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Cantorian Set Theory and Teaching Prospective Teachers

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Abstract: Infinity has contradictions arising from its nature. Since mind is actually adapted to finite realities attained by behaviors in space and time, when one starts to deal with real infinity, contradictions will arise. In particular, Cantorian Set Theory for it involves the notion of "equivalence of a set to one of its proper subsets," causes perceptual issues. Prospective mathematics teachers usually learn this concept via traditional lecture methods and experience possible problems later. Since Cantorian Set Theory is used in courses like Topology, Algebra, etc. and prospective teachers will teach it to students using appropriate methods, it is important that they comprehend it well. The purpose of this study is to assess the effects of teaching Cantorian Set Theory via an Active Learning Based Course on prospective students' achievement. The results indicate that prospective teachers who were in experimental group (taught by active learning based methods) were more successful than those who studied the traditional lecture method and the difference between the groups was significant ($p = 0.02$).

Key words: Achievement, Cantorian Set Theory, Countability, Equality, Infinity, 1-1 Correspondence

INTRODUCTION

Fifteen years ago it would have been prudent to discuss Because of its inferential nature, mathematics depends on situated primary ideas, axioms, and definitions. Definitions are often used in mathematics to introduce mathematical presence of some concepts (i.e., rational numbers, complex numbers, etc.) (Tirosh, 1999). However, history of mathematics shows that in many cases concepts introduced by this kind of definitions had not been accepted immediately. In fact, it takes mostly decades even centuries for these definitions to be accepted by the mathematics world (Fischbein, 1987).

The most extreme of these is probably the notion of infinite set. The reluctance to see the infinite sets as mathematical objects can be traced back to Aristotle who suggested that infinity was a potential not reality (Tirosh, 1991). This approach had been dominant in mathematics for 2000 years. Infinity in mathematics is seen clearly or implicitly in the works of pioneer mathematicians. However, the notion of infinity has caused contradictions arising from its own nature. Galileo and Gauss have concluded that real infinity cannot be included in logical and coherent implication. In 1831, Gauss stated that an infinite quantity would never to be used as a complete quantity. Kant referred to infinite space and time and suggested that human intelligence cannot understand finite or infinite nature of the world in terms of space and time. This, according to Kant, was evidence that space and time do not exist by themselves in the out world but instead they are concepts organized by our brains, which struggle to comprehend and organize (Fischbein, 2001).

Philosophers and mathematicians have distinguished between real and potential infinity. According to Aristotle, mathematical infinity was always potential infinity (Bagni, 1997). Infinity, regarded as difficult or even impossible to be comprehended by mind, is real as in these examples: "Infinity of the world, infinity of the points on a line." Mind is actually adapted to the finite realities we have attained with our behaviors in space and time. Mind can only deal with notions that are expressed with finite realities. Hence, when one starts to deal with real infinity, contradictions will arise.

In the end of the 19th century, Cantor formulated an infinite sets theory in which he defined an infinite set as a set, comparable via 1-1 correspondence with one of its proper subsets. By doing this, Cantor has dispelled the fundamental objections against perception of numbers as an infinite set at once. However, this theory has been harshly criticized by Kronecker, Poincaré, and their colleagues who were reluctant to dive into the world of real infinity where "a set could be at same length as its subset, a line could contain the same number of points as a line of half of its length, and infinite processes could be seen as complete things" (Rucker, 1982). On the other hand, Cantorian Set Theory had been accepted with great excitement by some famous mathematicians of that time such as Bertrand Russell and David Hilbert who regarded this theory as a great discovery.

Since it includes the notion of "the equivalence of a set to one of its proper subsets," Cantor's definition of infinite set constitutes a significant perceptual barrier. Since accepting this equality requires accepting the idea of "whole is greater than its parts" cannot be true for every set, it requires perceptual effort. Therefore, for this infinite set definition to be used naturally by

Table 1.Distribution of sample according to groups and gender

Gender	Active Learning Group	Traditional Learning Group	Total
Female	14	17	31
Male	16	13	29
Total	30	30	60

students who had never studied Cantorian Set Theory before has a very low probability (Tirosh, 1999).

From Cantor onwards the notion of infinity, which includes the mentioned difficulties, has become a field of study. Fiscbein and his colleagues (1979, 4-5), implicitly, has suggested that the work of Piaget and Inhelder (1956, p.125-149) be the beginning of studies about children's understanding of infinity (Monaghan, 2001). After this work involving repeated division of geometric figures, studies about the concept of infinity has continued.

According to Fischbein (1987), students' conceptions of infinity can be divided into two categories: First, there are personal experiences; second, formal education develops the idea of infinity. Moreover, studies have shown that students have some misunderstandings about the notion of infinity (Tall, 1990; Tsamir&Tirosh, 1994; Tsamir, 2002, Singer ve Voica, 2003).

In general, research has suggested that 8-year old children comprehend that natural numbers series have no end. Later, by the age of 11-12, children realize the dimensionless feature of points and then claim that line segments can be divided infinitely many times. In these studies, students were asked whether some processes would end or not. Researchers have assumed that students who claimed that the processes would end understood that the appearing set was infinite (Tirosh, 1999).

Studies including older students have indicated that students had difficulties in comprehending the Cantorian Set Theory (Tsamir1999, 2001, 2002). The concept is very important for the prospective mathematics teachers. Because, prospective teachers have to use the notion of infinity in courses such as Topology, Algebra, etc. and teach it to students taking into account their levels. Literature is scarce about studies investigating student success in Cantorian Set Theory. The subject is taught usually using traditional methods. The purpose of this study was to investigate the effects of teaching Cantorian Set Theory to prospective mathematics teachers using two different methods: Traditional teaching methods and a course designed with active learning methods (an active learning based course) (ALBC) on students' achievement.

METHODOLOGY

The effects of the teaching methods were assessed by comparing the performance of prospective teachers who participated in ALBC with those who participated in a formal, traditional course.

Subjects

Two groups of prospective secondary school mathematics teachers studying in a Turkish state university participated in the study. (a) Thirty prospective teachers participated in a 'formal course' for Cantorian Set Theory; and (b) thirty prospective teachers participated in the ALBC for Cantorian Set Theory.

In determining the groups totaling 60 students, Minimum Requirements Identification Test for understanding Cantorian Set Theory (MRIT), which includes the prerequisite concepts required to comprehend the Cantorian Set Theory such as "set, relation, function," has been used. According to the results of MRIT, students were ordered from top down using their achievement scores and first student was assigned to first group, second student to second group, third student to first group, forth student to second group and so on, and two groups were formed. Groups then were randomly assigned to one of the control and experimental groups. Distribution of groups is shown in Table 1

This study is a randomized pretest-posttest experimental design. Pretest-posttest research method with control group was employed in this study. Before teaching the subject to groups using two different methods, Cantorian Set Theory achievement test (CSTAT) was administered to both groups as the pretest. After the pretest, both groups were taught by the same instructor. In the control group, traditional, formal instructional methods were used, with time-to-time question-and-answer and whole class discussion sessions. After the teaching period, CSTAT was administered to both groups as the posttest.

Active Learning Based Course in Cantorian Set Theory

In order to determine the active learning methods to be used in teaching Cantorian Set Theory experts have been consulted. In the light of expert opinions, necessary teaching conditions and learning environments have been prepared. In the preparation of these methods, the subject was divided into four sections, which were (a) basic concepts and definitions about equivalence, (b) special equivalence theorems and proofs, (c) countability, and (d) cardinal numbers.

Finally, in the introduction part of the subject brainstorming method; in the second section, question-and-answer and discussion techniques, and computer animations; in the part (c), problem-based learning (PBL); and in the last section, group study techniques were decided to be used.

These techniques were then applied to the experimental group. The applications were as follows:

Application of brainstorming technique

learning the notion of equivalence.

Students were asked to define the concept of "equivalence." Students freely expressed their ideas and stated the definitions, which they regarded as true. These definitions were noted.

No critic or comment was made about students' definitions of equivalence. Definitions were noted exactly as stated by students. These definitions then discussed one by one together with students. Definitions, which were wrong, were discussed as to why they were wrong; definitions, which were short or insufficient, were discussed to correct them; and in the end, a common consensus about the definition of equivalence was reached and students constructed a common definition for equivalence.

Brainstorming technique was employed from time to time during the later stages of instruction.

The use of question-and-answer, discussion, and animation techniques

The notion of Cantorian Set Theory mainly deals with equivalence of two sets. When two sets are finite, it can be easy to see their equivalence; however, if they are infinite it is hard to show their equivalence to students.

Consequently, the concepts related to the equivalence of infinite sets and their proofs are difficult subjects for students to comprehend. To teach these subjects, question-and-answer and discussion techniques were employed in the experimental group. Moreover, computer animations were created for three proofs that were thought as important and rather abstract. These proofs were; the proofs of "the equivalence of N natural numbers and N2 set;" "the non-equivalence of N natural numbers set and interval (0, 1);" and "non-equivalence of R real numbers set and F functions set" theorems.

Animations were prepared using Macromedia Flash-mx vector-based Web enhanced animation program, Photoshop-6 image program, and Macromedia Director. The question-and-answer animations were produced so that students could manipulate the program with the help of a button, and necessary information was included where it is needed. Some examples of animation's interfaces are shown below.

Application of Problem-Based Learning Method in Countability

Countability is one of the concepts that constitute an important part in Cantorian Set Theory. Countable and uncountable sets are important concepts for mathematics students. For this reason, special attention is given to the teaching of this subject in this study.

In the experimental group, this concept is taught by using PBL. PBL is applied via a written scenario. PBL scenario is written after consulting experts.

PBL sessions are carried out with groups of 6-8 students and with a moderator guiding students

Figure 1. Interface of the animations in the introduction part

Figure 2 Animation related to $N^2 \sim N$

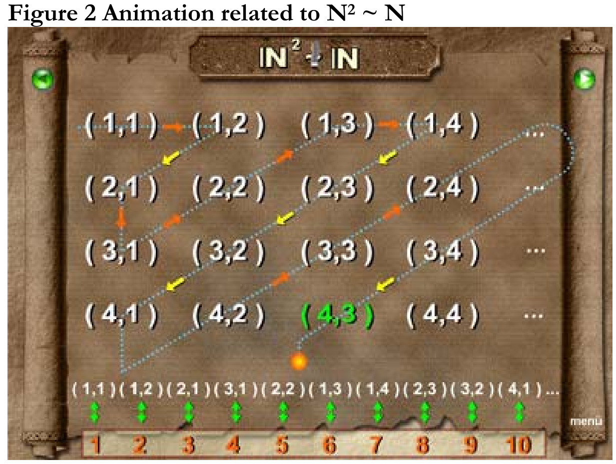
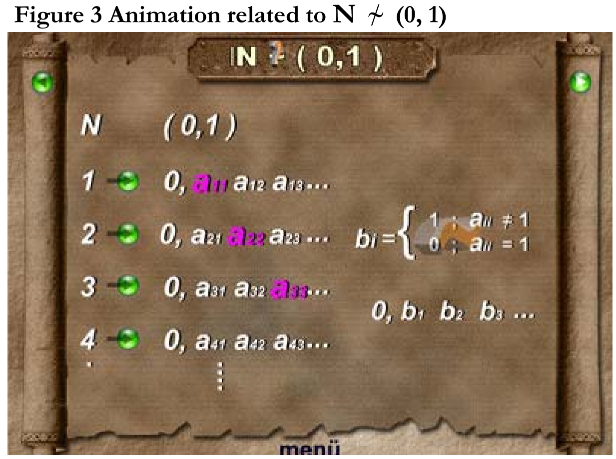
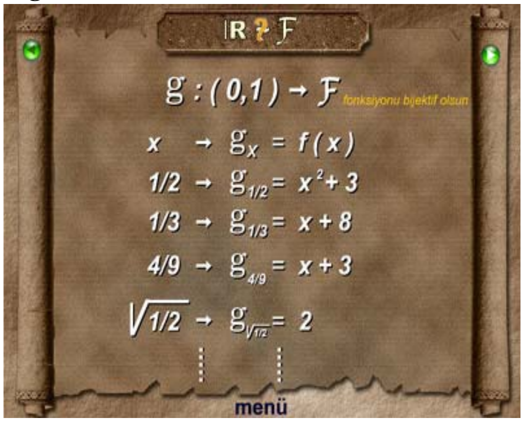


Figure 3 Animation related to $N \nsim (0, 1)$



(Abacıoğlu, 2002:15). As an active learning method, the basic principle of PBL is that the information that is assumed necessary and has professional importance is learned by doing research through learning objectives that are developed by students with curious and skeptical approach to problems and is applied to solve a problem (Abacıoğlu vd, 2002:25). In accordance with this situation, the story of Hilbert hotel was chosen as the scenario for this study. This story is interesting

Figure 4 Animation related to R $\nsim$ F



enough to draw and keep students' attention. The story is as follows:

"You have a hotel. The hotel has infinite number of rooms. Each room has a number: 1, 2, 3, 4, 5, 6… Thereby goes to infinity. There is no "last" room. There is not also a room, which numbered "infinite." Number of each room is finite, only number of rooms is infinite.

First Story: It is your lucky day; a bus full with customers arrives at your hotel. Infinitely many customers…. Customer names numbered as 1, 2, 3, 4, 5, 6….You assign a room to each customer. Room number 1 to customer 1, room number 2 to customer 2, and room number 3 to customer 3, and so on…

Just as you were thinking that everything was going all right, you saw one more customer came to your hotel. How are you going to arrange a room to this customer?

Second Story: Another lucky day, you have a bus full with customers, infinitely many…. They are named as a1, a2, a3, a4, a5, a6 …. You assign a room to each customer. Customer a1 to room 1, customer a2 to room 2, and so on…

Just as you were thinking, everything was going all right, all of a sudden….Surprise! Another bus full with customers parks in front of your hotel. There are infinitely many customers in that bus as well. They are named as b1, b2, b3, b4, b5, b6 …. You have no vacancy…. You have infinitely many new customers. How are you going to settle your new customers?

Third Story: This is your luckiest day, you have infinitely many busses every one of which full with infinitely many customers…. The busses are numbered: 1, 2, 3, 4, 5, 6…How can you arrange rooms to your customers?"

The story given above was written as a scenario to be used in PBL sessions to teach the notion of countability. Scenario was revised with corrections and additions in light of expert opinions.

Scenario has three sections, which develops together and taught in three sessions. First session is organized as to be able to teach the notion of infinite and countable set, properties of the union of finite sets and countable infinite sets and set properties such as difference and intersection of these sets. In the first session, the questions like "what is countability?, can an infinite set be countable? and is finite set countable?" were determined as learning objectives.

In the second session of the scenario, the union of two countable infinite sets and other set operations, differences between these operations and the operations that are carried out on finite sets, the union and set operations of more than two countable sets has been taught and a connection has been established with uncountable sets. The learning objectives in this session were: how can union of countable sets and the proof of countability of set operations be done. Is there an uncountable set? If so, what does that mean?

In the third session, the students studied the differences and similarities between finite and infinite sets, types of infinite sets, and the proof of countability of union of finite countable or infinite sets.

The sessions were done 3 days apart from each other because of students' time disagreement and the issue of finding moderators. The experimental group was divided into three groups of 10 students each. Each session lasted 90 minutes.

Employing the group work

In Cantorian Set Theory, the last section that comes after the countability is the notion of cardinal numbers. In this part, group work was employed in the experimental group.

In working groups, students plan to learn a subject, apply the plan, collect information, use that information to solve a complex problem, synthesize the solution, and put together their results (Açıkgöz, 2002:204).

Students have enough background from previous mathematics concepts they learned to do research on the cardinal numbers.

The experimental group was divided into 6 groups of 5 students each. Each group researched cardinal numbers and prepared to present their findings in class. During presentations, question-and-answer and discussion techniques were used. Each group submitted their findings as a report after the presentations.

Materials

Two achievement tests were developed for this study. "Minimum Requirements Identification Test for understanding Cantorian Set Theory (MRIT)" was developed to determine the experimental and control groups and "Cantorian Set Theory Achievement Test (CSTAT)" was developed to measure the achievement levels of the groups before and after the instruction.

Complying with the test preparation rules, MRIT was first written as 65 items. This version of the MRIT was administered to 401 first year mathematics students for item analyses, validity, and reliability calculations. Test was modified using the results of the first administration and the number of items was reduced to 40. The reliability coefficient was 0.85.

CSTAT was written as a 63-item test in its first version. This version was administered to 426 first year mathematics students for item analyses, validity, and reliability computations. After the first administration, the item number was reduced to 53 in accordance with the results of the application. Reliability coefficient was 0.91.

Application of the tests

Before starting to teach the Cantorian Set Theory, MRIT was administered to prospective teachers to compose the experimental and control groups. They were given 50 minutes to complete the test. After the groups have been determined, CSTAT was administered to both groups as a pretest before the instruction and as a posttest after the instruction. Each application of the CSTAT took 65 minutes.

Analyses of Data

Data were analyzed using SPSS 10.0 for Windows statistical analysis program. In the data analyses, means and standard deviations were calculated, frequency tables and percentage calculations were carried out. T-test was used to evaluate the differences between the groups. Moreover, paired-samples t-test was applied before and after instruction to investigate the development of each group within themselves (Hopkins, 1996).

RESULTS

Some definitions obtained using brainstorming method in the teaching of Cantorian Set Theory to the experimental group, are as follows:

If two sets are corresponding they are equivalent

If two sets are 1-1 corresponding they are equivalent

If there is a function between two sets they are equivalent

Sets having equal number of elements are equivalent

If two sets are subset of each other they are equivalent

If for every element of a set, there is a pair in the other set the sets are equivalent

Two infinite sets are equivalent

Sets having equal number of subsets are equivalent

Sets having equal number of proper subsets are equivalent

Sets, which have equal number of defined functions, are equivalent

While constructing these definitions students considered all the requirements of equivalence of sets. In order their definition of equivalence to be valid in infinite sets as well, they realized that they needed a definition other than "two sets with equal number of elements are equivalent." In the end, they constructed a definition, which was usable for infinite sets, and was broader than the definition used only for finite sets. The definition is as follows:

"Two sets having a 1-1 corresponding function between them are equivalent."

During PBL sessions designed for countability, observations and feedback obtained from students have shown that students achieved the learning objectives; they thought the scenario was written fluently, and they did not loose attention. Students stated that they found the PBL useful; they felt like they were in a university environment, and suggested that PBL could be used in other courses too.

After the groups determined by MRIT, CSTAT was administered before the instruction as a pretest. In order to compare the groups and see if there was a significant difference between them t-test was employed. The results are shown in Table 2.

When we look at Table 2 we see that according to CSTAT there is no significant difference between the control and experimental groups ($p = 0.128$). As a result, it can be concluded that the experimental and the control groups, determined according to MRIT results before the instruction, are homogeneous. Consequently, we can say that any differences that can be seen after the experiment will not be caused by initial behaviors.

After the instruction of the numerical equivalence concept CSTAT was administered to see if there was a significant difference between the groups. The results of t-test are depicted in Table 3.

The results of t-test show that mean CSTAT score of the experimental group is significantly higher than that of the control group ($p = 0.02$). Thus, active learning methods used in the instruction can be said to be effective.

Table 4 displays the paired-samples t-test results for the control group.

As can be seen in Table 4, in the control group significant difference was found between pre-and post-CSTAT mean scores ($p=.000$). This difference can be regarded as normal since students have almost no knowledge of numerical equivalence except for some basic equivalence definitions. Hence, before the instruction the mean of class achievement scores was relatively low, 5.72. After the instruction, however, the mean was 67.58. The difference between pre- and post-administration of CSTAT was significant.

Paired-samples t-test results of CSTAT for the experimental group are shown in Table 5.

Similar to control group, significant difference was found between pre-and post- CSTAT mean scores ($p=.000$) for the experimental group. Experimental group students had no knowledge of numerical equivalence concept expect for some basic equivalence definitions. Consequently, the observed difference was an expected result. In the experimental group, mean pretest score was initially 8.42 which has risen to 74.87 after the instruction. The mean posttest score of the experimental group was higher than that of the control group. According to this, it can be said that active learning methods applied in the experimental group to teach numerical equivalence were effective.

When we examine the achievement test, it is seen that the experimental group made generally less errors in questions. Control group made mistakes mostly in the following forms:

Stating that sets like Z whole numbers and {100, 101, 102,…} are equivalent to N natural numbers set, but not accepting that sets like $\left\{\sqrt[3]{n} : n \in Z\right\}$, indiced with whole numbers, are equivalent to N natural numbers set.

Not accepting the real numbers like $\{\sqrt[3]{n}$ | n∈R} or {n2: n ∈(1, 5)} or sets indiced with a set equivalent to real numbers equivalent to Real numbers set.

Errors in the proof of non-equivalence of R and F functions set

Not to be able to think of the results of non-equivalence of a set to its power set (non-equivalence of a set to its power set requires an important result which is there is a greater infinity than every infinity)

Not to be able to comprehend the relationships between countable finite and countable infinite sets

Not to be able to define the order relation in cardinal numbers set

Not to be able to define the symbols for transfinite numbers

Not to be able to comprehend the relationship between a finite set and cardinalities of its power set

Issues in operations with cardinal numbers

The experimental and control groups answered the following items correctly more or less at equal proportion:

defining the equivalence of finite or infinite sets using 1-1 correspondence,

the proof of non-equivalence of N natural numbers set and interval (0, 1),

the infinity of natural numbers set is smaller than that of real numbers,

the proof of equivalence of N2 and N natural numbers set,

the proof of equivalence of the interval (0, 1) and (0, 1)2 set,

the proof of non-equivalence of a set and its power set,

stating the Schroder-Berstein theorem,

defining the finite and infinite sets using notation,

defining the countable infinite set,

selecting the countable sets from a number of given sets, and

defining the cardinal number.

Moreover, there are important questions to which both groups replied wrongly at a high proportion. One

Table 2. T-test results of CSTAT administered before the instruction as pretest

	Mean		SD		t values		
Scores	Experimental	Control	Experimental	Control	t	sd	p
CSTAT Score	8.42	5.72	8.76	3.72	1.556	39.145	0.128

Table 3.T-test results of CSTAT administered after the instruction as posttest

	Mean		SD		t values		
Scores	Experimental	Control	Experimental	Control	*t*	*sd*	*p*
CSTAT Score	74.87	67.58	10.64	12.83	2.396	58	0.02

Table 4.T-test results of CSTAT progress for the control group

	Mean		SD		t values		
	pre	post	pre	post	*t*	*sd*	*p*
CSTAT Score	5.72	67.58	3.72	12.83	-31.083	29	0.000

Table 5. T-test results of CSTAT progress for the experimental group

	Mean		SD		t values		
	Pre	Post	Pre	Post	*t*	*sd*	*p*
CSTAT Score	8.42	74.87	8.76	10.64	-28.692	29	0.000

Table 6. Frequency table for question 43 according to groups

	Question 43						
	A*	B	C	D	E	Missing	Total
GROUP							
Experimental	10	10	4	1	2	3	30
Control	9	9	6	2	3	1	30
Total	19	19	10	3	5	4	60

of them regarded as important is the following:

Question 43: How many of the following statements are definitely true?

I. "No≤ c"

II. "if n is a finite cardinal number n < No"

III. "if A is countable #(A) < No"

IV. "if A is infinite No < #(A)"

V. "if A is uncountable No ≤ #(A)"

A) 1 B) 2 C) 3 D) 4 E) 5

Table 6 shows the frequency table according to groups for question 1. In this question about the relationships between concepts such as countability, uncountability, and cardinality of Cantorian Set Theory, approximately one-third of prospective teachers were mistaken. This may be caused by the fact that the examination made use of notations.

CONCLUSION

In this study, achievement of prospective mathematics teachers who taught with active learning based instruction was compared with that of prospective mathematics teachers who taught with traditional lecture style. Results showed that experimental group was more successful than the control group. The difference was significant. There are similar studies employed different teaching methods which showed that the experimental groups was more successful than the control group which was taught by traditional methods (e.g., Fischbein, Tirosh and Hess, 1979; Martin and Wheeler, 1987; Tirosh and Tsamir, 1996; Tsamir, 1999).

These studies have showed that when students, who had been given no instruction about Cantorian Set Theory, compared infinite sets they used the methods applied in case of finite sets. As shown in the result of brainstorming part of this study, when no instruction is given, students make intuitive definitions for the equivalence of infinite sets and transfer the equivalence definitions they use in finite sets (Tsamir, 1999). Statements such as "two sets that have the same number of proper subsets are equivalent; two sets that have the same number of subsets are equivalent, if two sets are corresponding they are equivalent" can be regarded as an indication of this situation. Moreover, as in "two infinite sets are equivalent," there seem to be students who have the basic misunderstandings in infinite sets. On the other hand, the definition, "if for every element of a set, there is a pair in the other set the sets are equivalent" may indicate the existence of students who has the idea of showing the equivalence of sets using 1-1 correspondence. As a result of brainstorming, students were able to construct the definition, "two sets having a 1-1 corresponding function between them are equivalent."

In both groups although reluctant at first, prospective teachers got used to the idea of equivalence of infinite sets. As indicated in the literature in time they revised their misunderstandings, which can be seen in every age group, such as "all infinite sets are equivalent; any two infinite sets can not be equivalent or they can not be compared" (Özgün and Narlı, 2006; Singer and Voica, 2003; Tall, 1990; Tsamir, & Tirosh, 1994; 1999; Tsamir 2002).

Nonetheless, even if they find the idea of 1-1 correspondence logical, they resisted considerably to the idea of equivalence of infinite sets like natural numbers set and whole numbers set. This may mean that although prospective teachers potentially accept the infinity, they have not yet comprehended the meaning of real infinity.

Results indicated that experimental group students could distinguish the sets equivalent to natural numbers or real numbers more easily than the control group students could. Moreover, it can be said that experimental group students better comprehended the relationships between countable finite sets and countable infinite sets and the operations on cardinal numbers than the control group students did. These results can indicate that the active learning processes have helped the students to better grasp the concepts related to infinite sets in their mind.

Furthermore, conversations after instruction have revealed that PBL sessions had a positive effect on students. Students found the Hilbert Hotel scenario as fluent and interesting and suggested that PBL sessions employing this kind of scenarios might be helpful in other classes as well. There are indications that PBL sessions were useful such that students embraced the PBL sessions about Cantorian Set Theory relationships between countable and uncountable sets and the experimental group was more successful in determining the relationships between countable finite and countable infinite sets than the control group. Haris, Marcus, McLaren and Fey (2001) have stated the need for new approaches to teach and learn mathematics for its vision and the curriculum materials supportive of these changes and suggested that PBL approach will achieve many objectives a learning process may ask for. PBL may provide this vision to students who come to university to find a different vision.

According to CSTAT results, the experimental group students are significantly more successful than the control group students are. This can be regarded as an evidence for ALBC in Cantorian Set Theory not only helps conceptual learning but also increases achievement. Teachers, who can better use the concepts related to infinite sets, can be more successful in courses such as Topology and Abstract Algebra. Moreover, achievement tests created for Cantorian Set Theory can be used to determine students' learning other than conceptual learning after instruction or as a pretest to determine pre-knowledge before instruction in courses like Topology. Furthermore, small number of achievement tests in Cantorian Set Theory could be increased.

In conclusion, it can be said that when we start to deal with the notion of infinity, like in other age groups prospective teachers experience problems. When one deals with a concept, primary intuition can cause some issues (e.g., Ball, 1990; Fischbein, 1987, 1993; Singer and Voica, 2003; Tall, 1990; Tall and Vinner, 1981; Tirosh, 1991). Hence, it can be said that during courses these primary intuitions cause them problems in accepting the equivalence of infinite sets. Considering the results of this study, it is reasonable to think that active learning methods alleviate these issues to some extent. Therefore, it is suggested that when preparing a mathematics course related to infinite sets or any other concept designing learning situations that uses and employs mental processes can be helpful.

IMPLICATIONS OF FINDINGS AND RECOMMENDATIONS

The findings of this study may have some implications for the teaching and learning of the concept of infinity. First, the use of technology in the teaching of the infinity concept may be said to be a new approach. The use of computer animations, which would provide visual instruction, would be helpful for the comprehension of hard to understand proofs. Results of this study indicate that animations can enhance the achievement. In the future, planning more active animations and providing online internet access to these animations during the instruction process may increase the efficiency of using technology in the teaching of cantorian set theory.

Another possible implication of the results is the indication that PBL may be an influential method in the teaching of the concept of countability. Since it leaves the student with problematic situations, PBL can enable students to internalize the concepts more easily. Students' approach towards PBL was observed to be more on the positive side during this research study. Different scenarios can be developed and used for the teaching of cantorian set theory.

In summary, the results of this study indicate that learning situations, which require students to be active participants of the process, may enhance the success in the teaching of cantorian set theory. In addition, further studies devoted to the stability of learning may provide alternative perspectives in this subject.

References

Abacıoğlu, H., Akalın, E., Atabey, N., Dicle, O., Miral, S., Musal, B., Sarıoğlu, S. (2002). *Probleme Dayalı Öğrenim*(First Ed.). İzmir: Dokuz Eylül Yayınları

Açıkgöz, K.Ü. (2002). *Aktif Öğrenme,* İzmir: Eğitim Dünyası Yayınları, Kanyılmaz Matbaası.

Ball, D.L.(1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education,* 21(2), 132–144.

Bagni, T. G. (1997). Didactics of Infinity: Euclid's proof and Eratosthenes' sieve Prime numbers and potential infinity in High School. *Didactics of Mathematics-Technology in Education* (1997), D'Amore, B. & Gagatsis, A. (Eds.), Erasmus ICP-96-G-2011/11, Thessaloniki, 209-218

Fischbein, E. (1987). *Intuition in science and mathematics*. Dodrecht, Holland: Reidel.

Fischbein, E. (1993). 'The interaction between the formal and the algorithmic and the intuitive components in a mathematical activity', in R. Biehler, R. W. Scholz, R. Straser and B. Winkelmann (eds.), *Didactic of Mathematics as a Scientific Discipline* (pp. 231–345). The Netherlands :Kluwer, Dordrecht.

Fischbein, E. (2001). Tacit Models and Infinity. *Educational Studies in Mathematics*, 48, 309-329.

Fischbein, E., Tirosh, D. and Hess, P. (1979). The intuition of infinity. *Educational Studies in Mathematics* 10, 3–40.

Haris, K; Marcus, R; McLaren, K; Fey, J. (2001). Curriculum Materials Supporting Problem-Based Teaching *School Science & Mathematics,* 101, Issue 6

Hopkins, Kenneth D., Hopkins, B. R., & Glass, Gene V. (1996). *Basic statistics for the behavioral sciences* (3rd ed). Boston: MS. Allyn and Bacon.

Martin, W.G. & Wheeler, M.M. (1987). Infinity concepts among preservice elementary school teachers. *Proceedings of the 11th Conference of the International Group for the Psychology of Mathematics Education*, France, pp. 362–368.

Monaghan, J. (2001). Young peoples' ideas of infinity. *Educational Studies in Mathematics,* 48, 239 – 257.

Ozgun, P.& Narli, S. (2006,September). *İlköğretim öğrencileri sonsuzluk fikrine sahip mi?* Paper presented at the 1. Ulusal Öğrenci Sempozyumu, Izmir, Turkey.

Piaget, J. and Inhelder, B. (1956). *The Child's Conception of Space*. London: Routledge and Kegan Paul, (originally published in 1948).

Rucker, R., (1982). *Infinity and the Mind.* Boston : Birkhaüser.

Singer, M., Voica, C. (2003, September). *Perception of infinity: does it really help in problem solving.* Paper presented at the meeting of the Mathematics Education into the 21st Century Project Proceedings of the International Conference The Decidable and the Undecidable in Mathematics Education Brno, Czech Republic.

Tall, D. (1990). Inconsistencies in the learning of calculus and analysis. *Focus on Learning Problems in Mathematics*, 12(3&4), 49-64.

Tall, D. and Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limit and continuity. *Educational Studies in Mathematics* 12, 151–169.

Tirosh, D. (1991). The role of students' intuition of infinity in teaching the Cantorian theory in D. Tall (ed.) *Advanced Mathematical Thinking* (pp. 199-214), Kluwer, Dordrecht,

Tirosh, D. (1999). Finite and infinite sets: definitions and intuitions. *International Journal of Mathematical Education in Science and Technology.,* 30 (3), 341-349

Tirosh, D. and Tsamir, P. (1996). The role of representations in students' intuitive thinking about infinity. *International Journal of Mathematics Education in Science and Technology* 27 (1), 33–40.

Tsamir, P., Tirosh, D. (1994). *Comparing infinite sets: intuitions and representations*. Proceedings of the 18th Annual Meeting for the Psychology of Mathematics Education (Vol. IV, pp. 345-352). Lisbon: Portugal.

Tsamir, P., Tirosh, D. (1999). Consistency and representations: The case of actual infinity. *Journal for Research in Mathematics Education. 30* (2), 213-219.

Tsamir, P. (1999). The Transitions From Comparisons Of Finite To The Comparison Of Infinite Sets: Teaching Prospective Teachers. *Educational Studies in Mathematics* 38, 209–234.

Tsamir, P. (2001). When 'The Same' Is Not Perceived As Such: The Case of Infinite Sets. *Educational Studies in Mathematics* 48, 289–307.

Tsamir, P. (2002). *Primary and Secondary Intuitions: Prospective Teachers' Comparisons Of Infinite Sets*, in CERME 2 Proceedings.

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