievement in some selected secondary schools in Southwestern Nigeria. *European Journal of Scientific Research,* 36(3), 336-341.

Picker, S. H., & Berry, J. (2000). Investigating pupils' images of mathematicians. *Educational Studies in Mathematics*, 43(1), 65-94.

Poffenberger, T. (1959). Factors in the formation of attitudes towards mathematics. *Journal of Educational Research,* 52, 171-176.

Ponte, J.P., Matos, J.F., Guimaraes, H.M., Leal, L.C., Canavarro, A.P. (1992). Students' views and attitudes towards mathematics teaching and learning: A case study of a curriculum experience. In D. Kirshner (Ed.). *Proceedings of 16th Annual Conference for the Psychology of Mathematics Education,* 2, 218-225.

Rensaa, R.J. (2006). Images of mathematics. *Philosophy of mathematics education*, 19, ISSN 1465- 2978. Online retrieved from http://people.exeter.ac.uk/PErnest/pome19/index.htm.

Richardson, F. C, & Suinn, R. M. (1972). The Mathematics Anxiety Rating Scale: Psychometric data. *Journal of Counseling Psychology,* 79,551-554.

Richardson, F. C, &. Woolfolk, R. L. (1980). Mathematics anxiety. In I. G. Sarason (Ed.), *Test anxiety: Theory, research and application* (pp. 271-288). Hillsdale, NJ: Erlbaum.

Sam, L.C. (1999). *Public images of mathematics*. An unpublished doctoral dissertation, University of Exeter, U.K.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer.

Spielberger, C.D., Gorsuch, R.L., & Lushene, R.E. (1970). *Manual for the state-trait anxiety inventory*. Palo Alto, CA: Consulting Psychologists Press.

Sterenberg, G. (2008). Investigating teachers' images of mathematics. *Journal of Mathematics Teacher Education*, 11, 89-105.

Strawderman, V.W. (2010). Mathematics anxiety model. Online publication retrieved on April 20, 2010 from http://www.mathgoodies.com/articles/math_anxiety_model.html

Suinn, R. M., Edie, C. A., Nicoletti, J., & Spinelli, P. R. (1972). The MARS, a measure of mathematics anxiety: Psychometric data. *Journal of Clinical Psychology, 28,* 373-37.

Swetz, F.J., Langgulung, H., & Johar, A.R. (1983). Attitudes towards mathematics and school learning in Malaysia and Indonesia: Urban-rural and male-female dichotomy. *Comparative Education Review*, 27(3), 394-402.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics*, *12*, 151-169.

Thompson, A.G. (1984). Relationship between teacher's conception of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, 15, 105-127.

Tobias, S. (1978). *Overcoming math anxiety*. Boston: Houghton Mifflin.

Tobias, S., & Weissbrod, C. (1980). Anxiety and mathematics: An update. *Harvard Educational Review, 50,* 63-70.

Vachon, J. (1984, December). *Attitudes: Their relationship with intelligence and achievement and their evaluation.* Paper presented at the NATO Scientific Affairs Committee Advanced Study Institute, Athens, Greece.

Wigfield, A., & Meece, J. L. (1988). Math anxiety in elementary and secondary school students. *Journal of Educational Psychology*, 80, 210-216.

Wood, E. F. (1988). Math anxiety and elementary teachers: What does research tell us? *For the Learning of Mathematics*, 8(1), 8-13.

Zakaria, E., & Nordin, N.M. (2008). The effects of mathematics anxiety on matriculation students as related to motivation and achievement. *Eurasia Journal of Mathematics, Science & Technology Education,* 4(1), 27-30.

Zan, R., & Di Martino, P. (2007). Attitude towards mathematics: overcoming the positive/negative dichotomy. *The Montana Mathematics Enthusiast, ISSN – 3440, Monograph 3*, 157-168.

Zemelman, S., Daniels, H., & Hyde, A. (1998). *Best practice: New standards for teaching and learning in America's schools* (2nd ed.). Portsmouth, NH: Heinemann.