

# Doing the Math:

## Are Maryland's High School Math Standards Adding Up to College Success?

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Examining the Alignment Among Maryland's Voluntary State Curriculum  
for High School Mathematics, the Algebra I High School Assessment,  
and the Accuplacer College Placement Tests

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# Executive Summary

The educational standards, instruction, and testing of mathematics remain a controversial subject in the United States. Nationally, there is a documented lack of alignment between the expectations of college professors and the mandates for high school mathematics educators. According to a 2008 report from the National Mathematics Advisory Panel, there is a “vast and growing demand for remedial mathematics education among arriving students in four-year colleges and community colleges across the nation.”<sup>1</sup> This trend has been reported in Maryland where nearly one-third of even the best-prepared high school graduates require mathematics remediation in college. Worse, the problem seems to be increasing: The need for remedial mathematics among Maryland students who take a college-preparatory curriculum in high school and attend Maryland colleges has increased from 23 percent in 1997 to 32 percent in 2007.<sup>2</sup> These rates vary across districts and student subgroups, and for some subgroups of students, the most recently reported remediation rate is as high as 69 percent.

It is, unfortunately, students who bear the brunt of the gap between the expectations of colleges and high schools. Colleges require remedial students to complete noncredit yet tuition-bearing classes in order to proceed with obtaining a degree. These classes are significant stumbling blocks for many students.

Maryland, like other states, has revamped its standards for mathematics and now offers guidance to educators with the Maryland Voluntary State Curriculum (VSC) for High School Mathematics. Furthermore, as of the graduating class of 2009, all Maryland public school students must pass the High School Assessment (HSA) for Algebra I/Data Analysis in order to earn a high school diploma. Simultaneously, Maryland’s community colleges utilize standardized college-placement testing, namely the College Board’s Accuplacer Test.

Given the significant and growing need for mathematics remediation, it is reasonable to question how well Maryland is preparing its high school students to succeed in post-secondary institutions. Is the state’s VSC for high school mathematics aligned with the skills that colleges are demanding? More specifically, are Maryland’s high school mathematics standards aligned with the placement testing for mathematics used by the state’s community colleges and many of the four-year public colleges and universities?

This evaluation examines the correlation between the skills required to perform well on the Accuplacer college-placement tests and the content covered by the related high school mathematics courses as determined by Maryland’s Voluntary State Curriculum. It also evaluates the relevancy of the recently mandated High School Assessment for Algebra I/Data Analysis.

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The remedial mathematics situation is particularly pronounced at Baltimore City Community College where nearly all incoming students are placed into remedial math courses. For this reason, Baltimore City Public Schools and Baltimore City Community College are examined in greater detail in order to provide additional insight about the statewide issue.

The overriding question addressed is: Does successful completion of mathematics courses and the Algebra HSA, as prescribed by the VSC, lead to mastery of the skills required for the Accuplacer tests?

With the exception of the recently updated Algebra II state curriculum, the answer is no.

The findings of this analysis conclude:

- The Voluntary State Curriculum for Algebra I/Data Analysis as currently presented does not prepare a student to perform at a minimally sufficient level on the Accuplacer Elementary Algebra Test.
- There is very little alignment (less than 6 percent) between the kinds of problems on the HSA for Algebra I/Data Analysis and the kinds of problem found on the Accuplacer Elementary Algebra Test.
- The current VSC for Algebra I/Data Analysis and Algebra II as currently presented does not form a particularly cohesive sequence for the learning or teaching of algebra.
- The newly revised VSC for Algebra II and the Accuplacer College Level Math Test are reasonably well aligned.
- Arithmetic proficiency, while not a focus of this study, is a component tested by the Accuplacer tests; students are often inadequately prepared to demonstrate mastery of these skills.

A series of recommendations aimed at reducing the need for post-secondary mathematics remediation is included. It calls for a closer alignment of college and high school mathematics standards, curricula, and testing instruments.

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# Introduction

## Background

Mathematics education is an enormously controversial subject in the United States. The quality of the education offered to elementary, middle school, high school, and college students is a subject of much debate. The two most salient topics at the heart of the debate are the role of arithmetic and the level of abstraction present in math curricula.

One faction in the debate was fueled largely by an influential, and now controversial, 1989 National Council of the Teachers of Mathematics publication entitled *Curriculum and Evaluation Standards for School Mathematics*. This group maintains that traditional mathematics education is overly focused on routine procedures and does not foster higher-order thinking skills and conceptual understanding. It believes that mathematics should be made relevant and engaging to students and that the ubiquity of calculators justifies a de-emphasis on arithmetic skills and procedures. Curricula were developed to implement these ideas, and in some of them the standard algorithms of arithmetic and memorization of basic math facts are neither emphasized nor practiced. The use of these kinds of curricula has become widespread, and this gave rise to the other side of the debate. Parents, who were taught traditional arithmetic, were shocked to find that their children were not learning basic math facts and procedures in school.

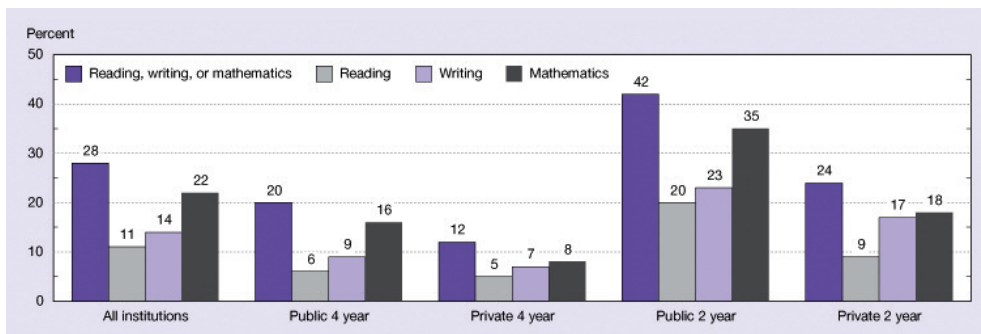
The second group—which now includes parents, teachers, mathematics professors, engineers, and scientists—favors the teaching of mathematics in more traditional forms, without relying on calculators to perform arithmetic. This group argues that mathematics has an internal, hierarchical logic, and that this structure needs to be preserved and emphasized in mathematics curricula. It believes that arithmetic fluency is an essential component of mathematics education, both for general computational literacy and for proceeding to the next level of mathematics. It points out that students who do not learn basic skills thoroughly are not prepared to study and understand more advanced mathematics, and that this may preclude them from pursuing careers in technical fields such as science and engineering. Equally deplored is American students' relatively poor performance on math tests that are administered on an international basis in order to compare math achievement in different countries.

Meanwhile, statistics show that there is a broad, national lack of alignment between the expectations of college professors and the mandates of high school educators. Many post-secondary institutions are testing incoming students and determining that many of them require remedial mathematics. According to a 2008 report issued by the National Mathematics Advisory Panel, there is a “vast and growing demand for remedial mathematics education among arriving students in four-year colleges and community colleges across the nation.”<sup>3</sup> For students, this means that successful completion of high school mathematics requirements does not necessarily lead to

success with mathematics courses in college. Remedial, or developmental, courses are now required by many colleges and are usually noncredit courses that do not count toward the acquisition of a degree from the institution. From the students' standpoint this means that they have to pay tuition to take required courses that do not count toward their degrees.<sup>4</sup>

As illustrated in the 2000 National Science Foundation chart below, there is a high prevalence of remedial education required in post-secondary institutions: More than one-fifth of students entering colleges with remedial mathematics courses are placed into such courses.<sup>5</sup>

**Figure 1 Freshmen enrolled in remedial courses, by subject area and institution type: Fall 2000**



Note: Includes only postsecondary institutions that offered remedial courses.  
 Source: B. Parsad and L. Lewis, *Remedial Education at Degree-Granting Postsecondary Institutions in Fall 2000*, NCES 2004-010, U.S. Department of Education, National Center for Education Statistics (2004).  
*Science and Engineering Indicators 2006*

It should be noted that there is no broad agreement from post-secondary institutions about remedial courses, or mathematics courses in general. Policies, practices, and procedures involving college mathematics courses and the criteria to designate a course as remedial vary from institution to institution, and from state to state. Many institutions make use of proprietary tests such as the SAT and ACT, and some high schools specifically prepare students to take these tests. However, these tests require a fee from the student and many students neither prepare for them nor take them. Thus, institutions must have their own mechanisms for placing incoming students into appropriate courses. Some develop their own placement tests, and some make use of proprietary placement tests such as the Accuplacer tests.

Amid all this controversy, many states have taken measures to revamp their educational approach to mathematics and to offer guidance for mathematics



educators at the state level. The Maryland Voluntary State Curriculum (VSC) for High School Mathematics is such a document. Maryland’s own explanation of the VSC for High School Mathematics is as follows:

*All high school students in the 21st century need to be mathematically competent and confident problem solvers if they are going to be able to be successful after graduation. The goal of the Maryland Voluntary State Curriculum (VSC) for High School Mathematics for College and Workplace Readiness is to provide high school students access to a curriculum that will achieve this goal by preparing graduating seniors for the first credit-bearing mathematics course in college and/or preparing them for employment in high-performance, high-growth jobs.<sup>6</sup>*

Despite this goal, Maryland follows the national trend; a significant number of Maryland high school graduates require remedial mathematics courses in college. According to data from the Maryland Higher Education Commission, college remediation rates for mathematics in Maryland are significant and increasing, even for those students who complete a college-preparatory curriculum in high school.

**Figure 2 Trends in Core and Non Core Curriculum Students Needing Math Remediation in College (By Major Jurisdiction)**

	1997-1998		1998-1999		1999-2000		2000-2001		2002-2003		2004-2005	
	Core	Non-Core	Core	Non-Core	Core	Non-Core	Core	Non-Core	Core	Non-Core	Core	Non-Core
Anne Arundel	22%	33%	22%	31%	19%	28%	24%	31%	31%	41%	32%	39%
Baltimore City	27%	54%	39%	63%	37%	53%	31%	54%	35%	59%	37%	60%
Baltimore	21%	26%	22%	35%	18%	22%	19%	22%	18%	21%	18%	20%
Frederick	30%	42%	32%	47%	24%	42%	25%	43%	26%	38%	26%	38%
Lower Shore	22%	30%	25%	40%	26%	41%	29%	41%	37%	55%	39%	45%
Mid Maryland	20%	31%	24%	34%	25%	34%	26%	34%	23%	35%	25%	36%
Montgomery	16%	31%	25%	39%	27%	41%	25%	35%	26%	38%	30%	41%
Prince George's	30%	40%	31%	41%	34%	45%	38%	47%	43%	51%	44%	54%
Southern Maryland	11%	16%	14%	21%	6%	14%	10%	17%	13%	20%	15%	18%
Susquehanna	28%	39%	28%	38%	33%	48%	34%	45%	31%	42%	40%	49%
Upper Shore	24%	37%	19%	43%	32%	45%	38%	45%	35%	47%	27%	47%
Western Maryland	30%	48%	41%	60%	34%	45%	37%	47%	27%	37%	31%	45%
<b>ALL MARYLAND</b>	<b>23%</b>	<b>36%</b>	<b>27%</b>	<b>41%</b>	<b>26%</b>	<b>38%</b>	<b>27%</b>	<b>38%</b>	<b>28%</b>	<b>40%</b>	<b>30%</b>	<b>41%</b>

	2006-2007	
	Core	Non-Core
Anne Arundel	31%	44%
Baltimore City	37%	69%
Baltimore	32%	45%
Frederick	24%	37%
Lower Shore	38%	51%
Mid Maryland	27%	42%
Montgomery	30%	42%
Prince George's	42%	55%
Southern Maryland	20%	32%
Susquehanna	40%	53%
Upper Shore	30%	46%
Western Maryland	28%	28%
<b>ALL MARYLAND</b>	<b>32%</b>	<b>49%</b>

Source: Maryland Higher Education Commission. SOAR report, March 2009 The core/ noncore designation is based on a curricular alignment of high school courses with admission requirements from the University System of Maryland (and includes two years of foreign language study) so that core may be thought of as “college preparatory.”

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As this chart indicates, one-third of even the best-prepared Maryland students are deemed by colleges to be unprepared for college-level mathematics. Other students fare even worse, and Baltimore City students fare among the worst of all.

Many Baltimore City public high school (BCPSS) graduates go on to attend Baltimore City Community College (BCCC). Mathematics remediation rates at BCCC are extremely high. One motivation for this evaluation can be summed up in the following statistic: In 2008, about 98 percent of BCPSS graduates attending BCCC required remedial mathematics courses. Moreover, almost half of these students were placed into a sequence of three remedial courses that begins with a course in arithmetic. The following chart shows the placement recommendations for 2007 high school graduates at BCCC by the number of remedial courses. While remediation rates are higher for BCPSS students, the chart illustrates that most other high school graduates (87 percent) are also placed into remedial mathematics courses.

Students' mathematics placement at BCCC and many other Maryland colleges is based on scores from a group of widely used proprietary college-placement tests, the Accuplacer tests. These tests are rigorous assessments of basic mathematical skills including arithmetic and basic algebra. As discussed, Maryland high school graduates are performing poorly on these tests, and one obvious question to pose is how well the material they are being taught in school aligns with the material on the tests. It is this question that is the primary focus of this evaluation.

There are three Accuplacer mathematics tests—Arithmetic, Elementary Algebra, and College Level Math. Baltimore City public high school students are currently required to pass courses in algebra I, algebra II, and geometry, in order to graduate from high school. (This is a more rigorous graduation requirement than the state's math requirement of algebra I, geometry, and a third, unspecified, math course.) The algebra courses should roughly correspond to the Accuplacer Elementary Algebra Test (Algebra I) and the Accuplacer College Level Math Test (Algebra II). The Voluntary State Curriculum (VSC) outlines the material that is required to be covered in these courses. The evaluation will examine the correlation between the skills required to perform well on the Accuplacer tests and the content covered by these courses as determined by the VSC. Therefore, the overriding question to be addressed is: Does successful completion of the courses prescribed by the VSC lead to mastery of the skills required for the Accuplacer tests?

### **The HSA and High School Algebra**

An additional factor included in the alignments is the state-mandated High School Assessment Test, or HSA. Students graduating from Maryland public high schools as of 2009 are now required to pass four HSA tests, or to satisfy some equivalent requirements, in order to receive a high school diploma. The mathematics HSA test

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is called the Algebra I/Data Analysis Test. The VSC for Algebra I/Data Analysis is closely aligned with this test. The inclusion of data analysis as part of algebra I is indicative of the broad, national tendency to emphasize applied mathematics over traditional, more abstract mathematics.

An Introduction/Rationale for the Algebra I/Data Analysis VSC is offered by the Maryland State Department of Education with excerpts that illuminate the basic curricular approach of the VSC for Algebra I/Data Analysis:

*As our society and technology changes, so too does the mathematics which is accessible to students and the way in which it is taught and learned.*

*The core learning goals call for a shift in emphasis from memorization of isolated facts and procedures and proficiency with paper-pencil skills to emphasis on conceptual understandings, multiple representations and connections, mathematical modeling, and mathematical problem solving.*

*Real-world applications are the backbone of this content and show students the inherent value and power of mathematics.*

*With the change in technologies, the mathematical processes change.<sup>7</sup>*

This clearly stated emphasis on applied mathematics has resulted in a de-emphasis on the formal skills associated with traditional algebra. This de-emphasis is pronounced enough that some Maryland college mathematics educators have criticized the VSC for Algebra I/ Data Analysis and the HSA for being practically devoid of what is traditionally thought of as algebra.<sup>8</sup> Moreover, the HSA Algebra I/Data Analysis test is designed to be taken with a calculator, so basic arithmetic fluency is not emphasized.

The inclusion of data analysis into algebra I courses creates an additional problem for the high schools attempting to follow the VSC. The focus on data analysis in the Algebra I/Data Analysis course may mean that much of the traditional material covered in an algebra I course, such as basic manipulation of expressions and equations, is not covered. This leaves students unprepared for their algebra II courses. One administrator in Baltimore wants to develop a “bridge” course to specifically address this disparity. In the absence of such a “bridge” course, much of the material covered in algebra II is necessarily the material that would traditionally have been covered in algebra I.

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## The Evaluation

Nationwide, and in Maryland, many colleges are grappling with the problem of remedial mathematics courses. The focus of this particular evaluation is Baltimore City public high school students, and the first post-secondary institution that many of them encounter is BCCC. For this reason, the evaluation includes an analysis of policies and procedures involving mathematics at BCCC.

For all Maryland community colleges there is an informal criteria used to designate remedial math placement. To avoid remediation, a student must achieve a score of 45 or more on the Accuplacer College Level Math Test. The use of the Accuplacer and the determination of the cut-off score is a result of an informal agreement of local colleges to try and standardize mathematics placement in Maryland in 1998.<sup>9</sup> It is not clear how or why this cut-off score on this section of the Accuplacer test was chosen.

The evaluation consists of three parts:

- An alignment of the VSC for Algebra I/Data Analysis and the HSA for Algebra I/Data Analysis with the Accuplacer Elementary Algebra Test.
- An alignment of the VSC for Algebra II with the Accuplacer College Level Math Test. The focus of this alignment is achievement of a score of 45 or better.
- An examination of mathematics placement, requirements, and procedures at BCCC.

Before explicating the alignments, here are some generalities of the Accuplacer tests and the VSC.

## The Accuplacer Tests

The Accuplacer tests are proprietary placement tests available for purchase from The College Board, a nonprofit membership association.<sup>10</sup> The tests are Internet-based, computer-adaptive multiple-choice tests. Scoring is immediate, and the scores are reported on a 120-point scale.

There are three mathematics tests: Arithmetic, Elementary Algebra, and College Level Math. Sample problems are available through the College Board website.<sup>11</sup> From a mathematical perspective, these tests assess basic, procedural math skills. The tests require knowledge of, and fluency with, basic mathematical procedures and the ability to use these procedures to solve problems. Problems are generally stated abstractly but word problems are also included. Calculators are not allowed, though on some problems a pop-up calculator may be provided. A full breakdown of the skills assessed and the meaning of the scores are available in the Coordinator's Guide.<sup>12</sup> Achieve, Inc. has also produced an independent analysis of the content and usage of the tests, and of other tests used by post-secondary institutions in its 2007 "Aligned Expectations" report.<sup>13</sup>

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It is important to note that the College Level Math Test assesses skills in intermediate algebra through precalculus. It includes material widely considered to be college-level material and is recommended in the Coordinator’s Guide as a placement test for courses up to and including introductory calculus. Because it assesses college-level material, it may not be appropriate for students who have not already studied mathematics at this level.

### **Maryland’s Voluntary State Curriculum (VSC) for High School Mathematics**

There are three state-provided VSCs for High School Mathematics: Algebra I/Data Analysis, Geometry, and Algebra II. Their organization and terminology is inconsistent. However, there are a few important general statements:

- Broad goals are stated and then further subdivided. The indicating statements are generally not skill specific and may include additional clarification under the headings of “Skill Statement” and “Assessment Limits.” These clarifications are vital to understanding the skills that are expected to be taught. **However, even with these clarifications, it is sometimes still not clear if, and to what extent, a particular skill is expected to be mastered.**
- Both VSCs for algebra include material that is entitled “Additional Topics Would Include,” or “Additional Topics.”<sup>14</sup> The publicly available material on the “Additional Topics” implies that they are, as their name suggests, optional topics. They are explained as “content that may be appropriate for the curriculum but is not included in the Core Learning Goals.” This statement seems to imply that coverage of these topics is optional, so this report treats them as if they are not necessarily included as part of the VSC. The “Additional Topics” are sometimes most illuminating in that a topic’s appearance in this category implies that it is not included as part of the VSC. For example, in the Algebra I/Data Analysis VSC, most of the material on factoring appears in this section. **The material included in “Additional Topics” forms the backbone of a usual algebra I course, so its lack of inclusion in the VSC itself is notable.**

As a rule, it is very difficult to read the VSC to determine whether or not a particular skill is included. The expectations are very broadly stated and are subject to interpretation on the part of the reader. For example, one indicator for Algebra I/Data Analysis states: *The student will recognize, describe, and/or extend patterns and functional relationships that are expressed numerically, algebraically, and/or geometrically.* This could include a wide range of particular skills spanning naïve to sophisticated levels of mathematics. Clarification of the meaning of the indicators may be provided in the “Assessment Limits” and “Skill Statement” that accompany it, but these are not always clear. For example, the “Assessment Limits” for the above indicator includes the following: *The student will not be asked to draw three-dimensional figures.* No further reference or clarification is given with regard to this remark or what it may imply about the indicator; the meaning of it remains unclear.

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An independent evaluation of the Maryland VSC is included in The Fordham Foundation's State of State Standards 2006 study. This report notes that Maryland's VSC for Mathematics is "odd" in structure and that the algebra standards are "...weak and include, strangely enough, several calculus topics too advanced for the grade level."<sup>15</sup> (It is possible that Fordham was considering only the Algebra I/Data Analysis VSC in the algebra evaluation.)

The VSC for Algebra II does not include data analysis and is noticeably more traditional in focus than the VSC for Algebra I/Data Analysis. As noted above, this may create an alignment issue in high schools, and different schools have probably addressed this issue in different ways. The content of both courses undoubtedly varies widely from school to school. However, because many schools use sequenced texts, it is probable that the applied approach heavily favored by the HSA and VSC for Algebra I/Data Analysis will be continued in the algebra II course.

### **The Methodology of the Alignments**

The basic methodology is to list the skills required for each of the Accuplacer tests in a chart and then determine to what extent the skills are covered in the VSC. For the Algebra I/Data Analysis alignment an additional component including the HSA is included. The degree of alignment is then examined in terms of the skill\score breakdown that is provided in the Accuplacer Coordinator's Guide.

In terms of a general approach to mathematics, the Accuplacer tests and the VSC are not well aligned. This reflects the national controversy between traditional and applied mathematics, and is particularly noticeable for the Algebra I/Data Analysis VSC and the Accuplacer Elementary Algebra Test. As outlined above, the VSC favors an applied approach to mathematics that de-emphasizes basic mathematical skills and procedures. The Accuplacer tests focus on basic skills and most of the problems are presented in formal, abstract terms. They also expect and assess facility in basic arithmetic, and only allow the use of a calculator on certain problems. This is in direct contrast to the HSA, which permits the use of a calculator.

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# The VSC for Algebra I/Data Analysis, the HSA for Algebra I/Data Analysis, and the Accuplacer Elementary Algebra Test

The full alignment is included in the appendix.

## **Conclusion:**

The VSC for Algebra I/Data Analysis does not prepare a student to perform at a minimally sufficient level on the Accuplacer Elementary Algebra Test.

The Accuplacer corresponds to traditional material typically covered in an algebra I course. It is largely concerned with the abstract manipulations required to solve equations and covers the basic mathematical skills necessary to do so. These skills include arithmetic skills as well as formal manipulations of arithmetic and algebraic expressions. The student is expected to be able to apply these skills to solve problems. There are a few problems that deal with real-world applications and interpretations, but most problems are stated in purely mathematical terms.

The VSC and the HSA do not focus on basic mathematical skills. The curricular approach, with its stated backbone of real-world applications, centers on making the material relevant to the student rather than on the acquisition of basic algebraic skills. The VSC and the HSA include many topics that are not required for the Accuplacer tests. In fact, only about 30 percent of the VSC for Algebra I/Data Analysis is found to be relevant to the Accuplacer Elementary Algebra Test. Conversely, the Accuplacer test assesses facility with arithmetic expressions and with formal manipulations of algebraic expressions, particularly polynomials, and much of this material is not included in the VSC. The following chart summarizes the findings:

Accuplacer	% of Applicable VSC Indicators	% of Comparable HSA Problems
12 problems	<30%	<6%

## **The Accuplacer Skills for Elementary Algebra**

The Accuplacer skills are straightforward and are generally the skills expected to be contained in traditional high school mathematics courses. The following description is from the Accuplacer Sample Questions for Students Guide:<sup>16</sup>

*There are 12 problems on this test. They are divided into problems of the following types.*

**Operations with Integers and Rational Numbers:** The first type involves operations with integers and rational numbers, and includes computation with integers and negative rationals, the use of absolute values, and ordering.

**Operations with Algebraic Expressions:** The second type involves operations with algebraic expressions using evaluation of simple formulas and expressions, and adding and subtracting monomials and polynomials. Questions involve multiplying and dividing monomials and polynomials, the evaluation of positive

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rational roots and exponents, simplifying algebraic fractions, and factoring.

**Solving Equations, Inequalities, and Word Problems:** The third type of question involves translating written phrases into algebraic expressions and solving equations, inequalities, word problems, linear equations and inequalities, quadratic equations (by factoring), and verbal problems presented in an algebraic context.

**Maryland’s VSC for Algebra I/Data Analysis**

The Maryland high school math VSCs for Algebra I/Data Analysis and Geometry are subdivided into three Core Learning Goals, or CLGs. CLG 1 is concerned with algebra, CLG 2 is geometry, and CLG 3 is data analysis. The CLGs are subdivided into expectations that are then subdivided into indicators. The VSC for Algebra I/Data Analysis includes CLG 1, which has nine indicators, and CLG 3, which has five indicators. The CLG 3 material does not include algebra, so only 64 percent of the VSC for Algebra I/Data Analysis is concerned with algebra.

A complete version of the VSC is included in the full alignment.

The indicators for the algebra portion of the VSC are:

- 1.1.1 The student will recognize, describe, and/or extend patterns and functional relationships that are expressed numerically, algebraically, and/or geometrically.
- 1.1.2 The student will represent patterns and/or functional relationships in a table, as a graph, and/or by mathematical expression.
- 1.1.3 The student will apply addition, subtraction, multiplication, and/or division of algebraic expressions to mathematical and real-world problems.
- 1.1.4 The student will describe the graph of a nonlinear function and discuss its appearance in terms of the basic concepts of maxima and minima, zeros (roots), rate of change, domain and range, and continuity.
- 1.2.1 The student will determine the equation for a line, solve linear equations, and/or describe the solutions using numbers, symbols, and/or graphs.
- 1.2.2 The student will solve linear inequalities and describe the solutions using numbers, symbols, and/or graphs.
- 1.2.3 The student will solve and describe using numbers, symbols, and/or graphs if and where two straight lines intersect.
- 1.2.4 The student will describe how the graphical model of a nonlinear function represents a given problem and will estimate the solution.
- 1.2.5 The student will apply formulas and/or use matrices (arrays of numbers) to solve real-world problems.

The “Additional Topics Would Include” topics of the VSC are included as subindicators of the above indicators, but these are not necessarily included as topics in the VSC.



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As discussed, reading the VSC to determine if a particular skill is covered is no easy task. The indicators listed above are too broadly stated to discern if a particular skill is included, so the “Assessment Limits” and “Skill Statement” must be carefully considered. Even upon consideration it is sometimes difficult to determine if a particular skill is covered. For contrast, one can consider the clearly stated and easily understood algebra I content standards developed in California.<sup>17</sup>

To further elaborate this difficulty, consider the Accuplacer sample problem: If  $2x - 3(x + 4) = -5$ , then  $x = ?$  Solving this problem involves several steps including simplifying the expression on the left. The algebra I standards in California include: Students simplify expressions before solving linear equations and inequalities in one variable, such as  $3(2x-5) + 4(x-2) = 12$ . This makes it obvious that a California student is expected to be able to solve the problem. In the Maryland VSC, it is difficult to conclude this. While indicator 1.2.1 clearly states that a student should be able to solve linear equations, the accompanying “Assessment Limits” and “Skill Statement” are confusing and prescribe a limited number of forms that the equation can have. This may preclude a student from solving a multi-step problem that involves a simplification such as the sample problem. There are no comparable problems on the HSA, and it seems likely that this sort of equation is not intended to be covered by the VSC.

In addition, indicator 1.1.3, regarding the application of addition, subtraction, multiplication, and/or division of algebraic expressions to mathematical and real-world problems, requires additional explication. It would, for example, seem to imply that a student could perform the simplification required in the sample problem above. Most of the problems on the Accuplacer Elementary Algebra Test assess similar skills with manipulating algebraic expressions, so this indicator is the single most applicable and important one for the alignment. However, the “Assessment Limits” for this indicator are extremely constrictive and the “Skill Statement” is not helpful. They are:

**Assessment Limits**

- The algebraic expression is a polynomial in one variable.
- The polynomial is not simplified.

**Skill Statement**

- The student will represent a situation as a sum, difference, product, and/or quotient in one variable.

These qualifications are so limiting that this indicator loses much of its applicability to the kinds of problems on the test. In fact, much of the material on the Accuplacer corresponds directly to the optional material included as “Additional Topics Would Include” subindicators of this indicator. Students who receive instruction on these optional topics are far better prepared for the Accuplacer tests than students who do not.

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### **The HSA**

The HSA is closely aligned with the VSC for Algebra I/Data Analysis and serves as a window into the types of problems that students are expected to solve. It should be noted that many of the problems on the HSA are not well posed in a mathematical sense. A full chart of the problems on the HSA is available in the complete alignment. The problems on the HSA require almost none of the classical, formal manipulations that are traditionally part of an algebra I course and that are required for the Accuplacer tests. The strong emphasis on applied mathematics may be relevant to the word problems on the Accuplacer tests; however, there are very few abstractly stated, nonsituational problems. Furthermore, most of the problems can be solved with minimal algebra skills. Thus, there is very little alignment between the kinds of problems on the HSA and the kinds of problems on the Accuplacer. According to the chart in the full alignment, only two out of the 37 HSA problems, or less than 6 percent, are found to be comparable to Accuplacer problems.

### **The Alignment**

The full alignment is included in the appendix.

As stated above, the VSC for Algebra I/Data Analysis and the Accuplacer Elementary Algebra Test are not well aligned; there is also no alignment between the Algebra I/Data Analysis HSA and the Accuplacer Elementary Algebra Test. A summary of the results is given in the following table.

### **Alignment of the Accuplacer Elementary Algebra Test compared with Maryland VSC and HSA**

Accuplacer Elementary Algebra	% of Applicable VSC Indicators	% of Comparable HSA Problems
12 problems	<30%	<6%

As discussed, only 64 percent of the VSC, the indicators for Core Learning Goals 1 (CLG 1), is concerned with algebra. This material includes many topics that are not required for the Accuplacer tests, such as graphing nonlinear functions and equations for lines. Of the nine indicators for CLG 1 only three or four appear to be directly applicable to the Accuplacer tests. This corresponds to less than 30 percent of the VSC for Algebra I/Data Analysis. Conversely, the Accuplacer test assesses facility with arithmetic expressions and with formal manipulations of algebraic expressions, particularly polynomials, and much of this material is not included in the VSC.

The three sections of the Accuplacer are analyzed separately below. Of the three categories of Accuplacer questions, only one of the categories, Solving Equations, Inequalities, and Word Problems, has any true alignment with the VSC. The other two

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categories involve operations with rational numbers and abstract manipulation, and are not well covered in the VSC.

There are two broad ways in which Accuplacer differs from the VSC and the questions included in the HSA. These reflect the national debate in mathematics education.

### *1) Arithmetic*

The Accuplacer is designed to be taken without a calculator (though it may offer a calculator in situ on some problems). It assumes and requires a certain level of arithmetic fluency. For example, the Accuplacer sample problem  $4 - (-6) =$   
-5

should not require a calculator; only the most basic arithmetic is involved. However, the VSC and the HSA do not include any indication that students should be expected to build upon and fully master the arithmetic skills they learned in elementary and middle schools. Thus, students may have long forgotten how to perform elementary calculations by hand. This sort of mastery is required for a traditional algebra class emphasizing abstract manipulation. However, given that the VSC favors a more applied approach depending on technology, it may be possible to perform well in a VSC-based algebra class without full mastery of basic arithmetic.

### *2) Level of Abstraction*

The problems on the Accuplacer are generally stated abstractly, and they emphasize mastery of particular mathematical skills. This is in direct contrast to the approach of the VSC for Algebra I/Data Analysis and the corresponding problems on the HSA, where nearly every problem is situational. Generally, the level of mastery of a particular mathematical skill required to solve an HSA problem is minimal.

## **The Accuplacer Categories for Elementary Algebra**

The following discusses the alignment within each subsection of the Accuplacer Elementary Algebra Test.

### *Operations with Integers and Rational Numbers*

Much of the material here should be prerequisite. However, as discussed above, some students may not be able to solve arithmetic problems without a calculator. Explicit mention of skills manipulating rational numbers is made in the VSC only under the “Additional Topics Would Include” section.

### *Operations with Algebraic Expressions*

The Accuplacer requires facility with general (more than one variable) polynomials, and that students be able to work with polynomials and other algebraic expressions to solve problems. These skills are not explicitly included in the VSC. The only directly applicable indicator is 1.1.3, which states: *The student will apply addition,*

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*subtraction, multiplication, and/or division of algebraic expressions to mathematical and real-world problems.* As discussed, the “Assessment Limits” for this indicator limit the polynomials to one-variable polynomials and also state: *The polynomial is not simplified.* This seems to imply that students are not expected to simplify and otherwise work with polynomial expressions to solve problems. There is an extensive list of topics under the “Additional Topics Would Include” section for this indicator. It is here that most of the relevant skills appear.

At the lowest Accuplacer scoring level of 25, the student must be able to multiply a whole number by a binomial. At the second lowest scoring level of 57, the student must be able to both multiply binomials and combine like terms. These skills are inadequately covered in the VSC.

*Solving Equations, Inequalities, Word Problems*

The skills in this category are at least partially covered by the VSC. However, the limiting statements in the “Assessment Limits” and “Skill Statement,” as well as a student’s potential lack of additional skills, may mean that the problems in this area cannot be solved by students.

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# The VSC for Algebra II and the Accuplacer College Level Math Test

The complete analysis is available in the appendix. In this analysis, the main focus is to determine if a student can achieve a score of 45 or more. This is the minimal score to avoid mathematics remediation at many Maryland colleges.

## **Conclusion**

The VSC for Algebra II and the Accuplacer College Level Math Test are reasonably well aligned. The Accuplacer includes one topic, trigonometry, which is not covered at all in the VSC for Algebra II. However, this should not preclude a student from obtaining a score of 45 points or more.

## **The Accuplacer Skills for College Level Math**

The Accuplacer skills are straightforward, and are generally the skills expected to be contained in traditional high school mathematics courses. The following description is derived from the description given in the Accuplacer Sample Questions for Students Guide:<sup>18</sup>

*There are 20 problems on this test. They are divided into problems of the following types.*

**Algebraic Operations:** The Algebraic Operations content area includes the simplification of rational algebraic expressions, factoring and expanding polynomials, and manipulating roots and exponents.

**Solutions of Equations and Inequalities:** The Solutions of Equations and Inequalities content area includes the solution of linear and quadratic equations and inequalities, systems of equations, and other algebraic equations.

**Coordinate Geometry:** The Coordinate Geometry content area presents questions involving plane geometry, the coordinate plane, straight lines, conics, sets of points in the plane, and graphs of algebraic functions.

**Applications and Other Algebra Topics:** The Applications and Other Algebra Topics content area contains complex numbers, series and sequences, determinants, permutations and combinations, factorials, and word problems.

**Functions:** The Functions content area includes questions involving polynomial, algebraic, exponential, and logarithmic functions.

**Trigonometry:** The Trigonometry content area includes trigonometric functions.

## **The VSC for Algebra II**

The organization of Maryland's VSC for Algebra II is similar to the organization for Algebra I/Data Analysis. However, some of the expectations have no indicators. The following list sums up the VSC for Algebra II.

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- 1.1.1 The student will determine and interpret a linear function when given a graph, table of values, essential characteristics of the function, or a verbal description of a real-world situation.
  - 1.1.2 The student will determine and interpret a quadratic function when given a graph, table of values, essential characteristics of the function, or a verbal description of a real-world situation.
  - 1.1.3 The student will determine and interpret an exponential function when given a graph, table of values, essential characteristics of the function, or a verbal description of a real-world situation.
  - 1.1.4 The student will be able to use logarithms to solve problems that can be modeled using an exponential function.
  - 1.2 Given an appropriate real-world situation, the student will choose an appropriate linear, quadratic, polynomial, absolute value, piecewise-defined, simple rational, or exponential model, and apply that model to solve the problem.
  - 1.3 The student will communicate the mathematical results in a meaningful manner.
    - 1.3.1 The student will describe the reasoning and processes used in order to reach the solution to a problem.
    - 1.3.2 The student will ascribe a meaning to the solution in the context of the problem and consider the reasonableness of the solution.
  - 2.1.1 The student will identify and use alternative representations of linear, piecewise-defined, quadratic, polynomial, simple rational, and exponential functions.
  - 2.1.2 The student will identify the domain, range, the rule, or other essential characteristics of a function.
  - 2.2.1 The student will add, subtract, multiply, and divide functions.
  - 2.2.2 The student will find the composition of two functions and determine algebraically and/or graphically if two functions are inverses.
  - 2.2.3 The student will perform translations, reflections, and dilations on functions.
  - 2.3 The student will identify linear and nonlinear functions expressed numerically, algebraically, and graphically.
  - 2.4 The student will describe or graph notable features of a function using standard mathematical terminology and appropriate technology.
  - 2.5 The student will use numerical, algebraic, and graphical representations to solve equations and inequalities.
  - 2.6 The student will solve systems of linear equations and inequalities.
  - 2.7 The student will use the appropriate skills to assist in the analysis of functions.
    - 2.7.1 The student will add, subtract, multiply, and divide polynomial expressions.
    - 2.7.2 The student will perform operations on complex numbers.
    - 2.7.3 The student will determine the nature of the roots of a quadratic equation

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and solve quadratic equations of the form  $y = ax^2 + bx + c$  by factoring and the quadratic formula.

2.7.4 The student will simplify and evaluate expressions with rational exponents.

2.7.5 The student will perform operations on radical and exponential forms of numerical and algebraic expressions.

2.7.6 The student will simplify and evaluate expressions and solve equations using properties of logarithms.

2.8 The student will use literal equations and formulas to extract information.

### **The Alignment**

Of the six topics designated by the Accuplacer College Level Math Test, three of them are well covered, two of them are covered incompletely, and one of them, trigonometry, is not covered at all in Maryland's Algebra II VSC. In the proficiency statements, trigonometry skills appear first at a score of 63. Students who lack knowledge of trigonometry are not precluded from scoring 45 or better on this Accuplacer.

### **The Accuplacer Categories**

#### *Algebraic Operations*

This area is well covered by the VSC for Algebra II.

#### *Solutions of Equations and Inequalities*

This area is well covered by the VSC for Algebra II.

#### *Coordinate Geometry*

Coverage here is somewhat uneven.

#### *Applications and other Algebra Topics*

Coverage here is also somewhat problematic.

#### *Functions*

This area is well covered by the VSC for Algebra II.

#### *Trigonometry*

Trigonometry does not appear as a topic in the VSC for Algebra II. A rudimentary understanding of trigonometry is explicitly mentioned in the proficiency statements to score about 63 on the Accuplacer. The skills mentioned to score about 40 do not include trigonometry.





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# Mathematics Placement, Requirements, and Procedures at Baltimore City Community College (BCCC)

## **Introduction**

In 2002 and 2004, The Abell Foundation published reports that focused on opportunities and challenges at BCCC.<sup>20</sup> These reports specifically addressed the dilemma of remedial education and extensively documented much of the following.

Nearly all of the Baltimore City public high school graduates entering BCCC are required to take remedial courses in mathematics. BCCC has a sequence of three remedial courses, and many students must pass all three courses to proceed with the goal of obtaining a degree. These courses are a tremendous obstacle to many students. They require tuition and attendance but do not apply to the awarding of a degree. Moreover, many students do not pass these courses, even after repeated attempts, and are thus effectively barred from obtaining a degree.

For students who are required to take the Accuplacer upon admittance, there is a single criterion to be placed in a remedial math course. This is failure to achieve a score of 45 or more on the Accuplacer College Level Math Test. This single cut-off score is used by many Maryland community colleges. It is unclear why or how this cut-off score was chosen. In particular, it is unclear why the College Level Math Test was chosen for a cut-off score rather than the Elementary Algebra Test. Because the College Level Math Test includes college-level material, it is likely that many students are inadequately prepared to take it. In the neighboring state of West Virginia, the designation of remedial math does not involve the College Level Math Test.

In the two previous reports about BCCC, students' admissions, processes, and experiences with taking the Accuplacer are explored. Many students do not understand the importance of the Accuplacer test as it relates to their course placement. They do not understand that poor Accuplacer scores will force them to take noncredit courses. Many students take it without any preparation whatsoever and in less than ideal circumstances; they are subsequently shocked to find that they are required to take math classes without receiving credit for them. The poor performance on the Accuplacer is undoubtedly related to these factors. Perhaps students would perform better on the Accuplacer tests with prior knowledge of the importance of their results. The Accuplacer tests cover a lot of skills, and even a basic review of math would be likely to affect many students' scores.

## **Accuplacer and Placement 2008**

Nearly all incoming BCCC students who are pursuing a degree or certificate are required to take the Accuplacer. Students are placed into courses depending on their Accuplacer test scores. According to one Accuplacer administrator, many students do not prepare for the Accuplacer. There is a booklet with some practice problems available outside the testing center where the test is administered. There is also some limited online material to review for the test. However, it seems that most

students take the test without preparing for it and without realizing the consequences of not performing well on it.

The general procedure is that placement in math courses is automatically generated when students take the Accuplacer. The Elementary Algebra Test is administered first. If the score is greater than or equal to 63, the College Level Math Test is administered. If the score is less than 35, the Arithmetic Test is administered. However, the Arithmetic score is not used in the placement criteria.

Placement, along with informal course names, is given in the following chart. The Math 80, 81, and 82 courses are all noncredit, remedial courses.

**Accuplacer Scores and Placement at BCCC**

Accuplacer Elementary Algebra Score	Accuplacer College Level Math Score	BCCC Course Placement
<35		Mat 80 (Arithmetic)
35-62		Mat 81 (Algebra I)
≥63	<45	Mat 82 (Algebra II)
≥63	≥45	College Credit Mat 107 (Statistics) 111, 115, 125 (Various) 128 (College Algebra/Precalculus)
≥63	≥63	See Chair

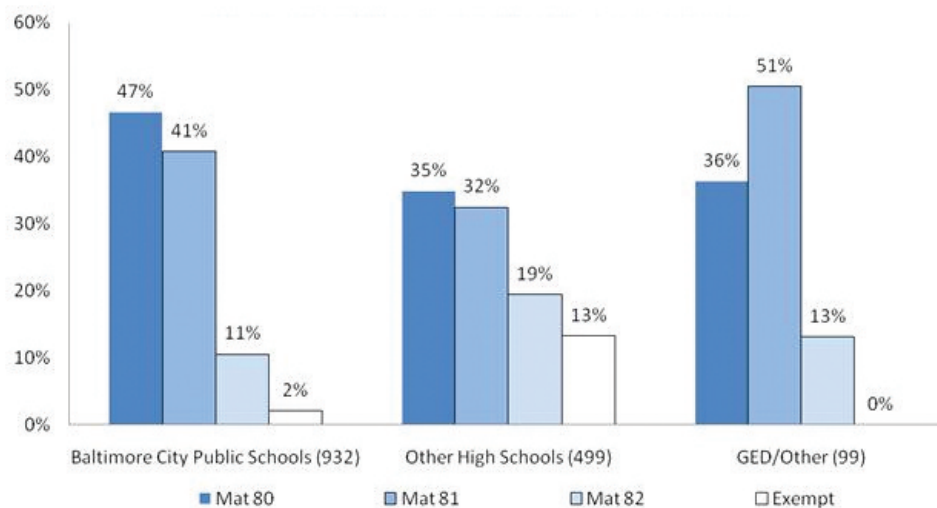
Source: *Baltimore City Community College, Institution Research Department*

**BCPSS Math Placement by Accuplacer**

The following chart indicates the placement results for incoming 2007 Baltimore City Public Schools graduates in relation to other incoming students. Note that 98 percent of Baltimore City graduates are placed into remedial courses. Almost half are placed into the lowest-level math class, Math 80, which is essentially arithmetic. A full 88 percent are placed into one of the two lower-level remediation courses. This means that 88 percent of them scored less than 63 on the Accuplacer Elementary Algebra Test. It is worth noting that the remediation rate at BCCC for high school graduates attending other Maryland school districts is also high at 87 percent.

**Figure 3 Placement Recommendations in Mathematics for 2007 High School Graduates based on Accuplacer scores through September 2008**

Source: *Baltimore City Community College, Institution Research Department*



Data reflect the recommendations made on the basis of the highest Accuplacer scores earned.

### The Remedial Courses at BCCC

There are three sequential remedial courses: Math 80, Math 81, and Math 82. All have departmental course outlines, course objectives, and final exams. The departmental final exam comprises 20 percent of a student's final grade. It is departmental policy that graphing calculators are not used in developmental courses. The use of a scientific calculator (a calculator without graphing capability) is permitted. The final exams may have calculator and noncalculator sections. Course descriptions from the course outlines are as follows.

#### Math 80

MAT 80 covers the following topics: whole numbers, fractions and mixed numbers, decimals, ratio, proportions and unit analysis, percents, and elements of geometry (perimeter, areas of rectangle, triangle, circle, and others). Word problems and the use of calculators to solve them are stressed throughout the course. Math 80 is a 0 credit course. It does not count toward graduation.

#### Math 81

MAT 81 meets for five contact hours per week, and counts as five billable hours. However it is a 0 credit course. It does not count toward graduation. MAT 81 covers the following topics: real numbers and operations on real numbers;

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absolute value; evaluations; grouping symbols; combining like terms; linear equations in one and two variables; literal equations; laws of exponents; scientific notation; graphs of linear equations in two variables; and finding equations of a line given slope and y-intercept, slope and a point, or two points. A discussion of slopes of parallel and perpendicular lines is presented. Operations on polynomials, factoring, and solutions to quadratic equations by factoring are also covered. Word problems and the use of calculators to solve them are stressed throughout the course.

### Math 82

MAT 82 meets for four contact hours per week, and counts as four billable hours. However it is a 0 credit course. It does not count toward graduation. MAT 82 covers operations on algebraic expressions; variation; rational equations; irrational equations; solutions to quadratic equations by completing the square, by the square root property, and by the quadratic formula; and solutions of quadratic inequalities. Absolute value equations and inequalities and finding equations of straight lines meeting specific criteria are emphasized. Graphs of linear inequalities in two variables, of linear inequalities on one variable, and of parabolas are also included. Functions and real-world applications of them are emphasized throughout the course.

Currently, the texts are a sequence of books by the same author. Students who place into one of these courses are required to complete the full sequence as a prerequisite to taking a first course for credit. The students have access to a “Math Lab” for help. They can obtain help in the “Math Lab” from other students and faculty.

### For-Credit Courses

There are many for-credit courses available to students to satisfy general college math requirements. However, most students would like to take either MAT 128 (College Algebra/Precalculus) or MAT 107 (Statistics) in order to fulfill requirements for their major. The Math 107 course is a goal for many of the students because it is required for most of the health-related majors, and this is a large component of the student body at BCCC. Course descriptions from the course catalog are as follows:

#### **MAT 107: Modern Elementary Statistics (three credits)**

*Meets Category IV General Education Requirements*

*45 lecture hours*

*Prerequisites: MAT 82; ENG 82 (For ESL: ELI 82W) or appropriate*

*ACCUPLACER scores*

Modern statistical methods with applications to the social and natural sciences are studied. The course focuses on descriptive statistics; probability; probability distributions; and estimation of statistical parameters from samples, hypothesis

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testing, and experimental design. It provides necessary statistical background for people interested in such diverse fields as psychology, sociology, computers, business, engineering, mathematics, and science.

**MAT 128: Precalculus I: College Algebra (four credits)**

*Meets Category IV General Education Requirements*

*60 lecture hours*

*Prerequisites: MAT 82; ENG 82 (For ESL: ELI 82W) or appropriate ACCUPLACER scores*

More advanced topics in algebra including functions and their graphs; inverse functions; polynomial, rational, exponential, and logarithmic functions; systems of linear and nonlinear equations; and inequalities are emphasized.

It is worth noting here that these topics are all included in the categories for the Accuplacer College Level Math Test. It, therefore, makes little sense to use performance on this test for placement into this course.

Many students successfully complete all requirements for their major before the math requirements. They are then faced with a potential two- or three-semester delay in the pursuit of their major in order to fulfill the math requirement. There has been some discussion in the department about eliminating Math 82 as a prerequisite for the statistics class Math 107, but this has not been pursued. There is, however, a condensed version of the Math 82 and Math 107 courses offered in the fall semester. It combines these two courses into a single semester.



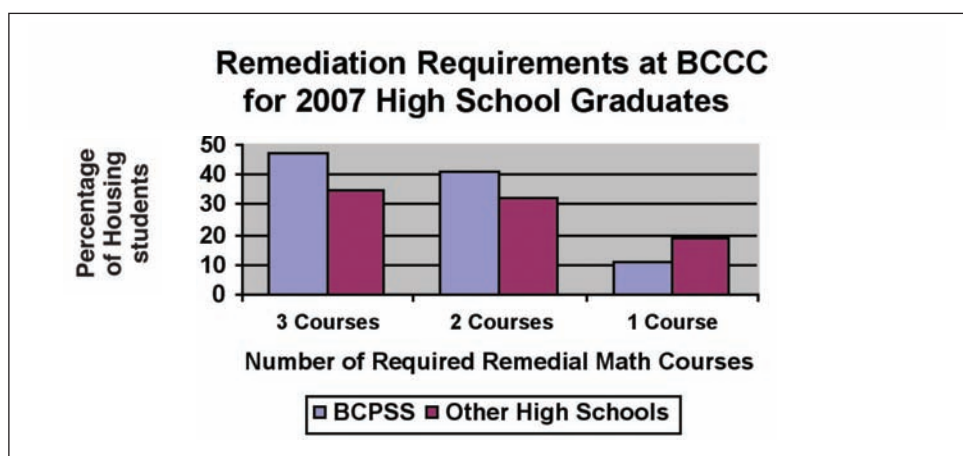
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# Recommendations

The primary purpose of this report is to review the alignment of Maryland high school mathematics mandates with the expectations of Maryland colleges, particularly community colleges, with a focus on graduates of Baltimore City Public Schools (BCPSS) who go on to college at Baltimore City Community College (BCCC). **This report finds that the state-mandated curriculum for mathematics does not adequately cover the material that colleges expect from incoming students.** This is illustrated by the fact that in 2008, about 98 percent of BCCC's incoming students from BCPSS were determined by BCCC to require remedial mathematics courses.

Many students' first experience with mathematics at BCCC is with an Accuplacer math test, used for determining the placement of students in their first mathematics course. Students are given the Accuplacer Elementary Algebra Math Test first. If they do well, they also take the Accuplacer College Level Math Test, and if they do not do well, they are given the Accuplacer Arithmetic Test. The following chart summarizes the placement of incoming students at BCCC by Accuplacer results.

This chart illustrates that approximately 88 percent of 2007 BCPSS graduates placed into two or more remedial math courses. Nearly nine in 10 city students failed to perform well on the Accuplacer Elementary Algebra Test, which is essentially a traditional algebra I test.



The following recommendations are made for the purpose of assuring that Maryland public high school mathematics requirements successfully prepare students for post-secondary institutions.

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**For the State Department of Education: High School Mathematics**

This report concludes that the Maryland VSC for Algebra I/Data Analysis and the accompanying state test, the HSA for Algebra I/Data Analysis, do not represent traditional algebra and so are not aligned with the Accuplacer Elementary Algebra Test. The VSC for Algebra I/Data Analysis and the HSA include material on precalculus and data analysis that is not required by colleges and do not include the basic arithmetic and algebraic skills that colleges do require.

The HSA for Algebra I/Data Analysis is now a state requirement for graduation and thus potentially determines the focus of math education in high schools. It currently focuses on naïve data analysis rather than on core mathematical skills. The kinds of problems on the HSA have almost no relevancy to the kinds of problems on the Accuplacer Elementary Algebra Test. This finding seems to be at the heart of Maryland high schools' failure to align with college expectations.

*For students to properly prepare for college:*

**Recommendation #1: The classical material of algebra I should be explicitly included in the state's VSC for Algebra I/Data Analysis. At a minimum, the "Additional Topics" portion of the VSC should be explicitly required as part of the Algebra I VSC.**

**Recommendation #2: The HSA for Algebra I/Data Analysis should be redesigned to focus on core mathematical material in algebra.**

The preoccupation of the VSC for Algebra I/Data Analysis and the HSA with data analysis instead of basic algebraic skills has led not only to a misalignment with college expectations, but also to a potential misalignment of Algebra I with Algebra II in high schools. The current VSCs for Algebra I/Data Analysis and Algebra II do not form a cohesive sequence for the learning or teaching of algebra. The Algebra II VSC is generally well aligned with college expectations and should be largely retained. However, the sequencing of high school algebra as a whole should be examined.

**Recommendation #3: The state should review and revise, if necessary, both of the VSCs for algebra. Together, they should constitute a clear and focused sequence for the teaching and learning of algebra.**

Arithmetic proficiency is a component of the Elementary Algebra Test. This is appropriate because algebra can be described as a generalization of arithmetic. Traditional algebra has its foundations in the laws of arithmetic, and it should not be formally studied without a thorough knowledge of arithmetic. At BCCC, about 47 percent of BCPSS students graduating in 2007 were placed into the first sequenced



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remedial course, which is Arithmetic. Although this study concentrates on algebra and high school, these results reveal a problem with students' arithmetic skills and thus in their preparation for algebra.

It is possible that the Accuplacer Arithmetic Test is the first rigorous arithmetic test that many students have ever encountered. This is not fair to the students. To ensure that students are adequately prepared for both college and algebra:

**Recommendation #4: Students should master arithmetic before they take an algebra I course.**

**Recommendation #5: All Maryland high school graduates should be proficient in arithmetic without the use of a calculator. Proficiency includes adding, subtracting, multiplying, and dividing fractions and decimal numbers as well as being able to work with percents and ratios.**

*These lead to the following recommendations:*

**Recommendation #6: Maryland should review the K-8 VSC to ensure that arithmetic skills are thoroughly covered.**

**Recommendation #7: The state should have in place some assessment of arithmetic proficiency that could help determine if students are prepared for algebra. State assessments are the logical tool, and they could be reviewed to ensure that they adequately assess arithmetic skills. Alternatively, another test (such as the now-defunct Maryland Functional Math Test) could be developed or adopted.**

Many states have worked on developing statewide curricula, and all states can potentially benefit from the results. Maryland, with its national visibility and high proportion of STEM employment opportunities, could be a national leader in the development and implementation of exemplary mathematics education. The State Department of Education should review mathematics standards of other well-regarded states, and perhaps even countries, and incorporate and/or adopt some of the material that has already been developed. States that have highly rated math standards include California and Massachusetts, so the standards of these states and their accompanying state assessments should be carefully considered.

Maryland has committed to ensuring that its high school graduates are prepared for college-level mathematics courses. This commitment implies that high school graduates should not be relegated to remedial status by colleges. Ideally, the pre- and post-secondary communities in Maryland should work together to align expectations

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and offer students a clear graduation route that avoids remediation in college. One way to accomplish this would be to redesign the HSA to conform more closely to college expectations and encourage colleges to use HSA scores to make placement decisions.

Meanwhile, in the absence of state guidance, students need to be informed that college mathematics requirements are likely to differ significantly from high school mathematics. They need to know that many colleges will require them to take a rigorous mathematics placement exam. So long as local colleges require Accuplacer testing, it would be of enormous benefit to students if high schools provided guidance and preparation for the Accuplacer tests. High schools should help students achieve the required fluency with arithmetic and basic algebra to perform well on the Accuplacer Arithmetic Test and the Accuplacer Elementary Algebra Test. This could help to ensure that Maryland high school graduates are not required to take remedial mathematics courses in college.

**Recommendation #8: High schools should provide guidance for preparing for college mathematics. Because the Accuplacer is the instrument used by many local colleges, high schools should develop appropriate tools and/or courses to help students prepare for the Accuplacer tests. High schools should consider administering the Accuplacer in 12th grade, provided that the scores are accepted as valid by local colleges.**

#### **For BCCC and other Maryland Post-Secondary Institutions**

A student's first experience with mathematics at BCCC, other Maryland community colleges, and many of the state's public universities is with the Accuplacer math tests. Students are typically not aware of the importance of the tests and the effects they will have on their college careers.

**Recommendation #9: Students should be informed in advance about the importance of their Accuplacer scores, be provided with appropriate guidance on preparing for the Accuplacer tests, and have access to material and/or guided review sessions to help them prepare.**

Students are given the Accuplacer Elementary Algebra Math Test first. If they do well, they also take the Accuplacer College Level Math Test. As the name suggests, this test is generally considered to be college-level mathematics. At BCCC, the material covered in the for-credit college algebra course is also covered by the Accuplacer College Level Math Test. This illustrates that it is inappropriate to use the College Level Math Test to determine the necessity for remediation.

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**Recommendation #10: The Accuplacer College Level Math Test should not be used to determine whether or not a student requires remediation in mathematics.**

Unlike BCCC, the University of Maryland College Park campus allows students to take the placement test again after five weeks of review in the remedial courses. If the students pass, the course changes into a credit-bearing course for them. This is a model that could help many BCCC students.

**Recommendation #11: BCCC should consider having students retake the placement test after a few weeks of remediation rather than locking them into remediation for a whole semester or more.**

**For Baltimore City Public Schools**

The state of Maryland requires that all students complete high school mathematics courses in algebra I, geometry, and one other unspecified course. Currently, Baltimore City has the more stringent requirement that all BCPSS students are required to take algebra I, geometry, and algebra II for graduation. Theoretically, this requirement should result in satisfactory scores on the Accuplacer. However, as is demonstrated by college remediation rates for BCPSS students, this does not turn out to be the case. It seems likely that students are not benefitting much from their algebra II courses and would be better served by the option of taking a more basic or other alternative math course. This leads to:

**Recommendation #12: Baltimore City should align its math graduation requirement with the state requirement. Algebra II should not be required unless the state requires it.**



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# About the Authors:

· **Gabrielle Martino** received her Ph.D. in mathematics from The Johns Hopkins  
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· **W. Stephen Wilson** is a professor and former chair of the mathematics department at  
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· 1972. In 2006, he spent eight months as the Senior Advisor for Mathematics at the  
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· Education, and was one of the coauthors of the Fordham Foundation Report, “The  
· State of State MATH Standards, 2005.” More recently, Wilson helped revise the  
· Washington State K-12 mathematics standards and evaluated textbooks for the state.  
· He has written more than 60 research papers in mathematics, specializing in algebraic  
· topology.



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- <sup>14</sup> Both VSCs for algebra include material that is entitled “Additional Topics Would Include,” or “Additional Topics.” The indicators for the “Additional Topics” contain many of the skills necessary for the Accuplacer, so, for the purposes of the alignment, it was necessary to determine whether or not to include these topics as part of the VSC. Their name suggests that they are optional topics and a close reading of the publicly available material verifies this interpretation. In the publicly available draft of the VSC, the state stipulates that all students must successfully complete an Algebra I/Data Analysis course and a Geometry course in which the Core Learning Goals are part of the curriculum, but it does not mention the “Additional Topics” ([http://mdk12.org/share/hsvsc/source/VSC\\_algebra\\_hs.pdf](http://mdk12.org/share/hsvsc/source/VSC_algebra_hs.pdf)). Additional explication of the “Additional Topics” is that they are “content that may be appropriate for the curriculum but [are] not included in the Core Learning Goals” (<http://mdk12.org/instruction/hsvsc/algebra/standard1.html>). Together, these statements imply that coverage of these topics is, indeed, optional, so this report treats them as if they are not necessarily included as part of the VSC.
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- <sup>17</sup> “Mathematics Content Standards for California Public Schools,” *California State Board of Education*, December 1997. Retrieved from <http://www.cde.ca.gov/be/st/ss/documents/mathstandard.pdf>.
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# Appendices

## I. ACCUPLACER ELEMENTARY ALGEBRA TO VSC ALGEBRA I ALIGNMENT

### A. Accuplacer Elementary Algebra Skills Chart

The Accuplacer categories and skills in this chart were compiled from two sources: the Accuplacer Sample and the Accuplacer Coordinator's Guide. Both are available online:<sup>1</sup>

The sample problems are from the Accuplacer Sample.

Accuplacer Skill	Sample Problem	VSC Algebra I/ Data Analysis	HSA Comparable Problem	VSC Additional Topics	Notes
Operations with integers and rational numbers				1.1.1.4 The student will read, write, and represent rational numbers. 1.1.1.6 The student will add, subtract, multiply, and divide rational numbers.	This is largely prerequisite material. However, neither the HSA nor the VSC stresses fluency with arithmetic, or the basic laws governing arithmetic such as the distributive law.
Computation with integers	$-3(5-6)-4(2-3)=$				
Computation with negative rationals	$\frac{4-(-6)}{-5} =$				
The use of absolute values				1.1.3.2 The student will evaluate expressions containing absolute value. 1.1.3.1 The student will locate the position of a number on the number line, know that its distance from the origin is its absolute value, and know that the distance between two numbers on the number line is the absolute value of their difference.	
Ordering				1.1.1.5 The student will compare order and describe rational numbers.	

<sup>1</sup> [http://www.collegeboard.com/prod\\_downloads/student/testing/accuplacer/accuplacer-sample.pdf](http://www.collegeboard.com/prod_downloads/student/testing/accuplacer/accuplacer-sample.pdf);  
[http://www.collegeboard.com/prod\\_downloads/highered/apr/accu/coordinatorsGuide.pdf](http://www.collegeboard.com/prod_downloads/highered/apr/accu/coordinatorsGuide.pdf)

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Accuplacer Skill	Sample Problem	VSC Algebra I/ Data Analysis	HSA Comparable Problem	VSC Additional Topics	Notes
<p>Operations with algebraic expressions</p> <p>Full description: The second type involves operations with algebraic expressions using evaluation of simple formulas and expressions, and adding and subtracting monomials and polynomials.</p>		<p>1.1.3 The student will apply addition, subtraction, multiplication, and/or division of algebraic expressions to mathematical and real-world problems.</p> <p><b>Assessment Limits</b> The algebraic expression is a polynomial in one variable. The polynomial is not simplified.</p>			<p>The “Assessment Limits” specifying one variable for polynomials is extremely limiting. The Accuplacer requires familiarity with two-variable polynomials and some basic manipulations of them. The statement that “the polynomial is not simplified” is unclear. Does it mean that the student is required to solve a problem with an unsimplified polynomial, or that the student is not required to simplify a polynomial expression? The HSA problem and the VSC toolkit material available online indicate the latter. This is very limiting in terms of simplifying expressions and solving equations. The Accuplacer requires some proficiency in basic algebraic manipulations. Some of the more advanced problems involve several steps, which include simplifying expressions.</p>
Evaluation of simple formulas and expressions	What is the value of the expression $2x^2 + 3xy - 4y^2$ when $x = 2$ and $y = -4$ ?				<p>Evaluating formulas appears in indicator 1.2.5: The student will apply formulas and/or use matrices (arrays of numbers) to solve real-world problems. The real-world specification seems to rule out an abstractly given problem such as the sample problem.</p>

Accuplacer Skill	Sample Problem	VSC Algebra I/ Data Analysis	HSA Comparable Problem	VSC Additional Topics	Notes
Multiplying and dividing monomials		1.1.3		1.1.3.3 For one- or two-variable polynomials: The student will add, subtract, and multiply polynomials.	
Multiplying and dividing polynomials	$(3x+2y)^2=$	1.1.3		1.1.3.3 For one- or two-variable polynomials: The student will add, subtract, and multiply polynomials.	
Evaluation of positive rational exponents	$\sqrt{2} \times \sqrt{15} =$			1.1.3.6 The student will use the laws of exponents, including negative exponents, to simplify expressions. 1.1.3.7 The student will simplify radical expressions with or without variables.	
Simplifying algebraic fractions	If $x > 2$ , then $\frac{x^2 - 2x + 6}{x^2 - 4} =$	1.1.3			Certainly the sample problem, which involves factoring, is not included in the VSC. However, a simpler problem may be.
Solving equations, inequalities, word problems		1.2 The student will model and interpret real-world situations using the language of mathematics and appropriate technology.			The VSC includes extensive material on lines, such as equations and graphing, that is not included in the Accuplacer, which is only concerned with solving linear equations and inequalities, and being able to do word problems involving these skills.

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Accuplacer Skill	Sample Problem	VSC Algebra I/ Data Analysis	HSA Comparable Problem	VSC Additional Topics	Notes
Solving linear equations	If $2x - 3(x + 4) = -5$ , then $x = ?$	<p>1.2.1 The student will determine the equation for a line, solve linear equations, and/or describe the solutions using numbers, symbols, and/or graphs.</p> <p><b>Assessment Limits</b> Linear equations will be given in the form <math>Ax+By=C</math>, <math>Ax+By+C=0</math>, or <math>y=mx+b</math>.</p>			While 1.2.1 does include solving linear equations, the “Assessment Limits” and “Skill Statement” are confusing. It is not clear how general a facility is expected. It is not clear from the “Assessment Limits” what acceptable values are for A, B, and C. It is also not clear whether a student could solve a linear equation that is not given in the forms stated in the “Assessment Limits.” The sample problem from the Accuplacer is a multi-step problem that requires some basic algebraic manipulation to obtain the specified form.
Solving linear inequalities	Which of the following expressions is equivalent to $20 - (4/5)x \geq 16$ ?	<p>1.2.2 The student will solve linear inequalities and describe the solutions using numbers, symbols, and/or graphs.</p> <p><b>Assessment Limits</b> Inequalities will have no more than two variables with rational coefficients.</p>	<p>Ichiro plans to spend no more than a total of \$60 for both lunch and dinner each day during his vacation. Complete the following in the Answer Book:</p> <ul style="list-style-type: none"> <li>Write an inequality that models this relationship. Let <math>x</math> represent the amount, in dollars, that Ichiro spends on lunch. Let <math>y</math> represent the amount, in dollars, that Ichiro spends on dinner.</li> </ul>		

(continued on next page)

Accuplacer Skill	Sample Problem	VSC Algebra I/ Data Analysis	HSA Comparable Problem	VSC Additional Topics	Notes
Solving linear inequalities <i>(continued)</i>			<ul style="list-style-type: none"> <li>Graph the inequality on the grid provided in the Answer Book.</li> <li>On Thursday, Ichiro spent exactly \$60 on lunch and dinner. He spent three times as much on dinner as he spent on lunch. How much did Ichiro spend on lunch? How much did Ichiro spend on dinner? Use mathematics to explain how you determined your answers. Use words, symbols, or both in your explanations.</li> </ul>		
Solving quadratic equations (by factoring)				1.1.4.2 The student will solve quadratic equations by factoring and graphing.	Factoring is not in the VSC or the HSA.
Verbal problems presented in an algebraic context, including geometric reasoning and graphing	In the figure below, both circles have the same center, and the radius of the larger circle is $R$ . If the radius of the smaller circle is 3 units less than $R$ , which of the following represents the area of the shaded region?				Geometry is essentially missing from the VSC and the HSA. A problem like this one could go either way.
Translation of written phrases into algebraic expressions	If $A$ represents the number of apples purchased at 15 cents each, and $B$ represents the number of bananas purchased at 10 cents each, which of the following represents the total value of the purchases in cents?	1.1.3 The student will apply addition, subtraction, multiplication, and/or division of algebraic expressions to mathematical and real-world problems.  <i>(continued on next page)</i>	Three students sold pizzas to raise money. Dwayne sold $x$ pizzas. Tamara sold $x+20$ pizzas. Rueben sold $3(x+20)$ pizzas. Which of these expressions represents the total number of pizzas that all three students sold?		The one-variable limit in the “Skill Statement” for 1.1.3 is problematic. However, many of the other indicators include context in their “Assessment Limits” and “Skill Statement.”

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Accuplacer Skill	Sample Problem	VSC Algebra I/ Data Analysis	HSA Comparable Problem	VSC Additional Topics	Notes
Translation of written phrases into algebraic expressions <i>(continued)</i>		<p><b>Skill Statement</b>            The student will represent a situation as a sum, difference, product, and/or quotient in one variable.            1.2.1 The student will determine the equation for a line; solve linear equations; and/or describe the solutions using numbers, symbols, and/or graphs.</p> <p><b>Assessment Limits</b>            Linear equations will be given in the form <math>Ax+By=C</math>, <math>Ax+By+C=0</math>, or <math>y=mx+b</math>.            The majority of these items should be in real-world context.            Functions are to have no more than two variables with rational coefficients.</p> <p><b>Skill Statement (Paraphrased)</b>            Given a written description of a situation that can be modeled by a linear function, the student will write the equation.</p>			

### B. The VSC for High School Math

The Maryland VSC for high school math is subdivided into three Core Learning Goals, or CLGs. CLG 1 is concerned with algebra, CLG 2 is geometry, and CLG 3 is data analysis. The CLGs are subdivided into expectations, which are then subdivided into indicators. The naming scheme is thus as follows: 1.1.1 corresponds to CLG 1, expectation 1, indicator 1. The indicators may be provided with additional clarification by the use of the “Assessment Limits” and “Skill Statement” subheadings. These statements are necessary because the indicators are not skill-specific. For example, indicator 1.1.1 states: *The student will recognize, describe, and/or extend patterns and functional relationships that are expressed numerically, algebraically, and/or geometrically.* This could include a very wide range of particular skills, spanning naïve to sophisticated levels of mathematics. The necessary clarification as to the meaning of the indicators is supposed to be provided in the “Assessment Limits” and “Skill

Statement” that accompany it. However, these are not always clear. For example, the “Assessment Limits” for indicator 1.1.1 states: *The student will not be asked to draw three-dimensional figures.* No further reference or clarification is given with regard to this remark, or what it may imply about the indicator, and the meaning of it remains subject to interpretation.

There is also a section of the VSC labeled “Additional Topics Would Include,” and each indicator may have several apparently optional subindicators of the form 1.1.1.1 (additional topic 1 for 1.1.1). The publicly available material for the VSC states that the additional topics are “content that may be appropriate for the curriculum but is not included in the Core Learning Goals.” The state further stipulates that students must complete courses in algebra 1/data analysis and geometry that include the CLG but fails to mention the “Additional Topics.”<sup>2</sup> This seems to imply that the VSC does not necessarily include these topics, so the alignment treats them as if they are, indeed, optional. The “Additional Topics” portion of the standards are then sometimes most illuminating in terms of what the corresponding indicator does *not* include. For example, the factoring skills that form a large part of classical algebra are included only as “Additional Topics” in the VSC, leading one to conclude that they are not de facto part of the VSC.

The Accuplacer skills are quite straightforward, and are generally the skills expected to be contained in traditional high school mathematics courses. However, reading the VSC to determine if a particular skill is covered is no easy task. The indicators are too broadly stated to discern if a particular skill is included, so the “Assessment Limits” and “Skill Statement” must be carefully considered. Even upon consideration, it is sometimes difficult to determine if a particular skill is covered. For contrast, one can consider the clearly stated and easily understood algebra I content standards developed in California.<sup>3</sup>

### C. The VSC for Algebra I/Data Analysis

The VSC for Algebra I/Data Analysis includes CLG 1 and CLG 3. The content of the CLG 3 is not relevant to the Accuplacer exam. It is included below for completeness.

Note: In the following, all material numbered with four numerals is not part of the VSC. It is instead included under the heading, “Additional Topics Would Include.” The “Additional Topics” for CLG 3 are not included below.

*CLG 1 The student will demonstrate the ability to investigate, interpret, and communicate solutions to mathematical and real-world problems using patterns, functions, and algebra.*

*1.1 The student will analyze a wide variety of patterns and functional relationships using the language of mathematics and appropriate technology.*

1.1.1 The student will recognize, describe, and/or extend patterns and functional relationships that are expressed numerically, algebraically, and/or geometrically.

#### **Assessment Limits**

The given pattern must represent a relationship of the form  $y = mx + b$  (linear),  $y = x^2 + c$  (simple quadratic),  $y = x^3 + c$  (simple cubic), simple arithmetic progression, or simple geometric progression with all exponents being positive.

The student will not be asked to draw three-dimensional figures.

Algebraic description of patterns is in indicator 1.1.2.

<sup>2</sup> Retrieved from: [http://mdk12.org/share/hsvsc/source/VSC\\_algebra\\_hs.pdf](http://mdk12.org/share/hsvsc/source/VSC_algebra_hs.pdf)

<sup>3</sup> California algebra I content standards. Retrieved from: <http://www.cde.ca.gov/BE/ST/SS/mthalgebra1.asp>.

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### *Skill Statement*

Given a narrative, numeric, algebraic, or geometric representation description of a pattern or functional relationship, the student will give a verbal description, or predict the next term or a specific term in a pattern or functional relationship.

Given a numerical or graphical representation of a relation, the student will identify if the relation is a function and/or describe it.

1.1.1.1 Additional Topics: The student will define and interpret relations and functions numerically, graphically, and algebraically.

1.1.1.2 Additional Topics: The student will use patterns of change in function tables to develop the concept of rate of change.

1.1.1.3 Additional Topics: The student will multiply and divide numbers expressed in scientific notation.

1.1.1.4 Additional Topics: The student will read, write, and represent rational numbers.

1.1.1.5 Additional Topics: The student will compare, order, and describe rational numbers.

1.1.1.6 Additional Topics: The student will add, subtract, multiply, and divide rational numbers.

1.1.1.7 Additional Topics: The student will identify and extend an exponential pattern in a table of values.

1.1.2 The student will represent patterns and/or functional relationships in a table, as a graph, and/or by mathematical expression.

### *Assessment Limits*

The given pattern must represent a relationship of the form  $mx + b$  (linear),  $x^2$  (simple quadratic), simple arithmetic progression, or simple geometric progression with all exponents being positive.

### *Skill Statement*

Given a narrative description, algebraic expression, graph, or table, the student will produce a graph, table, algebraic expression of the form  $mx + b$  (linear) or  $x^2$  (simple quadratic), or equation.

1.1.2.1 Additional Topics: The student will be able to graph an exponential function given as a table of values or as an equation of the form  $y = a(b^x)$ , where  $a$  is a positive integer,  $b > 0$ , and  $b \neq 1$ .

1.1.3 The student will apply addition, subtraction, multiplication, and/or division of algebraic expressions to mathematical and real-world problems.

### *Assessment Limits*

The algebraic expression is a polynomial in one variable.

The polynomial is not simplified.

### *Skill Statement*

The student will represent a situation as a sum, difference, product, and/or quotient in one variable.

1.1.3.1 Additional Topics: The student will locate the position of a number on the number line, know that its distance from the origin is its absolute value, and know that the distance between two numbers on the number line is the absolute value of their difference.

1.1.3.2 Additional Topics: The student will evaluate expressions containing absolute value.

1.1.3.3 Additional Topics: For one- or two-variable polynomials: The student will add, subtract, and multiply polynomials.

1.1.3.4 Additional Topics: For one- or two-variable polynomials: The student will divide a one- or two-variable polynomial by a monomial.



1.1.3.5 Additional Topics: For one- or two-variable polynomials: The student will factor polynomials using greatest common factor; the form  $ax^2+bx+c$ ; and special product patterns  $a^2-b^2$ ,  $(a-b)^2$ , and  $(a+b)^2$ .

1.1.3.6 Additional Topics: The student will use the laws of exponents, including negative exponents, to simplify expressions.

1.1.3.7 Additional Topics: The student will simplify radical expressions with or without variables.

1.1.4 The student will describe the graph of a nonlinear function and discuss its appearance in terms of the basic concepts of maxima and minima, zeros (roots), rate of change, domain and range, and continuity.

#### ***Assessment Limits***

A coordinate graph will be given with easily read coordinates.

“Zeros” refers to the x-intercepts of a graph; “roots” refers to the solution of an equation in the form  $p(x) = 0$ . Problems will not involve a real-world context.

#### ***Skill Statement***

Given the graph of a nonlinear function, the student will identify maxima/minima, zeros, rate of change over a given interval (increasing/decreasing), domain and range, or continuity.

1.1.4.1 Additional Topics: The student will describe the graph of the quadratic, exponential, absolute value, piecewise, and step functions.

1.1.4.2 Additional Topics: The student will solve quadratic equations by factoring and graphing.

### **1.2 The student will model and interpret real-world situations using the language of mathematics and appropriate technology.**

1.2.1 The student will determine the equation for a line, solve linear equations, and/or describe the solutions using numbers, symbols, and/or graphs.

#### ***Assessment Limits***

Functions are to have no more than two variables with rational coefficients.

Linear equations will be given in the form:

$Ax + By = C$ ,  $Ax + By + C = 0$ , or  $y = mx + b$ .

Vertical lines are included.

The majority of these items should be in real-world context.

#### ***Skill Statement***

Given one or more of the following:

the graph of a line,

written description of a situation that can be modeled by a linear function,

two or more collinear points, or

a point and slope,

the student will do one or more of the following:

write the equation,

solve a one-variable equation for the unknown,

solve a two-variable equation for one of the variables,

graph the resulting equation,

interpret the solution in light of the context,

evaluate the equation for a given value,

create a table of values, or

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find and/or interpret the slope (rate of change) and/or intercepts in relation to the context.  
Any correct form of a linear equation will be acceptable as a response.

1.2.1.1 Additional Topics: The student will graph systems of linear inequalities and apply their solution to real-world applications.

1.2.2 The student will solve linear inequalities and describe the solutions using numbers, symbols, and/or graphs.

### ***Assessment Limits***

Inequalities will have no more than two variables with rational coefficients.

Acceptable forms of the problem or solution are the following:

$Ax + By < C$ ,  $Ax + By \leq C$ ,  
 $Ax + By > C$ ,  $Ax + By \geq C$ ,  
 $Ax + By + C < 0$ ,  $Ax + By + C \leq 0$ ,  
 $Ax + By + C > 0$ ,  $Ax + By + C \geq 0$ ,  
 $y < mx + b$ ,  $y \leq mx + b$ ,  
 $y > mx + b$ ,  $y \geq mx + b$ ,  
 $y < b$ ,  $y \leq b$ ,  $y > b$ ,  $y \geq b$ ,  
 $x < b$ ,  $x \leq b$ ,  $x > b$ ,  $x \geq b$ ,  
 $a < x < b$ ,  $a \leq x \leq b$ ,  $a \leq x < b$ ,  $a < x \leq b$ , and  
 $a < x + c < b$ ,  $a \leq x + c < b$ ,  $a < x + c \leq b$ ,  $a \leq x + c \leq b$ .

The majority of these items should be in real-world context.

Systems of linear inequalities will not be included.

Compound inequalities will be included.

Disjoint inequalities will not be included.

Absolute-value inequalities will not be included.

### ***Skill Statement***

Given a linear inequality in narrative, algebraic, or graphical form, the student will graph the inequality, write an inequality and/or solve it, or interpret an inequality in the context of the problem.

Any correct form of a linear inequality will be an acceptable response.

1.2.2.1 Additional Topics: The student will graph systems of linear inequalities and apply their solution to real-world applications.

1.2.3 The student will solve and describe using numbers, symbols, and/or graphs if and where two straight lines intersect.

### ***Assessment Limits***

Functions will be of the form:  $Ax + By = C$ ,  $Ax + By + C = 0$ , or  $y = mx + b$ .

All coefficients will be rational.

Vertical lines will be included.

Systems of linear functions will include coincident, parallel, or intersecting lines.

The majority of these items should be in real-world context.

***Skill Statement***

Given one or more of the following:

- a narrative description,
- the graph of two lines, or
- equations for two lines,

the student will do one or more of the following:

- determine the system of equations and/or its solution,
- describe the relationship of the points on one line with points on the other line,
- give the meaning of the point of intersection in the context of the problem,
- graph the system, determine the solution, and interpret the solution in the context of the problem, or
- use slope to recognize the relationship between parallel lines.

Any correct form of a linear equation will be an acceptable response.

1.2.3.1 Additional Topics: The student will determine if two lines in a plane are parallel, perpendicular, or neither.

1.2.4 The student will describe how the graphical model of a nonlinear function represents a given problem and will estimate the solution.

***Assessment Limits***

The problem is to be in a real-world context.

The function will be represented by a graph.

The equation of the function may be given.

The features of the graph may include maxima/minima, zeros (roots), rate of change over a given interval (increasing/decreasing), continuity, or domain and range.

“Zeros” refers to the x-intercepts of a graph; “roots” refers to the solution of an equation in the form  $p(x) = 0$ .

Functions may include step, absolute value, or piecewise functions.

***Skill Statement***

Given a graph that represents a real-world situation, the student will describe the graph and/or explain how the graph represents the problem or solution and/or estimate a solution.

1.2.4.1 Additional Topics: The student will describe the graph of the quadratic and exponential functions.

1.2.4.2 Additional Topics: The student will identify horizontal and vertical asymptotes given the graph of a nonlinear function.

1.2.4.3 Additional Topics: The student will solve, by factoring or graphing, real-world problems that can be modeled using a quadratic equation.

1.2.5 The student will apply formulas and/or use matrices (arrays of numbers) to solve real-world problems.

***Assessment Limits***

Formulas will be provided in the problem or on the reference sheet.

Formulas may express linear or nonlinear relationships.

The students will be expected to solve for first-degree variables only.

Matrices will represent data in tables.

Matrix addition, subtraction, and/or scalar multiplication may be necessary.

Inverse and determinants of matrices will not be required.

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### *Skill Statement*

Given a formula, students will substitute values, solve, and interpret solutions in the context of a problem.  
Given matrices, the students will perform operations and interpret solutions in real-world contexts.

1.2.5.1 Additional Topics: The student will solve literal equations for a specified variable.

***CLG 3: The student will demonstrate the ability to apply probability and statistical methods for representing and interpreting data and communicating results, using technology when needed.***

3.1 The student will collect, organize, analyze, and present data.

3.1.1 The student will design and/or conduct an investigation that uses statistical methods to analyze data and communicate results.

### *Assessment Limits*

The student will design investigations stating how data will be collected and justify the method.

Types of investigations may include: simple random sampling, representative sampling, and probability simulations.

Probability simulations may include the use of spinners, number cubes, or random number generators.

In simple random sampling each member of the population is equally likely to be chosen and the members of the sample are chosen independently of each other. Sample size will be given for these investigations.

### *Skill Statement*

The student will design an investigation and justify their design.

The students will describe how they would do an investigation, select a sampling technique, and justify their choice.

The student will demonstrate an understanding of the concepts of bias, sample size, randomness, representative samples, and simple random sampling techniques.

3.1.2 The student will use the measures of central tendency and/or variability to make informed conclusions.

### *Assessment Limits*

Measures of central tendency include mean, median, and mode.

Measures of variability include range, interquartile range, and quartiles.

Data may be displayed in a variety of representations, which may include: frequency tables, box and whisker plots, and other displays.

### *Skill Statement*

The student uses measures of central tendency and variability to solve problems, make informed conclusions, and/or display data.

The student will recognize and apply the effect of the distribution of the data on the measures of central tendency and variability.

3.1.3 The student will calculate theoretical probability or use simulations or statistical inferences from data to estimate the probability of an event.

### *Assessment Limits*

This indicator does not include finding probabilities of dependent events.

***Skill Statement***

Using given data, the student determines the experimental probability of an event.

Given a situation involving chance, the student will determine the theoretical probability of an event.

**3.2 The student will apply the basic concepts of statistics and probability to predict possible outcomes of real-world situations.**

3.2.1 The student will make informed decisions and predictions based upon the results of simulations and data from research.

***Skill Statement***

Given data from a simulation or research, the student makes informed decisions and predictions.

3.2.2 The student will interpret data and/or make predictions by finding and using a line of best fit and by using a given curve of best fit.

***Assessment Limits***

Items should include a definition of the data and what it represents.

Data will be given when a line of best fit is required.

Equation or graph will be given when a curve of best fit is required.

***Skill Statement***

The students will find a line of best fit, use it to interpolate and extrapolate, and/or interpret slope and intercepts.

The student will use a curve of best fit to interpolate and extrapolate.

The student's response will be in the context of the problem.

3.2.3 The student will communicate the use and misuse of statistics

***Assessment Limits***

Examples of "misuse of statistics" include the following:

misuse of scaling on a graph,

misuse of measures of central tendency and variability to represent data,

using three-dimensional figures inappropriately,

using data to sway interpretation to a predetermined conclusion,

using incorrect sampling techniques,

using data from simulations incorrectly, and

predicting well beyond the data set.

***Skill Statement***

The student will analyze and identify proper and improper use of statistical data and/or statistical methods.

## **D. The Algebra I/Data Analysis HSA 2008**

**HSA Table of 2008 Online Test**

This chart was compiled from the online material for the Maryland 2008 HSA.<sup>4</sup> All problems are multiple choice except ones labeled BCR, for brief constructed response, and RG, for response grid. (The grading of the BCR items is a definite issue.)

# APPENDICES

Included here are only the assessors for the VSC Core Learning Goal 1. The Core Learning Goal 3 content is data analysis and is not included in the Accuplacer exam. Of the 37 problems on the HSA, 24 of them, or 65 percent, assess the Core Learning Goal 1 material, so the following chart represents only 65 percent of the HSA.

Calculator note: This test is designed to be taken with access to a calculator. In multiple-choice problems, the judicious use of a calculator to check the answer choices may allow problems to be solved without using the usual formal manipulations of algebra. See problem 12 below.

Indicator	Problem number	Problem <i>Note: many of the problems have accompanying charts that are not shown. A few are shown as examples.</i>	Descriptive Notes	Notes
				Many of the problems below are not well posed in a mathematical sense. Parameters and assumptions are often not fully explicated. A few, but not all, instances will be pointed out.
1.1.1			1.1.1 The student will recognize, describe, and/or extend patterns and functional relationships that are expressed numerically, algebraically, and/or geometrically.	Many “pattern matching” problems are not well posed because they can have more than one correct answer.
	1	Look at the pattern below. 324, 108, 36, 12, . . . If this pattern continues, what is the next term?		
	9-RG	The table below shows a relationship between $x$ and $y$ . What is the value of $y$ when $x$ is 10?	Table gives the correspondence 1 to 2, 2 to 5, 3 to 10, and 4 to 17. Desired answer is $10 \cdot 10 + 1 = 101$ .	The given table represents many functions, and not just the one corresponding to the pattern that is desired in the answer.
	14--BCR	Jared wants to rent a carpet cleaner. The table below shows the cost of renting a carpet cleaner. Complete the following in the Answer Book: Complete the table to show the cost of renting a carpet cleaner for 5 and 6 hours if this pattern continues. Write an equation to represent the relationship between the cost of renting a carpet cleaner and the number of hours that a carpet cleaner is rented.  <i>(continued on next page)</i>		It should be stated that the student may assume that the function is linear. This is then a fine problem on linear cost functions. The equation of the line relating cost to hours is explicitly requested. However, the subsequent questions can actually be solved without the equation, using trial and error and simple arithmetic.

<sup>4</sup> It should be stated that the student may assume that the function is linear. This is then a fine problem on linear cost functions. The equation of the line relating cost to hours is explicitly requested. However, the subsequent questions can actually be solved without the equation, using trial and error and simple arithmetic.

Indicator	Problem number	Problem <i>Note: many of the problems have accompanying charts that are not shown. A few are shown as examples.</i>	Descriptive Notes	Notes														
(continued)		<p>If Jared has \$60, can he rent the carpet cleaner for 9 hours? Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation. The store also sells the same carpet cleaner for \$165, including tax. What is the maximum number of hours that the cost of renting the carpet cleaner is less than the cost of buying the carpet cleaner? Use mathematics to justify your answer.</p> <table border="1"> <thead> <tr> <th>Number of Hours</th> <th>Cost</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>\$16</td> </tr> <tr> <td>2</td> <td>\$22</td> </tr> <tr> <td>3</td> <td>\$28</td> </tr> <tr> <td>4</td> <td>\$34</td> </tr> <tr> <td>5</td> <td>?</td> </tr> <tr> <td>6</td> <td>?</td> </tr> </tbody> </table>	Number of Hours	Cost	1	\$16	2	\$22	3	\$28	4	\$34	5	?	6	?		
Number of Hours	Cost																	
1	\$16																	
2	\$22																	
3	\$28																	
4	\$34																	
5	?																	
6	?																	
		<p>The table below shows a relationship between x and y. What is the value of y when x is 7?</p> <table border="1"> <thead> <tr> <th>x</th> <td>-2</td> <td>-1</td> <td>0</td> <td>1</td> <td>2</td> </tr> </thead> <tbody> <tr> <th>y</th> <td>7</td> <td>4</td> <td>1</td> <td>-2</td> <td>-5</td> </tr> </tbody> </table>	x	-2	-1	0	1	2	y	7	4	1	-2	-5		Again, there are infinitely many possible answers.		
x	-2	-1	0	1	2													
y	7	4	1	-2	-5													
1.1.2			1.1.2 The student will represent patterns and/or functional relationships in a table, as a graph, and/or by mathematical expression.															
	3	Mr. Smith's pool is filled with water to a height of 48 inches. It has developed a slow leak. At the end of the first day, after the leak started, the height of the water decreased by 0.5 inches. If this rate continues, which of these tables represents the height of the water at the end of the second, fifth, and seventh day?		This problem requires only arithmetic and careful reading.														
	14-BCR	See above.																
	32	The table below shows a relationship between x and y. Which of these equations represents this relationship?	The x values are 2, 3, 4, and 5, and the answer is $y=2x+1$ .	Can solve by checking—this is actually easier than coming up with the equation for the line.														
1.1.3			1.1.3 The student will apply addition, subtraction, multiplication, and/or division of algebraic expressions to mathematical and real-world problems.	This is where the usual abstract, symbolic manipulations of algebra ought to appear. There are only two questions, both word problems. The level of algebraic sophistication required is minimal.														

# APPENDICES

Indicator	Problem number	Problem <i>Note: many of the problems have accompanying charts that are not shown. A few are shown as examples.</i>	Descriptive Notes	Notes
	19	Marina has \$20 in a savings account. She wants to deposit \$10 each week for $x$ weeks into her savings account. If she does not withdraw any money, which expression below represents the total amount of money, in dollars, she will have in her savings account in $x$ weeks?	Answer is $10x+20$ . There are four algebraic expressions as choices.	
	36	Three students sold pizzas to raise money. Dwayne sold $x$ pizzas. Tamara sold $x+20$ pizzas. Rueben sold $3(x+20)$ pizzas. Which of these expressions represents the total number of pizzas that all three students sold?	Answer is $x+(x+20)+3(x+20)$ .	Answer is not simplified, rendering this a trivial problem in terms of solving problems using algebra. This reflects that the "Assessment Limits" for this indicator do not require simplification.
1.1.4			1.1.4 The student will describe the graph of a nonlinear function and discuss its appearance in terms of the basic concepts of maxima and minima, zeros (roots), rate of change, domain and range, and continuity.	
	18	Look at the function that is graphed below. What is the range of this function?	Graph is centered at the origin with maximum $x$ value of 5. Function looks like a cubic with all local extrema visible. Range choices are intervals with correct choice of $-2 \leq y \leq 4$ .	
	26	Which function graphed below is not continuous?	Pictures of functions—all continuous except a presumably hyperbolic.	Continuity is usually taught at a higher level than algebra 1. With an informal definition of continuity, no real skill is needed to answer this question correctly.
1.2.1			1.2.1 The student will determine the equation for a line; solve linear equations; and/or describe the solutions using numbers, symbols, and/or graphs.	The problems here do not adequately assess general facility with solving linear equations.



Indicator	Problem number	Problem <i>Note: many of the problems have accompanying charts that are not shown. A few are shown as examples.</i>	Descriptive Notes	Notes
	8--BCR	<p>Ichiro plans to spend no more than a total of \$60 for both lunch and dinner each day during his vacation.</p> <p>Complete the following in the Answer Book:</p> <ul style="list-style-type: none"> <li>• Write an inequality that models this relationship. Let <math>x</math> represent the amount, in dollars, that Ichiro spends on lunch. Let <math>y</math> represent the amount, in dollars, that Ichiro spends on dinner.</li> <li>• Graph the inequality on the grid provided in the Answer Book.</li> <li>• On Thursday, Ichiro spent exactly \$60 on lunch and dinner. He spent three times as much on dinner as he spent on lunch. How much did Ichiro spend on lunch? How much did Ichiro spend on dinner? Use mathematics to explain how you determined your answers. Use words, symbols, or both in your explanations.</li> </ul>		This is a good, classical algebra problem. It explicitly requests equations. However, the subsequent questions can actually be solved without the equation, using trial and error and simple arithmetic.
	11--RG	A car rental company has two rental plans. Plan A charges \$49.00 per day. Plan B charges \$25.00 per day, plus \$0.10 per mile. How many miles must Teri drive in one day for Plan A to cost the same as Plan B?		Careful reading is required.
	21	Sean's movie rental company charges a monthly fee of \$5.00 plus an additional cost of \$1.25 per movie rental. Which of these equations represents the total monthly cost ( $c$ ) of renting $x$ movies?	Answer is $c=1.25x+5.00$ .	
	34	Look at the function that is graphed below. Which of these equations represents this function?	Answer is $y=2x+4$ .	Answer can be pulled out from $y$ intercept alone.
1.2.2			1.2.2 The student will solve linear inequalities and describe the solutions using numbers, symbols, and/or graphs.	The problems here do not adequately assess general facility with solving algebraic linear inequalities.
	6	Kaila and Joey are starting a lawn-mowing company. They have to buy a lawn mower for \$250. They will charge \$15 per lawn. Which of these inequalities represents the number of lawns ( $l$ ) that they need to mow to earn at least \$800 after they pay for the lawn mower?	Answer is $l \geq 54$ . All choices are similar. However, two choices have $l$ as a lower bound and can be ruled out immediately.	Careful reading is essential. This problem can be solved using basic arithmetic in place of algebra, especially with a calculator.
	8--BCR	See 1.2.1		

# APPENDICES

Indicator	Problem number	Problem <i>Note: many of the problems have accompanying charts that are not shown. A few are shown as examples.</i>	Descriptive Notes	Notes
	35	The yearbook club washes cars to raise at least \$600. The club charges \$3 for each car, $c$ , that they wash. Which of these inequalities models this situation?	Answer is $3c \geq 600$ . All choices are similar.	
1.2.3			1.2.3 The student will solve and describe using numbers, symbols, and/or graphs if and where two straight lines intersect.	
	4	Look at the system of equations below. $12x - 4y = 8$ $3x - y = 2$ Which of these statements describes the graph of this system of equations?	Answer is: two lines that intersect only at $(0, -2)$ .	
	23	William charges \$4 per hour to babysit. LaRhonda charges \$10, plus an additional \$2 per hour to babysit. Both William and LaRhonda work the same number of hours. After how many hours will they earn the same amount of money?	Choices are 2 hours, 2.5 hours, 4.5 hours, and 5 hours.	Choices are easily checkable. Can solve this with simple arithmetic and no knowledge of lines.
	29--RG	For a party, Simon has pizza delivered to his home. Pizza House charges \$8 per pizza plus an additional \$12 for delivery. Spaghetti World charges \$10 per pizza with no delivery charge. If Simon orders the same number of pizzas from each store, how many pizzas must be delivered for the total cost to be the same for Pizza House and Spaghetti World?		This can be solved by checking 1, 2, and the answer, 3.
1.2.4			1.2.4 The student will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.	
	13	The graph below models the relationship between time, in minutes, and the volume of water, in gallons, in a tub. What is the rate, in gallons per minute, at which the tub is being filled?		This is graph reading rather than classical algebra.
	24	The graph below shows the distance, in miles, that the Campbell family drives on the first day of their vacation. What is the total number of hours that the Campbell family stopped during the first day?		This is graph reading rather than classical algebra.

Indicator	Problem number	Problem <i>Note: many of the problems have accompanying charts that are not shown. A few are shown as examples.</i>	Descriptive Notes	Notes
1.2.5			1.2.5 The student will apply formulas and/or use matrices (arrays of numbers) to solve real-world problems.	The use of matrices below seems gratuitous. Students are perfectly capable of interpreting charts without learning matrix terminology. The matrix problems seem to be as much about careful reading and simple arithmetic as about algebra.
	12	The following formula can be used to find the wind-chill temperature ( $w$ ) when the wind speed is 20 miles per hour. Which of these is the <u>actual</u> air temperature if the wind-chill temperature is $-12^\circ$ ? $w = -39 + 3/2 t$ ( $t$ =actual air temperature).	Choices are $-57$ , $-21$ , $18$ , and $41$ degrees.	As the underline indicates, careful reading is required. Without a calculator, and without solution choices, this problem would involve solving a linear equation with rational coefficients using algebra and arithmetic. However, using general knowledge and a calculator to check the choices, it is possible to solve this without using algebra.
	30	Brooke and Josh each took a two-day road trip. The matrix below shows their average speeds, in miles per hour, for each day that they traveled. If they each traveled 6 hours per day, what is the longest distance, in miles, traveled by either Josh or Brooke in a single day?	Matrix is 2-by-2—boys as rows, days as columns.	This problem requires careful reading. It is then a simple rate problem. The student just needs to find the maximum entry and multiply it by 6.
	33	The matrices below show the sales information for three different Fast Food Unlimited stores over a two-week period. The district manager will present an award to the store with the single highest-selling item over this two-week period. Using the matrices above, determine which store received the award and for which highest-selling item.	A 3-by-3 matrix for week one and week two—three food items as rows, three stores as columns.	There is no algebra whatsoever involved in this problem.

### HSA Analysis for the Accuplacer

The problems in the HSA require almost none of the classical, formal manipulations that are traditionally part of an algebra I course, and that are required for the Accuplacer tests. The strong emphasis on applied mathematics may be relevant to the word problems in the Accuplacer exam. However, except for problem four, there are no abstractly stated, nonsituational problems. Furthermore, most of the problems can be solved with minimal algebra skills.

### **E. Proficiency Statements For Accuplacer Elementary Algebra Test**

The following proficiency statements are from the Accuplacer Coordinator's Guide.<sup>5</sup> A total of 12 questions, divided into three types, are administered in this test. The first type involves operations with integers and rational numbers, and includes computation with integers and negative rationals, the use of absolute values, and ordering. These questions test minimal skill levels of the student.

A second type involves operations with algebraic expressions, evaluation of simple formulas and expressions, and adding and subtracting monomials and polynomials. At all skill levels, questions involving multiplying and dividing monomials and polynomials, the evaluation of positive rational roots and exponents, simplifying algebraic fractions, and factoring are provided.

The third type of question involves the solution of equations, inequalities, and word problems.

#### **Total Right Score of About 25**

Students at this level have minimal pre-algebra skills. These students demonstrate:

- a sense of order relationships and the relative size of signed numbers, and
- the ability to multiply a whole number by a binomial.

#### **Total Right Score of About 57**

Students scoring at this level have minimal elementary algebra skills. These students can:

- perform operations with signed numbers,
- combine like terms,
- multiply binomials, and
- evaluate algebraic expressions.

#### **Total Right Score of About 76**

Students at this level have sufficient elementary algebra skills. By this level, the skills that were beginning to emerge at a Total Right Score of 57 have been developed. Students at this level can:

- add radicals, add algebraic fractions, and evaluate algebraic expressions,
- factor quadratic expressions in the form  $ax^2 + bx + c$ , where  $a = 1$ ,
- factor the difference of squares,
- square binomials, and
- solve linear equations with integer coefficients.

#### **Total Right Score of About 108**

Students at this level have substantial elementary algebra skills. These students can:

- simplify algebraic expressions,
- factor quadratic expressions where  $a = 1$ ,
- solve quadratic equations,
- solve linear equations with fractional and literal coefficients and linear inequalities with integer coefficients,
- solve systems of equations, and
- identify graphical properties of equations and inequalities.

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<sup>5</sup> Accuplacer Coordinator Guide. Retrieved from:  
[http://www.collegeboard.com/prod\\_downloads/highered/apr/accu/coordinatorsGuide.pdf](http://www.collegeboard.com/prod_downloads/highered/apr/accu/coordinatorsGuide.pdf).

## Analysis

In general, the VSC and the Accuplacer exams are not well aligned. Of the 14 total indicators for the VSC, only about three or four appear to be relevant to the Accuplacer. This corresponds to less than 30 percent of the VSC.

The Accuplacer corresponds to traditional material covered in an algebra I course. It is largely concerned with the abstract manipulations required to solve equations and covers the basic mathematical skills necessary to do so. These skills include arithmetic skills, as well as formal manipulations of arithmetic and algebraic expressions. The student is expected to be able to apply these skills to solve problems. There are a few problems that deal with real-world applications and interpretations, but most problems are stated in purely mathematical terms.

The VSC, in general, does not focus on basic mathematical skills. The curricular approach centers on making the material relevant to the student rather than the acquisition of basic algebraic skills. For example, a simply stated skill such as *solve a linear equation with rational coefficients* is buried in the VSC under general material on lines. The “Assessment Limits” and “Skill Statement” do not adequately explain how proficient a student is expected to be at this skill.

The VSC has two Core Learning Goals, and one of them, concerned with data analysis, is entirely missing from the Accuplacer exam. The CLG 1 for the VSC also includes many topics that are not required for the Accuplacer exam, such as graphing nonlinear functions, and equations for lines. Of the nine indicators for CLG 1, only three appear to be directly applicable to the Accuplacer. Conversely, the Accuplacer exam assesses facility with arithmetic expressions and with formal manipulations of algebraic expressions, particularly polynomials, and much of this material is not included in the VSC.

The three sections of the Accuplacer are analyzed separately below. In general terms, the VSC and the HSA do not prepare students to do well on this test. Of the three categories of Accuplacer questions, only one of the categories, Solving Equations, Inequalities, and Word Problems, has any true alignment with the VSC. The other two categories involve operations with rational numbers and abstract manipulation, and are not covered in the VSC.

There are two broad ways in which Accuplacer differs from the VSC and the questions included in the HSA.

### *Arithmetic*

The Accuplacer is designed to be taken without a calculator (though it may offer a calculator in situ on certain problems). It assumes and requires a certain level of arithmetic fluency. For example, the problem  $\frac{4}{-5}(-6) =$  should not require a

calculator; only the most basic arithmetic is involved. However, the VSC and the HSA do not include any indication that students should be expected to build upon and fully master the arithmetic skills they learned in elementary and middle schools. Thus, students may have long forgotten how to perform elementary calculations by hand. This sort of mastery is required for a traditional algebra class emphasizing abstract manipulation. However, given that the VSC seems to favor a more applied approach depending on technology, it may be possible to perform well in a VSC-based algebra class without full mastery of basic arithmetic.

### *Level of Abstraction*

The problems on the Accuplacer are generally stated in a purely abstract fashion, and they emphasize mastery of particular mathematical skills. This is in direct contrast to the problems on the HSA, where nearly every problem is situational. Generally, the level of mastery of a particular mathematical skill required to solve an HSA problem is minimal. The VSC also seems to de-emphasize mathematical skills in favor of more applied contexts. It is, in fact, difficult to interpret the VSC in terms of mastery of the abstract skills that are required for the Accuplacer.

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The following discusses the alignment within each subsection of the Accuplacer exam.

### **Operations with Integers and Rational Numbers**

Of the four skills listed, none is mentioned in the VSC, nor tested for in the HSA. However, many of the questions in this section are likely to be prerequisite.

The first two skills listed are computation skills and are largely prerequisite. However, the lack of emphasis in the VSC and HSA on computation is likely to leave students ill-prepared to perform even simple arithmetic tasks.

The other two skills, involving order and absolute value, are mentioned only as “Additional Topics.”

### **Operations With Algebraic Expressions**

All of the Accuplacer skills in this section are largely missing from the VSC.

The Accuplacer requires some facility with general (more than one variable) polynomials. However, the VSC only includes one-variable polynomials, and it is unclear as to how much manipulation a student is expected to be able to perform with even one-variable polynomials. The usual polynomial skills expected by the Accuplacer are covered in the “Additional Topics” portion of the VSC.

At the lowest scoring level of 25, the student must be able to multiply a whole number by a binomial. At the second-lowest scoring level of 57, the student must be able to both multiply binomials and combine like terms. These skills are inadequately covered in the VSC.

Rational exponents and simplifying algebraic fractions are not covered by the VSC.

### **Solving Equations, Inequalities, Word Problems**

There are two skills involving linear equations and linear inequalities. Both are at least partially covered by the VSC. However, the limiting statements in the “Assessment Limits,” as well as a student’s potential lack of additional skills, may mean that the problems in these areas cannot be solved by students.

The skill involving translation of phrases into algebraic expressions is well covered.

The verbal-problems skill may be well covered if no geometric skills are required for a particular problem.

The factoring-quadratics skill is not covered.

### **Conclusion**

**The VSC does not prepare a student to perform at a minimally sufficient level on the Accuplacer exam.**

In the proficiency statements for the Accuplacer, a score of 76 is considered to indicate sufficient elementary algebra skills. The skills specified at this score level are:

- add radicals, add algebraic fractions, and evaluate algebraic expressions,
- factor quadratic expressions in the form  $ax^2 + bx + c$ , where  $a = 1$ ,
- factor the difference of squares,
- square binomials, and
- solve linear equations with integer coefficients.

Three of these skills are concerned only with polynomials and are not covered in the VSC. This would seem to preclude students from performing at this level.

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## F. Achieve Analysis of Elementary Algebra Accuplacer

Achieve Analysis: “Aligned Expectations”<sup>6</sup>

### *Content by category*

Number: 2%

Algebra: 94%

Geometry/Measurement: 3%

Data: 1%

### *Algebra content by classification*

Pre-algebra: 75%

Basic Algebra: 14%

Advanced Algebra (Algebra II): 11%

### *Cognitive demand*

Recalling: 3%

Routine Procedures: 79%

Nonroutine Procedures: 9%

Formulating Problems/Strategizing Solutions: 10%

Advanced Reasoning: 0%

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<sup>6</sup> “Aligned Expectations: A Closer Look at College Admissions and Placement Tests,” Washington, D.C.: Achieve, Inc., 2007. Retrieved from [http://www.achieve.org/files/Admissions\\_and\\_Placement\\_FINAL2.pdf](http://www.achieve.org/files/Admissions_and_Placement_FINAL2.pdf).

# APPENDICES

## II. ACCUPLACER COLLEGE LEVEL MATH SKILLS TO VSC ALGEBRA II ALIGNMENT

Accuplacer Skill	Sample Problem	VSC Algebra II	Additional Topics	Notes
Algebraic operations				
Simplifying rational algebraic expressions		2.5 The student will use numerical, algebraic, and graphical representations to solve equations and inequalities. 2.7.1 The student will add, subtract, multiply, and divide polynomial expressions.		This one is prerequisite and implicit.
Factoring		2.7.1 The student will add, subtract, multiply, and divide polynomial expressions. 2.7.3 The student will determine the nature of the roots of a quadratic equation and solve quadratic equations of the form $y = ax^2 + bx + c$ by factoring and the quadratic formula. 2.2.1 The student will add, subtract, multiply, and divide functions.		2.2.1 Items involving factoring will be restricted to quadratics or the sum or difference of two cubes.
Expanding polynomials		2.7.1 The student will add, subtract, multiply, and divide polynomial expressions. 2.2.1 The student will add, subtract, multiply, and divide functions.		
Manipulating roots and exponents	$2^{5/2} - 2^{3/2}$	2.7.4 The student will simplify and evaluate expressions with rational exponents. 2.7.5 The student will perform operations on radical and exponential forms of numerical and algebraic expressions.		
Solutions of equations and inequalities				Well covered
Solution of linear and quadratic equations and inequalities		2.5 The student will use numerical, algebraic, and graphical representations to solve equations and inequalities. 2.7.3 The student will determine the nature of the roots of a quadratic equation and solve quadratic equations of the form $y = ax^2 + bx + c$ by factoring and the quadratic formula.		<b>Skill Statement</b> Given an equation or inequality, the student will find the solution and express the solution algebraically.
Equation systems		2.6 The student will solve systems of linear equations and inequalities.		
Other algebraic equations	If $a \perp b$ and $\frac{1}{x} + \frac{1}{a} = \frac{1}{b}$ , then $x =$	2.5 The student will use numerical, algebraic, and graphical representations to solve equations and inequalities.		



Accuplacer Skill	Sample Problem	VSC Algebra II	Additional Topics	Notes
Coordinate geometry				
Plane geometry	If the two square regions in the figures below have the respective areas indicated in square yards, how many yards of fencing are needed to enclose the two regions?			Somewhat missing; There is a set of VSC for geometry
The coordinate plane		2.1.2 The student will identify the domain, range, the rule, or other essential characteristics of a function. 2.4 The student will describe or graph notable features of a function using standard mathematical terminology and appropriate technology. 2.7 The student will use the appropriate skills to assist in the analysis of functions.		The coordinate plane is implicit in graphing—describe the domain assumes knowledge of the plane.
Straight lines	An equation of the line that contains the origin and the point (1, 2) is	1.1.1 The student will determine and interpret a linear function when given a graph, table of values, essential characteristics of the function, or a verbal description of a real-world situation.		
Conics			1.1.2.1	Not in VSC
Sets of points in the plane		2.1.2 The student will identify the domain, range, the rule, or other essential characteristics of a function. 2.4 The student will describe or graph notable features of a function using standard mathematical terminology and appropriate technology. 2.7 The student will use the appropriate skills to assist in the analysis of functions.		This is again implicit.
Graphs of algebraic functions		2.1.2 The student will identify the domain, range, the rule, or other essential characteristics of a function. 2.2 The student will perform a variety of operations and geometrical transformations on functions. 2.2.3 The student will perform translations, reflections, and dilations on functions. 2.4 The student will describe or graph notable features of a function using standard mathematical terminology and appropriate technology.		Well covered
Applications and other algebra topics				Coverage here is somewhat weak.
Complex numbers		2.7.2 The student will perform operations on complex numbers.		

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Accuplacer Skill	Sample Problem	VSC Algebra II	Additional Topics	Notes
Series and sequences			2.9.0.1 2.9.0.2 2.9.0.3 2.9.0.4	Not in VSC
Determinants				Missing—may be included in 2.6. The student will solve systems of linear equations and inequalities.
Permutations and combinations				Missing—may be prerequisite and/or as part of 2.9.0.1-2.9.0.4.
Factorials				Missing—may be prerequisite and/or as part of 2.9.0.1-2.9.0.4.
Word problems	An apartment building contains 12 units consisting of one- and two-bedroom apartments that rent for \$360 and \$450 per month, respectively. When all units are rented, the total monthly rental is \$4,950. What is the number of two-bedroom apartments?	1.1 The student will model and interpret real-world situations, using the language of mathematics and appropriate technology.		Some issues with technology. It is specified that linear models should be able to be done without calculators, but not elsewhere.
Functions		2.1 The student will be familiar with basic terminology and notation of functions. 2.2 The student will perform a variety of operations and geometrical transformations on functions. 2.4 The student will describe or graph notable features of a function using standard mathematical terminology and appropriate technology. 2.7 The student will use the appropriate skills to assist in the analysis of functions.		Well covered
Polynomial functions		2.7.1 The student will add, subtract, multiply, and divide polynomial expressions.		2.1.2 <b>Assessment Limits</b> Essential characteristics of a polynomial function include degree, intercepts, end behavior, and symmetry of even or odd power functions.
Algebraic functions		2.2.1 The student will add, subtract, multiply, and divide functions.		2.1.2 <b>Assessment Limits</b> Rational functions should have denominators that are linear quadratic sum and/or difference of two cubes in factored form.

Accuplacer Skill	Sample Problem	VSC Algebra II	Additional Topics	Notes
Exponential functions		1.1.3 The student will determine and interpret an exponential function when given a graph, table of values, essential characteristics of the function, or a verbal description of a real-world situation. 2.7.5 The student will perform operations on radical and exponential forms of numerical and algebraic expressions.		
Logarithmic functions	If $\log(x)=3$ then $x=$	1.1.4 The student will be able to use logarithms to solve problems that can be modeled using an exponential function. 2.7.6 The student will simplify and evaluate expressions and solve equations using properties of logarithms.		
Trigonometry				Missing
Trigonometric functions	If $q$ is an acute angle and $\sin(q) = 1/2$ then $\cos q =$			

## A. College-Level Mathematics Proficiency Statements

These guidelines come from the Accuplacer Coordinator's Guide.

### Total Right Score of About 40 or Less

These students should take the Elementary Algebra Test before any placement decisions are finalized.

### Total Right Score of About 40

These students can:

- identify common factors,
- factor binomials and trinomials, and
- manipulate factors to simplify complex fractions.

These students should be considered for placement into intermediate algebra. For further guidance in placement, have these students take the Elementary Algebra Test.

### Total Right Score of About 63

Students scoring at this level can demonstrate the following additional skills:

- work with algebraic expressions involving real number exponents,
- factor polynomial expressions,
- simplify and perform arithmetic operations with rational expressions, including complex fractions,
- solve and graph linear equations and inequalities,
- solve absolute value equations,
- solve quadratic equations by factoring,
- graph simple parabolas,
- understand function notation, such as determining the value of a function for a specific number in the domain,
- have a limited understanding of the concept of function on a more sophisticated level, such as determining the value of the composition of two functions, and
- have a rudimentary understanding of coordinate geometry and trigonometry.

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These students should be considered for placement into college algebra or a credit-bearing course immediately preceding calculus.

### **Total Right Score of About 86**

Students scoring at this level can demonstrate the following additional skills:

- understand polynomial functions,
- evaluate and simplify expressions involving functional notation, including composition of functions, and solve simple equations involving:
- trigonometric functions,
- logarithmic functions, and
- exponential functions.

These students can be considered for a precalculus course or a nonrigorous course in beginning calculus.

### **Total Right Score of About 103**

Students scoring at this level can demonstrate the following additional skills:

- perform algebraic operations and solve equations with complex numbers,
- understand the relationship between exponents and logarithms and the rules that govern the manipulation of logarithms and exponents,
- understand trigonometric functions and their inverses,
- solve trigonometric equations,
- manipulate trigonometric identities,
- solve right-triangle problems, and
- recognize graphic properties of functions such as absolute value, quadratic, and logarithmic.

These students should be considered for placement into calculus.

## **Analysis**

Of the six topics designated by the Accuplacer, three of them are well covered, two of them are covered incompletely, and one of them, trigonometry, is not covered at all. In the proficiency statements, trigonometry skills appear first at a score of 63. Students who lack knowledge of trigonometry are not precluded from scoring 45 or better on this Accuplacer.

### **Algebraic Operations**

All of the four subtopics appear to be well covered.

### **Solutions of Equations and Inequalities**

All of the three subtopics appear to be well covered.

### **Coordinate Geometry**

Coverage here is somewhat uneven. Of the six subtopics, three of them are well covered, one of them is partially covered, and one of them is covered only in the “Additional Topics” portion of the VSC.

The following subtopics are implicit in functional analysis, and so appear to be well covered:

- The Coordinate Plane,
- Straight Lines, and
- Graphs of Algebraic Functions.

The coverage of the two remaining subtopics is uneven:

Sets of Points in the Plane: This topic is somewhat assumed in functional analysis. However, there could be sets of points prescribed that the student is unfamiliar with such as conics, and

Conics: included in the “Additional Topics” portion of the VSC.

### Applications and Other Algebra Topics

Coverage here is also somewhat problematic. Of the six topics, two of them are covered. Of the remaining four, one is covered in the “Additional Topics” portion of the VSC. The others are not explicitly mentioned in the VSC, though they may be prerequisite and/or implicit.

Complex Numbers

Word Problems

These are both covered in the VSC.

Series and Sequences are included in the “Additional Topics” section of the VSC. Coverage of Series and Sequences may include coverage of:

Permutations and Combinations,

Factorials, and

Determinants: not mentioned in the VSC, though it may be covered in the teaching of solutions of systems of equations.

Note: Lack of knowledge of the four problematic subtopics mentioned here may not prevent a student from performing well on the Accuplacer. They do not appear in any of the sample problems, and they do not explicitly appear in the proficiency statements.

### Functions

All of the four topics are well covered by the VSC.

### Trigonometry

Trigonometry does not appear as a topic in the VSC. A rudimentary understanding of trigonometry is explicitly mentioned in the proficiency statements to score about 63 on the Accuplacer. The skills mentioned to score about 40 do not include trigonometry.

## B. Algebra II VSC

Note: In the following, all material numbered with four numerals is not part of the VSC. It is instead included under the heading “Additional Topics.”

**Algebra II Goal 1: Integration into Broader Knowledge: The student will develop, analyze, communicate, and apply models to real-world situations using the language of mathematics and appropriate technology.**

**1.1 The student will model and interpret real-world situations, using the language of mathematics and appropriate technology.**

1.1.1 The student will determine and interpret a linear function when given a graph, table of values, essential characteristics of the function, or a verbal description of a real-world situation.

#### *Assessment Limits*

The majority of these items should be in context.

Essential characteristics are any points on the line, x- and y-intercepts\*, and slope\*.

#### *Skill Statement*

Given one or more of the following:

a verbal description,

a graph,

a table of values\*,

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an equation\*,  
two or more essential characteristics, or  
an absolute value equation,

the student will be able to do each of the following:

write and/or solve an equation or an inequality that models the situation,  
graph the function, and  
find and/or interpret the meaning of any essential characteristics in the context of the problem.

\*Students should be able to perform these skills with and without the use of a graphing calculator.

1.1.2 The student will determine and interpret a quadratic function when given a graph, table of values, essential characteristics of the function, or a verbal description of a real-world situation.

### *Assessment Limits*

The majority of the items should be in context.

Essential characteristics are zeros, vertex (maximum or minimum), y-intercept, and increasing and decreasing behavior.

A table of values must include rational zeros and at least one other point.

All have real zeros.

### *Skill Statement*

Given one or more of the following:

a verbal description,  
a graph,  
a table of values, or  
a function in equation form,

the student will be able to do each of the following:

find one or more of the essential characteristics,  
write the function in equation form,  
graph the function,  
approximate the value of  $f(x)$  for a given number  $x$ , and  
determine  $x$  for a given value of  $f(x)$ .

Additional Topics: 1.1.2.1 The student will determine and interpret information from models of simple conic sections. (Note: There are assessment and skill statements here—equations of circles, ellipses, and hyperbolas—GM.)

1.1.3 The student will determine and interpret an exponential function when given a graph, table of values, essential characteristics of the function, or a verbal description of a real-world situation.

### *Assessment Limits*

The majority of the items should be in context.

Essential characteristics are y-intercepts, asymptotes, and increasing or decreasing

For  $f(x) = a b^x$ ,  $b > 0$ ,  $a$  and  $b$  are rational numbers,  $b$  is not 1.

The y-values for  $x = 0$  and  $x = 1$  will be given.

### *Skill Statement*

Given one or more of the following:

a verbal description,  
a graph,  
a table of values, or

a function in equation form,  
the student will be able to do each of the following:  
find one or more of the essential characteristics,  
write the function in equation form,  
graph the function,  
approximate the value of  $f(x)$  for a given number  $x$ , and  
determine  $x$  for a given value of  $f(x)$ .

1.1.4 The student will be able to use logarithms to solve problems that can be modeled using an exponential function.

***Assessment Limits***

The majority of the items should be in context.  
Properties used to solve problems may include the product, quotient, and/or power properties of logarithms.

***Skill Statement***

Given verbal descriptions and formulas in exponential form, the student will be able to use the properties of logarithms to solve problems such as exponential growth and decay.

**1.2 Given an appropriate real-world situation, the student will choose an appropriate linear, quadratic, polynomial, absolute value, piecewise-defined, simple rational, or exponential model, and apply that model to solve the problem.**

***Assessment Limits***

The majority of the items should include a verbal description of a real-world situation.

***Skill Statement***

Given a scatter plot of approximately linear data, the student will write an equation of best fit and/or use that equation to find values for  $x$  or  $f(x)$  using a graphing calculator.

Given a verbal description and/or a table of values of a function, the students will recognize that the function is linear, quadratic, polynomial, absolute value, piecewise-defined, simple rational, or exponential, and/or write the appropriate equation that models the situation.

1.2.0.1 The student will communicate when it is appropriate to use a line of best fit to make predictions based on its correlation coefficient.

**1.3 The student will communicate the mathematical results in a meaningful manner.**

1.3.1 The student will describe the reasoning and processes used in order to reach the solution to a problem.

***Assessment Limits***

This indicator is assessed through the implementation of the Core Learning Goal rubric for the constructed response items.

Additional Topics: 1.3.0.1 The student will compute and interpret summary statistics for distributions of data including measures of center (mean, median, and mode) and spread (range, percentiles, variance, and standard deviation).

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Additional Topics: 1.3.0.2 The student will interpret the meaning of the characteristics of the Gaussian normal distribution (bell-shaped curve).

1.3.2 The student will ascribe a meaning to the solution in the context of the problem and consider the reasonableness of the solution.

### *Assessment Limits*

This indicator is assessed through the implementation of the Core Learning Goal rubric for the constructed response items.

**Algebra II Goal 2: Mathematical Concepts, Language, and Skills: The student will demonstrate the ability to analyze a wide variety of patterns and functional relationships using the language of mathematics and appropriate technology.**

**2.1 The student will be familiar with basic terminology and notation of functions.**

2.1.1 The student will identify and use alternative representations of linear, piecewise-defined, quadratic, polynomial, simple rational, and exponential functions.

### *Assessment Limits*

These items are not in context.

### *Skill Statement*

Given one or more of the following:

- a verbal description,
- a graph,
- a table of values,
- an equation, or
- two or more essential characteristics,

the student will be able to do each of the following:

- find a value for  $x$  or  $f(x)$ ,
- find real roots,
- find maximum and/or minimum, and
- find intervals on which the function is increasing and/or decreasing.

Given an absolute value function, the student will graph the function and/or calculate a numeric value of the function.

2.1.2 The student will identify the domain, range, the rule, or other essential characteristics of a function.

### *Assessment Limits*

Vertical and horizontal lines are included.

Functions with restricted domain and/or range are included.

Absolute value, step, and other piecewise-defined functions are included.

Rational functions should have denominators that are linear quadratic sum and/or difference of two cubes in factored form.

Essential characteristics of a polynomial function include degree, intercepts, end behavior, and symmetry of even or odd power functions.



***Skill Statement***

Given one or more of the following:

- a graph of a linear or nonlinear function or relation including polynomial functions,
- an equation over a specified interval,
- a written description of a real-world situation with a restricted domain, or
- a simple rational function,

the student will be able to do each of the following:

- describe the domain,
- describe the range,
- describe the end behavior of a polynomial function,
- describe the symmetry of even or odd power functions, and
- describe the interrelationship between the degree of a polynomial function and the number of intercepts.

Given the equation of a function, the student will produce the graph and describe the domain and range using inequalities.

**2.2 The student will perform a variety of operations and geometrical transformations on functions.**

2.2.1 The student will add, subtract, multiply, and divide functions.

***Assessment Limits***

Items involving factoring will be restricted to quadratics or the sum or difference of two cubes.

Long division is restricted to linear, binomial, or monomial terms in the denominator.

2.2.2 The student will find the composition of two functions and determine algebraically and/or graphically if two functions are inverses.

***Assessment Limits***

Functions given in equation form can include linear, quadratic, exponential, logarithmic, or rational functions such as  $f(x) = (ax+b)/(cx+d)$ .

***Skill Statement***

Given a function in equation form, the student will find the inverse function in equation form.

Given a one-to-one function as a graph, the student will graph the inverse of the function.

Given a function as a table of values, the student will determine the domain and/or range of the inverse of the function.

2.2.3 The student will perform translations, reflections, and dilations on functions.

***Assessment Limits***

Translations are either vertical or horizontal shifts.

Dilations either shrink or stretch a function.

This indicator assesses recognition of translations, reflections, and dilations on functions.

Transformations for absolute value functions are restricted to translations and reflections. They do not include dilations.

Exponential functions are restricted to translations.

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### *Skill Statement*

The student will describe the effect that changes in the parameters of a linear, quadratic, or exponential function have on the shape and position of its graph.

Given a verbal description of a transformed linear, quadratic, or exponential function, the student will write the function in equation form.

Given a transformed linear, quadratic, or exponential function in equation form, the student will give a verbal description of the transformation.

### **2.3 The student will identify linear and nonlinear functions expressed numerically, algebraically, and graphically.**

#### *Assessment Limits*

Functions can include linear, quadratic, exponential, logarithmic, or functions such as  $f(x) = (ax + b)/(cx + d)$ .

The items may have no real-world context given.

Graphs may include piecewise functions.

#### *Skill Statement*

Given one or more of the following:

a table of values, or

a graph,

the student will be able to do each of the following:

choose the correct equation or graph from the same family of functions, and

choose the correct equation or graph from a variety of families of functions.

Additional Topics: 2.3.0.1 The student will expand powers of binomials by using Pascal's triangle and the binomial theorem.

Additional Topics: 2.3.0.2 The student will use the binomial theorem to determine the probability of an event.

### **2.4 The student will describe or graph notable features of a function using standard mathematical terminology and appropriate technology.**

#### *Assessment Limits*

Essential characteristics of a linear, quadratic, or exponential function are those listed for 1.1.1, 1.1.2, and 1.1.3.

Transformations for an absolute value function in one variable are restricted to translations and reflections. They do not include dilations.

#### *Skill Statement*

Given one or more of the essential characteristics of a function, the student will graph the function.

Given the equation form of a linear, quadratic, or exponential function, the student will find one or more required essential characteristic and/or graph the function.

### **2.5 The student will use numerical, algebraic, and graphical representations to solve equations and inequalities.**

#### *Assessment Limits*

Equations may be in one or two variables.

Quadratic equations and inequalities are included.

Higher-order polynomial equations will be factorable.

Absolute value equations and inequalities are single variable and may be linear or quadratic.

Radical equations will lead to a linear or quadratic equation.

Rational equations will lead to a linear or quadratic equation.

Simple rational inequalities will lead to a linear inequality.

Exponential equations are either of the form  $f(x) = a b^x$ ,  $b > 0$ ,  $a$  and  $b$  are rational numbers,  $b$  is not 1, or the form  $c n^x + d = g m^x + f$ , where  $c$  and  $g$  are powers of the same base.

***Skill Statement***

Given an equation or inequality, the student will find the solution and express the solution algebraically and graphically. For constructed response items, students will also justify their method and/or solution.

**2.6 The student will solve systems of linear equations and inequalities.**

***Assessment Limits***

Systems of linear equations will be  $2 \times 2$  or simple  $3 \times 3$ , which do not take too much time to solve without a calculator.

Systems of linear inequalities will be  $2 \times 2$ .

***Skill Statement***

Algebraically and graphically solve  $2 \times 2$  systems of linear equations and algebraically solve simple  $3 \times 3$  systems of linear equations.

Solve systems of two linear inequalities in two variables and graph the solution set.

Interpret the solution(s) to systems of equations and inequalities in terms of the context of the problem.

**2.7 The student will use the appropriate skills to assist in the analysis of functions.**

2.7.1 The student will add, subtract, multiply, and divide polynomial expressions.

***Assessment Limits***

Rational expressions may include monomials, quadratics, and the sum and difference of two cubes.

2.7.2 The student will perform operations on complex numbers.

***Skill Statement***

The student will represent the square root of a negative number in the form  $bi$ , where  $b$  is real, and simplify powers of pure imaginary numbers.

The student will add, subtract, and multiply complex numbers.

The student will simplify rational expressions containing complex numbers in the denominator.

2.7.3 The student will determine the nature of the roots of a quadratic equation and solve quadratic equations of the form  $y = ax^2 + bx + c$  by factoring and the quadratic formula.

***Assessment Limits***

The solutions may be real or complex numbers.

2.7.4 The student will simplify and evaluate expressions with rational exponents.

2.7.5 The student will perform operations on radical and exponential forms of numerical and algebraic expressions.

***Assessment Limits***

Denominators in problems requiring rationalizing the denominator are restricted to square roots.

Radicals containing a numerical coefficient are restricted to square roots and cube roots.

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### *Skill Statement*

The student will convert between and among radical and exponential forms of expressions.

The student will add, subtract, multiply, and divide radical expressions.

The student will apply the laws of exponents to expressions with rational and negative exponents to order and rewrite in alternative forms.

2.7.6 The student will simplify and evaluate expressions and solve equations using properties of logarithms.

### *Assessment Limits*

Properties of logarithms include the change-of-base formula; property of equality for logarithmic functions; and the product, quotient, and power properties of logarithms.

### **2.8 The student will use literal equations and formulas to extract information.**

#### *Assessment Limits*

Problems may include addition/subtraction and multiplication/division properties of equality, factoring a common factor, and terms that are rational.

Additional Topics: 2.9.0.1 The student will represent the general term of an arithmetic or geometric sequence and use it to determine the value of any particular term.

Additional Topics: 2.9.0.2 The student will represent partial sums of an arithmetic or geometric sequence and determine the value of a particular partial sum.

Additional Topics: 2.9.0.3 The student will find the sum of an infinite geometric series whose common ratio,  $r$ , is in the interval  $(-1, 1)$ .

Additional Topics: 2.9.0.3 The student will recognize and solve problems that can be modeled using a finite arithmetic or geometric series.

## **C. Achieve Analysis of College Level Algebra Accuplacer**

Achieve Analysis: "Aligned Expectations"

Algebra content by classification

Pre-algebra: 33%

Basic Algebra: 7%

Advanced Algebra (Algebra II): 60%