Focus on Basics

CONNECTING RESEARCH & PRACTICE

IN THIS ISSUE:

Numeracy

Using Part-Whole Thinking in Math

by Dorothea Steinke

The teachers at the West Side Learning Lab in Denver had been using a new approach in their math instruction for about six months when Victoria took her General Educational Development (GED) math test. She passed on the first try with a score near 600. That's well above the 430 minimum passing score on the 200-point to 800-point score range. "It's all about parts and whole," Victoria said about the test.

Since August, 2006, the West Side Learning Lab of the Community College of Denver, CO, has seen 92 percent (48 of 52) of its GED students pass the GED math test on the first try. These students attend class as part of a

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Focus on Basics is a publication of the US Division of World Education, Inc. It presents best practices, current research on adult learning and literacy, and how research is used by adult basic education teachers, counselors, program administrators, and policymakers. *Focus on Basics* is dedicated to connecting research with practice, to connecting teachers with research and researchers with the reality of the classroom, and by doing so, making adult basic education research more relevant to the field.

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Welcome!

Numeracy, more than prose and document literacy, is the skill most associated with employability, according to a national literacy survey as quoted by Myrna Manly, a numeracy expert based in California. Numeracy matters. It matters to individuals and to the nation. Manly artfully lays out the case for strengthening numeracy in adult basic education (ABE) in the article that begins on page 8.

Algebra matters: it's the gatekeeper into college. ABE students, regardless of their skill level, can and should start developing their algebraic thinking today. Tricia Donovan, Massachusetts, provides suggestions on how to do this in the article that starts on page 35.

Even the lowly equal sign matters. Many students think it signals only that "the answer is" rather than indicating the broader concept of equality. In our cover article, Dorothea Steinke, Colorado, describes how to change students' thinking about this sign, and about approaching math in general.

But what if your students are resistant to change? What if you're resistant to change? Kate Nonesuch, British Columbia, shares her techniques for introducing new types of activities into her math classroom (see page 20). She also gives us a peek at how she coaxes herself into trying these activities.

In the classroom, numeracy may seem limited to solving problems on a page, but using numeracy in real life requires the coming together of all sorts of cognitive and affective processes, explains Lynda Ginsburg, New Jersey (see page 14). What are these processes and how do we enable ABE students to develop them? Lynda examines many current math-teaching practices and suggests ways to alter them to ensure that students have more well-rounded skills in numeracy.

Teaching numeracy involves starting where students are, and moving on from there. If your students are mathematically in a place you've never been, you are faced with a challenge. Joanne Kantner, Illinois, reminds us that math is not exactly universal: different cultures think differently about mathematical concepts; they use different symbols and notation; and the contextual knowledge they bring to the classroom varies widely. Joanne talks with *Focus on Basics* about how she works with students from various cultures (see page 26).

To learn how to teach new approaches to numeracy involves first learning the approaches yourself, and also learning how to teach them. Researchers Mary Jane Schmitt, Massachusetts, and Beth Bingman, Tennessee, have been leading a research and development process called Teachers Investigating Adult Numeracy (TIAN), testing an intensive approach to professional development in six states. They share their model and what they have learned from TIAN in the article that begins on page 39. Beverly Wilson and Roberto Morales, of the Department of Education in Arizona, write about the decision to bring TIAN to Arizona, and how it was implemented across the state (see page 44).

Yes, numeracy matters. To share your thoughts on the many aspects of numeracy explores in this issue, join the Adult Numeracy Network discussion list; see page 43 for information on how to subscribe. Or, send us an email at FOB@worlded.org. We would love to hear from you.

Sincerely,

Barbara Garner

Barbara Garner
Editor

Part-Whole Thinking

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requirement to receive Temporary Assistance for Needy Families (TANF) support. They are mainly Hispanic, under age 40, and female. Previously, the overall success rate for the Learning Lab's students on the GED math test was around 50 percent after multiple attempts at the test.

The teaching team attributes the improved student success rate to teaching math from a problem-solving approach based on the part-whole concept. It is an approach that I developed over a number of years and shared with the team.

I use the same part-whole approach myself in tutoring math at a halfway house for offenders. After I introduce the learners to this novel (to them) way of looking at math, I hear comments like this: "Why didn't they teach me that in kindergarten? I wouldn't have had a problem [with math] all through my schooling."

The Part-Whole Concept

The part-whole concept is the foundation for number sense. Number sense is the ability to deconstruct quantities, keep track of the parts, put the parts back together in a different way to solve the problem, and know that the answer makes sense. The part-whole concept is a way of thinking about number relationships. Figure 1 shows how those who lack the concept of part-whole coexistence think of numbers compared to those who have the concept.

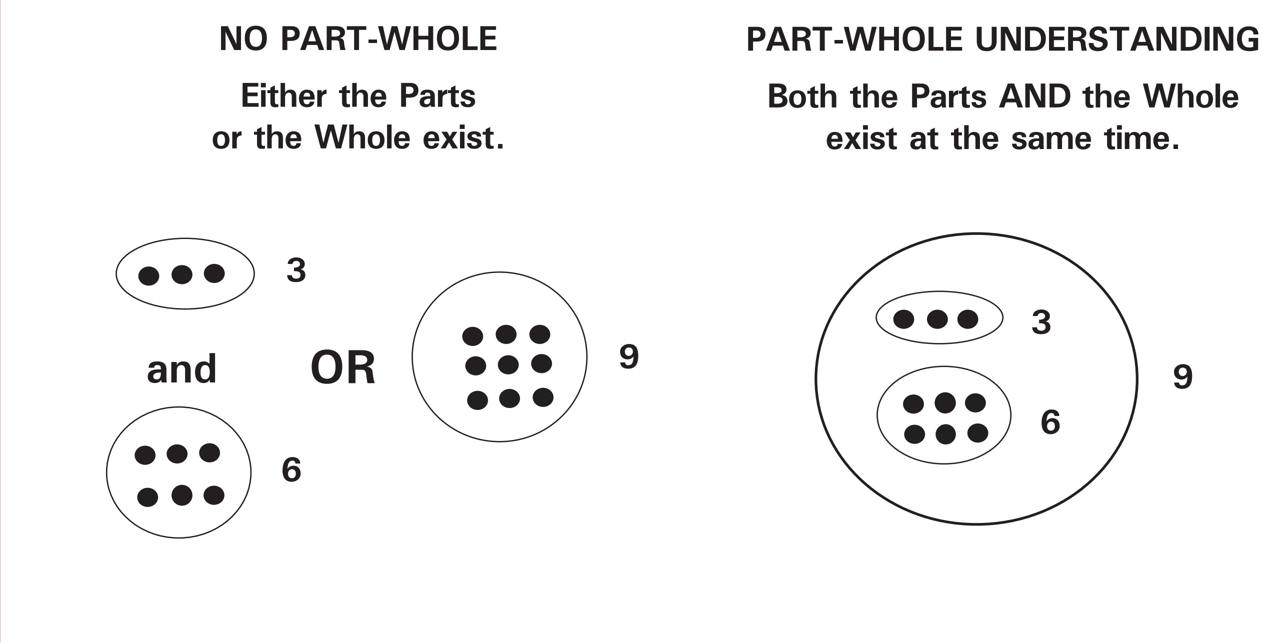
When I think of parts and whole in either/or terms, I have trouble with "mental math." If I try to subtract eight from 72 in my head, I repeatedly guess what the number might be: 61? 65? I have to add eight to every number I guess until I find the correct number. I use this same "guess and check" strategy with word problems. It takes me a long time to find the right answer.

If I have a sense of part-whole coexistence, I do the same problem (72 - 8 = ?) one of several ways: I may mentally break 72 into 60 and 12 and keep track of all the parts. I think "12 minus 8 is 4", and add the four to the remaining 60 for my answer. I may think, "8 and 2 is 10. 72 minus 10 is 62. Add back the extra two I took off. I have 64". Again, I keep track of all the parts. Whichever way I do the problem, I know the answer is right and I can tell you why.

Why the Concern about Part-Whole?

When I first began looking at how people understand math, I used the model for children's part-whole understanding developed by Steffe and Cobb (Steffe & Cobb with von Glasersfeld, 1988). I tested the concept with a group of volunteer community college students. To my surprise, four of my original 11 adult volunteers

Figure 1



(36 percent) showed the same behaviors as the children who lacked part-whole understanding in Steffe and Cobb's study. So did eight of the 12 students (67 percent) in a community college GED class (Steinke, 1999). The more I looked, the more evidence I found that many adults lacked part-whole thinking.

The 1992 National Adult Literacy Survey (Kirsch et al., 1993) tested the reading and math skills of 13,600 individuals who had been randomly selected to represent the adult population in the United States. Of those tested, 22 percent missed a single-step subtraction word problem. Such problems require a grasp of part-whole coexistence. Only nine percent missed a similar-difficulty level addition word problem. Addition word problems can be understood with an either/or sense of parts and whole because addition puts parts together to form the whole.

In an analysis I did of more than 2,900 Test of Adult Basic Education (TABE) placement tests given over a period of five years at a community college (Steinke, 2000), more than a quarter of entering students missed problems of the form of those in Figure 2. Such problems are difficult for those who lack part-whole coexistence; they make sense to those who have part-whole coexistence.

Over a period of four months this past autumn, I tested 116 new clients (90 percent of the total) entering the halfway house where I currently tutor. Of those, 55 clients (47 percent) lacked a sense of part-whole coexistence. That breaks down to 86 percent (18 of 21) of those without a GED or high school diploma; 43 percent (25 of 58) of those with a GED; and 32 percent (12 of 37) of those with a high school diploma.

I categorized both this recent group and those in the 1999 study for part-whole understanding using the type of questions and observable behaviors documented by Steffe and Cobb with children (Steffe & Cobb with von Glasersfeld, 1988). When behaviors did not clearly indicate whether a person had or lacked the part-whole concept, I counted that person in the with part-whole concept group. That means the percentage of people I found lacking part-whole concept may be too low rather than too high.

Figure 2

Examples Similar to TABE Questions

Form A (Advanced) 4 ☐ =(4x8)+(4x6)

answer choices:
A) x (8 + 6)
B) x (8 x 6)
C) +(8 + 6)
D) +(8 x 6)

Form A
424 Students
38% missed

Form D (Difficult) 4 x 2 x 7 = ☐ x 7

answer choices:
A) 6
B) 8
C) 21
D) 42

Form D
1,066 Students
25% missed

Form M (Medium) 4 +2 = 2 x ☐

answer choices:
A) 3
B) 4
C) 8
D) 12

Form M
1,118 Students
26% missed

Part-Whole Current Materials

When I began tutoring GED math, I knew I needed to teach the part-whole concept, so I looked for materials that featured it. I found that the most commonly used GED math texts (Steck-Vaughn and Contemporary) assume adults grasp part-whole. I say this because the first lessons in those books offer only the most cursory statement about the value of a digit in each "place" and no explanation of the relationship of one "place value" column to another. Even a recent and student-friendly text (*EMPower Math*) waits until fractions are introduced to mention parts and whole.

The part-whole approach as I present it is a change from these texts. My materials overtly and consciously present part-whole thinking as a framework for problem-solving starting with the simplest number sentences and word problems. This puts students in a part-whole mindset from the outset. When fractions are introduced, they are an extension of what students already understand about parts and whole rather than an entirely new process. What follows is a brief look at how my materials introduce the concept.

Stumbling Block: the Equal Sign

As children, we may have thought of the symbol "=" as meaning "the answer is" (Falkner et al., 1999; Witherspoon, 1999) because the equal sign comes at the end of a one-step equation and usually has only one number after it. As adults, we may continue to lump the equal sign, a relationship symbol, with the action symbols for the four basic operations: + x - ÷ .

This misconception of "=" is reinforced by the use of calculators. As one young woman pointed out to me, when we use a calculator, "equal" involves a physical action. We push the "=" key, and the screen changes to provide the answer. That gives calculator users the sense that numbers "become" or "turn into" something else. This feeling goes along with the either/or sense of number relationships held by people who lack the part-whole concept. A major step in moving students toward understanding part-whole coexistence is changing their sense of "=" from one of action to one of relationship. An incorrect understanding of the equal sign interferes with understanding the part-whole relationship in simple number sentences and in thinking about equations in algebra (Lubinski & Otto, 2002).

I teach the equal sign by starting with people's names. Putting the familiar symbol "=" in a different context helps students think about it in new ways. Here's the basic strategy.

People use different names for me based on my connection to their lives. I am Dorothea to my friends, Grandma Dee to my grandchildren, Mrs. Steinke to business associates. First I write down my names (or a student's names), and then put equal signs between them like this:

Dorothea = Grandma Dee = Mrs. Steinke

The point I make to students is that I (or they) do not physically change when someone uses a different name for me (or them). The key phrase is "different name, same person." With this connection to their personal experience, students almost immediately see the equal sign as a relationship symbol rather than as an action or agent of change.

From this new understanding of "equal" we proceed to "different names" for the same number. "How many ways can you show eight using the fingers on three hands?" I ask them. After students realize that they can use a partner's hand with their own (and laugh a bit at not immediately seeing that option), they come up with a half dozen or more ways to show eight, allowing permutations to count for different ways. We draw two large circles. One circle has eight dots inside it, representing the whole. The other circle also has eight dots. These we make into three or four groups, representing the parts (Figure 3).

After the pictorial representation, we move on to the abstract. We put the equal sign between the different names for eight (written as number expressions) (Figure 4) and start the mantra, "different name, same thing." Yes, it is more mathematically accurate to say "same amount" or "same size" instead of "same thing." But "different name, same thing" feels more colloquially comfortable.

"Different name, same thing" applies to every math topic: equivalent fractions, fractions and their equivalent decimals and percents, numbers written in expanded form or in scientific notation. The presence of the equal sign in all these concepts connects to the equal sign between "my different names."

Figure 3

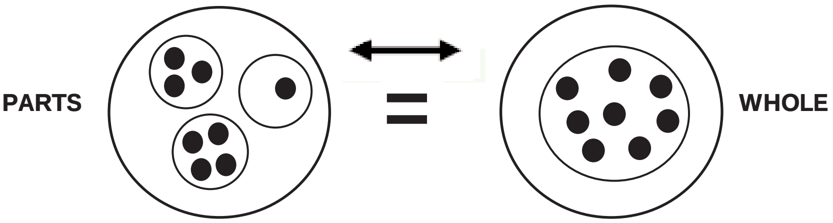


Figure 4

$$4 + 1 + 3 = 2 + 2 + 2 + 2 = 5 + 2 + 1 = 3 + 2 + 3 = 8$$

Understanding Number Relationships

Tying a new understanding to a personal, concrete experience (like "my different names") is the key to helping learners make new conceptual connections. Steffe and Cobb (1988) pointed out that young children learn math starting from personal experiences, then move on to concrete objects (perceptual stage), followed by representations of quantities (figurative stage), and eventually to dealing with numbers as abstractions (abstract stage). Adults go through the same perceptual then figurative then abstract sequence in developing new understandings in math. Because they have more life experiences to connect ideas to than children do, adults are likely to move through the sequence faster than children. Still, new concepts have meaning only when tied to the known and familiar. That is why I introduce the critical missing understanding in adults, part-whole coexistence, with that most personal experience, the human face.

I show the students a picture of a Halloween trick-or-treater dressed as a ghost. Only the eyes peek out from two holes in the sheet (Figure 5). "What can't you see on the face because of the costume?" I ask. When the students have given me a few responses (for example, nose, hair, mouth, ears), I continue: "How do you know those parts of the face are there?"

Figure 5

After a pause to find the right words, someone will say, "We know what a face looks like." "So you expect the parts to be there even when you can't see them?" I ask.

"Yes," they answer.

We continue through the perceptual/ figurative /abstract learning sequence using other common physical examples of parts and whole: "When I have a dozen eggs, do I have 12 at the same time?" (Some students say "no" at first.) Students name objects in the room (for example, a chair) and describe why all the parts have to be there all the time with the whole (the chair would fall down without legs).

Next, students set up simple number sentences from pictures where a part is missing (Figure 6). We use circles with an equal sign once again as an intermediate representation that the parts are a "different name" for the whole. Students see the number relationship from the relationship in the circles and are able to write the number sentence (or equation).

New Model for Word Problems

From pictures of soda six-packs and other everyday groups with a part missing, we move to simple word problems about money. Using everyday, real-life experiences appeals to learners' interests and allows them to apply their existing skills to math problems. We start with questions about making change. Calculating correct change is difficult for many adults. Making change is a straightforward "find the part" problem. Making change is then contrasted to "How much do I owe at the checkout counter?", which is a "find the whole" problem.

After they have done a few examples, I ask them: "What operation did you use to find the missing part?"

"Subtraction," they say.

I ask next, "What operation did you use to find the whole?"

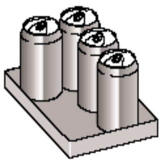
"Addition," they reply.

Then I deliver the punch line.

"And those are the only two kinds of math problems there are, from kindergarten to calculus. Either you know the parts and have to find the whole with addition. Or you know the whole and a part and have to find that hidden or missing part with subtraction. And that hidden part is always there, like the nose and mouth and ears that you couldn't see on the child in the ghost costume."

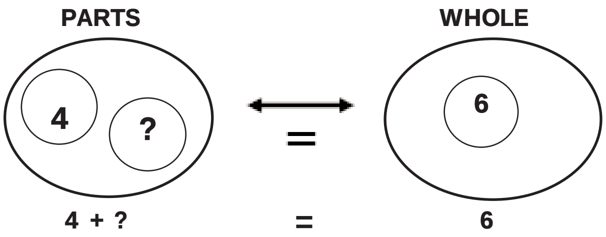
Figure 6

6-PACK OF SODA



You see a **PART** of the complete soda pack.

Put the number for the cans you see (a **PART**) and the number for the **WHOLE** number of cans in a complete park in the small circles.



Some students immediately say "Oh" in response. Others are silent as they begin to make the new mental connection. Generally someone protests, "But what about multiplication and division?"

I write out a multiplication problem as repeated addition. I demonstrate a division problem as repeated subtraction. I repeat: "There are only two kinds of math problems: "find the whole" or "find the part"."

Figure 7

Find the Whole	**Find the Part**
Part + Part = Whole	Whole - Part = Part
12 + 6 = ?	17 - 5 = ?

Even in multi-step problems (which are introduced later), each step is either "find the part" or "find the whole". From examples and problems, I lead students to uncover one additional fact they will need to set up word problems using the part/whole strategy:

"In addition and multiplication "find the whole", the whole is by itself on one side of the equal sign. In subtraction and division "find the part", the whole comes directly before the operation sign in the number sentence. That means the whole is usually the first number in the number sentence." (Figure 7)

Now students have a new way of looking at number relationships (Is that number a part or the whole?) and a practical, consistent model for writing that relationship in mathematical symbols. With the groundwork laid, we go on to label each number in word problems as a part or the whole. We write number sentences using addition or multiplication for "find the whole" problems and subtraction or division for "find the part problems.

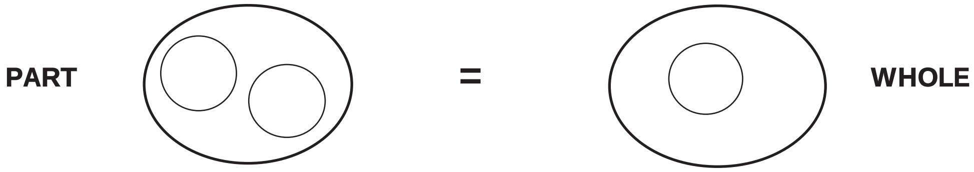
The proof of students' new understanding comes in later lessons with word problems like the one below (Figure 8). For the time being, we continue to use a question mark [?] to stand for the unknown.

Without a sense of part-whole coexistence, many students rely on vocabulary "keywords," such as "times," and write the problem as a multiplication: 102 x 6 = ? Students who use the part-whole model know to ask questions about the number relationships. They identify 102 as the whole. They identify six as a part. They write: 6 x ? = 102. They know that when they already have the whole, the problem is "find the part". Because they are focusing on the number relationships instead of the operation, they write the problem correctly: 102 ÷ 6 = ?

The part-whole model for problem solving offers students a different way of approaching word problems, one that is based on number relationships. In the classroom, I get immediate feedback from students that the part-whole model is working for them. I see the eyes go wide and hear the inadvertent "Ohhh" of understanding escape from lips.

Figure 8

Miguel has 102 baseball cards.
That is six times as many cards as Steve has.
How may cards does Steve have?



THINK: Who has more cards, Miguel or Steve?

What is 102, the **WHOLE** or a **PART**? _______________

Write that number inside the small circle where it belongs.

What is 6, the **WHOLE** or a **PART**? _______________

Write that number inside the small circle where it belongs.

What do you need to find, the **WHOLE** or a **PART**? _______________

What the number sentences: _______________

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When students grasp part-whole coexistence, they understand number relationships differently. They know why, not just how. They get math. They have number sense. For these students, math becomes just what Victoria said: "It's all about parts and whole."

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About the Author

Dorothea Steinke has tutored adults preparing for their GED test in math for several years. She holds a K-12 teaching credential in math and a Colorado Literacy Instruction Authorization for GED instruction. She is a frequent presenter at workshops and is a board member of the Literacy Coalition of Colorado. She is completing an adult-learner math book that is rooted in the principles outlined in this article. For information about the book, contact her through the Web site *www.numberworks4all.com* ❖

Numeracy Matters

by Myrna Manly

What is the situation in the United States with respect to numeracy skills? Information from two studies that have been implemented nationally, the Adult Literacy and Lifeskills Survey (ALL) and the Adult Education Program Study (AEPS), provide important insights (see the box on page 13 for more on these studies). Both studies included a numeracy domain that was represented by tasks that demand mathematical understanding to interpret or solve. Each task was embedded in a situation that would be more or less familiar to adults in their everyday lives as workers, citizens, or learners. The findings from the two studies show that the majority (58.6 percent) of the US population in general (ALL) and that most (91.6 percent) of the learners in US adult education programs (AEPS) are not likely to have the numeracy skills necessary to function successfully in society today (Statistics Canada, 2005, p. 50; Tamassia et al., 2007, p.16). The percentages indicate the proportion of each population whose skill level was below level 3, mid-way in the range of five levels of difficulty that were represented in the survey.

The table on page 9 presents a better idea of the meaning of numeracy as it was characterized by the tasks at each of the five levels used in the ALL and AEPS. Tasks at level 1 are the easiest, those at level 5 are the most difficult, and level 3 skills are considered to be the minimum for coping effectively with the demands of today's society.

In this article I describe how numeracy, as a distinct domain of adult basic education, is important for a number of reasons. Not only does numeracy have an impact on our nation's potential for economic growth, it also affects individuals' chances for economic success and daily well-being. And as the data just described reveal, we as a country are weak in this area and our students in adult education are especially at risk.

A Nation of Numerate Citizens Has Potential for Leadership and Growth

Evidence shows that, historically, an increase in the skill level of a nation's population precedes the economic growth of that country. Moreover, raising the lower levels of proficiency is even more significantly related to long-term economic progress than honing the skills of the most proficient (Coulombe et al., 2004). The International Adult Literacy Survey (IALS) data used for the report, drawn from 14 countries, suggests that a large proportion of the population with level 1 skills acts as an inhibitor or drag to the system, limiting the rate at which companies can adopt more productive technologies. While the IALS study uses skill levels from three forms of literacy—prose, document, and

Level	Description of Tasks
1	Simple tasks require understanding basic numerical ideas in concrete, familiar contexts where the mathematical content is explicit with little text. Tasks consist of simple, one-step operations such as counting, sorting dates, performing simple arithmetic operations or understanding common and simple percents such as 50 percent. *Example: Tell how many bottles of soda are pictured in a photo of two full cases of soda bottles stacked on top of each other.*
2	Fairly simple tasks relate to basic mathematical content that is quite explicitly embedded (few distractors) in familiar contexts. Tasks include one-step or two-step processes and estimations involving whole numbers, benchmark percents and fractions, interpreting simple graphical or spatial representations, and performing simple measurements. *Example: Tell how much gas is remaining in a 48-gallon tank by interpreting a gas gauge whose indicator is positioned halfway between the tic-marks for full and half-full.*
3	Tasks require understanding mathematical information represented in numbers, symbols, maps, graphs, texts, and drawings and involve undertaking a number of processes. They involve number and spatial sense, knowledge of mathematical patterns and relationships and the ability to interpret proportions, data and statistics embedded in texts where there may be distractors. *Example: Insert a mark that indicates the level to which liquid must rise for a container to be 1/3 full.*
4	Tasks require understanding more abstract mathematical information represented in texts of increasing complexity or in unfamiliar contexts. They involve undertaking multiple steps to find solutions to problems and require more complex reasoning and interpretation skills, including comprehending and working with proportions and formulas or offering explanations for answers. *Example: Determine which percent of change (shown in a bar graph describing dioxin levels in breast milk of North European women over a span of 20 yrs) is greater and explain your answer.*
5	Tasks require understanding complex representations and abstract and formal mathematical and statistical ideas, possibly embedded in complex texts. Respondents may have to integrate multiple types of mathematical information, draw inferences, or generate mathematical justification for answers. *Example: Test the accuracy (and justify your answer) of an advertising claim that one can double an investment of $1000 in seven years when a fixed annual interest rate of 10% is paid.*

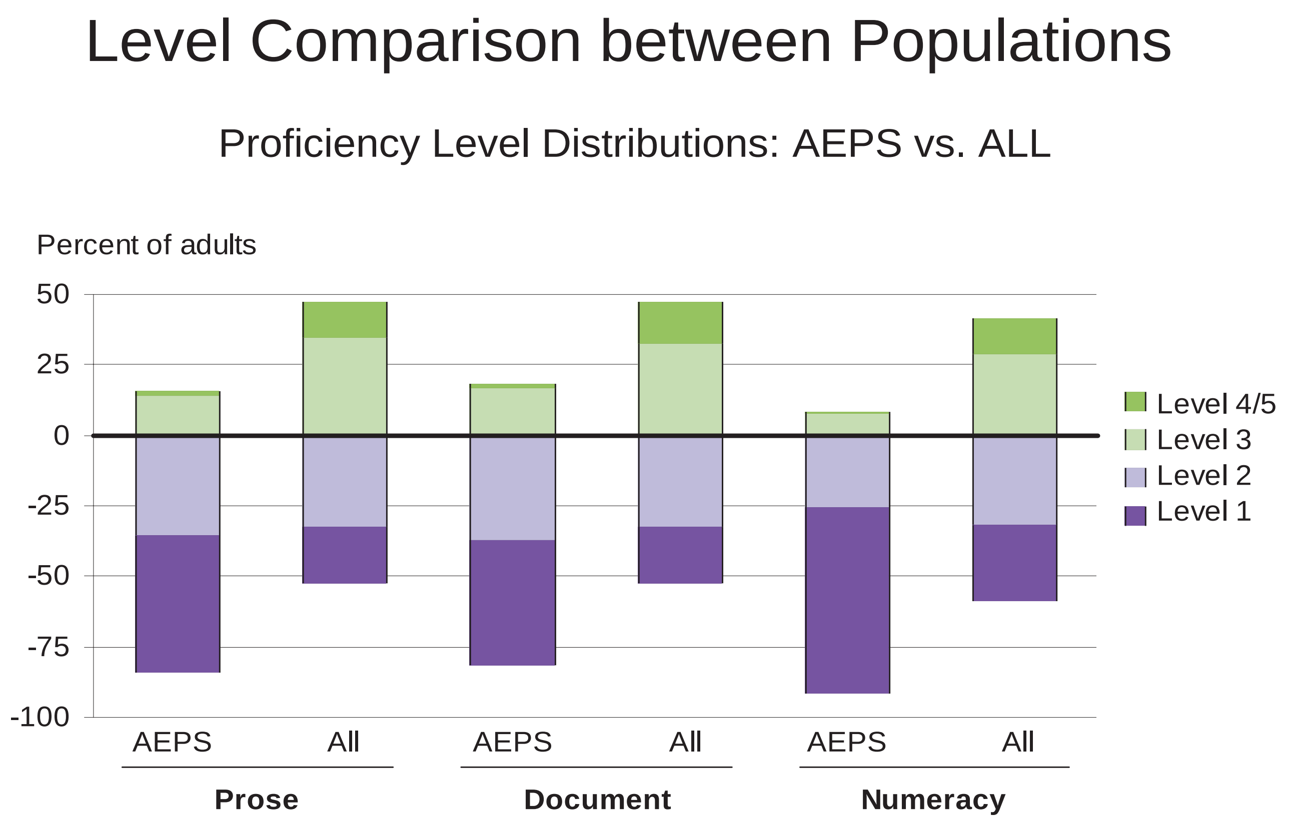
quantitative—we will focus on how this finding has implications for numeracy instruction in adult education by looking again (and more closely) at the results from the ALL and AEPS surveys. The graph below compares the percentages of the US population (ALL) that perform at each level of proficiency in each of the domains the (prose and document literacy and numeracy) to the percentages of the adult learner population (AEPS) at these levels. It is to be expected that the proficiency of adult learners would be lower than that of the population as a whole and the graph confirms that to be true.

Note that the percentages of adult learners (AEPS) that performed at level 3 or above (shown above the line in the graph) are low in prose literacy (15.6 percent) and in document literacy (18.2 percent), but even lower in numeracy (only 8.4 percent). Adult learners are less likely to have the required numeracy skills than to have the literacy skills that are needed today. Further, when we look at the percentage of learners who performed at the lowest level (level 1) in each of the domains, we see more evidence of a serious deficit. In numeracy, 66.4 percent of the adult learners performed at level 1 compared to 48.8 percent and 44.3 percent in prose and document literacy. When coupled with the information about the national economic importance of reducing the proportion of the population at skill level 1 from the Coulombe study mentioned above, these numbers should send a clear message both to national policy makers and adult educators: Numeracy matters.

Greater Employment Opportunities

I have always noticed that adult educators around the country show extraordinary motivation to help their students succeed. Thus it is likely that they are driven more by the difference that mathematics knowledge could make in an individual student's life than by what it could contribute to the competitiveness of the nation. Information from the ALL survey as well as the workplace provides corroboration that the impact of being competent in numeracy is an important asset for our students as individuals.

In the workplace, technological advances have changed the task demands, replacing human workers with computers and robotic machines for many routine tasks, and also requiring workers at all levels to have the knowledge previously expected only of managers. For example, quality control monitoring using visual displays of information has become an integral part of the job across the spectrum, from factory workers to



home health care workers. Factory line workers who have the authority to make a decision to stop the production line and home health workers who monitor the care of patients far from a hospital cannot rely merely on recalling a memorized procedure; they need to understand the nature of data collection, the mathematics involved in interpreting graphical trends, and the implications the trends have for the situation at hand before they make a judgment.

> “Evidence shows that, historically, an increase in the skill level of a nation's population precedes the economic growth of that country. Moreover, raising the lower levels of proficiency is even more significantly related to long-term economic progress than honing the skills of the most proficient.”

Career demands for greater conceptual understanding and problem solving ability have influenced many states to require all students to successfully complete at least one algebra course for graduation from high school. In turn, at least one algebra course has become a key ingredient for entrance into most of the high-growth careers which require post-secondary certificates or degrees of some kind. On a Web site sponsored by the U.S. Department of Labor (www.careerinfonet.org accessed December 30, 2007), a list of careers predicted to have the highest growth in the years from 2004 to 2014 shows that only 12 of the 50 careers can be entered without some postsecondary education. Moreover, with the exception of hazardous waste removal, these 12 careers are all at the lowest salary levels, paying less than a living wage for a family in most parts of the country. Further validating this point, a check of a separate list at the same site shows that all the careers predicted to have the least demand in that time period are ones for which on-the-job training is the only educational requirement.

In the ALL report, numeracy is the one skill revealed to be highly associated with employability. In the United States, individuals with numeracy skills below level 3 are 2.6 times more likely to be unemployed than those whose skills are above that level (p. 124). It is not surprising then to note that, even after adjusting for education, age, gender, and household income, the results also show that those with numeracy skills below level 3 are more than three times more likely to receive social assistance payments (p.178).

In the United States, where many adult basic education leaders tend to think of numeracy as a subset of literacy, it is significant to note that the findings of the ALL Survey show that the construct of numeracy acted independently from literacy. That is, proficiency in prose or document literacy was not a good predictor of proficiency in numeracy. The fact that literacy does not subsume numeracy is also apparent in two reports from Great Britain that examined the effects of poor literacy and numeracy in the labor market.

The two papers, "Does Numeracy Matter?" (1997) and "Does Numeracy Matter More?" (2005) used longitudinal data from the U.K. following aspects of the lives of two cohorts of individuals, one born in 1958 and another in 1970. In these studies, the level of literacy and numeracy skills that was considered was at the level of basic skills. They found, in general, that the negative impact on employment of a low level of skills has increased over the years as the availability of unskilled jobs has decreased. More particularly, they found that the effects of a poor level of numeracy (irrespective of literacy level) were more significant for women than for men. With regard to the labor market, women with low numeracy skills were more likely to be unemployed or to work in less satisfactory careers. The writers interpret this result as being associated with the fact that women with low skills have traditionally found employment in offices where there were many routine jobs available. However, with the growth of technology-based jobs in modern offices, a person without information technology (IT) skills (closely allied with numeracy skills) cannot function effectively. Additionally, women with poor numeracy skills were found to be more likely to have low self-esteem and poor physical health, and to report that they feel a lack of control over their lives. The writers of the study warn that when a group finds it difficult to function effectively, there is a risk of social exclusion, a threat to the cohesion of society at large (1997, p.15). A similar warning is voiced by Kirsch and colleagues in *America's Perfect Storm: Three Forces Changing our Nation's Future*, who note that because the wide disparity in literacy and numeracy skills in the United States follows lines "defined by race, ethnicity, nativity, and socioeconomic status", its economic impact serves to drive the country's populations apart (2007).

Employment in Day-to-Day Transactions

Computers have made it so easy to generate graphs and other visual forms of information that they appear in articles in every section of the newspaper these days, making articles seem incomplete without them. Each month gives rise to new airline on-time arrival reports, each week of the presidential campaign generates a new wave of polling information, and each game in the football season leads to new rankings based on the data. The numbers seem to lend credibility and importance to the information, even though the topic may be trivial or the data-gathering techniques may be suspect. A mathematical common sense, or numeracy, is critical to sift through the chaff to find the grains of valuable information that help to make important decisions.

> *"…at least one algebra course has become a key ingredient for entrance into most of the high-growth careers which require post-secondary certificates or degrees of some kind."*

> *"…numeracy is the one skill revealed to be highly associated with employability."*

This vast amount of information, both quantitative and substantive, was not available to lay people in the past. Nor were we challenged to be involved in as many decisions that are now required of us. Numeracy skills are demanded when we participate in health decisions (care plans, treatment options) and when we need to understand complex directions for determining certain drug dosages. Reasoning with proportions and percents helps to make sense of nutrition labels in order to maintain a healthy diet. Perhaps even more critical is the numeracy required to make sound financial decisions. Creating household budgets one can live with, deciding if borrowing from a payday loan company is the best option, and weighing the risks and rewards of making purchases with delayed payment schedules demand individual competence, especially in today's world of predatory lenders. Autonomous individual competence is also a necessity for the nation's health as a democracy. Participating in civic affairs involves being able to analyze issues such as budgets, tax rates, social security suggestions, and universal health care that are debated with quantitative arguments.

Summary and Suggestions

This article has shown that numeracy does matter; it noted both economic and social benefits for an individual with competent numeracy and long term economic benefits for a nation that makes consistent efforts towards improving the literacy and numeracy of those with low skills. It cited survey results that show that a large proportion of the US population needs improvement in numeracy skills before they can participate fully in today's changing society. Moreover, it revealed that for adult education, the deficit in numeracy skills may be more significant than in literacy skills.

What does the situation suggest for adult education? At the national level and program level, it is important that numeracy be recognized as a distinct domain, not subsumed under literacy, and be given the attention it deserves. Efforts that take positive steps toward that goal have already begun. This issue of *Focus on Basics*, devoted entirely to numeracy, will raise its profile in the adult education community. Another promising development is the series of OVAE-sponsored numeracy initiatives, one which surveyed the existing research and scanned the field for promising practices in professional development in the field of adult numeracy (AIR, 2006-7). Following that groundwork another initiative, which is in its initial stages, will convene an adult numeracy panel that is charged with building on the National Math Panel's report on K-8 mathematics education to craft a document of principles for adult numeracy instruction. In addition, the initiative provides for the development of a model for numeracy professional development to be recommended for use across the country. Recognizing that adult education teachers are often asked to teach outside their field of expertise, it is critical that their mathematics and mathematical pedagogy knowledge be upgraded if mathematics instruction to adults in the United States is to be improved.

To keep adult learners in the classroom long enough to make a meaningful difference in their lives, instruction in adult numeracy cannot be merely a rehash of K-12 mathematics instruction. Using contexts that are applicable to adults' needs, focusing on important mathematics content, and recognizing that reasoning and problem solving abilities are critical can convince learners themselves of the truth of our premise: Numeracy matters.

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About the Author

Myrna Manly, a long-time mathematics educator and founding member of the Adult Numeracy Network, was a member of the international team (along with Iddo Gal from Israel, Mieke van Groenestijn from the Netherlands, Mary Jane Schmitt from Massachusetts, and Dave Tout from Australia) that wrote the numeracy framework and constructed the tasks that would be included in both the ALL and AEP surveys. She is presently working as a consultant to the Adult Numeracy Instruction (ANI) project sponsored by the federal Office of Vocational and Adult Education (OVAE).❖

The Studies

Adult Literacy and Lifeskills: The ALL Study

The Adult Literacy and Lifeskills Survey (ALL) was implemented in seven economically developed countries or regions in 2003. It included a numeracy domain in addition to the prose and document literacy domains. For this survey, numeracy was defined to be the knowledge and skills required to effectively manage the mathematical demands of diverse situations. Along with test booklets that measured skill proficiency in each domain, the survey included a background questionnaire that recorded social, educational, and health factors that are thought to be instrumental in skill development.
www.statcan.ca/english/freepub/89-603-XIE/2005001/summary.htm

Adult Education Program Survey The AEPS

The Adult Education Program Survey (AEPS) involved two surveys: the Program survey, which collected information about the characteristics of adult education programs and the services they offered and the Learner Survey, which assessed the skills of learners in a sample of adult education programs and also collected information about their characteristics with respect to language background, educational background and experiences, labor force participation and other activities, and general demographic information such as gender and age. The learner survey was administered from March through June, 2003.
www.ets.org/Media/Research/pdf/ETSLITERACY_AEPS_Report.pdf.❖

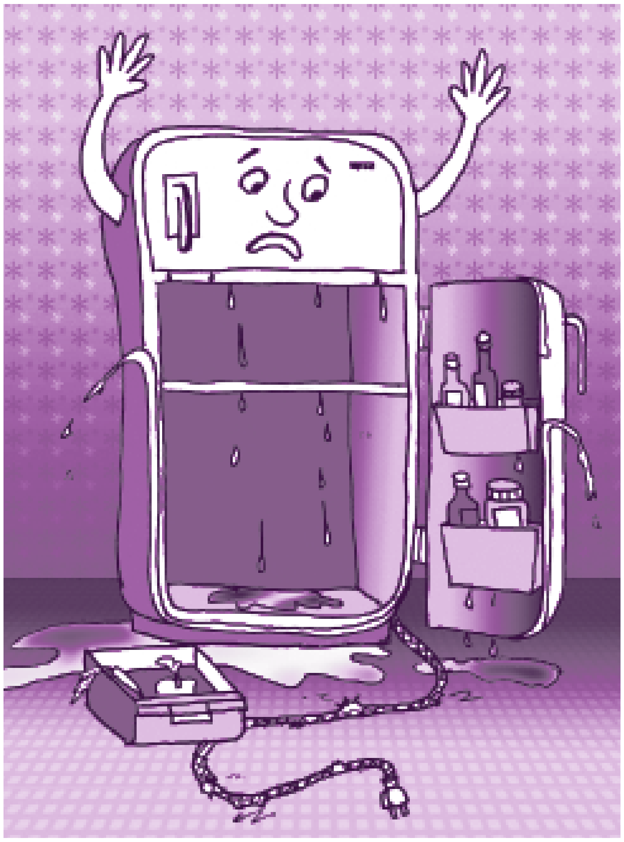
Designing Instruction with the Components of Numeracy in Mind

by Lynda Ginsburg

I need to replace my refrigerator. Among other things, I will have to determine the exact size of the space available for the refrigerator because I have a pretty small kitchen, I will have to comparison shop for the appropriate combination of features and price, and I will have to decide among alternative methods of paying for the refrigerator. To accomplish the task of replacing my refrigerator, I will need:

1. *a conceptual understanding of measurement (including fractions), percent (for discounts, interest), and volume (to compare features of different refrigerators),*
2. *to reason about particular situation, identifying the demands of the task (determine fit, pay the least for the most value, minimize interest payments), and then determining which strategies will be useful (spatial reasoning, measuring dimensions, comparing different discounts and rebates, charting different financing options with different interest rates)*
3. *to formulate mathematical problems given the situation and figure out how to solve them,*
4. *to perform necessary calculations and measurements (and decide when exact answers are needed, when estimates will suffice, if numbers should be rounded up, etc.),*
5. *a willingness to undertake this task; a belief that mathematics is a problem solving tool not just a school subject; a belief that I can manage to do this myself; and knowing that if I get stuck, I can rethink and still succeed.*

Without developing all of these cognitive and affective processes, knowing the mathematical content, and understanding the features of the context, I will not be prepared to deal with this situation in a powerful, competent way. The specific content knowledge needed for this task as well as others, along with the five processes described, enable a person to handle situations effectively. Numeracy comprises much more than arithmetic skills (had I known how to calculate a percent of a quantity, that would hardly have been sufficient); it requires the coming together of different kinds of knowledge and competence. The knowledge of context and content along with an ability to use the cognitive and affective processes are the components of numeracy. For a description of the components, see the box on page 15; for the paper in which the components were originally proposed, see: www.ncsall.net/fileadmin/resources/research/op_numeracy.pdf.



Adult educators are charged with helping learners acquire the numeracy skills to function competently and proficiently. This means attending to *all* of the components of numeracy. Questions to consider are: How should educators go about helping adult learners progress towards becoming numerate? What instructional strategies are most likely to pay off for learners given their goals for themselves? What decisions do educators need to consider about how to organize their classes, what to teach, and how to teach it? Without addressing all of the components of numeracy, while addressing these questions, we are limiting what our learners will be able to do with what they are learning.

Examining "Risky Practices" in Numeracy Instruction

It is worthwhile to stand back and, through the lens of the components of numeracy, examine five instructional practices that can be observed in many adult education programs. Each of these practices has useful and beneficial elements and reflects educators' sincere expectations that the practices will contribute to learners' success. Yet, each practice also is risky in that it narrowly defines the numeracy terrain and severely limits adult learners' opportunities to become numerate in a competent and functional way. Our primary goal as numeracy educators should be to consider how to create a balance among multiple goals, practices, and outcomes.

Risk #1. Primarily emphasizing calculation skills

Probably the most common instructional strategy found in adult education settings is a strong emphasis on mastery of arithmetic calculation skills. This approach is clean and easy: identify the arithmetic skill that the learner is "lacking," provide workbooks, start at the appropriate page and monitor the learner as he or she completes practice on consecutive topics. Generally, the particular operation (such as subtract one fraction from another) is clearly specified and modeled for the learner. The goal is for the learner to gain procedural fluency and be able to calculate accurately, efficiently, and quickly. Teachers perceive that this goal is closely aligned with some existing assessments such as the mathematics computation test of the Test of Adult Basic Education (TABE).

While learners do progress from topic to topic, this type of instruction is often not very effective in helping learners make meaning of what they are doing. Most math teachers have heard learners say when confronted with a contrived word problem with fractions, "I know how to do this. Just tell me if I should multiply or divide." Similarly, learners say, "I know the

Components and Subcomponents of Numeracy

CONTEXT - the use and purpose for which an adult takes on a task with mathematical demands

Family or Personal—as a parent, household manager, consumer, financial and health-care decision maker, and hobbyist

Workplace—as a worker able to perform tasks on the job and to be prepared to adapt to new employment demands

Further Learning—as one interested in the more formal aspects of mathematics necessary for further education or training

Community—as a citizen making interpretations of social situations with mathematical aspects such as the environment, crime and politics

CONTENT - the mathematical knowledge that is necessary for the tasks confronted

Number and Operation Sense—a sense of how numbers and operations work and how they relate to the world situations that they represent

Patterns, Functions and Algebra—an ability to analyze relationships and change among quantities, generalize and represent them in different ways, and develop solution methods based on the properties of numbers, operations and equations

Measurement and Shape—knowledge of the attributes of shapes, how to estimate and/or determine the measure of these attributes directly or indirectly, and how to reason spatially

Data, Statistics and Probability—the ability to describe populations, deal with uncertainty, assess claims, and make decisions thoughtfully

COGNITIVE AND AFFECTIVE-the processes that enable an individual to solve problems and, thereby, link the content and the context

Conceptual Understanding—an integrated and functional grasp of mathematical ideas

Adaptive Reasoning—the capacity to think logically about the relationships among concepts and situations

Strategic Competence—the ability to formulate mathematical problems, represent them, and solve them

Procedural Fluency—the ability to perform calculations efficiently and accurately by using paper and pencil procedures, mental mathematics, estimation techniques, and technological aids

Productive Disposition—the beliefs, attitudes, and emotions that contribute to a person's ability and willingness to engage, use, and persevere in mathematical thinking and learning or in activities with numeracy aspects

From Ginsburg, L., Manly, M., & Schmitt, M. J. (2006). *The Components of Numeracy* [NCSALL Occasional Paper]. Cambridge, MA: National Center for Study of Adult Learning and Literacy. Available at www.ncsall.net/fileadmin/resources/research/op_numeracy.pdf steps, but I can't remember which I do first." And what teacher has not seen the strange results of a learner's computation with a misplaced decimal point, because the learner has not stopped to consider if the solution "makes sense"? Extensive practice with procedures and focusing on attaining the one right answer, does not necessarily help learners develop either number sense, estimation skills, or the reasoning skills that should be used to monitor computation and also contribute to sense making and everyday mathematics activity.

There is little reason to assume learners will be able to apply their computational procedures in productive ways if the procedures have only been practiced in isolation as drills on worksheets, workbooks, or computer programs. Without also developing learners' conceptual understanding of the meaning of the procedures and of the relationships among them, as well as providing practice seeing how the procedures are applied in real life situations, learners have only learned to do what a calculator can do efficiently and have only been prepared for a computation assessment. Seeing the connections between operations on fractions and actually measuring wood or material for a home project, or deciding whether to use a "$10 off" rather than a "30% off" coupon, requires more than computational skill. We should not assume that learners will be making the connections themselves and will know when and how to apply their computational skills.

Risk #2. Focusing on the language aspects of word problems

Most workbooks include word problems, meant to provide examples of real world applications of computational procedures. The word problems usually follow sets of computational exercises that provide practice on the procedures needed for the word problems. Sometimes, the procedures needed to solve word problems are less obvious to learners and they must decide what computation to perform.

Some teachers look upon the word problems as a special kind of task to master: one requiring a particular set of problem solving skills and strategies. However, rather than focusing on the mathematical relationships among the elements of the problem situation described, the instructional focus is on the language clues within the word problem. Examples of this include pointing to words and phrases such as "in all," "all together," "greater than," "difference," or "each." While the words in the problem obviously contribute to one's understanding of the situation, focusing on particular words as guides to computational procedures does not promote numeracy understanding, real problem solving, or growth in being able to apply learning in real situations. Rarely do problems in everyday or work life contain "clue words". More often than not, they are situations that need to be understood, represented, and addressed with mathematical reasoning and then computation.

> “Learners need help to build such a repertoire, chances to practice multiple strategies, and benefit from opportunities to discuss their thinking.”

Risk #3. Attempting to dissipate math anxiety

Many adults returning to school fear having to deal with numbers or mathematics, especially in a school-like context. They feel that they are unable to learn what they need to know, often basing these feelings on past educational experiences, messages they have received from teachers and family members, and a cultural assumption that mathematics is hard for anyone who is not a "math person" with innate math talent that allows math learning to come easily.

Adult educators want to empower their learners to see themselves as capable and competent. They strive to create a learning environment that is safe. Some, however, assume that to be safe, a learner should not have to struggle, be wrong, or be challenged. They break mathematical content into very small, discrete pieces that learners can master without difficulty. This approach reinforces learners' learned helplessness.

In reality, for adults to function in a competent way in numeracy situations in and out of school, they need to develop a productive disposition which involves more than simply not being anxious about math. For example, encountering frustration when not being immediately able to solve an equation, find a computational error, or figure out how to find the maximum area a fence can enclose is normal. Such frustration is not a symptom of mathematical inability but is rather a part of the problem solving process. Everyone goes down blind alleys and hits walls when tasks are hard or seem complex to them. A good way to deal with this frustration is to recognize its basis and to have a repertoire of strategies available to use to attack the problem. These might include simplifying the problem by substituting small or round numbers for the numbers in the problem or by focusing attention on one aspect of the problem at a time. Other strategies include working backwards, drawing a diagram or making a table, or talking through a situation. Learners need help to build such a repertoire and chances to practice multiple strategies, and they benefit from opportunities to discuss their thinking. Once learners develop such strategic approaches to problem solving, they are less likely to feel anxious and panicky and more likely to feel control and power.

Risk #4. Primarily dividing math content into distinct, non-overlapping topics

To manage our complex lives, we often categorize or separate what we do into stand-alone chunks. In mathematics, we frequently talk about operations with whole numbers; dealing with fractions, decimals, and percents; algebra; geometry; and statistics. And then, many teachers and most workbooks address each topic in isolation, as if one had pretty much nothing to do with the others. Learners are somehow expected to make connections among them independently, if that is a goal at all.

Indeed, much evidence indicates that assumptions established and reinforced when studying one narrow topic are easy to apply but difficult to modify when studying another topic. For example, when multiplying whole numbers, people come to assume that "multiplication always makes larger" and is best imagined as repeated addition. This assumption leads to confusion when multiplying fractions or when conceptualizing the area of a shape (Fischbein et al., 1985).

Since adults have limited time to participate in formal educational activities, narrowly focusing on only one topic with few opportunities to make connections across topics does not really benefiting them. In this instructional model, adults have few opportunities to make connections between new learning and their patchy knowledge from earlier learning. And, in the everyday world, numeracy activity is rarely defined and delimited according to school-based topics. Deciding whether to pay the rent, heat, or electricity does not fit neatly into one topic.

Risk #5. Only embedding instruction within real-life contexts

Some adult educators embed mathematics instruction within contexts and problem situations that are meaningful for their learners. Whether they are real situations or simulations, they present opportunities for learners to see and experience mathematics challenges that they might encounter outside the classroom. These contexts or situations generally are less well defined than textbook word problems, require learners to make decisions, solve a consequential problem, and present and justify a solution. They provide rich opportunities for involving learners in mathematizing, or formulating math problems from within situations, and for developing problem solving skills. Learners feel that they are engaging in real problem solving, and they are. A good example can be found in Captured Wisdom's Restaurant Problem (www.ncrtec.org/pd/cw/adultlit.htm).

Researchers have found that adolescents who learn math embedded in real world, problem-based activity often need time and help from teachers to make the mathematics explicit and pull the mathematics out of the situation for it to be accessible for use in other settings or for a standardized, decontextualized test (Boaler, 2000; Stone et al., 2005). Conceptual understanding might be developing, but it is tightly linked to the context within which it was learned and practiced.

For example, in a construction context, fractions appear in a very rich way in measurement, as does proportional reasoning in reading blueprints, yet the mathematics may be encountered within specific, practice-based procedures that do not require conceptual understanding. One can use a ruler by learning the names of the lines between inch marks (i.e., ½, 5/8, 15/16) without gaining an understanding that ½, 2/4, 4/8, 8/16 are equivalent, or being able to find the distance between 8½ inches and 12^3/8 inches. And would 8.3 inches really be a useful or meaningful representation within this context? Embedding math learning deep in a real world context has benefits for helping learners master the mathematical practices of the context and solve problems within the constraints of the context, but the instructor needs to also locate the mathematical ideas within broader contexts as well so that learners can "carry" their knowledge and skills to other places.

Instruction with All Components

No one ideal way to teach mathematics meets the needs of all learners, is within the reach of all teachers, and can be supported by all programs. However, all teachers should challenge themselves to help learners acquire the range of skills and knowledge that are necessary for proficient numeracy practice. And this should lead them to question their current practices. It also might well lead them to realize that they may need to upgrade their own depth of understanding, knowledge, and skills so that they can best meet the needs of their learners.

All Components at Once

Among the practices that adult educators might consider is addressing all the components at once, not in succession. No evidence indicates that extensive practice adding fractions, multiplying decimals, or finding percent discounts will lead to conceptual understanding of fractions, decimals, or percents; an understanding of how they are related and different from each other; or an ability to decide how they might be used when representing or solving problems.

For example, when studying fractions, consider spending time talking about the relative size of fractions, i.e., what happens when numerators increase while denominators stay the same and what happens when denominators increase while numerators stay the same or which fractions are close to 0, ½, or 1 and how to know. Discuss what it means and what happens when you multiply two fractions each less than 1, or multiply a whole number by a fraction that is less than 1 or greater than 1. Draw diagrams to represent different situations. Use rulers and tape measures to add or subtract fractional quantities. Estimate answers and discuss situations in which estimates would or would not be adequate.

> "Over the years, many teachers have told me that they came to understand mathematics so much better once they began teaching because they had to figure out how to explain it in a meaningful way to someone else. Why not provide the same learning opportunity to adult learners?"

Encourage and celebrate the use of alternative strategies to solve problems, represent situations (in diagrams, tables, graphs, etc.), and do calculations. "Can anyone do it another way?" is not only asking for alternative thinking but is sending the message that there is always more than one way to approach a problem or computation. Discussions of alternatives, right or wrong ones, lead to deeper understanding. I'm assuming that the classroom is a safe environment for learners, which should be the norm in adult education.

I have seen that learners pick up the mantra of "Can anyone do it another way?" trying to outdo each other to come up with as many alternative solution paths as possible. In the course of this, learners are free to opine why an alternative strategy will or will not work, which strategy makes most sense to them, or which strategy seems most efficient. They start noticing patterns and see that math can be a creative enterprise. Learning math is no longer passive, but an active endeavor. I have been amazed at something a learner said, often in the context of, "Why couldn't it be ...?" and have had to really think about whether or not a particular suggestion always worked and why. Sometimes, we end up identifying one particularly useful or clever method as "Tara's method" and the nickname remains for the rest of the year.

Not surprisingly, different procedures are better in different situations or with different numbers. In addition, different procedures work better for different people. Why not provide a rich assortment of procedures and strategies from which learners can choose the most meaningful or "easiest" for themselves?

Talk about Meaning

Make certain that learners can talk about the meaning of what they are doing: what, why, and how. In explaining their thinking to someone else, whether to another learner or a teacher, people have to really understand what they are doing. This does not mean rattling off a sequence of steps or procedures, but rather explaining why they are doing what they are doing. Sometimes, when challenged to do this, people realize that they don't actually understand what they are doing. If people don't understand or "own" their mathematical ideas and skills, they will just lose them, just as happened to them before.

Over the years, many teachers have told me that they came to understand mathematics so much better once they began teaching because they had to figure out how to explain it in a meaningful way to someone else. Why not provide the same learning opportunity to adult learners?

In addition, communicating about numerical information is a necessary task in the workplace and for everyday life. Developing mathematical vocabulary, not from worksheets or from vocabulary lists, but from meaningful use is empowering.

Problem Solving Processes

In class, solve real problems that may be complex and messy and worry less about getting a "correct" solution than about the problem solving processes used. Talk about alternative strategies, the choices that could be made, and the benefits and drawbacks of each.

Most of the time, numerical information and real problems do not appear in the terse language of word problems. People need to learn how to make numerical decisions when the numbers don't come out even, and even when all the information is not clearly defined. Being a flexible problem solver means knowing what to try when the obvious doesn't work or seems to lead to a dead end. Learners need to amass a repertoire of strategies and experiences that can be available when needed.

Consider Student Goals

Consider the goals of students. For those who plan to continue their education at a community college, consider making algebra the center of instruction, building in work on other topics as needed. For those who have other aspirations, build in appropriate contextual work, building on embedded math but relating it to other concepts.

There really is not a rigid sequence of topics that must be followed. Adult learners all bring a collage of mathematical knowledge with them from earlier schooling, everyday life, work experience, and even the advertising they see around them. As educators, we should be helping them make connections among all that they already know and that which they seek to know. I believe that we should start with what they want or need to learn and fill in the gaps as they become apparent. For example, an algebra class can provide ample opportunities for discussions of fractions when considering the values that lie between plotted points on a graph. Similarly, for a group preparing to enter construction trades, measuring walls, doors, and windows (including fractions or decimal measurements), using proportional reasoning for drawing and also reading blueprints, and exploring geometry strategies for ensuring that walls are perpendicular are both useful skills and mathematical concepts that should be understood and examined.

Number Sense and Estimation Skills

Develop number sense and estimation skills in addition to procedural (computational) fluency by exploring the arithmetic processes rather than solely focusing on getting correct answers. The advent of calculators makes these skills even more important for monitoring accuracy.

Often, actual computation is cumbersome and an estimate is sufficient for many real world situations. This is even true for eliminating obviously incorrect answers on standardized tests. There are strategies that people use to estimate, including focusing on round, friendly numbers while attending to the order of magnitude (consider 50 rather than 53, 14,000 rather than 13,789, etc.) and using body parts for estimating measurements. Most people seem to develop their own estimation strategies, whether for use at the store or when determining a tip in a restaurant. Why not make these mental processes apparent to learners so they can use them without having to first develop them themselves? Most learners do have some estimation strategies that they use; however, they often don't bring them into class, wrongly perceiving that in math class one must only find the one, accurate answer (even if it makes no sense).

Other times, mental math procedures are easy and accurate. Students are often amazed to find that they can do most percent problems mentally by decomposing numbers. Virtually every adult learner knows that 50 percent of a number means half of it (Ginsburg, et al., 1995), and through discussion and experimentation most come to realize that it is easy to know how much 10 percent of any number is without having to write the calculation (I have been amazed at how many people leave adult basic education without knowing this). Then, many percent calculations are possible (60% is 50% + 10%; 15% is 10% + half of 10%; etc.). Adult learners feel empowered when they are able to do such feats mentally, and they are not doing meaningless tricks that will easily be forgotten.

Build Productive Dispositions

Build productive dispositions towards mathematics and towards problem solving by demystifying all the processes and procedures and by sharing reasoning strategies. Productive dispositions refer to a learner's willingness to engage, use, and persevere in mathematical thinking and learning or in activities with numeracy aspects. Emphasizing sense-making is much more likely to help learners develop confidence than practice with procedural routines that might easily be forgotten or remembered inaccurately. Through reasoning about mathematical tasks, practicing solving problems and engaging in opportunities to see that everyone (including teachers) goes down wrong solution paths at times, learners can feel satisfaction when it "works" and better manage frustration when it doesn't. But first, they need productive tools and experiences. They gradually replace inhibiting fatalistic beliefs such as "I am not a math person" with a willingness to persevere and an expectation that meaning can be found.

Group Work

Encourage group problem solving. Multiple heads are better than one and having to justify reasoning to others helps pinpoint areas of understanding and of confusion. It is also more fun. Most of us find comfort and help when we work together. Many adult education classes develop a sense of community, with learners providing emotional and intellectual support for each other. Yet, many times, when it comes time for math, learners retreat to individualized, solo work. Collaborative work promotes discussions of alternative strategies and makes necessary the need to explain and justify ideas and solution paths. Mathematics then becomes an engaging, interactive learning process, with the teacher not necessarily the source of all information.

Ask Yourself...

All teachers can benefit from standing back and reflecting on whether the instruction being provided really helps learners become numerately competent. It is difficult to completely change deeply embedded instructional patterns, but it is easy to be curious and ask, "What will happen if I ..." noting how changes affect learners' skills, knowledge, understanding, and dispositions. This approach asks more of teachers but potentially improves outcomes for learners. If competent performance in numeracy is really a goal for our learners, then they must have opportunities to develop all of the components of numeracy and also have opportunities to put it all together.

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About the Author

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Changing Practice, Expanding Minds

It's hard enough to change your own practice, how do you get students to change along with you?

by Kate Nonesuch

When I began teaching basic math to adults 15 years ago, I inherited a set of workbooks and tests and a self-paced method of delivery. In the spirit of things, I tried to carry on with what I had been given, but I very quickly realized it would not work. Students often remembered enough to pass the chapter test, but promptly forgot it all again; many were mainly bored and often frustrated; they sometimes cheated to get the right answers; they seemed not to expect to understand anything, only to memorize algorithms; they dropped out; they failed the final tests. Many others managed to pass the course, but they were usually the ones who came into the class already knowing most of the material, needing only a review to get them ready for the next level. The ones who needed to learn the material were not learning it by the methods I was using. So I began to change my teaching practice, slowly introducing the kinds of activities outlined by Lynda Ginsburg and Iddo Gal (1996) and Gal and colleagues (1994): I started to pay attention to emotions and attitudes, to use manipulatives, diagrams, and models, to do group work and real-life activities. I replaced the workbook I had inherited with a better one, then threw the workbook out entirely, and was later glad to read Mary Jane Schmitt (2006) on the subject. Although I didn't have a regular community of colleagues with whom to exchange ideas, like the New York City Math Exchange (Brower et al., 2000), I went to workshops and conferences to learn techniques so students would understand what they were doing when they were practicing algorithms, otherwise known as teaching for understanding. When I turned to research, I found that Beth Marr (2000, 2001) and Alison Tomlin (2002) confirmed my experience and extended my understanding.

I hoped that the changes in my practice would make my teaching and my students more successful. What happened? I ran into a wall of student resistance. Before I could successfully introduce such new activities, strategies, and techniques, I had to learn to honor and respect student resistance, to work with it, and to bring students on board with my reforms. At the same time I learned to recognize and deal with the various shapes resistance takes, so that old ideas, habits, and emotions would not stop me from improving the teaching and learning of math.

Moreover, I continue to deal with student resistance in every group of students I teach. Although the years have convinced me that these strategies are worthwhile, every new term brings a new group of students who expect to be taught the old way. My class meets for about three hours a week, with another three to five hours per week available for students to do independent work with a teacher available for consultation. Most students take six months to complete the course. Nearly all students also take classes in English and computer skills; the program is a storefront literacy center, and includes other activities as well, running for four hours a day, four days a week. Since nearly every adult who enrolls in my math class has years of unsuccessful experience as a math student, it stands to reason that my students have a firm idea of what math class should be and what success in math looks like. They expect me to give them sheets of questions and some tricks to help them remember algorithms. They believe that activities such as using manipulatives, doing group work, going on field trips, and, especially, aiming for understanding, just delay them on the road to the real thing. Often they will tell me such activities are "not real math."

Honoring Student Resistance

My colleague, Evelyn Battell, says "If you truly respect students, you will honor their resistance. You have to set aside that niggling belief that they'll learn it if only you can force them to take part." When I respect student resistance and encourage its expression, students are more likely to be able to stay present and attend to the work. When Arleen Pare (1994) did some research in my classroom, she found a positive correlation between student expression of resistance and student retention. The more complex and open their resistance to me and my teaching, the more likely they were to continue to attend regularly; she concludes: These results suggest a positive association between conscious, active resistance and regular attendance. It also suggests that the more that conscious resistance is encouraged, the more likely it is that regular attendance will result (Pare, p. 115).

As an example, take the student who keeps her coat on, sits silently at the back of the room or near the door, and whose body language says, "I'm not here." Pare found that this student is more likely to drop out than the student who says, "Why do we have to do this stuff anyway?" and gets a response that takes seriously her concerns about not wasting her time.

I have developed several strategies to respect and encourage the expression of resistance and to make students (and me) more conscious of their resistance: acknowledge feelings, acknowledge that students are in charge of their learning, help them learn about themselves as learners, get them on board, invite them into the decision-making process, and find allies in the process. I will discuss each of these strategies in the sections that follow.

> *"Before I could successfully introduce such new activities, strategies, and techniques, I had to learn to honor and respect student resistance, to work with it, and to bring students on board with my reforms."*

Acknowledge Feelings

When I introduce a new type of activity, I start by acknowledging that what I am asking them to do may be new to them, and it might feel strange. A comment in passing is often enough. If I see a student sitting with his hands in his lap and a set of fraction manipulatives in front of him, I'll sit down and say, "When I first started to use these, I felt clumsy trying to keep track of all the little pieces," or "Sometimes students tell me they feel silly, like they're back in kindergarten, when they use these things." The remard usually gets me a smile or a grunt of acknowledgement, and I have opened a conversation about the use of manipulatives, during which the student will decide whether or not to use them.

Students resist for reasons that are not always apparent. A colleague speculated that her students thought working in groups was inefficient and time-consuming. After leading discussions in two different classes, she reported:

Well, I was wrong about thinking it was a time issue! At least for the students who responded in these two classes, the common denominator was that **taking part in some of these activities involves interaction, and during interaction, others can see that they are dumb, stupid, and not as smart. Fear of exposing ignorance seems to be the motivator for these folks to want to do their own individual work.** (Iris Strong, e-mail, February 23, 2006, my emphasis).

"I'm bored," is easier—and "cooler"—to say than "I'm scared," or "I feel stupid," or "It's humiliating to be put at this low level." In fact, students will very rarely say any of these latter things in class, but if they are feeling scared or stupid or humiliated, and not acknowledging it, they cannot fully participate in learning, and I have to guess what is going on with them. However, if they say, "I'm bored with this," I have a chance to change the situation.

I'll respond, "I'm so glad you said that. I could see that you were not really into what we are doing, and I get bored, too, when the class is flat. So let's see—what is boring about this piece? Too hard? I know frustration gets boring for me after about 50 seconds. Too easy? So far out of your normal life that it might as well be on Mars? What can we do to make it more interesting?"

I announce early every term that the first rule in my class is to refuse to be bored. Students who expect math class to be an uncomfortable mix of boredom and crisis see that I have different expectations, and I make it clear that I will respect expressions of resistance to activities and content. Furthermore, when refusing to be bored is part of the culture of the class, expressing boredom is participation, not resistance. It means we are all on the same team, trying to make the class interesting.

My Own Resistance

I had to deal with the voices in my own head, too, when I changed my teaching practice to focus on teaching for understanding, using group work, models and manipulatives, real life math, and so on. I listened to those voices, and I talked back to them.

The inner voice said....	I answered back...
"This is not real math..."	The standard way—teaching a tiny algorithm and giving two pages of questions requiring identical operations—now that is not real math!
There is so much material to cover, you don't have time for these "frills."	Maybe if I take enough time at the beginning so people really understand, we won't need to spend so much time on drill and practice and memory work.
You don't have time to redesign your whole course—and it probably won't work anyway.	I'll start small, with one tiny section, next week.
These new methods will not help students pass the exam.	They can't hurt.
You know that your students have huge holes in their math backgrounds. The only way to get through the material is to stick to the path. You're creating more holes by doing it this way, but it's the only way.	Go back to the big concepts: place value, the meaning of operations, the sanctity of the equals sign. Don't try to fill in the holes by shoveling stuff down from above, rather go down to the bottom of the hole and build up.
You're the only one making these changes. Why do you have to be out of step?	I'm one of a growing number. I'm going to find my allies and stick with them.

Acknowledge Learners are in Charge of Their Learning

At the most basic level, students are always in charge of their learning. Even in the most authoritarian classroom, students decide whether to participate in class, how much they will co-operate with or challenge the instructor, what they will do to make the class work for them, and finally they will decide whether or not to continue. I need to acknowledge that learners are in charge, so I can figure out how to work with them to change my practice for our mutual benefit.

"Do we *have* to?" This question is a common way that students express their resistance to a teaching strategy. My answer is always "no." No matter what decisions the class has made, or the general agreement in the group to try new ways of learning math, every individual is free to choose whether or not to take part in any activity at any given time. Fundamental to my stance as a teacher is a refusal to get into a power struggle with a student about the way learning will take place. If I insist, if I answer this question with a list of reasons why they have to, we enter into a power struggle and both lose. If the student continues to refuse and I continue to insist, then one of us has to leave, and it won't be me! If the student agrees unwillingly to participate because I say so, his attention will not be on the math we are doing, but rather on his feelings about being coerced. When teacher and student struggle for power, there are no "teachable moments." The student's right to choose is the only position from which we can both truly win.

Help Them Learn About Themselves as Learners

Often I ask someone from student advising or the learning resource center to do a session on learning styles and multiple intelligences. I give students some learning styles and multiple intelligence (MI) inventories (sources are included at the end of the article). I like them to do more than one inventory of each type, because different inventories give different results; the purpose is not to attach a label, but rather to give them the opportunity to think about themselves as learners. Later, I ask students to do an activity that helps them find out who else in the room has similar styles and strengths. For example, I might ask them to get into groups according to their dominant learning style—visual, auditory, or kinesthetic—and discuss such questions as "What do you do when you have to remember something?" "What is the best way for you to learn a new skill?" "How do you make sure you really understand how something works?" and make a presentation of their discussion to the whole group.

When everyone has this background, I can talk about math activities and assignments in terms of different learning styles and MI theory. Students see that some activities that they previously resisted may be exactly suited to their own strengths, or that they may be exactly what someone else in the class needs. They can then choose whether or not to participate from a position of knowledge. "Group work is good for me because talking about what I'm doing helps me learn," or "I'm a body [kinesthetic] learner, and moving the pieces and piling them up helps me remember."

Get Students on Board

I bring a teacher's expertise to the class: I know math; I have experience with helping many different students; and I have access to lots of material. I also bring a teacher's goals: I want student retention and student success; and I want the time I spend planning for class to be fruitful. These new strategies—making math real, using manipulatives, working in various groupings—all require more complex facilitation skills than simply lecturing or working on-on-one with students. At the end of the day, or at least at the end of the week, I want to feel satisfied with my own performance as a facilitator. I can't reach those goals unless the students engage with my methods of teaching. Getting the students on board is the ultimate strategy to deal with resistance, the culmination of the strategies I've mentioned so far.

I ask the class to tell me all the ways they have tried to learn math. What methods have past teachers used? What kinds of activities helped? What did they hate? What did they have fun with? Then I ask, "Does anyone know one single way to learn math that really works?" Invariably, nobody does because they have all been previously unsuccessful. This conversation makes the students partners in designing their own learning. The discussion about past methods of learning math, an evaluation of what parts were more and less useful, and the conclusion that something new needs to be tried, means that they are helping to decide what form teaching will take.

Invite Students into the Decision-making Process

Before I introduce a teaching strategy that I think might meet with resistance, I present the strategy to the group and give my reasons for thinking it will be valuable. For example, I might want to ask them to use manipulatives to demonstrate that their answers to fractions problems are correct. I expect resistance here because not only am I asking them to use manipulatives, I am asking them to change other parts of their usual way of working. Instead of doing all the questions and then handing them in or checking their answers, they will have to set up a demonstration of each problem, show it to me and talk to me about it. The pace of the work is different, with more interruptions and more waiting time, and showing me and talking to me is more stressful than handing the work in and hoping for the best. The resistance to the change of pace and added stress may be greater than the resistance to using manipulatives.

In this instance, I might introduce the idea for their consideration, and start by explaining some of the advantages of this activity: that people usually get the answers right, that manipulatives show the reasons behind all the mumbo-jumbo of the algorithms, that they connect the paper and pencil work to real life, that they help slow the pace of work so students have more time to think, and that they reduce the need for pages and pages of drill on the algorithms. I ask for their reactions, then propose that we try it out for a reasonable length of time, for example, three weeks, and that we evaluate it briefly at the end of the first week, and more thoroughly after the trial period. I make it clear that I will act on the decisions made at this final evaluation, and that I will stop using the strategy if most students don't like it or don't find it useful.

If the class agrees at this point to try the strategy, I explain it in a little more detail, with examples. I ask the class to predict what effects the new strategy might have. How might it be useful to visual learners? Kinesthetic? Auditory? How might it increase understanding, improve memory, or lead to better test scores? How might it make math class more enjoyable or interesting? All this discussion gives us something to watch for as we begin to use the strategy. At the end of the first week, a brief discussion reminds people what to look for, allows students to give an initial response, and encourages everyone to keep with it for the rest of the trial period. Sometimes we get some insight into how to tweak the strategy slightly to suit the students' needs.

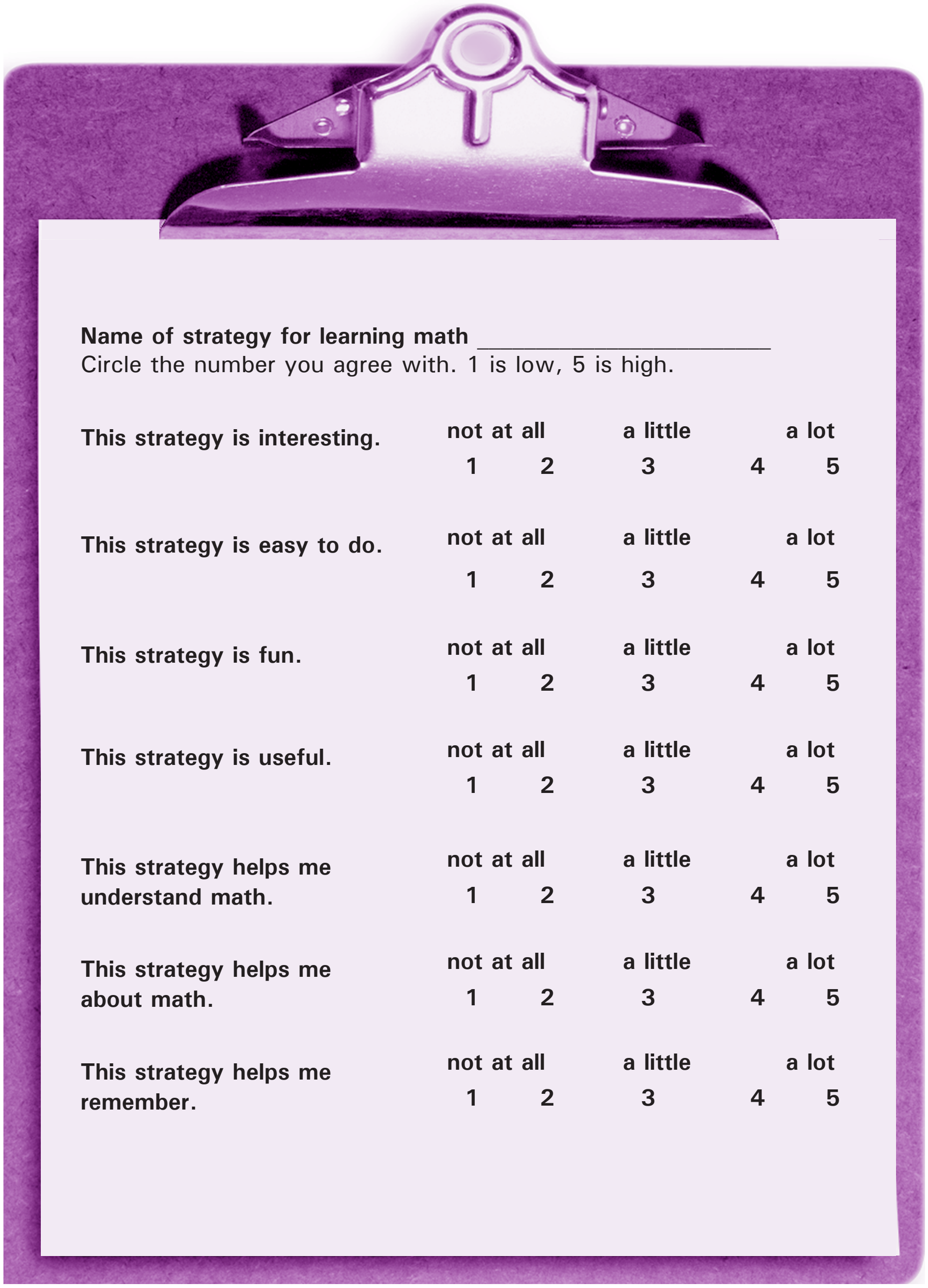
At the end of the trial period, I do a more thorough evaluation. Sometimes I use an evaluation sheet like the one here, and sometimes I ask students to generate a list of questions they would like answered, and I turn that into an evaluation sheet. Collating and analyzing the answers is an interesting activity for the whole class, or for a small group of students.

I always keep my part of the bargain—that is, I abide by the evaluation of the class. Why would I try to force people to do something they believe is not useful?

Find Allies

Often, the students who resist new ways are the loudest or the most

A Simple Tool to Evaluate a Teaching Strategy



Name of strategy for learning math ________________________
Circle the number you agree with. 1 is low, 5 is high.

This strategy is interesting.	**not at all** **1**	**2**	**a little** **3**	**4**	**a lot** **5**
This strategy is easy to do.	**not at all** **1**	**2**	**a little** **3**	**4**	**a lot** **5**
This strategy is fun.	**not at all** **1**	**2**	**a little** **3**	**4**	**a lot** **5**
This strategy is useful.	**not at all** **1**	**2**	**a little** **3**	**4**	**a lot** **5**
This strategy helps me understand math.	**not at all** **1**	**2**	**a little** **3**	**4**	**a lot** **5**
This strategy helps me about math.	**not at all** **1**	**2**	**a little** **3**	**4**	**a lot** **5**
This strategy helps me remember.	**not at all** **1**	**2**	**a little** **3**	**4**	**a lot** **5**

problematic in terms of classroom management, while the students who are happy are quiet about their feelings. To get some feedback, I ask the students to brainstorm a list of the teaching strategies I've been using, such as group work, work with manipulatives, working at the board, lectures, field trips, or tests. I ask them to consider how useful each strategy is in helping them learn math, and list the three they find most useful and the one they least like to do. Students hand in their lists anonymously, and I calculate the responses for each strategy and report back the next class session.

> "Sometimes the tide of student resistance seems too strong to swim against, so I move slowly, I celebrate small successes, and I pay attention to evidence that the strategies really work."

I often find that most of the class thinks that a particular strategy is helpful, even though a few protest when I bring it out. Knowing I have allies in the class, albeit silent ones, helps me persevere in using a given strategy, in spite of groans and sighs from a few. Furthermore, reporting back to the class, so everyone knows where we stand on the issue, helps too. I can be firmer about using the strategies many students find useful, in spite of resistance from the few, and it gives me a chance to acknowledge the discomfort and resistance I know are there. For example, I might say to the class, "On the feedback sheets yesterday, $^{2}/_{3}$ of you said working at the board was useful, although five people said it was the thing they liked the least. So I'll keep on asking you to go to the board two or three times a week. I'm hoping that those of you who don't like it very much will put up with it because other people find it useful. If you decide not to go to the board, please do the work at your desk and follow the discussion."

Changing Practice, Expanding Minds

To change my practice, I had to expand my students' views about how math should be taught. To do so, I gradually learned and honed the strategies I've outlined here: acknowledging feelings, acknowledging that students are in charge of their learning, helping them learn about themselves as learners, getting them on board, inviting them to participate in making decisions, and finding my allies. Over the years, it has become second nature to use these strategies to honor and work with student resistance, but it did not come quickly or easily to me. I had also to deal with my own resistance to giving up control and to trying something new. Sometimes the tide of student resistance seemed too strong to swim against, so I moved slowly, I celebrated small successes, and I paid attention to evidence that the strategies really worked. I don't have quantitative evidence that changing my practice has meant more student success, but many indications: a more cheerful and more purposeful atmosphere in my classes; a willingness to participate; the care students take with manipulatives; the pride they show in their work; their desire to pass on what they have learned to their children, who are often struggling with the same math; and comments such as "Why didn't they teach us this way in the first place?"

Changing the Way We Teach Math: A Manual

What do the experts say about how to teach basic math to adult students? Most people who teach math have heard about most of their recommendations, yet putting those recommendations into practice is harder than it seems. Kate Nonesuch consulted adult basic education and literacy practitioners in British Columbia about adapting their teaching practices in the light of research findings about adult numeracy instruction, and wrote *Changing the Way We Teach Math* as a result. Both her manual and her literature review are available free, on-line.

Changing the Way We Teach Math: A Manual for Teaching Basic Math to Adults
www.nald.ca/library/learning/mathman/mathman.pdf

More Complicated Than It Seems: A Review of Literature about Teaching Math to Adults www.nald.ca/library/research/morecomp/morecomp.pdf ❖

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About the Author

Kate Nonesuch has been teaching adult literacy and numeracy for 20 years, most of that time at Malaspina University-College, Cowichan Campus, Duncan, British Columbia. Her own education (a four-year BA from Carleton University and a teaching certificate earned at the University of Saskatchewan) informs her teaching practice less than her politics of inclusion and the lessons her students have taught her. Learner autonomy has always been a strong focus of her work.❖

Resources

Multiple Intelligences Inventories

Birmingham Grid for Learning www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks3/ict/multiple_int/index.htm

lsquared02.tripod.com/Multiple_Intelligences/MultipleIntelligences.htm - testfaculty.usiouxfalls.edu/arpeterson/multiple_intell.htm

LD Pride.net www.ldrc.ca/projects/miinventory/miinventory.php

Learning Styles Inventories

North Carolina State www.engr.ncsu.edu/learningstyles/ilsweb.html

Memletics www.learning-styles-online.com/inventory/

University of South Dakota www.usd.edu/trio/tut/ts/stylest.html❖

A Conversation with FOB...

Is Math Universal?

Math is math around the world, isn't it? No, math teacher Joanne Kantner reminds us, it's not. Culture plays a role in an individual's number sense and ways of thinking about and doing mathematics; these differences are further accentuated by the amount of formal schooling in math someone has. Ms. Kantner, who coordinates the Adult Student Connections Program and teaches math at Kishwaukee College in Malta, Illinois, talked with Focus on Basics about how she began to realize this and what it means for the adult basic education (ABE) classroom.

FOB: Tell us a little about yourself.

Joanne: My teaching background is in community college developmental math. I also do some training for ABE, adult secondary education (ASE), and English for speakers of other languages (ESOL) instructors, and have a Doctorate in adult and higher education.

FOB: How did you get interested in cultural aspects of math?

Joanne: I was teaching developmental math at the community college when two ESOL students were placed into my class. These students were paying for advanced ESOL classes and needed additional credit hours to be full time students. Someone in the registrar's office put them into developmental math thinking that math is universal. But it became very apparent within the first weeks that the two students would not be able to understand me or interact in group work.

FOB: It sounds like despite their advanced ESOL status these students had language barriers that kept them from participating...

Joanne: Yes. So the college provided them with an interpreter in class and a tutor. But that did not help much.

One day I was helping a student from Southeast Asia. We were working on three-digit subtraction, subtracting 487 from 623. When exploring the student's written work I was confused by how she wrote and figured out the problem. I recognized her first step: she borrowed to subtract 7 from 13; but she didn't repeat this procedure. Instead, she wrote "1"s underneath the digits of the second number. When I asked her why she placed these markings under the number 487, she said it was easier to add one to each of the numbers to be subtracted than to remember to borrow and carry over between place values. As she explained her computation, I understood that she was using a different algorithm for subtraction than I use. I was born and educated in the United States. Her method made it easier to do mental computations, which she described as the sign of a good mathematics student. It dawned on me that computation algorithms, mathematics procedures, and the values related to mathematics are not universal across cultures and nations.

FOB: What do you mean by "not universal"?

Joanne: Mathematics depends upon building on what people know. How do we teach the mathematics our students need to know, building on what they know, if we're not familiar with what they *do* know? It's not specific to mathematics, writing has similar differences in how a culture presents information. But many people seem to think mathematics is exempt from those concerns. Our registrar thought that. But it's not. There's a whole field called ethno mathematics and one of the components is how different cultures learn and use math.

The method—the algorithm—might not be universal, as her example demonstrates; even how to understand the problem might not be universal. For example, when you give a word problem, it is a cultural event. The context, such as covering a living room floor with carpet, is only universal to those who share the same experiences.

Notations are different. Whether the person considers the "big picture" or the "details" is cultural. Of course, there are differences among learners no matter what the culture, but you have to factor culture in as well.

FOB: What about differences among students who have had formal schooling in other countries? For example, you started by talking about differences in notation and algorithms. Some of my former students lay out their division problems differently than I do.

Joanne: Daniel Orey, via something called the Algorithm Collection Project, has identified four variations of the division algorithm which are grouped as North American, Franco-Brazilian, Russo-Soviet, and Indo-Pakistani styles [see Figure 1]. Various processes, notations, and symbols are used in each division style. The long division sign used in the United States can be interpreted as the square root symbol by students educated in the Russo-Soviet method. The placements of the dividend and the divisor change in the different forms as does the position of the quotient.

Figure 1

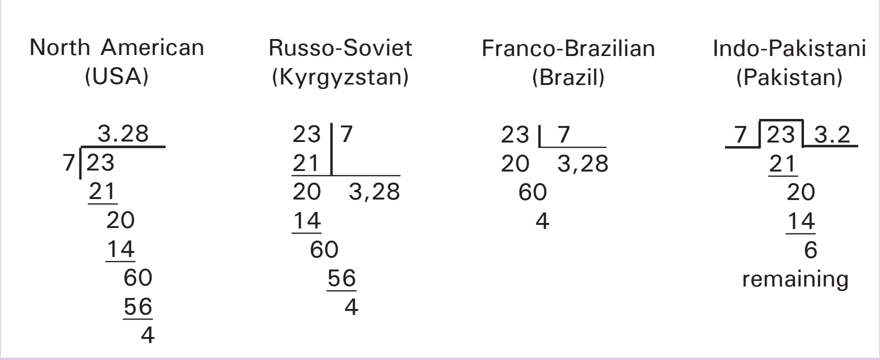

Figure 1, Variations of Division Algorithm (Orey, 1999)

Joanne: The other issue is not just procedures—how they divide—but notation: symbols. Points and commas are used differently. We may not be aware of this and mark something wrong. Particularly on the GED, this will trip them up. Spaces are used differently as well.

FOB: I've noticed this in foreign newspapers.

Joanne: Some cultural groups—this tends to fall by country—use the comma in place of the decimal point and others use periods instead of commas to separate multiples of hundreds. According to the International System of Units, the decimal marker is the period in English-speaking countries. In most other places, the decimal marker is the comma. To avoid confusion, the International System of Units (SI) represents large numbers by groups of three digits separated by narrow spaces and not commas or decimals.

FOB: Those don't seem very hard for teachers to recognize, once they're aware of them.

Joanne: It is important to be aware of them. If teachers are not, they may mark something wrong, particularly on the GED practice tests. Nonetheless, these elements are pretty easy for students to switch. The conceptual differences are harder.

It is hard enough for a teacher to discern whether American students are parodying what they have memorized or really understand what they are doing. If a student is from another culture, you must also think about whether a cultural difference is causing miscommunication.

FOB: Can you give us an example?

Joanne: Think about logical steps in deductive reasoning: If a = b and b = c then a = c. Cognitive psychology has shown that countries with collectivist cultures have a different concept of what is a valid logical conclusion. In general, collectivist cultures based reasoning more on realism than on rules of mathematical logic.

Let's apply this, using this "if a = b and b = c, then a = c" set of sentences: If you are healthy you don't get sick. If you don't get sick, you will be smoking. Following a logic model you could say, "If you are healthy, you will smoke." Someone in an individualistic culture such as the culture of the United States would say that even if we don't agree, it makes sense mathematically. But in collectivist cultures they'd say it doesn't make sense mathematically. They look at the credibility of the conclusion, not whether the conclusion is drawn from the logic model. This example comes from a study of Koreans and Korean-Americans. Those raised in Korea said that those who smoke could not be healthy and were more apt to say that was not a logical conclusion. Korean Americans, raised in a different culture, could follow the logic model, saying that even though it doesn't follow what we know, following the if a = b, and b = c, then a = c smoking could be equated with health in this model.

> *"If anything, you're translating into a third language for your students: American math."*

FOB: Can you relate that to the classroom for us?

Joanne: In our classrooms we need to remember that what we assume our students see as important and logical may in reality appear unimportant and irrelevant logically. As cultural psychologist Geoffrey Saxe points out, deductive reasoning does not universally equate to logical thinking in all cultures.

FOB: Let's go back to the question of word problems. The context, such as covering a living room floor with carpet, is only universal to those who share the same experiences. What does this mean for numeracy education?

Joanne: With ABE learners, we're trying to build on everyday math: to what experiences in our learners' lives can we relate this math concept? Our learners' everyday experiences are based on culture, rather than on formal instruction.

Translating this into classroom practice, it's hard to figure out what an authentic situation is for your students if they're from many different cultures. For example, if you think of reading advertising circulars, they might not understand what the math issue is because they don't understand how a discount coupon works or even what an advertising circular is. Without that knowledge, how can they begin to find the math in a particular problem?

This happens with textbooks: authors assume some common experience when they give their examples or their representations. But our students' are not coming from shared experiences.

FOB: At the risk of stating the obvious, it's useful to remember that socioeconomic class cultures can be as different as ethnic class cultures, especially when it comes to choices involving finances. By the way, have you found any math books to be good at handling the cultural aspects of math?

Joanne: Multicultural math books present math problems suitable for different cultures. For example, an Islamic way to divide a will: certain people in a family get a certain percent. So there might be a problem like that in percent.

Multicultural math books are useful for teachers who are looking for some examples their students can relate to. They're also good for teachers to understand cultural awareness and diversity. Claudia Zaslavsky's textbook, *Multicultural Math Classroom: Bringing in the World*, provides an overview of multicultural mathematics education and curriculum ideas such as counting with words, numeral symbols, geometry, and data analysis. I often refer to the Ethnomathematics Digital Library (www.ethnomath.org/index.asp). It can be searched by subject, geographical area, or cultural group.

FOB: I am hearing a couple different issues. One is that teachers need to know that students who have been schooled in other countries may know different ways to do math than students who have been schooled in the United States. Given their different backgrounds, how students make math meaning will be different.

Joanne: And as an instructor, you have to find examples and problems that help these students bridge two cultures: the one we're trying to teach them, because that's what their GED is written in, and what they were taught formally or the way they use math in everyday life.

FOB: Is anyone's "different" math background an asset to them?

Joanne: Definitely. For example, in the case of the Vietnamese student I mentioned earlier, she showed me how she did borrowing. For her, good math was about doing something mentally. The algorithm she knew allowed her to do good mental math. She had good math practices. The only snag was when that student made an error. Then I had to have her walk me through her work to see what was causing her error.

FOB: So it sounds like teachers need to consider possible differences in procedure, symbolism, and concepts based on what each student bring to the classroom.

Joanne: And language differences as well. For example, although "least common denominator" is a phrase, a student may look up each word separately. Without a good dictionary or a mathematical dictionary, the meaning can be lost. So vocabulary lists are helpful.

As a math teacher with ESOL students I work one-on-one with students, spending extra time with them. I devote a lot of office hours to students who are asking me about word problems. Sometimes I check language versus math understanding. I feel like I'm an interpreter between whatever culture they came from and the one I'm trying to get them to be able to function in easily.

My advice? Assume cultural differences. The same way you think cultural differences impact other subjects, assume they impact math. If anything, you're translating into a third language for your students: American math.

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Numeracy at the Downtown Learning Center

An alternative assessment process, four numeracy packets, and well-trained tutors result in successful math learners

by Avril DeJesus

Three times a year, anywhere from 150 to 350 students, in small groups, walk into a meeting hall in the Downtown Learning Center in Brooklyn, New York. The hall is furnished with seven round tables, each seating eight, and relaxing music plays in the background. The students are greeted warmly; their math skills are going to be assessed.

The Downtown Learning Center (DLC) is an eight-year-old church-based program funded by the Brooklyn Tabernacle Church and private donations. Begun as a program to teach reading to church members who wanted to follow services in their Bibles, participants also requested help in preparing for the tests of General Educational Development (GED). The program started with no paid staff; even the director was a volunteer. Now it has four full time staff, including a coordinator for English for speakers of other languages (ESOL), a GED/numeracy coordinator (myself), an administrator, and a director. We also employ a part time Adult Basic Education (ABE) coordinator and a part time receptionist. About 100 active volunteers, 35 of whom focus on numeracy, do most of the tutoring. Students can choose to attend morning, afternoon, or evening sessions, two to three times per week. Instruction is in small groups, generally between six to 18 students per group.

Located in downtown Brooklyn, the DLC has moved around a bit. First it was in a rented church basement, then the basement dinning hall of our sponsoring church, and now in its own facility. With ABE, GED, ESOL, SAT®, business, parenting, decorative arts, and leadership classes open to the community, the student population of approximately 500 reflects the African American, Caribbean, Hispanic, and Asian communities of the neighborhood and the variety of faiths they represent. The programs are free.

We were happy with our assessment tools for reading and writing, but fumbled with our approach to math placement. My experience prior to joining the DLC and since is that some adult students have severe math anxiety, so a standard math test might not convey their real knowledge and ability, but rather bring back bad math memories and fears. I knew of students who had their own contracting businesses or were incredible professional seamstresses, but froze when asked to solve simple decontextualized math problems. They wanted to know the formulas. Our goal was to have the students relax and show us what they know. I hoped that a more "user friendly" assessment tool, with familiar problems and resource materials, would enable students to feel comfortable and demonstrate the math skills they use everyday. We could then place them into groups with others of the same comfort level with numbers.

A Different Process

So, on assessment day, students start with reading and writing. If those skills are strong enough, they move into the hall to take the math assessment. Every eight minutes, six additional students join the assessment as the students at table one move to table two, table two people move to table three, and so on. At the prep table, the new students receive instruction on the process and purpose of the 50 minute exercise they are about to begin. We remind them that it is not a test and they will not receive grades. The information gathered will be used to place them in a small group with students at their level of math skill. They are each given a blank answer sheet on which they will answer all the questions they encounter at tables one to five. Everything is in English.

At the prep table they also receive a "Math and I..." sheet that asks them to write, draw, or express in some way how they feel about math. I got this form from a New York Math Exchange Group workshop (see *Focus on Basics* 4B, www.ncsall.net/?id=317). They complete this while waiting the three to seven minutes before moving to the next table where they will begin to answer questions. Usually at least a few of the six people at the table express some apprehension at the task before them and surprise that others feel the same way.

Learners are also informed at the prep table that "guesstimates" are a good idea and partial answers are considered. Some questions have answers that are correct if they fall within an acceptable range. To encourage them to estimate the way they would when working, shopping, or homemaking, some questions ask "about how many" of something are needed.

The next six tables are labeled "1" through "6". In front of each chair at tables one through five is list of questions. Prominent on the tables are tools such as calculators, rulers, play money, and scrap paper that learners can use to estimate or work out answers. Each table has questions relating to one of the five general GED math categories the way we approach them at the DLC: basic number sense, less than one whole, geometry, algebra, and measurements and data.

On the geometry table, for example, a question asks about the shape of a tile, the area and perimeter of the tile, and finally, about how many tiles would it take to cover the entire table. The table is strewn with actual ceramic tiles, paper cut out tiles, rulers, tape measures, and calculators. We often see some students get up and lay out the tape measure, while others lay out a few tiles and "guesstimate" based on the few. Still others find the circumference of the table and press away on the calculator until they find an answer that satisfies them.

Constantly Evolving

Since our numeracy assessment differs from what adults returning to school tend to expect, some are disappointed. They want a real test. They expect the test and school to be hard, painful; for some it still is. This is why the last table, the one marked "6", is for debriefing. A volunteer counselor seated there listens to student feedback, asking: How was this process for you? What did you like most? What would you change? He or she also assesses whether anyone is discouraged or upset. If everything seems okay, the counselor congratulates students for completing the process and assures them that everyone has passed.

The feedback gathered at table six is usually positive, but some people do not like having to move from table to table or that the time is limited. A few say the music is distracting. We review these comments for ideas about how to improve the process. For example, we used to allow 10 minutes at each table to solve as many of the problems as possible, with an additional two minutes for everyone to rotate from one table to the next. That made the process last from 80 to 90 minutes. Student feedback was that it was too long, especially since they had already done the reading and writing assessments and an interview before the numeracy assessment. Now everyone gets eight minutes at each table and one minute to rotate. For ease and speed of scoring there are now only five questions at each table, graduated in difficulty. The tutor at the table observes the students. When learners finish with time left, they write the amount of time left on their answer sheets.

Assignments

While students are discussing their experiences with the volunteer counselor, a second volunteer tallies

Table 2: *Shapes and Angles*

Part 1

Look at the tile.

1) About how many of these tiles would you need to cover the table?
2) About how big is the tile?

Part 2

1) Draw a square with a 2 inch side.
2) Draw a rectangle 2 inches by 1 inch.
3) Draw one 90 degree and one 45 degree angle.

their answer sheets while a third volunteer enters the number of answers they had correct into the database. The answer sheets are returned to the students as they leave the numeracy assessment area. The students put their answer sheets into their assessment folder with their reading and writing assessments and interview report, all of which are reviewed by the director. Students are placed in numeracy groups based on the total number of correct answers they have on the test. This puts them with students who are just about as comfortable working with math as they are. So far this has worked.

Materials

When students come to their first numeracy class they can purchase the packet of materials from which the group will be working for two dollars. They need to buy the packet within the first few classes so as not to fall behind. Tutors, who work with groups rather than individuals, collect the monies for their group. We used to distribute materials at no cost, but have found that the minimal fee helps to insure that students remember to bring the packet to school and that they do not get lost as frequently. It also helps us offset the cost of reproducing them. Most of the problems and activities in the packet have been created by our tutors or me. We have received permission from publishers to use the sections that are reproductions.

We have four unit packets: "Geometry: Those Wonderful Lines, Circles, Polygons, Surfaces and Solids!" "Less Than One Whole: Fractions, Decimals and Percents," "Introduction to the Operation, Language and Love of Algebra," and "Measurement and Data Analysis." As we go through these packets we also work on foundation skills if need be. The packets are intended to help facilitate both group and individual work, since participants are expected to do some work at home. Students have told me that they like seeing, in a nutshell, what the curriculum is that they will complete over the next several weeks. (We are hoping to offer our packets with our tutor training to outside organizations beginning in September, 2009.)

The units are not necessarily sequential. Since we work with adults who use numbers everyday, we can start anywhere. This September we started with the geometry unit and were into "Less Than One Whole" by mid November. When we started classes in January we went right into the algebra unit. We have new student registration three times a year, in

Geometry Projects

Be as creative as you like, but remember it is all about the math. You may work on your own, with a partner or a group. Your final projects represent your learning in the unit and will be judged on accuracy as well as creativity. Below are some ideas to get you started.

1. Design a quilt. Draw your design on grid paper. Name and measure the shapes.
2. Determine how many square feet are in your apartment. Draw your floor plan on grid paper to scale. Calculate perimeter and area of each room.
2. Construct a scale model of a structure and provide all the measurements.
3. Complete pages 272-331 in the GED Mathematics book. Show all your work.
4. Write a story about the shapes that you have been studying. Be as creative, funny or dramatic as you like, but make sure that your understanding of the material is evident.
5. Write a poem, rap or song about rectangles, triangles and/or circles. Present it.
6. Present 3 different right triangles (having sides of different lengths) and calculate the Pythagorean Theorem for each.
7. Using the "rectangle method", show how to find the area for at least 4 different triangles: a right triangle, an equilateral triangle, an isosceles triangle, and a scalene triangle.
8. Knit or crochet something circular. Show numeric values and how you find them.
9. What are concentric circles? Display both man made and natural examples.
10. Create a collage of round items (buttons, pasta, etc.). Give their numeric values.
11. What is the circumference of the earth, and what has Eratosthenes to do with it? Write an essay.
12. Make designs with a compass. What math can you give us from that? Explain and show the math.

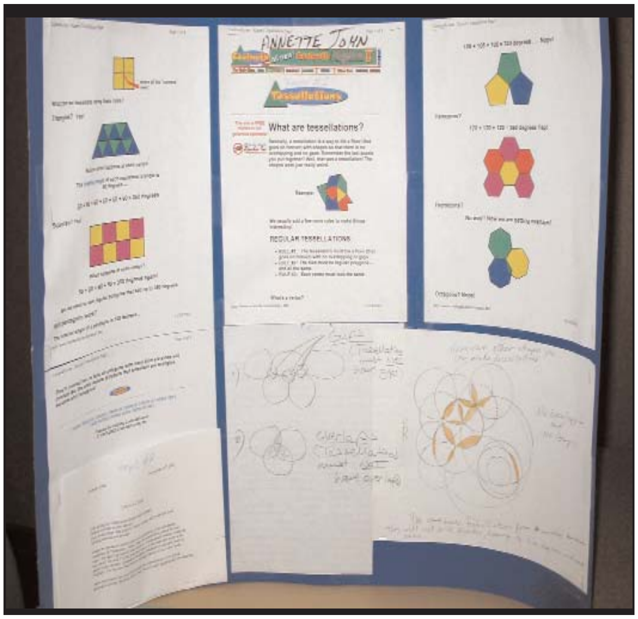
September, January, and April, and we usually start a new unit with each registration. New students are not totally lost since each term is like a new beginning for all the students.

The numeracy unit packets help to keep us all on track. Every pre-GED and GED group in our center uses the same material. Students who want to move ahead faster than the rest of their group can do so; we do not hold them back. At the same time, those who are absent do not have to fall behind because they can continue to work in the unit at home.

We try to make sure the material is relevant to all. There are graphs, word problems, and some activities that require hands on materials. Intermediate students, those found to need more reading, writing, and math work when assessed, need more tutor support than some of their fellow students and often repeat the packets in the advanced group the following year, if they remain with us.

Advanced groups get through the packets more quickly and move on to the recommended textbook pages that are printed on the front of the packets. The textbook pages are only required for advanced students. These students are also encouraged to take a timed GED preparation test to get ready to take the actual GED math test.

When students complete their packets, final project, and journal entries, their tutor and the program coordinator or director sign the certificate of completion at the back of the packet. Students usually display their certificates during their portfolio presentations in June. Completing a packet is a good way for students to see that they are making progress. They know that their time in school has been time well spent.



Numeracy Fairs

Our numeracy fairs are another way that we celebrate progress. To complete a packet, a student has to complete a project, demonstrating how the topic can be used in real life. The students display their projects during the four numeracy fairs we hold each school year. Each fair culminates with an awards presentation.

Students can get ideas for projects from the page-long list of suggested projects located on the second page of each unit packet. We try to include projects for every type of learner (see the box on page 31). Of course, students can come up with their own ideas and have their tutors approve them.

During the last numeracy fair for the "Less Than One" unit, a trophy-winning project included a gorgeous Christmas quilt. The student calculated the different percents in her quilt based on color, and she provided an additional chart with the conversions to decimals and fractions. Another student baked a cake and then cut it to show various portions in fractions, decimals, and percents. These winners also provided work sheets showing their measurements with percents, decimals, and fractions and how to make the conversions.

Other projects included a survey of the DLC students about their ethnicities and a pie chart to show the percentages of various ethnic groups within our learning center. Students sometimes elect to do research on the development of mathematical theorems or formulas. Students have built tables and houses to show how they use what they have learned. One student, Joe, once wrote a "Math Rap" (see page 33) which was published in one of our annual center publications and in an external publication.

Work Acknowledged

Every participant receives a ribbon; we award first, second, and third place trophies. Students must stand beside their projects and explain, defend, or answer questions about them. They need to demonstrate that they understand what they have done. Projects are judged on mathematical accuracy and creativity. Winners often accept their trophies with tears, exclamations of thanks to their tutors, program administrators, fellow students, and God. We have called it our own "Academy Awards."

Though some groups may not be finished with the unit, the numeracy fair signifies that we are preparing to move on to a new unit and encourages continued learning and application. Also, we get to have a celebration; and we love a good celebration. When we rejoice in our students, we affirm not just their work, but the fact that they can and are learning. Many of our students have never had their intelligence or success at school affirmed before.

Some students and tutors have balked at the prospect of having to do the projects, but most do them anyway, and everyone agrees that it is rewarding and valuable. The awards certificate at the end of the unit is incentive for completing all the unit requirements. It is signed only if the project is done. And every student gets a ribbon for their project entry at the fair. No one wants to be left out of that.

The numeracy fairs also encourage the tutors. When students win at a fair, the tutors' faces reveal how rewarding it is for them as well as for their students. Some of our tutors have even jokingly gotten into competing over which group has the best projects or the most trophy winners. They see that they are effective as volunteers. When winning students accept their trophies they often thank their tutors. Sometimes after a student has won a trophy, his or her tutor proudly tells me the story behind the project and the work he or she did to help the student.

Effective Tutor Support

By encouraging the use of manipulatives, real life examples, and student involvement, our materials are intended to support good practice. The tutors are able to use the material within the packets in the order they choose and bring in additional manipulatives, such as pizza, cake, or candy to help make the sessions more concrete. We do not generally encourage additional instructional material, however, because our material is intended to be comprehensive and the use of additional materials can mean that the students will not have time to complete the packet. If the tutor feels that something is missing from the packet, we do review it and sometimes incorporate it.

We support tutors through ongoing support sessions with a coordinator, other experienced tutor, or special guest who shares methodology, ideas for using the materials effectively, or an inspirational message. We have these sessions about once a month, often holding them during our daily half hour of Sustained Silent Reading (SSR). Although we like tutors to model SSR by reading alongside the students, sometimes we use this period for our monthly tutor support session because it enables us to have a captive audience of tutors. The time is short but effective; I usually like to use the session to model something that the tutors then use in class immediately.

Activities in the packets require use of hands-on materials. If a tutor thinks it is silly or childish to use rods or whatever manipulative is called for, the student notices and also feels childish, so it is important that the tutors are comfortable using these tools. I have to admit that this is an area that we need to work on. Just last week in a tutor support session, I asked tutors how they could encourage students to use Tangrams and Cuisenaire ® Rods. They decided to use them at least once each class session.

Usually during the summer, when class is out, the about four to seven of our 35 math tutors and I meet to review and improve our materials. We may meet weekly for four weeks depending on what we are trying to accomplish. One summer, for example, we removed an algebra puzzle that the majority of the numeracy tutors thought was not a good use of limited class time. We have removed some exercises, actually whole sections of the packet, that I thought were terrific but many tutors found either difficult to use or ineffective.

Some tutors still think that they need to use a textbook more often. We prefer that the students complete the packet first, but also encourage students who find using a textbook helpful to do so. Our experience is that students who complete the four units pass the exam to receive their GED diploma. I don't have hard data, but lots of happy souls.

Conclusion

As I prepare for our next registration and assessment, I think about Joe, Becca, and Allisa, who have been in the program for a long time and still have just the math section to pass. Hopefully this will be their year. Below is a poem Joe wrote a couple of years ago.

Math Rap

There is always some type of hesitation
When dealing with addition, subtraction and multiplication
You can't keep calm even though you try to be
There are many ways to deal with all that anxiety
Take a deep breath and count to ten
Don't give up try that problem again
We all have problems that we can't get
Knowing how to do it makes us less anxious
Perimeter, diameter, area, base times height
Gets us confused and fills us with fright
We won't give up, gotta continue to fight

----Joseph Wyatt

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We are still here to support him and all the others who come for help in their quest to improve their numeracy ability and achieve their goals. We are blessed to have committed staff, students, and volunteers in our community of learners working together to grow a learning center in Brooklyn. We average about 75 GED graduates per year out of about 90 who attempt it. Some of them, like Juvon, now a senior in college, have become terrific tutors, teaching others some of the math that once stood between them and their dreams.

About the Author

Avril Osborne-DeJesus, who has 13 years of experience in ABE, is the Downtown Learning Center's GED and Numeracy Coordinator. A GED holder herself, she went back to college at age 30 and graduated cum laude from Brooklyn College.❖

DLC GEOMETRY: 2007-2008

Name:____________________ **Date**______________

Activity & Page	Work Log & Journal
1. Geometry Projects Page 2 Date __________	Review and select a project to be completed prior to the Numeracy Fair. Write it here. Why did you choose that one?
2. Let's Get Started Page 3 Date __________	Were you tempted to go to a dictionary? Are you glad to be getting started? What do you think is the most important thing for you to learn in this unit?
3. Hands on Learning Page 4 Date __________	Did working with your hands increase your understanding? Do you like working with manipulatives (tools that help you visualize the math)?
4. Angles Pages 5 - 17 Date __________	Which of these pages did you like working on the most? Do you have a good understanding of how to measure angles now? Do you think you can look at any angle and give a good estimate of the angles measurement?
5. Three Important Drawings Page 18 Date __________	Did you write at least 3 characteristics for each drawing? Do you think you will remember their names?
6. Congruent Figures Pages 19 Date __________	Did you cut out the figures? Do you think you might need to know this information?
7. Measuring Lines, Angles & Figures Page 20 - 22 Date __________	Did you already know most of this? How do you think knowing these definitions will help you?
8. Word Problems Page 23 - 24 Date __________	Do you like these problems? Did you draw pictures to help you? The GED exam is loaded with word problems, and you can use scrap paper.

The Importance of Algebra for Everyone

by Tricia Donovan

"Finishing a math course beyond Algebra II more than doubles the odds that a student will get a bachelor's degree, and in 2000 the median income of someone with a bachelor's degree was nearly twice that of someone with a high school diploma," wrote educator Jonathan Osler (2007) in a paper he posted on the Web site "Radical Math," which provides resources for educators interested in introducing issues of social and economic justice into their math classes and curricula. Algebra matters!

Bob Moses, founder of the Algebra Project, an organization that uses mathematics as an organizing tool to ensure quality public school education for every child in America, sums up much of the problem when he says, "The most urgent social issue affecting poor people and people of color is economic access... [and that] depends crucially on math and science literacy." Math literacy education must be offered to everyone, child or adult, cognitively gifted, average, or slow. We must champion math literacy (numeracy) as vociferously as we advocate for language literacy. Both numeracy and literacy contribute to the personal, social, and economic success of citizens in the 21st century. And given its pivotal importance in college-level mathematics, algebra I in particular must be made accessible, especially to adult basic education (ABE) students.

The challenge before us as adult educators is to teach all students to think algebraically and to use the tools of algebra effectively. We must acknowledge that the roots of algebra are situated in 'lower level' mathematics, and those roots need to be cultivated in classes for ABE, pre-GED and English for speakers of other languages (ESOL) students.

Big Ideas in Algebra

To nurture algebraic thinking it helps to understand what we mean when we refer to algebra. "For most adults, notation such as use of variables, operations, and equal signs is the chief identifying feature of algebra," according to Susan Jo Russell of TERC, a not-for-profit education research and development organization dedicated to improving mathematics, science, and technology teaching and learning. However, algebra is more than x's and y's. The notation expresses rules about how operations work; rules that students can reason out for themselves. This reasoning —about how numbers can be taken apart and put together under different operations—is the primary work of elementary students in algebra (Russell, 2007, p. 2). Russell describes four areas that form the foundations of algebra: generalizing and formalizing patterns, representing and analyzing the structure of number and operations, using symbolic notation to express functions and relations, and representing and analyzing change.

Others name three big ideas central to algebra: equality, function, and generalization (Driscoll, 1999; NCTM, 2000). Equality can be thought of as representing and analyzing the structure of number and operations and using symbolic notation to express functions and relations. Understanding equality helps set the stage for what Mark Driscoll of Educational Development Center, a not-for-profit education research and development organization, calls the 'doing and undoing' actions that are inherent in simplifying expressions and solving equations. Function relates to representing and analyzing change and working with patterns. According to Driscoll, those who "focus on the important role that functions play in algebra, ...may characterize algebraic thinking as the capacity to represent quantitative situations so that the relations among variables becomes apparent." Generalization refers most commonly to abstracting from computation the regularities that exist independent of particular numbers (for example, the fact that order does not matter when totaling a series of numbers) and to the creation of equations that represent the general rules of a pattern.

> "...the roots of algebra are situated in 'lower level' mathematics..."

This article shares one way to approach the teaching of algebraic thinking and expression that can facilitate ABE students' understanding of the purposes, meanings, and uses of algebra, starting with adding single digit numbers.

The Equal Sign

How many of us have bothered to ask students what the equal sign means? One Massachusetts ABE teacher, Marilyn Moses, did just that. She read an article by E.J. Knuth and colleagues entitled *Does Understanding the Equal Sign Matter? Evidence from Solving Equations* (2006), then she read equal sign materials shared by the author at the 2007 National College Transitions Network conference in Providence, RI. Knuth had asked eighth graders what the equal sign means. They discovered that almost all students, even those who had been taught some algebra, retained an operational view of the equal sign. That is, they saw the equal sign as meaning such things as:

"What the problem's answer is"

"The total"

"How much the numbers added together equal"

"Do something"

When Moses asked her adult students the same question (see activity below) she was surprised to discover that her students held the same limited conception. She thought for certain she would hear statements like:

"It means that what is to the left and right of the sign mean the same thing."

"The expression on the left side is equal to the expression on the right side"

"The left side of the equals sign and the right side of the equals sign are the same value."

Moses was surprised. All of her students saw the equal sign as a trigger to give the answer. Her adult students did not know the relational meaning of the equal sign, knowledge that she and most math teachers realize is fundamental to algebraic work. An accomplished math teacher, she was disconcerted to think that she had assumed an understanding of a basic mathematical symbol that did not exist. Such a mathematically limited understanding, she understood, makes the whole idea of balancing equations or doing and undoing operations within expressions feel like a new rule in math, a rule that contradicts earlier learning.

Below are the questions Knuth and his colleagues asked students to understand their conceptions of the equal sign:

The following questions are about this statement:

$$3 + 4 \underset{\uparrow}{=} 7$$

(a) The arrow above points to a symbol. What is the name of the symbol?

(b) What does the symbol mean?

(c) Can the symbol mean anything else?

What value of m will make the following number sentence true?

$4m + 10 = 70$

[Editor's note: We deleted a second number sentence.]

Our ABE students, like most students who received conventional math instruction, learn early to think that equal sign signifies "find the answer." Without thinking, we, and textbook publishers, frequently expose low-literacy students to equations such as:

5 + 7 =
12 + 8 =
6 - 3 =
17 - 9 =
5 x 3 =
8 ÷ 2 =

With these sorts of equations, we look only for a total or an answer; we do not look for the relationship between numbers.

Fostering Relational Understanding

To foster the relational understanding of the equal sign, we can use equations such as these, which are appropriate at the same levels:

4 + 2 = 3 + ___
___ = 0 + 5
___ x 3 = 3
36 ÷ 2 = 18 ÷ ___
___ + 7 = 9 + 3 + 7

Not only do such equations force attention to both sides of the equal sign, they can set the stage for discourse about general rules of arithmetic central to algebraic manipulation of equations, for example: commutativity $a + b = b + a$; identity $1 \times a = a$; associativity $a + b + c = a + (b + c)$. Students' understanding of these rules at the arithmetic level is achievable, as I witnessed in my GED classes years ago. Learners can be exposed to many examples of a rule then asked, for instance, what the examples show them about the order of numbers in addition problems and then asked if the same rule holds true for subtraction. Or, they can be asked what a different set of examples shows them about multiplying a number by one.

As students come to understand, *Oh, yes, any time I multiply a number by one, I get the same number (the identity property)* or *Any time I divide a number by one, I get the same number*, they grasp a fundamental mathematical truth. Then, as students learn that $^4/4$ or $^2/2$ or $^a/a$ equals one, they gain a valuable tool for later use in algebra. The apparently simple knowledge that 1 x 4 = 4 because any number multiplied by one equals itself combined with the knowledge of the various representations for 'one' helps later when we move to situations where we might do this:

$$\frac{X^2}{X} + 4 = 8$$

$$\frac{X(X)}{X} + 4 = 8$$

$$X + 4 = 8$$

Equations emphasizing the relational meaning of the equal sign also nurture number sense by encouraging students to reason about values and their relationships. For instance, in the equation 24 ÷ 4 + 6 = 12 ÷ 4 + ___, we can, of course, perform the operations in order to find a total on the left and then determine the total on the right and what we need to add to make both sides equal. However, we can also reason that 12 divided by four will be half of 24 divided by four, so the value added to the right must be half again as big as the six on the left; therefore, the missing number must be nine. This type of reasoning provides the mental gymnastics that prepare learners for the tasks of algebra.

The Equal Sign
=

The equal sign was first recorded in 1557 by Welsh physician and mathematician Robert Recorde in *The Whetstone of Witte*, a book for teaching arithmetic. He is reported to have used the symbol to indicate equality because he believed that nothing could be more equal than two parallel lines. The equal sign was not popularly used until centuries later.❖

This earlier work in reasoning, generalizing and working with notation prepares students to work on equations such as:

$$\underline{\quad\quad} = X + 0$$

$$X \times Y = \underline{\quad\quad} \times X$$

$$2X + 5 = 5 + \underline{\quad\quad}$$

$$\frac{10}{2} + 5 = \frac{20}{X} + 5$$

> *"Equations emphasizing the relational meaning of the equal sign also nurture number sense by encouraging students to reason about values and their relationships."*

The recognition of a developmental process in algebraic thinking paves the way for ABE teachers to retool our early level mathematics' instruction. Thinking developmentally, we act on the knowledge that adult students are complex individuals with prior knowledge that can be built upon, or in some cases, needs to be 'unlearned' in order to move forward. We also apply the knowledge that presenting ideas at a concrete level before moving to the abstract solidifies understanding. From our developmental perspective, we recognize that we must first uncover misconceptions and then scaffold learning to help students rebuild mathematical notions on more solid ground, ground that will support later algebraic work.

Helping students pass the GED by better understanding mathematics and algebra, in particular, and helping to prepare students for transitions to next steps, begins long before they arrive in a GED or college transitions course. It begins, mathematically speaking, as students learn to add and subtract, learning inverse operations and the conceptual meaning of symbols. When we introduce the equal sign, we can begin to prepare students for algebra by staying conscious of the relational meaning of the symbol =. Everyone can access algebra given thoughtful conceptual development and examination of teaching assumptions, and every adult student deserves the chance to succeed provided by conscious mathematics instruction at all ABE levels.

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Always, Sometimes, Never True

The following exercise is best used in a GED or College Transitions Course, though it can be adapted for lower level classes by replacing equations with variables with equations like the first one, labeled " a)". Students work independently on the problems then share responses with a partner. The whole class convenes to discuss each statement and the reasoning behind classifications as 'always, sometimes, or never true.' Follow-up for this exercise might include asking students to create their own examples.

Are the equations below always, sometimes, or never true? Share your thinking with a partner. Do you both agree? For the same reasons?

a) $4 + 5 = 7 + 2$

b) $r2 = r \times r$

c) $3x + 1 = x + 7$

d) $x + 4 = x + 6$

e) $x - 7 = 7 - x$

f) $x \cdot 1/x = 1$

g) $x = y + 2$ therefore $2y + 4 + x = 3y + 6$

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About the Author

Tricia Donovan has been active teaching GED preparation and developing curriculum: she is one of the authors of the adult education *EMPower* Math Series published by Key Curriculum Press (adultnumeracy.terc.edu/EMPower_home.html). Tricia has presented nationally and throughout Massachusetts on math-related topics and is currently the Massachusetts ABE Math Initiative Coordinator. She earned her doctorate in education, specializing in curriculum development and curriculum reform and serves as a staff development specialist for the Massachusetts' System for Adult Basic Education Support, based at World Education, Boston. ❖

Algebra-Related Resources

For materials that focus on ways to develop a notion of linear functions that emerges from more basic work through pattern generalizations, check out these publications:
EMPower text, Seeking Patterns, Building Rules: Algebraic Thinking, at: adultnumeracy.terc.edu/EMP_books.html.
Math for All Learners: Algebra, by Pam Meader and Judy Storer, offers worksheets and directions for contextual pre-algebra and algebra problems suitable for the GED class. Published in 2002 by Walch Publishing.
Myrna Manley's book *The GED Math Problem Solver: Reasoning Skills to Pass the Test*, offers a nice way to introduce algebraically-based notation and properties at the whole numbers level of mathematics. ❖

TIAN: A Professional Learning Model for ABE Math Teachers

by Beth Bingman & Mary Jane Schmitt

Research has found that mathematical proficiency includes being fluent with procedures and much more. It indicates that effective math learning and teaching should also attend to conceptual understanding, strategic competence, adaptive reasoning, and a productive disposition (Kilpatrick, et al., 2001). While such research refers to children learning math, we, the principal investigators of the Teachers Investigating Adult Numeracy (TIAN) project, believe this definition of math proficiency holds true for all ages. Here, we describe the professional learning model we believe is needed to support teachers who wish to base their instruction on this definition, and we present some of the initial data about the impact of the model on teachers involved in the pilot year.

Here's the problem as we see it: We tend to teach the way we were taught. That means teachers who were taught only math procedures, now teach only procedures, and those who were taught skills, concepts, and strategies in an environment that encouraged reasoning, communication, and problem-solving, now teach that way. While some of us did quite well in the rather uni-modal, procedurally-focused (and silent) math learning environment of our youth, many more of us did not. Many of us became uncomfortable with math, or just stopped taking it beyond what was required. While not true for all adult basic education (ABE) math teachers, we find this to be true for many. And as a result, in visits to ABE classrooms —and we include ABE through preparation for the tests of General Education Development in this—, we are much more likely to see teachers experimenting with innovative approaches to instruction in reading, writing, and social studies than in math. In testing a professional learning model via TIAN, our central question is: What does it take to help the adult education workforce move closer in belief and practice to mathematics instruction that addresses fluency with procedures, conceptual understanding, strategic competence, adaptive reasoning, and a productive disposition?

> "What does it take to help the adult education workforce move closer in belief and practice to mathematics instruction that addresses fluency with procedures, conceptual understanding, strategic competence, adaptive reasoning, and a productive disposition?"

The TIAN Project

TIAN's primary focus is on teachers' learning. A total of 40 Massachusetts and Ohio teachers participated in the pilot phase of TIAN in 2005-2006, and 76 teachers from Arizona, Kansas, Louisiana, and Rhode Island took part in the field test in 2006-2007. In 2008, teachers from all six states are taking part in a variety of activities to support and extend and share what they have learned. The goals of TIAN are

- to increase and deepen teachers' mathematical content knowledge;
- to increase the number and range of teachers' instructional approaches; and
- to increase teachers' knowledge and use of state mathematics content standards.

TIAN's mathematical content centers on two strands of mathematical proficiency: algebra and data. While a comprehensive instructional program in ABE mathematics must also include the development of number and operation sense and geometry and measurement, we chose to focus on algebra and data analysis for several reasons. Algebra, the "gatekeeper" subject, is, as Robert Moses (2001) believes, essential for full citizenship. Understanding the presentation of basic statistics in the media is also essential. Moreover, algebra and data analysis have received added emphases on the most recent edition of the GED exam and in the most recent sets of adult-focused standards. However, we have found both to be areas with which current teachers are uncomfortable, and which are often taught only to high level students. TIAN intends to help teachers build their confidence and competence in algebra and data by involving them in doing math as well as in learning how to teach math.

> *"The institutes and teacher meetings are structured in ways that ask teachers to be learners."*

TIAN gives participating teachers opportunities to learn new instructional approaches, including:

- working collaboratively on open-ended investigations;
- sharing strategies and knowledge orally and in writing;
- justifying answers in multiple ways;
- using contexts that are meaningful to adults; and
- exploring a variety ways for entering and solving problems.

These instructional approaches are intended to increase students' opportunities to learn and are supported by research on principles of effective teaching (Brophy, 1999; Bransford et al., 2000; Hiebert & Grouws, 2007).

One of the challenges of beginning to use new approaches to instruction, particularly approaches that are not based on rigidly sequenced published materials, is assuring that necessary content is covered at appropriate levels. Adult education mathematics content standards and curriculum frameworks can provide that structure. TIAN training includes each state's standards, and teachers are helped to connect their instruction to their state standards.

Professional Learning Model

The professional development provided by TIAN is both extensive and intensive. Two processes for teacher change that have been shown to be effective in mathematics education play a central role in the model. The first is the opportunity for teachers to do mathematics themselves with an emphasis on learning with understanding (Ball, 2000; Hill, et al., 2005). In the first year, participating teachers attend three institutes; in each, they spend two days doing mathematics together, sharing their work, and analyzing how they can apply what they learn in their classrooms. They experience new approaches first-hand (see the box below for an example). The institutes and teacher meetings are structured in ways that ask teachers to be learners. The second process effective in math education is the opportunity to conduct close examination and discussion of student work. Between institutes, the participants teach lessons on data and algebra that they adapt from math materials they develop and from a resource series called EMPower, developed by TERC (see adultnumeracy.terc.edu/EMPower_home.html). The teachers document their planning and

Snapshot: A TIAN Institute

Potatoes are sitting in a bowl next to some vegetable peelers; a pile of pennies with coin wrappers are on a nearby table; on another table a bunch of envelopes are waiting to be stuffed. Twenty ABE teachers break into teams of four and rush to one of the stations to begin to do a "sample of work" to determine how long it would take to help out at a community event by peeling 50 pounds of potatoes for a huge potato salad, rolling 10,000 pennies, and stuffing 1,000 envelopes. Everyone is left to their own devices, and all five teams take different tacks: some have one team member do the work, while another records how long it takes to stuff of 10 envelopes; others count how many envelopes can be stuffed in a minute; others test out what can be done in two or five minutes. Some build up to 10,000 by calculating in their heads, others round numbers with confidence. Some use good old-fashioned paper and pencil computation, others punch numbers into calculators. Everyone is on-task and having a good time.

Once they have completed the tasks at each station, the groups post their results on newsprint. The facilitator asks them to describe strategies and why the strategies work or don't. People seem amazed that there are so many ways to arrive at a reasonable answer. If one estimate is way off, the whole class focuses on why. The facilitator pushes the participants to compare, contrast, and make connections between the various strategies.

The teachers have "lived" the lesson they will be trying out in their classes. Next, the teachers examine student work. They read a classroom vignette which describes a dilemma that came up for a group of students and are asked, "What would you do next as a teacher that would be helpful?" If you compare this active open-ended exploration of ratio and proportion with the typical way proportion is presented — setting up two ratios and cross-multiplying —, you get a sense of what goes on in a TIAN Institute as well as the extent to which we are encouraging teachers to stretch their mathematical understandings and classroom practices. ❖

instruction in two detailed samples in which they describe what they have done and why and how three students at different levels responded to the instruction. Some of these samples have been posted on the website to illustrate to other teachers how the lessons played out in an adult education classroom (see adultnumeracy.terc.edu/TIAN_worksamples.html). Groups meet four times during the year at between-institute regional teacher meetings.

Insights from the Pilot Year

In an effort to understand the effectiveness of TIAN, we have gathered a variety of data from the participating teachers. We also conducted observations in many classes. We are still collecting and analyzing data, but we have looked closely at the experiences of the teachers from Ohio and Massachusetts, who were part of the pilot phase of the project. In this article, we share some of what we learned from questionnaires these teachers completed before and after the institutes in the pilot year and from interviews conducted with 22 of these teachers a year later in the spring, 2007.

The 40 Massachusetts and Ohio teachers who participated in the TIAN pilot told us about their teaching situations in an initial questionnaire. Three-quarters of them taught in open-entry, open-exit programs sponsored by a school district, city, or community-based organization. Their classes met from one to five days a week, with the majority (59 percent) meeting for six or fewer hours in a week. Of that time, they reported their students spent an average of 3.6 hours a week doing math. Most—90 percent—of these teachers taught other subjects as well as math. They had a range of four to 18 students per class with an average of nine students. Eight of them had classes with 50 percent or greater students for whom English was not a first language.

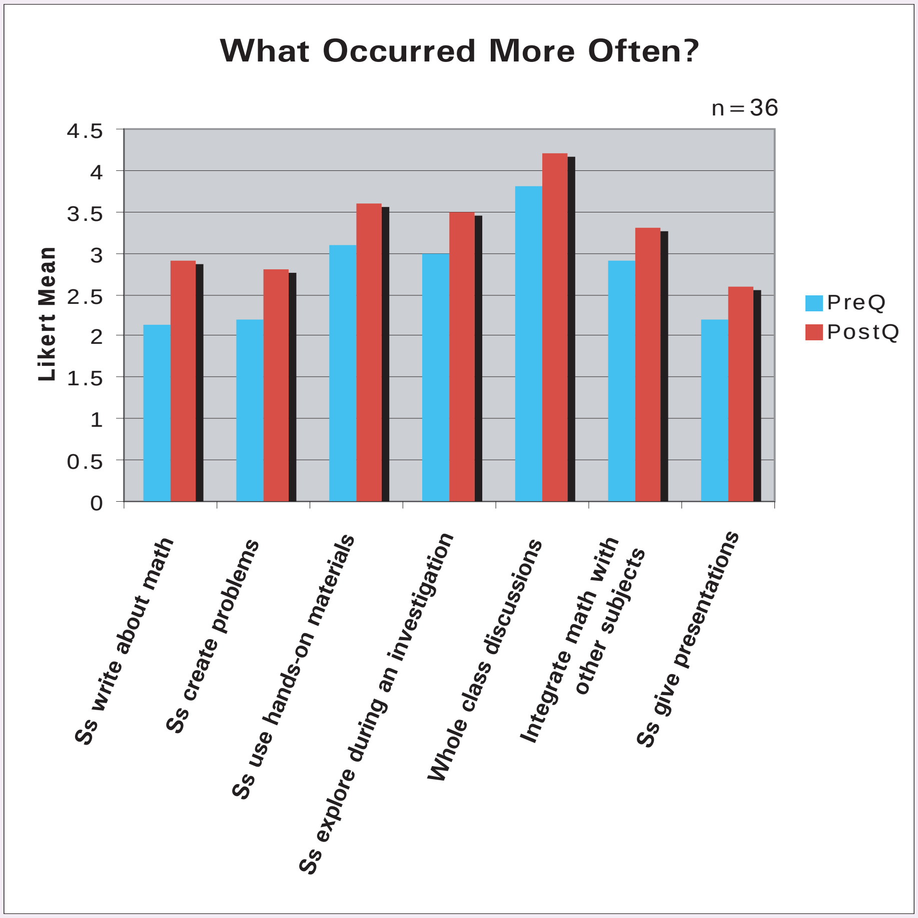
All but three of the teachers in the pilot group had taken at least one college-level math class and nine had graduate level courses in math. A slight majority of these teachers (55 percent) described themselves as "very comfortable" with their level of math knowledge.

Changes

We are interested in the impact of the professional development intervention, such as what kinds of changes occurred and whether the direction of the change was promising or problematic, especially with respect to the project goals and objectives. In a pre-questionnaire, the teachers supplied information about their assessment of their math content knowledge, beliefs, teaching practices, style, philosophy, comfort, and how they determined what to teach. After taking part in the pilot institutes and implementing some suggested approaches in their classes, the teachers completed a comparable post-questionnaire. Comparing a section of the pre- and post- questionnaires reveals some interesting change in instructional practice.

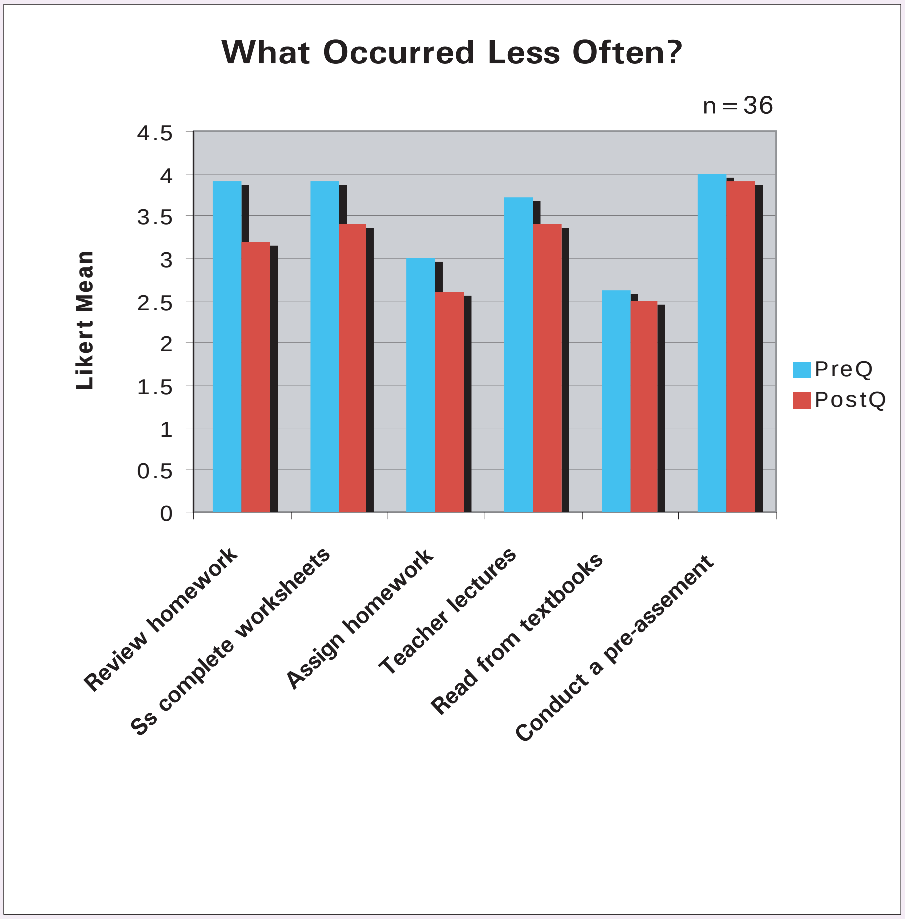
The teachers were asked to estimate how often they used various types of activities in their math classes. They rated the frequency of occurrence on a Likert scale as "never = 1, seldom = 2, sometimes = 3, usually = 4, or always = 5." Our analysis of the results is that some change to a more active, reflective experience for students occurred.

As shown in the graph on this page, teachers reported that students wrote more often about the math they were learning (the greatest change), created more of their own problems to solve, and used more hands-on materials. Also, students gave more presentations, explored problems in groups, and had more whole class discussions. Math was more often integrated with other subjects. All of these actions were encouraged by the TIAN initiative in various ways.



What occurred less often is also interesting. The graph on this page shows that teachers reported that overall they did less lecturing, that students used fewer worksheets, and that there was less reading from textbooks. We are pleased that teachers seem to have added more strategies to their teaching repertoire, so are less dependent on lecture, texts, and worksheets. We cannot account for the change of less homework and homework review.

The data displayed in these two graphs comes from the self-reported data provided by the teachers who participated in the pilot; it reflects what they were thinking immediately after they had finished participating in the intensive year of institutes and small group work. While this information is valuable, we will not really understand the lasting impact of TIAN on participants until we gather and analyze data again after more time has passed.



A Year Later

Nearly a year after the final pilot institutes, we invited the pilot teachers to take part in a follow-up interview. Of the 40 pilot teachers, 22 participated in these semi-structured phone interviews. All 22 were still teaching and 20 were teaching ABE math. We learned that most of them were using ideas and approaches introduced in TIAN. We also learned about the challenges they were facing as they implemented TIAN in their classes.

One set of interview questions focused on teachers' assessments of what they had learned. What these teachers remembered as most useful about TIAN were: learning math content (6), working through math anxieties (6), learning new pedagogical approaches (5), and taking part in hands-on math activities in the institutes (5). When asked about changes in their understanding of math, six reported their understanding had improved or deepened, three spoke of specific math knowledge they had gained, and five mentioned new ways of thinking or approaches to solving problems. Five teachers reported no change in their math understanding. All but one of these 20 teachers reported changes in how they understood teaching math, including eight who talked about using "real-life" materials or activities and seven who reported new understanding of the importance of focusing on mathematical concepts and problem-solving.

Another set of questions focused on how these teachers were using what they had learned. When they described a typical class, 17 of the 20 teachers mentioned using at least one of the features that TIAN supports. A dozen spoke of using materials or activities from the EMPower series or that had been modeled in TIAN institutes, for example having student develop frequency charts and bar graphs to report on the country of origin for the clothes they were wearing that day. Seven had provided hands-on activities for their students, four had used a real-life context as part that class, and four had involved students in small group work. Two teachers described classes that had combined math with another subject, for example having students write about their driving habits after they had done a lesson on gas prices.

Nearly all the teachers interviewed reported implementing at least some aspect of what they had learned and experienced with TIAN. However, nearly all also reported problems or challenges in doing so. The most-reported challenges included finding the material to be too difficult for their students, not having enough of their own preparation time, having students resist new instructional approaches, and what Beder and Medina (2001) have described as enrollment turbulence: students entering and leaving in the middle of a term (or class) and erratic attendance. Factors other than teachers' understanding of mathematics content and pedagogy impacted the extent to which they were able to implement new approaches.

In Conclusion

This article provides an introduction to the TIAN project and some initial indications of what teachers are learning from it. When we look across the results we report here, we see strong indications of change in the number and range of participating teachers' instructional approaches in mathematics. At the end of their year's involvement in investigating math, they had moved away from lots of drill, a strict sequence of skills, and the exclusive use of workbooks. Nearly all the teachers we interviewed reported that they now use more hands-on activities, have students explore possible solutions, and increased communication about math.

While we are still in the process of developing TIAN and seeking resources to conduct a fuller impact study, we believe that much of what we are learning indicates that our techniques are useful and can be helpful to other professional developers working with teachers on mathematics. Our advice is to go for depth rather than breadth; working with a smaller group of teachers over time gives them opportunities to experience and then implement different approaches to learning and teaching math. Then, increase the numbers with whom you are working by using successful teachers as catalysts to lead the work. They might lead study groups, facilitate regional meetings, serve as mentors in their programs. Use multiple formats over time—regional teacher meetings, work with students, individual reading and reflecting, and on-line discussions—to continue to support and involve old and new teacher participants. Collect and analyze data. Programs like TIAN are not cheap, and funders and participants need evidence of its impact. In addition to professional development, look at other factors that may affect the ability of teachers to facilitate effective math learning. Teachers need time to plan and to carefully examine student work. Managed enrollment may lead to more stable student groupings. TIAN is a model of professional development in mathematics for adult education teachers that encourages reasoning, communication, and problem-solving in mathematics learning and in mathematics teaching. Join us in this effort by visiting the TIAN website at adultnumeracy. terc.edu/TIAN_home.html.

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About the Project

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ANN

The Adult Numeracy Network (ANN) is a community dedicated to quality mathematics instruction at the adult level. Members support one another, encourage collaboration and leadership, and influence policy and practice in adult math instruction.
Visit the ANN Web site at literacynet.org/ ann/index.html
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Arizona's Professional Learning Journey through the Teachers Investigating Adult Numeracy Project

by Beverly Wilson & D. Roberto Morales

For many GED candidates, the thought of taking and successfully passing the mathematics portion of the tests of General Educational Development (GED) may be quite intimidating, and with good reason. According to the 2006 GED Testing Program Statistical Report, published by the American Council on Education, the lowest mean score on any sub-test from 2003-2006 was observed for the mathematics test. Arizona's data also reflected this national trend. Concerned with the mathematics achievement of adult learners, the Arizona Department of Education, Adult Education Services professional learning unit, partnered with the Center for Literacy Studies at the University of Tennessee and TERC, a Cambridge, MA, educational research center, to implement the pilot project, Teachers Investigating Adult Numeracy (TIAN), in the fall, 2006.

Arizona's adult education instructional delivery system consists of 27 adult education providers representing community colleges, local education agencies, community based programs and probation departments, and 35 GED Testing Centers. Teachers use state content standards to guide their instruction of adult basic education (ABE) and adult secondary education (ASE) students (see www.ade.az.gov/adult-ed/). In addition, the Arizona Department of Education has adopted the National Staff Development Council (NSDC) Standards for Staff Development (see www.nsdc.org), which address three important questions:

- What are all students expected to know and be able to do?
- What must teachers know and do in order to ensure student success?
- Where must staff development focus to meet both goals?

A 2006 survey of Arizona's adult education teaching force revealed that only 40 percent of the 183 teachers who responded hold teacher certification in elementary, secondary, or special education. These teaching certificates require the completion of college courses in teaching methodology, instructional strategies, and subject matter content, as well as successful scores on the state administered content proficiency assessments. The data collected in the survey indicated that the majority of Arizona adult education teachers do not have a background in teaching methodology or mathematics content and need the opportunity to build their skills and knowledge.

Project TIAN seemed to be a good match for Arizona for a number of reasons. State staff member Beverly Wilson, a former Arizona ABE program director, had identified the need for improved math instruction for ABE teachers in her local program based on student assessment data from the Test of Adult Basic Education (TABE) and the GED Practice Test. The state GED data also indicated that math was a need. In addition, the TIAN goal of increasing the knowledge and skills of ABE teachers via a job-embedded, standards-based, and results-driven model matched Arizona's Adult Education Services (AES) goals. Furthermore, state staff determined the TIAN model could be replicated and disseminated to all ABE teachers in the state after the completion of the pilot project. Timing was right: TIAN was looking for pilot states. So we, Arizona's AES staff, submitted a proposal to TIAN and were selected to participate.

The Beginning

State staff and TIAN researchers together identified a target audience, set dates for three mandatory two-day institutes, chose strategies for communicating with the field, and established roles and expectations of both the TIAN and state staff. (See the table for a list of state and TIAN roles). We also discussed how to apply the Arizona adult education mathematics content standards during the project. We decided that the pilot project would be limited to a maximum of 20 teachers to ensure that we could manage the project effectively within the allocated budget.

To participate in the project, teachers had to be currently teaching ABE or ASE math. They had to make a two-year commitment to the project during which time they promised to actively participate in three two-day TIAN institutes, complete homework assignments, and attend regional meetings with fellow TIAN participants in between the institutes. In addition, educators could earn up to 45 professional development hours to be applied toward their teacher re-certification.

Interested teachers completed an application that requested information on their teaching experience and attitudes about teaching and learning math. Each applicant's program administrator was required to review and sign the application to ensure that the employer—the ABE program—understood the project requirements and would provide paid time and/or release time so the teacher could participate fully. The application process did not exclude teachers based on their responses. Twenty adult educators applied to participate in the first year of TIAN.

Of the 20 applicants, one was excluded for not being employed by a state-funded adult education program. The other 19 applicants, all of whom were selected, were from a cross section of regions around Arizona and represented community colleges, local education agencies, and community-based adult education programs. This regional diversity was a goal, we were also happy that seven of the participants were from the two largest adult education programs in the state. These programs collectively serve about 13,000 adult learners annually, representing 60 percent of the adult learners enrolled in state funded programs each year. We knew that teachers from these programs could help influence math instruction to substantial numbers of ABE learners in the state.

Three state staff also attended the TIAN institutes and regional learning community meetings as participants to learn the about the TIAN process, content, and methodology, and provide technical assistance when appropriate.

Implementation and Results

The first of the three face-to-face TIAN institutes was held in the fall, 2006. The institute focused on increasing teachers' understanding of data and algebra related math content, such as using and analyzing data sets. The second institute focused on

TIAN Implementation Responsibilities Phase 1

State Staff	TIAN Staff
Recruiting ABE math teachers	Planning and facilitating three institutes
Ongoing communication with teachers & program administrations	Collecting and analyzing participant assignments and student assessment data
Logistics of institutes	Phone calls and email communications with participants regarding assignments and project tissues
Travel arrangements for teachers and administrators	Arranging multiple classroom site visits for teachers
Financial management	Completion of research project requirements
Monthly phone calls with TIAN researchers	Updating state staff on logistical or participant issues

algebraic thinking and patterns, and linear and non-linear functions. The third and final institute covered proportional reasoning, and building number and operations sense. The TIAN researchers administered pre- and post-assessments to the teachers on their mathematical skills and concepts.

In addition to the institutes, teachers designed and taught six to 12 math lessons from the *EMPower* book series (www.terc.edu/ work/644.html), provided student release forms and TABE assessment data from their enrolled students, participated in four regional learning community meetings, submitted work samples, and read and responded to two on-line articles on teaching math. The TIAN staff also worked with teachers to design classroom lessons using the specific math operations from each institute that aligned to the AZ math content standards. Based on their research criteria, TIAN staff selected and observed in six of the 19 teachers' classrooms.

Of the 19 teachers who started Project TIAN, one left the adult education field, one moved out of state, and 17 successfully completed the all the activities. At the end of the year-long pilot, state staff examined the original applications submitted by the phase I teachers and the feedback from the teachers that was collected by the principal investigators. At the beginning of the TIAN project, teachers described math instruction as difficult because the students lacked prior mathematical knowledge. One teacher wrote: "Many students have a poor foundation in math. As high school students, they either were not attending school or not paying attention to math instruction."

Another teacher wrote, "Students often have memory problems, and what might be learned one day, is not necessarily stored and recalled the next day." Student self-efficacy and math anxiety were perceived as significant challenges. One teacher reported that a primary focus of her job was helping students cope with math phobia and to develop confidence in doing math. Additional perceived challenges included attrition, self-efficacy, prior knowledge, and helping students learn mathematical procedures or the step-by-step math process.

> *"Overall, Project TIAN participants perceived more confidence in their own knowledge of mathematics, how to listen to students to better understand the process the students are following in solving math problems, and how to refine their instruction to help students be successful."*

Feedback from the culminating survey indicated that teachers who completed phase I reported that TIAN helped them to focus on student learning, and the process of solving math problems rather than just solving the math problem and getting the right answer. One teacher stated that, "TIAN had value for me because it forced me to stop and think about the process and the students' thinking and not the problem. AND it forced me to think about how I was going to reach the student and connect (and build on) what they already know to what they did not."

Another teacher reported that she very much benefited from the TIAN project. "It changed the way I want to teach math and want students to learn math, and it has reminded me forcefully that we all may be 'seeing' things quite differently."

Due to the intensity and duration of Project TIAN, and the regional learning community model, teachers reported they formed some very strong personal and professional relationships. In addition, instructors perceived being more confident about their math instruction, content knowledge, and teacher self-efficacy. One teacher reported that, "The most important influence this project had on me was expanding my view and ability to teach data analysis and algebra."

Another wrote, "... participation in Project TIAN had helped us become more comfortable with math in general and in teaching math."

Overall, Project TIAN participants perceived more confidence in their own knowledge of mathematics, how to listen to students to better understand the process the students are following in solving math problems, and how to refine their instruction to help students be successful.

TIAN researchers also gathered written data from teachers regarding their perceptions of student learning. According to this qualitative data, teachers reported that students increased specific math skills, improved problem solving skills, and some students developed multiple approaches to solving math problems. In addition, TIAN teachers reported that their students were more confident in math, had increased interest in mathematics, and were making real life connections from the math they were learning in the classroom to application outside the classroom.

Sustaining the Initiative

In determining if and how TIAN would continue in Arizona after the pilot phase, we focused on several key issues as well as the results from the pilot project. The key issues included ensuring the project was in alignment with the NSDC Professional Development Standards as well as budget considerations. After much reflection, we decided to continue the project. To begin planning for phase 2, the TIAN and state professional learning staff met to discuss options. We decided to build on the regional learning communities model, creating learning communities that would be facilitated by trained TIAN teachers and open to new participants. Teacher demographic data collected in January, 2008, indicated that 220 teachers statewide teach ABE/ASE classes. Of those 220 teachers, 16 percent indicated they taught specialized math classes for students with learning differences. The phase 2 initiative could add up to 30 more teachers to the TIAN regional learning communities to collaboratively build their skills and knowledge to improve math instruction.

Of the 17 TIAN participants who completed phase I, seven were identified as prospective TIAN learning community facilitators for phase 2, based on their work during the project, their location within the state, and their interest in continuing with the project. Responsibilities would include participation in initial planning meetings and facilitator training provided by TIAN staff, planning and facilitating monthly regional professional learning communities; and participation in team meetings. State staff and TIAN staff would also provide support and technical assistance.

Of the seven teachers identified as prospective facilitators, four agreed to join phase 2. One of those who did not become a facilitator had accepted a high school principal position, another became an acting program director, and the third was so inspired by Project TIAN that she took a leave from teaching and enrolled full-time in a university to earn a master's degree in mathematics. One of the teachers with Pima College Adult Education Program in Tucson, who agreed to be a facilitator in phase 2, had already formed a successful TIAN learning community for the ABE teachers in her program before phase 2 was even launched.

Project TIAN, Phase 2, was officially launched as a day long pre-conference session at the 2007 Arizona Adult Education State Conference in December, 2007. A total of 24 adult educators from around Arizona attended. The TIAN facilitators from Arizona provided a brief reflection on the TIAN model and shared their personal stories on how their involvement in TIAN changed their practice and understanding of mathematics instruction. New teachers were introduced to the TIAN strategies through small group mathematics activities using the TIAN Math Bundles and the *EMPower Mathematics* Series, published by Key Curriculum Press.

Participants then selected a learning community that was the most convenient to their program location. Each learning community began to complete a team charter form that included the name and contact information for each team member, the goals of the learning community, and a communications plan. The learning communities also discussed tentative dates for their initial meetings to be held in January or February, 2008.

Lessons Learned

As a result of our participation in TIAN, we have learned some valuable lessons regarding research partnerships. First, all parties involved in the research project need to establish written agreements at the outset of the project with clearly delineated roles and responsibilities for each project member, the outcomes and evaluation requirements of the project, deliverables needed by each partner, and information about what data will be collected during the project and when it will be available to each party. Deliverables should include enough data to allow the partners analyze the project outcomes to determine the implications and sustainability of the project for the state.

Another lesson learned was the importance of timing in rolling out a new initiative. The phase 2 kick-off event took place as a pre-conference session at the Arizona Adult Education State Conference in mid-December, just prior to the holidays and winter break for many adult education programs and staff. As a result, TIAN facilitators reported losing momentum after the conference and delays in forming their regional learning communities. To reinvigorate the implementation of phase 2, state staff and the TIAN facilitators facilitated a TIAN session during a regional adult education conference in April, 2008.

Summary

In summary, the implementation of significant and sustainable professional development initiatives that will result in teachers changing their practice takes time. Nonetheless, in Arizona, we believe that nurturing and growing teacher instructional practice in mathematics through TIAN is a sustainable and important effort.

Furthermore, the framework of TIAN enables teachers to improve their own math skills through the use of learning communities. This professional development model allows teachers to collaborate in reviewing data, examining and refining their math skills and instructional practice to improve math instruction for Arizona's adult learners. The adult learners we serve deserve no less than our best efforts.

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New England Literacy Resource Center

The Change Agent is published twice a year in March and September. Issue #26, March 2008, of this social justice magazine for adult learners and adult educators focuses on democracy in action. An election year guide, this issue doesn't just explain voting and advocacy but embraces the multitude of ways that ordinary people can have an impact in their communities - all year, every year, in large ways and small. For more information visit nelrc.org/changeagent/.

Educating the Public and Elected Officials about Adult Education: Report on Adult Education Advocacy Efforts in New England
This new report takes stock of the program, policy and legislative context for adult education in each of the six New England states and includes local and statewide advocacy strategies by adult educators. The findings show that adult education advocacy efforts in New England are multi-faceted, and growing in sophistication and reach. Principal challenges and related promising strategies are grouped into four areas:
1) Visibility; 2) Framing the Message;
3) Student Involvement; and 4) Increasing and Sustaining Advocacy Efforts. Available for downloading from www.nelrc.org

National College Transition Network Guides

The College Transition Toolkit is a comprehensive guide to program planning and implementation that draws on the expertise of practitioners from The New England ABE-to-College Transition Project and around the country. It contains detailed information to help adult educators and administrators plan for the needs of students interested in pursuing postsecondary education and training. Chapter titles include: Program Models; Partnerships and Collaborations; Recruitment; Assessment; Counseling; Curriculum and Instruction; Planning; and Using Data for Program Development. The toolkit also provides templates that you can download and adapt for use in developing your college transition program, links to a variety of online resources, and supplementary printable resources.

Integrating Career Awareness into the ABE and ESOL Classroom curriculum guide provides guidance to adult educators on how better to equip students with career awareness and planning skills through lessons and activities correlated to the SCANS competencies. The curriculum is available in CD form, with handouts and worksheets that can be downloaded and modified. A new addition is available, published by NCTN in collaboration with the Massachusetts System for Adult Basic Education Support (SABES).

Contact Leah Peterson at literacy@worlded.org or 617-385-3740 to order a copy of the *College Transition Toolkit* ($75.00 plus $5.00 for shipping/handling) or the *Integrating Career Awareness Curriculum Guide* (the CD is free, but costs $5.00 to ship).

System for Adult Basic Education Newsletters

The SABES Math Bulletin. This bulletin is a vehicle for sharing math/numeracy research and professional literature in an accessible, abbreviated platform. It is published electronically quarterly. Funded by a Massachusetts DESE grant, Volume 2, Issue 3, April, 2008, is now available at www.sabes.org/resources/publications/mathbulletin/math-bulletin-apr2008.pdf

The Problem Solver. This serves adult basic education (ABE) practitioners by offering math activity outlines, math problems, Web links, and stories about ABE math-related events in Massachusetts and around the United States. A new issue will be posted on the SABES web site in June. Past issues are available at: www.sabes.org/resources/publications/problemsolver/index.htm

Field Notes is a quarterly, theme-based publication in which Massachusetts adult basic education practitioners share innovative and reliable practices, resources, and information. Published by the System for Adult Basic Education Support (SABES) and funded by the Massachusetts Department of Education, *Field Notes* is also of interest to readers outside the state. The most recent issue (spring 2008) focuses on learning disabilities and includes articles on universal design, tools for the classroom, and listings of Web and print resources. Past themes have included numeracy, social justice, assessment, technology, student leadership, and workforce education. Find back issues at www.sabes.org/resources/publications/fieldnotes/index.htm

Fundamentals of Assessment

Standardized assessments are used everywhere in modern life, but few people really know how they are put together, how they should best be used, and what underlying principles govern their design. This course, the first in a cluster of assessment-related courses, is a layperson's guide to concepts such as reliability, validity, bias, and standard error of measurement. A basic knowledge of these concepts will not only equip you to better understand and deal with standardized assessments but also help you as a teacher to develop better in-class assessments.

This facilitated, online course will open with a Webinar on June 2, 2:00 p.m. (EST). After the opening Webinar, you will engage in self-paced activities and readings, as well as asynchronous discussions with the facilitators and course participants. The course will close with an additional Webinar on June 16, 11:00 a.m. (EST). You may complete the final exam after the June 16 Webinar. The fee is $99.
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