The effects of constructivist learning environment on prospective mathematics teachers' opinions

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Abstract: To explore the effects of constructivist learning environment on prospective teachers' opinions about "mathematics, department of mathematics, discrete mathematics, countable and uncountable infinity" taught under the subject of Cantorian Set Theory in discrete mathematics class, 60 first-year students in the Division of Mathematics Education at the Department of Science and Mathematics in Buca Education Faculty at Dokuz Eylul University were divided into two homogenous groups. In order to do this segmentation, Minimum Requirements Identification Test was developed and used by the researchers. This test includes concepts like "set", "correlation" and "function", which are required to understand Cantorian Set Theory. While the control group was taught by traditional methods, a teaching method based on a constructivist approach was applied to the experimental group. Data were gathered by an open-ended questionnaire administered to total 40 students, 20 from the each group. Collected data were evaluated through content analysis. In the end, despite the minor differences, no statistically significant difference was found between the opinions of control and experimental groups about mathematics ($\chi^2_{calculation}$=2.578, *SD*=3, $p>0.05$), department of mathematics ($\chi^2_{calculation}$=3.185, *SD*=3, $p>0.05$) and discrete mathematics ($\chi^2_{calculation}$=4.935, *SD*=3, $p>0.05$) after the instruction. However, opinions about Cantorian Set Theory were significantly differentiated between experimental and control groups after the instruction ($\chi^2_{calculation}$=13.486, *SD*=2, $p<0.05$).

Key words: infinity; finite and infinite sets; equality

1. Introduction

Could part be equivalent to the whole?—This question leads to contradictions in regard to equivalence of sets. While equivalent sets could be defined as "sets which have the same cardinal number", they have also been described as sets "whose elements can be put into one-to-one correspondence with each other".

If equivalence is represented with the symbol of "~", equivalent sets could be defined as follows:

$$\{f \,/\, f: A \to B,\ f bijective\} \neq \varnothing \Leftrightarrow A \sim B$$ (A is equivalent to B) (Güney, 1993)

That constitutes the very basis of equivalence. Though it may not be significant for finite sets, it is indispensable for the equivalence of infinite sets, since the definition of "sets with the same cardinal number" is useless for the equivalence of infinite sets. For this case, the second definition of equivalent sets as "whose elements can be put into bijective correspondence with each other" is functional.

By using this definition, Cantor formulated his theory which postulates the equivalence of infinite sets whose

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elements can be put into bijective correspondence. For example, let us consider set A={46, 47, 48, …} and set B={100, 101, 102, ...}. In terms of algebra, it is clear that $A \supset B$. However, when we look from the point of equivalence, the equivalence between two sets will be determined according to the existence of bijective function between them. Let us think about the function f: $A \rightarrow B$, f(x) = x + 54. This function would put set A and set B into one-to-one correspondence. For this reason, set A is equivalent to set B from Cantor's point of view.

In this respect, the sets like "the set IN of natural numbers, the set Z of integers, the set P of prime numbers, the set Q of rational numbers, IN^2, Z^2, etc." which have one-to-one correspondence are equivalent sets according to Cantorian Set Theory. Furthermore, one-to-one correspondence can be established between the sets like "the set R of real numbers, the set I of irrational numbers, (a, b) interval, etc.", thus they are also equivalent sets.

Cantorian Set Theory was originally regarded as utter nonsense by some mathematicians, while some others celebrated it as a masterwork (Güney, 1998). The theory might cause cognitive difficulties, since it rejects the notion that "whole is greater than its parts".

As a matter of fact, infinity had long been a challenging concept for human mind. The reluctance to see infinite sets as mathematical objects can be traced back to Aristotle who conceived infinity only as potential and not actual (Tirosh, 1991). This conception had prevailed in mathematics for 2000 years. The notion of infinity could be found, either implicitly or explicitly, in the works of pioneering mathematicians. Nevertheless, the notion of infinity has led to contradictions emanating from the very essence of the notion itself. Galileo and Gauss reached to the conclusion that actual infinity could not be incorporated into logical and coherent thinking. In 1831, Gauss stated that it was never permissible to use infinite magnitude as completed quantity. As for Kant, who referred to spatial and temporal infinity in his works, asserted that human mind could not comprehend spatial-temporal finiteness or infiniteness. According to him, this proves that time and space are mental constructions of our mind trying to perceive and organize the outer world, thus they do not have an existence apart from human mind (Fischbein, 2001).

Philosophers and mathematicians have distinguished between actual and potential infinities. According to Aristotle, mathematical infinity is always a potential infinity (Bagni, 1997). The infinity that human mind hardly comprehends or found it impossible to grasp is actual infinity, as in the examples like "the infinity of world, the infinity of points in a line segment". As a matter of fact, mind is originally adapted to the finite realities that we gained through our actions in time and space. Logic could deal with the notions consistently, only if they are expressed in finite realities. For this reason, contradictions begin to arise as soon as mind starts to deal with actual infinity.

At the end of 19th century, Cantor formulated a theory of infinite set, which he defined as a set in one-to-one correspondence with one of its proper subsets. With this formulation, Cantor dispelled the deep rooted objections against using numbers as an infinite magnitude at once. Nevertheless, this theory has been harshly criticized by Kronecker, Poincaré and their colleagues, who were reluctant to dive into the world of actual infinity where "a set could be at same length as its subset, a line could contain the same number of points as a line of half of its length, and infinite processes could be seen as complete things" (Rucker, 1982). On the other hand, some other great mathematicians of that time, such as Bertrand Russell and David Hilbert who regarded it as a great discovery, greeted the theory with enthusiasm.

Since it includes the notion of "the equivalence of a set to one of its proper subsets", Cantor's definition of infinite set constitutes a significant cognitive barrier. Since accepting this equality requires accepting the idea of "whole is greater than its parts" cannot be true for every set, it requires cognitive effort. Therefore, it is not very probable for students, who had never studied Cantorian Set Theory, to use this infinite set definition naturally on their own (Tirosh, 1999).

From Cantor onwards the notion of infinity, which includes the mentioned difficulties, has become a field of study. Fiscbein and his colleagues (1979, pp. 4-5), implicitly, have suggested that the work of Piaget and Inhelder (1956, pp. 125-149) be the beginning of studies about children's understanding of infinity (Monaghan, 2001). After this work involving repeated division of geometric figures, studies about the concept of infinity has continued.

According to Fischbein (1987), students' conceptions of infinity can be divided into two categories: First, there are personal experiences; second, formal education develops the idea of infinity. Moreover, studies have shown that students have some misunderstandings about the notion of infinity (Tall, 1990; Tsamir & Tirosh, 1994; Tsamir, 2002; Singer & Voica, 2003).

Since the article of Fishbein, Tirosh and Hess (1979) about intuitions in regard to infinity, educators of mathematics have dealt with this subject in terms of teaching and learning and tried to find out ways to study it experimentally.

Tall (1980) published a study scrutinizing the intuition of infinity in relation to the infinity of real numbers. Furthermore, with Vinner (1981), he introduced the terms of "Concept images" and "Concept definition" in order to explain the difficulties in learning limit and continuity. Duval (1982) interpreted students' obsessions of infinite sets in relation to the difficulty in assigning different roles to mathematical objections (For example 4 as a real number, 4 as a square and 4 as a even number). In 1986, at the Conference of PME (Psychology of Mathematics Education), Falk and his colleagues (1986) discussed students' reactions to the non-existence of very big natural numbers. In 1987, Sierpinska analyzed the types of problems in regard to conceptualizing limit. The same year, Waldegg (1987) introduced the findings of historic-epistemological and cognitive research on the response schemas of the epistemic-subject and the psychological-subject, when faced with typical problems of actual infinity. Furthermore, Moreno and Waldegg (1991) reported their study on high-school students. This study showed the similarities of students' response schemas when confronted with the contradictions while approaching actual infinity by using the intuitions of finite sets and historical developments.

Subsequently, Tall (1992) introduced transition from primitive mathematical thinking into sophisticated mathematical thinking through discussing notions of limit, function, mathematical proof and infinity. The same year, Tsamir and Tirosh (1992), at the 16th PME Conference, presented their study on students' comprehension of contradictory ideas. In 1993, Nunez made his psycho-cognitive study based on infinity and Waldegg (1993b) examined the comparison of infinite sets by people who gained intuition based on their previous experiences rather than their formal conceptions. Falk (1994) published his research on the cognitive challenge of comprehending infinity and Gonzalez (1995) studied the infinity intuitions of the Hispanic students in the USA.

Mura and Louce (1997) were the first to take the problem of infinity from the perspective of teachers in education and practice. In 1999, Arrigo and D'Amore analyzed the answers of the students with respect to the equivalency of a square and the points on one edge of the same square. Garbin (2000) tried to identify the contradictions of the high school students through their conceptual schemas about actual infinity.

Journal of Educational Studies in Mathematics allocated its 48th volume to research about infinity. Here, Monoghan (2001) and Tsamir (2001) published their researches which include opinions of the young people about actual infinity.

Waldegg (1988) elaborated on Cantor's studies about the collocation of actual infinity with an historical outlook. In his later studies, the author discussed Aristotle's definition of infinity in Aryan Greek culture (1993a) and analyzed Bolzano's existential definitions (2001). As for Horng (1995), he examined the connections between Greek and Chinese mathematics with respect to infinity.

In addition, paradoxical infinities, which include potential equivalence of infinite sets, became an object of research interest (Waldegg, 2005; Mamolo & Rina, 2008; Dubinsky, et al., 2005).

In general, research has suggested that 8-year-old children comprehend that natural numbers series have no end. Later, by the age of 11-12, children realize the dimensionless feature of points and then claim that line segments can be divided infinitely many times. In these studies, students were asked whether some processes would end or not. Researchers have assumed that students who claimed that the processes would end understood that the appearing set was infinite (Tirosh, 1999).

Studies including older students have indicated that students had difficulties in comprehending the Cantorian Set Theory (Tsamir, 1999, 2001, 2002; Narli & Baser, 2008). The concept is very important for the prospective mathematics teachers, since they use the notion of infinity in the classes like topology, algebra, etc. In addition to this, prospective teachers may need to evoke the notion of infinity in their professional life when teaching infinite sets like natural numbers and real numbers or when talking about infinite rational numbers in a finite interval.

In this study, which is about the subject where above mentioned problems may be encountered, it is assumed that prospective mathematics teachers can experience confusion and their perspective on mathematics may be affected. In the literature, there are some studies which include the effect of the teaching of this subject on the students' opinions. Therefore, this study may contribute to the literature. In the study, the research question is determined as "Does the applied method make a difference between the opinions of students in experimental and control groups?" and its resolution is sought.

2. Methods

This study is based on an experiment. Prospective mathematics teachers were divided into two groups and Cantorian Set Theory was introduced to them by using two different methods: traditional teaching method and method based on a constructivist approach (MBCA). Both at the beginning and the end of instruction, the opinions of each group were gathered via student opinion questionnaire (SOQ) and the effects of constructivist approach on their opinions have been assessed after evaluating the results through the content analysis.

This research methodology is in line with the interview technique constructed on qualitative research methods. Constructivist interview technique has structural similarities with questionnaires or attitude indexes in which participants have responded to the questions in specific categories (Türnüklü, 2000). Here, the purpose is to identify similarities and differences between participants by comparing them (Yildirim & Şimşek, 2000). The researcher asks the same questions to each participant in the same manner with the exact wording. The answers of participants are close-ended. Hence, constructed interviews produce quantitative results similar to questionnaires.

However, in this research, questions were asked in written and answers were also taken in written form, not verbally. Then, opinions, those under four headlines in the form, were categorized. This operation was important, since it was a process of simplification, summarization and transformation through reducing data, selecting the essential parts of abundant raw information and focusing on specific points.

2.1 Subjects

First-year students of mathematics education at the Department of Science and Mathematics in Buca

Education Faculty at Dokuz Eylul University participated to this research. The study has been conducted with an experimental group of 30 students and a control group of 30 students. In total 40 responses to the questionnaire, 20 from each group, were evaluated.

Minimum Requirements Identification Test (MRIT), including concepts like "set", "correlation" and "function", which are required to understand Cantorian Set Theory, was used in order to designate experimental and control groups. The students were graded in a descending order according to their level of success, and the groups were formed by selecting one student after another, the first student was assigned to the first group, the second to the second, third to the first and forth to the second and so on. Then, one group was set as control group, the other as experimental group by random selection.

After administering SOQ to both groups, the researcher taught the subject to the students. Subsequent to the instruction, the opinions of both groups were collected through SOQ once more. In the control group, traditional and formal instructional methods were used, with time-to-time question-and-answer and whole class discussion sessions.

2.2 Constructivist learning environment

Firstly, we consulted to the experts in order to specify the active learning methods to be used in teaching Cantorian Set Theory. Necessary teaching conditions were prepared in line with the experts' views. For the preparation, the subject matter was divided into four categories: basic concepts and definitions about equivalence, special equivalence theorems and their proofs, countability and cardinal numbers.

Finally, it has been decided to use brainstorming technique for teaching introductory part related to "the basic concepts and definitions about equivalence"; question-and-answer, discussion and animation techniques for "special equivalence theorems and their proofs"; problem based learning method for "countability" and group work for teaching "cardinal numbers".

Above mentioned techniques were applied to the experimental group. A more detailed about constructivist learning environment can be found in Narli and Baser (2008).

2.3 Materials

In this research, MRIT was used in order to assign control and experimental groups and SOQ was used for gathering opinions. These tools are outlined below:

2.3.1 Developing MRIT

First of all, the number of questions to be asked in regard to each category (knowledge, comprehension, application, analysis, synthesis and evaluation) in the Cognitive Domain of Bloom's Taxonomy, was determined in order to specify target behaviors for the subjects of set, correlation and function. The table of test characteristics was prepared and the number of questions was specified according to the opinions of experts and target behaviors set by Ministry of National Education. 65 questions were written in line with the specified target behaviors. These questions were presented to seven experts in order to find out whether they measure target-behaviors effectively. The necessary corrections were made in accordance with expert opinions and hence the validity of scope was ensured.

Thereby, the first draft of MRIT as 65-item test has been completed. In this first draft of MRIT, there were 14 questions in regard to the category of knowledge, 20 questions pertaining to comprehension, 22 in the category of application, 3 in synthesis and 2 in related to evaluation.

The first draft of 65-item MRIT was administered to the group comprised of 50 first-year students of mathematics education from the Department of Science and Mathematics in Buca Education Faculty at Dokuz

Eylul University. The duration of test was set as 80 minutes. During the administration, observations were made in regard to students' attention to the test, their speed of solving questions and how long did it take for them to finish the test. Then, the students were interviewed after the test.

The results of observations and interviews showed that students found enough time to answer the questions and their attention was not distracted. Plus, the students stated that the test was not very difficult.

This 65-item test was administered to 401 junior students from mathematics departments of different universities for validity, reliability and item-analyses. After item-analyses, a test of 40 items was derived.

Next to the specification of items experts, by looking at the distribution of the questions according to the target behaviors, confirmed that the first design of the test was not deformed.

Split-half method and KR-20 formula were used in order to measure the reliability of the test. For measuring reliability by Split-half method, the test was divided into two halves, one with odd numbers, and other with even numbers. After grading these two halves separately, reliability coefficient was found as r_{12}=0.74. Then, overall reliability of the test was measured by applying Spearman-brown formula. Overall reliability coefficient was r_{xx}=0.85.

For this research, KR-20 formula was also been used in order to find out the reliability, and KR-20 reliability coefficient was found as r=0.85. While this result showed that items were sufficiently coherent for the test, it also supported the result of "Half-split" method.

After the evaluation of its validity and reliability, MRIT had reached to its final form.

2.3.2 Developing SOQ

The SOQ was designed in a way that it asked opinions about four categories: mathematics, department of mathematics, discrete mathematics and equivalence. Sub-interrogative sentences like "their expectations, fears, whether they like it and understand it, if it is difficult for them, etc." were added next to the questions in the form.

Since all the questions in the opinion form were open-ended, only validity evaluation was in the process of validity and reliability analyses (Türnüklü & Şahin, 2004). Validity of scope was determined by resorting to the opinions of professors at the Faculty of Education.

As to the reliability analysis, the analysis was made only for the researcher who conducted the analysis and not for the form. Before coding the data into the previously prepared categories, it was necessary to check the reliability of people who would do the coding. For this operation, below formula could be used (Türnüklü, 2000):

Reliability = Number of agreements / Total number of agreements and disagreements

For the research, categorization of data was also made by an expert other than the researcher. When the percentage of agreement was checked, the reliability was found as 0.85 for the category of "opinions about mathematics"; 0.78 for the category of "opinions about department of mathematics"; 0.75 for "opinions about discrete mathematics"; 0.71 for "equivalence".

After setting the categories, codings were made for experimental and control groups separately.

2.4 Analyses of data

Data were analyzed by using qualitative research methods. χ^2 compatibility test was used in order to test the difference among the categories.

3. Findings and interpretation of results

Data were divided into four categories in regard to the opinions about mathematics, department of mathematics, discrete mathematics, and equivalence. Then these categories were put further into sub-categories and evaluated separately each by each.

3.1 Opinions about mathematics

Opinions about mathematics were put into four sub-categories: dimension of fun, dimension of difficulty, dimension of fear and liking, dimension of necessity. Control group's opinions pretest and posttest could be found in Table 1 below.

Table 1 Control group's opinions about mathematics

Categories	Opinions	Before the instruction		After the instruction	
		f	%	f	%
Dimension of fun	Mathematics is fun	11	55	12	60
	Mathematics is boring	-	-	1	5
	It was fun at the high school but not here	2	10	2	10
	Sometimes fun, sometimes not	5	25	2	10
	It is boring when we learn it by heart; it is fun when we are taught by reasoning	2	10	2	10
Total		20	100	20	100
Dim. of difficulty	Mathematics is difficult	4	20	2	10
	Mathematics is easy	1	5	3	15
	Sometimes easy, sometimes difficult	6	30	5	25
Total		11	55	10	50
Dim. of fear and liking	Mathematics is sometimes frightening	5	25	4	20
	Mathematics is not frightening	5	25	4	20
	I like mathematics	3	15	3	15
	I do not like mathematics	1	5	2	10
Total		14	70	13	65
Dim. of necessity	Mathematics is necessary	1	5	-	-
	It is important for other sciences	1	5	-	-
	It is part of life	5	25	2	10
	Mysterious and interesting	1	5	2	10
	It enhances the capability of abstract thinking	2	10	2	10
Total		10	50	6	30

When the results are evaluated statistically, it is not possible to find a significant change of opinions for the students in the control group, before and after the instruction ($\chi^2_{calculation}$=0.7359, *SD*=3, p>0.05). As it could be seen from Table 1, there is no significant change of opinions for the students in the control group. The percentage of students who do not find mathematics frightening is 25%. Furthermore, though the percentage of students who think that mathematics is easy is quite low as 5%, more than half of the students still think that mathematics is fun. Since the students could not find the relevance of mathematics with other sciences (5%) and life (25%), they do not think that mathematics is necessary. This finding is striking when we consider that these students are specially chosen for the Department of Mathematics.

When the results are examined statistically, there is no significant change found in the opinions for the experimental group before and after the instruction ($\chi^2_{calculation}$=0.789, *SD*=3, p>0.05). However when the Table 2 is scrutinized, it can be seen that though the students find mathematics difficult (60%), they do not find it frightening (20%), moreover they think that it is fun (60%). This situation has not been changed much after the experiment. The opinions of students in the experimental group are similar to those in the control group in regard to the necessity of mathematics (20%), since they could not establish the connection of mathematics with life

(20%) and other sciences (10%), either.

Table 2 Experimental group's opinions about mathematics

Categories	Opinions	Before the instruction		After the instruction	
		f	%	f	%
Dimension of fun	Mathematics is fun	12	60	13	65
	Mathematics is boring	5	25	3	15
	It was fun at the high school but not here	-	-	-	-
	Sometimes fun, sometimes not				
	It is boring when we learn it by heart; it is fun when we are taught by reasoning	1	5	2	10
Total		18	90	18	90
Dim. of difficulty	Mathematics is difficult	12	60	11	55
	Mathematics is easy	5	25	4	20
	Sometimes easy, sometimes difficult	-	-	-	-
Total		17	85	15	75
Dim. of fear and liking	Mathematics is sometimes frightening	4	20	3	15
	Mathematics is not frightening	5	25	5	25
	I like mathematics	-	-	-	-
	I don't like mathematics	-	-	-	-
Total		9	45	8	40
Dim. of necessity	Mathematics is necessary	4	20	1	5
	It is important for other sciences	2	10	2	10
	It is a part of life	4	20	4	20
	Mysterious and interesting	-	-	-	-
	It enhances the capability of abstract thinking	3	15	1	5
Total		13	65	8	40

Furthermore, it is also not possible to observe a statistically significant difference of opinions between students in experimental and control group, before the instruction ($\chi^2_{calculation}$=2.831, *SD*=3, *p*>0.05). Thus, it can be said that before the instruction students' opinions and attitudes about mathematics were similar to their attitudes shown in preliminary attitudes.

As it was the case before the administration of the test, a statistically significant difference between both groups could not be found after the experimental study as well ($\chi^2_{calculation}$=2.578, *SD*=3, *p*>0.05). This might show that deep rooted opinions of students cannot be changed significantly with short-term studies.

3.2 Opinions about department of mathematics

Opinions about Department of Mathematics are studied under four sub-categories: dimension of fun, dimension of difficulty, dimension of contentment, and dimension of course. Control group's opinions about Department of Mathematics are given in Table 3 below.

A statistically significant difference could not be found in control group's opinions after the instruction when it was compared to their opinions before the instruction ($\chi^2_{calculation}$=1.994, *SD*=3, *p*>0.05). When Table 3 studied, one might conclude that the students did not think the Department of Mathematics fun and they found it difficult. However, 60% of students stated that they were happy to be in the Department of Mathematics. This might have been caused by the present popularity of Department of Mathematics and wide range of job opportunities it provided.

Table 3 Control group's opinions about the department of mathematics

Categories	Opinions	Before the instruction		After the instruction	
		f	%	f	%
Dimension of fun	Department is fun	1	5	2	10
	It is not fun	8	40	2	10
	Sometimes fun, sometimes not	4	20	2	10
	Classes are boring	1	5	1	5
Total		14	70	7	35
Dim. of difficulty	It is a difficult department	10	50	5	25
	Not difficult	3	15	5	25
	Sometimes difficult, sometimes easy	1	5	-	-
Total		13	65	8	40
Dim. of contentment	I am happy in my department	12	60	12	60
	I am not happy	8	40	8	40
Total		20	100	20	100
Dimension of course	Courses are not learnt by heart as it was in the high school	1	5	1	5
	Courses are tedious	3	15	-	-
	Subject matters are taught too fast	1	5	-	-
	Courses are not relevant to life	-	-	1	5
	High school topics must be taught	-	-	3	15
Total		5	25	5	25

Opinions of experimental group about the Department of Mathematics are shown in Table 4 below.

Table 4 Experimental group's opinions about department of mathematics

Categories	Opinions	Before the instruction		After the instruction	
		f	%	f	%
Dimension of fun	Department is fun	2	10	7	35
	It is not fun	3	15	-	-
	Sometimes fun, sometimes not	4	20	5	25
	Classes are boring	-	-	-	-
Total		9	45	12	60
Dim. of difficulty	It is a difficult department	9	45	13	65
	Not difficult	3	15	2	10
	Sometimes difficult, sometimes	-	-	-	-
Total		12	60	15	75
Dim. of contentment	I am happy in my department	14	70	13	65
	I am not happy	4	20	3	15
Total		18	90	16	80
Dimension of course	Courses are not learnt by heart as it was in the high school	5	25	2	10
	Courses are tedious	-	-	-	-
	Subject matters are taught too fast	-	-	-	-
	Courses are not relevant to life	3	15	-	-
	High school topics must be taught	-	-	-	-
	Classes taught with an active group are better	-	-	3	15
Total		8	40	5	25

As it could be seen in Table 4, similar to those in control group, students in experimental group did not think the Department of Mathematics was fun and they found it difficult before the presentation. 70% of students stated that they were happy in their department. However, contrary to the control group, in the experimental group, the number of students who found the Department of Mathematics was fun, increased after the instruction. This indicates that the motivation of the students in the experimental group might have been raised by the method applied. Furthermore, in the experimental group, the number of students who found the mathematics difficult, also increased after the instruction. This finding, which was not observed in the control group, shows that the method contributed to the conceptual learning of students in the experimental group and made them realize the depth of mathematics. Nevertheless, when we look at statistically, there is no significant difference in experimental group's opinions about the Department of Mathematics before and after the instruction ($\chi^2_{calculation}$=1.55, *SD*=3, $p>0.05$).

When we compared pretest opinions of control and experimental groups about department of mathematics, a significant difference could not be found. This finding may indicate that, besides the similarity of their opinions about mathematics, the opinions of control and experimental groups were also similar about Department of Mathematics. There is no statistically significant difference of opinions between the groups after the instruction as well ($\chi^2_{calculation}$=3.185, *SD*=3, $p>0.05$). This result shows a parallel with the result obtained from the category of "opinions about mathematics". In another words, it can be seen that deep rooted and long established opinions of students do not significantly change by short-term applications.

3.3 Opinions about discrete mathematics

One of the main categories for finding opinions of students, the category of "opinions about discrete mathematics", was also studied under four sub-categories: dimension of fun, dimension of difficulty, dimension of meaningfulness, dimension of fear. Control group's opinions about discrete mathematics are outlined in Table 5 below.

Table 5 Control group's opinions about discrete mathematics

Categories	Opinions	Before the instruction		After the instruction	
		f	%	f	%
Dimension of fun	This course is fun	2	10	4	20
	It is not fun	1	5	1	5
	Sometimes fun, sometimes not	1	5	3	15
	More enjoyable than other courses	2	10	-	-
	I like discrete mathematics	3	15	3	15
	I do not like discrete mathematics	3	15	1	5
	I changed my opinions after numerical equivalence (I liked it)	-	-	2	10
Total		12	60	14	70
Dim. of difficulty	It is a difficult course	2	10	3	15
	Not difficult	5	25	1	5
	It is a complicated course	5	25	5	25
	This course requires reasoning instead of memorizing	3	15	6	30
Total		15	75	15	75
Dim. of meaningfulness	It is a meaningful course	4	20	2	10
	It is a meaningless course	6	30	3	15
	It brings new perspectives	-	-	-	-
Total		10	50	5	25
Dim. of fear	A frightening course	3	15	3	15
	It is not a frightening course	3	15	1	5
Total		6	30	4	20

As it can be seen from Table 5, the percentage of students, who found discrete mathematics fun, is quite low and the number of students who regarded discrete mathematics meaningless is higher than those regarded it meaningful. 40% of students stated that discrete mathematics was a complicated course that required reasoning instead of memorizing. In statistical terms, the opinions of students pertaining to discrete mathematics have not changed significantly before and after the instruction ($\chi^2_{calculation}$=1.917, *SD*=3, *p*>0.05). However, the number of students, who found discrete mathematics fun, increased after the instruction, albeit slightly. Moreover, the number of students, who thought that discrete mathematics required reasoning instead memorizing, also increased. This might have been caused by the fact that students found numerical equivalence even more abstract and notational.

Experimental group's opinions about discrete mathematics are given below in Table 6.

Table 6 Experimental group's opinions about discrete mathematics

Categories	Opinions	Before the instruction		After the instruction	
		f	%	f	%
Dimension of fun	This course is fun It is not fun Sometimes fun, sometimes not	6 4 3	30 20 15	5 - 3	25 - 15
Total		13	65	8	40
Dimension of difficulty	It is a difficult course Not difficult It is a complicated course This course requires reasoning instead of memorizing	6 2 5 5	30 10 25 25	3 - 10 4	15 - 50 20
Total		18	90	17	85
Dim. of meaningfulness	It is a meaningful course It is a meaningless course It brings new perspectives	8 - 3	40 - 15	4 2 4	20 10 20
Total		11	55	10	50
Dim. of fear	A frightening course It is not a frightening course	3 7	15 35	2 7	10 35
Total		10	50	9	45

More than half of the students in the experimental group stated that discrete mathematics was a complicated and difficult course which required reasoning instead of memorizing. Still, students were not afraid of discrete mathematics and only 20% of them thought that it was not fun. At the same time, almost 50% of students in this group also thought discrete mathematics as a meaningful course which required reasoning instead of memorizing. Nevertheless, it is not possible to find a statistically significant difference in the experimental group's opinions about discrete mathematics before and after the instruction ($\chi^2_{calculation}$=0.654, *SD*=3, *p*>0.05). However, the number of students, who found discrete mathematics difficult and not fun, decreased after the presentation. This finding might indicate that the applied method has raised the motivation of students.

There is no statistically significant difference in control and experimental groups' pretest opinions about discrete mathematics ($\chi^2_{calculation}$=0.5114, *SD*=3, *p*>0.05). This might show the closeness of both groups' attitudes towards discrete mathematics before the instruction. Yet, when we compared the attitudes of both groups after the experimental study, a statistically significant difference could also not be found ($\chi^2_{calculation}$=4.935, *SD*=3, *p*>0.05). This finding is in line with the results obtained from the categories of "Mathematics and Department of Mathematics". Deep-seated opinions of students do not change significantly with short-term applications.

3.4 Opinions about equivalence of finite and infinite sets

The last category of student opinions, the category of "opinions about numerical equivalence" is divided into three sub-categories: dimension of fun, dimension of difficulty, dimension of meaningfulness. The opinions of students in control group about Cantorian Set Theory are given below in Table 7.

Table 7 Control group's opinion about Cantorian Set Theory

Categories	Opinions	Before the instruction		After the instruction	
		f	%	f	%
Awareness	I have no information on this subject	20	100	-	-
Dimension of fun	This subject is fun			3	15
	It was nonsense at first, but now enjoyable			3	15
	Not fun			-	-
	Sometimes fun, sometimes not			-	-
Total				6	30
Dim. of difficulty	It is a difficult subject			4	20
	Not difficult			1	5
	It is a complicated subject			3	15
	It is a subject that requires reasoning instead of memorizing			4	20
Total				12	60
Dim. of meaningfulness	This subject is illogical			7	35
	I was understanding at first, cannot understand now			1	5
	Neither an utter nonsense nor a masterpiece				
	I understood			1	5
Total				9	45

Experimental group's opinions about numerical equivalence are as follows (see Table 8):

Table 8 Experimental group's opinions about Cantorian Set Theory

Categories	Opinions	Before the Instruction		After the Instruction	
		f	%	F	%
Awareness	I have no information on this subject	20	100	-	-
Total		20	100		
Dimension of Fun	This subject is fun			10	50
	It was nonsense at first, but now enjoyable			3	15
	Not fun			-	-
	Sometimes fun, sometimes not			3	15
	Being in the experimental group made me like the subject more			6	30
	It changed my concept of infinity completely			2	10
	Magic Hotel was very nice			6	30
Total				30	150
Dim. of Difficulty	It is a difficult subject			5	25
	Not difficult			1	5
	It is a complicated subject			-	-
	It is a subject that requires reasoning instead of memorizing			-	-
Total				6	30
Dim. of Meaningfulness	This subject is illogical			5	25
	I was understanding at first, cannot understand now			2	10
	Neither an utter nonsense nor a masterpiece			-	-
	I understood			6	30
Total				13	65

As it could be seen from Table 7 and Table 8, students in experimental and control groups had no prior information on numerical equivalence before the instruction. This could be regarded as normal, since the equivalence of infinite sets, except the simple equivalency of sets, has not got a place in the curriculums of high schools. After the study, 30% of students in the experimental group found the subject of numerical equivalence difficult, 60% of them stated that numerical equivalence was a complicated and difficult subject which required reasoning instead of memorizing. Furthermore, the number of students who found the subject nonsensical was more in the control group.

As to the dimension of fun, there are significant differences of opinions between the experimental and control groups. While the percentage of students who found the subject fun, is 15% in the control group, this percentage is 50% in the experimental group. Thirty percent of students in the experimental group stated that being in the experimental group made them like the subject more and some of the students from experimental group also indicated that their concept of infinity has completely changed after studying numerical equivalence. Moreover, the percentage of students, who liked the magic hotel (Hilbert Hotel) which was taught as a part of problem-based learning (PBL) method in the experimental study, is 30%. This emphasis of PBL among other methods used in experimental study might be indicating that this method was effective and, at least, it increased students' motivations.

When it was scrutinized statistically, a significant difference of opinions in regard to numerical equivalence was also found between experimental and control groups ($\chi^2_{calculation}$=13.486, *SD*=2, $p<0.05$). So we can conclude that the applied method made the subject more comprehensible and funnier, and hence it increased the motivations of students and contributed to their success.

4. Conclusion and discussion

According to the findings of the research, there is no significant difference in the opinions of both experimental and control groups, pretest and posttest, in regard to the categories of "mathematics, department of mathematics and discrete mathematics". This indicates that short term applications do not influence students' deep-rooted opinions significantly.

Besides, students do not believe the necessity of mathematics since they cannot correlate mathematics with other sciences and life. Nevertheless, this circumstance has not changed significantly before or after the instruction in both experimental and control groups and this is thought-provoking fact considering that these students are specially selected for the Department of Mathematics. As for mathematics, which can be regarded as life itself, this result might be an indicative of the fact that mathematics has not been taught by reasoning but by heart in elementary and high school education.

According to the qualitative results, students do not find Department of Mathematics fun and they regard it very difficult. In spite of this fact, the percentage of students who are happy in their department is quite high. This result shows that availability of job opportunities after the university influences the students' choice of department to a great extent.

In contrast with the control group, the number of students who find mathematics fun increased in the experimental group after the instruction. It can be said that applied method enhanced the motivation of students.

In both groups, the number of students who think that discrete mathematics is fun and requires reasoning, increased after the instruction. This might indicate that the numerical equivalence could be an important subject which may influence the students' views about discrete mathematics. Furthermore, after the instruction, the number of students who regarded discrete mathematics difficult and complicated increased in the experimental group. That is to say, the students in experimental group comprehended the depth of discrete mathematics.

When students' opinions about numerical equivalence are examined, significant differences are found between experimental and control groups. While students in both groups stated that they had no opinion about numerical equivalence before, this has changed after the instruction. The number of students who found the subject fun is higher in experimental group, while students in the control group found it difficult and nonsensical. This is a pleasing result, since it can be taken as another indicator of the efficiency of applied method.

Moreover, students' special reference to PDL, which was used for the research, makes us think that PDL can also be a method for teaching mathematics, a method that students may accept and prefer. Different research results also showed that PDL increased the motivations of students and could be used for teaching mathematics (Feikes, 1995; Torp & Sage, 2002; Roh, 2003; Hämäläinen, 2004; Hmelo-Silver, 2004; Javier & Cepeda, 2005; Günhan, 2006; Özgen, 2007; Ozgen & Pesen, 2008).

5. Recommendations

In experimental group, it was observed that students paid special attention to the computer animations. The animations prepared for some of the very abstract proofs enabled the students to understand these proofs and their underlying conceptual structures more easily. Thus, animations could be prepared for other courses and subjects as well, and even for elementary and high schools to visualize the mathematical subjects. In fact, the preparation of animations related to the mathematical concepts can be given as homework to the students with computers, in order to help them learn mathematical subjects thoroughly and make them like mathematics more.

(1) Teachers should set an example to students by using technology in teaching mathematics, and educational institutions should encourage them to do that.

(2) Teachers should be taught in a way that they would use the new teaching technologies effectively, and they should be informed about the advantages and disadvantages of teaching technologies.

(3) Teachers should be open to new teaching technologies and they should try to organize their classes accordingly.

It was observed that brain-storming technique used in the introduction of teaching equivalence, has brought very different perspectives. Creativity increases in an environment in which opinions are expressed freely without fear of criticism. For this reason, brainstorming technique can be used for teaching all mathematical concepts, especially in the introduction.

PDL has generally been considered as a teaching method which required a lot of effort. However, with proper scenarios, its contribution to the learning is worth all the efforts. The researcher observed that student paid most attention to PDL among all other techniques used in the experimental study. PDL can fulfill the expectations of students who expect different learning environment from the university than that of elementary and high school education provided. Considering the fact that PDL can channel students' interests directly into the problems, it can be used in Departments of Mathematics, or at least pilot studies can be made.

Any learning environment, which makes learning a responsibility of a student and enables the student to reach conclusions by herself/himself, thus, in which mental activity is accelerated, would enforce learning. This approach, which is the very essence of active learning, should be explained especially to the students in Faculty of

Education. It should be made clear to the teachers and prospective teachers that active learning can take place even without proper equipment, a laboratory or basic facilities.

The teachers of national education should be given in-service courses to make them understand the nature of active learning. These courses should be prepared in a way that they would serve to their purpose, teachers should understand why they must use active learning methods in order to raise a generation of creative students who can think, analyze and synthesize.

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