**THE USE OF COMPUTERS IN MATHEMATICS EDUCATION: A PARADIGM SHIFT FROM “COMPUTER ASSISTED INSTRUCTION” TOWARDS “STUDENT PROGRAMMING”**

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**ABSTRACT**

The purpose of this study is to review the changes that computers have on mathematics itself and on mathematics curriculum. The study aims at investigating different applications of computers in education in general, and mathematics education in particular and their applications on mathematics curriculum and on teaching and learning of mathematics. There are three broad categories of the applications of computers in the field of mathematics education: computer assisted instruction (CAI), student (educational) programming and general purpose educational tools such as spreadsheets, databases and computer algebra systems (CAS). This study presents a historical background and attempts to describe the paradigmatic shift in the use of computers in mathematics education from behavioristically oriented CAI movement towards constructivist based student programming movement.

**INTRODUCTION**

The educational history of computers began in sixties with the realization of its potential to teaching and learning. That was a period of enchantment. According to Suppes et al. (1968) the change that was to come through computers could only be compared to the fact how books had changed the way of people looking at the world. Computers would change the face of education in a very short period of time by eventually removing the teacher from the classroom scene. Looking in retrospect, can it really be said that his predictions were realized? Or is it just a “techno-romantism” (Underwood and Underwood, 1990) to believe that the computer is a panacea for all of the problems in education. Although not having been able to solve all of the problems of education by itself, this powerful machine, no doubt, will continue to occupy a very central place in education.

A major figure in the history of computers, besides Suppes, in education is Seymour Papert who is famous for his work “Mindstorms” (1980) which presents quite revolutionary ideas about the place of the computers in education. He, like Suppes argued that the computer would change the face of education, but unlike Suppes he advocated the use of the computer not as a teaching machine but as a device to develop learners’ intellectual skills through writing their own programs to direct the computer and not let the computer direct themselves.

It seems that the effect of computer technology on education is greater in mathematics than in any other discipline. This may be because of the close links between the two disciplines. In fact the computer science was a part of mathematics and afterwards gained independence as a sole discipline.

The purpose of this study is to review the changes that computers have on mathematics itself and on mathematics curriculum. The study aims at investigating the following questions:

1. What are different applications of computers in education in general, and mathematics education in particular?
2. What are the effects of these applications on mathematics curriculum and on teaching and learning of mathematics?

**COMPUTERS IN MATHEMATICS EDUCATION**

Mathematics instruction is among the most explored research area in education. There have been considerably varied computer applications in instruction (Hatfield, 1984). The teachers of mathematics are confused with the extensive amount of suggestions on how to teach mathematics with a computer. Teachers’ attitudes towards computers vary mostly as a function of teachers’ age or years in service. Complete ‘ignorance’ attitude towards computers still continues, although its magnitude is weaker compared to past years This attitude is mostly shared by teachers who had had their training before the start of the computer age who have the most negative attitudes towards its pedagogical use and who insist on using the traditional modes of teaching. Second major attitude is not being able to abandon their traditional habits completely foreseeing its potential for the future of education.

Most prevalent and widening attitude is the realisation and acceptance of the importance of computers for education.
There are three broad categories of the applications of computers in the field of mathematics education:

- computer assisted instruction (CAI)
- student (educational) programming
- general purpose educational tools such as spreadsheets, databases and computer algebra systems (CAS).

This survey of literature revealed that, this categorization is also a historical one, although it cannot be said that there were sharp shifts from one movement to another. Another important note is that the CAI movement is not as popular in the Europe as it is in United States.

1. Computer assisted instruction (CAI) and its effects on mathematics curriculum

1.1. Behavior modification programs before CAI

There are two major events that had a great influence on education in general and mathematics education in particular in 1958 (Dick, 1986). The first one is the Sputnik event, the satellite launched by the Russians. The other one is the paper presented by Skinner, an influential and famous neo-behaviorist, on programmed instruction.

Behaviorism is considered to be the theory underlying CAI. Hence, it is understandable that the CAI programs are mainly behavioral control programs (Hartley, 1981). An example is Skinner’s Programmed Instruction (PI) which was designed to change the behavior of the learners. Fundamental approach of Skinner was to identify the desired behaviors, then to prepare situations in which successive approximations of the behavior would be reinforced. All the students study the so called “linear text”, the instructional material used by Skinner. When students complete the text, they were assumed to have acquired the behaviors required from them. The basic characteristic of programmed instruction is the small steps approach, meaning the division of the task into small manageable units, and the immediate feedback given to students from each response they give.

The teaching machine is the box designed to expose the programmed instruction text one frame at a time. It is commonly considered to be ancestor of the device called the “computer” to be used for educational purposes. Skinner’s programmed instruction formed a basis for the computer assisted instruction movement (Dick, 1986).

Among several other trends Skinner’s PI became more widespread than the others. Other major movements in that tradition which followed PI chronologically were Glaser’s individually prescribed instruction (IPI) and Keller’s personalized system of instruction (PSI). Both approaches contributed to the individualization of instruction movement in similar ways. In Keller’s (1968) PSI there were self-paced courses in which students were required to master successive unit tests. Bloom’s (1976) is considered to be the last widespread individualized approach to instruction, before computer assisted instruction movement. The method involve the mastery of certain subject area (e.g. trigonometry) before passing to another.

According to Hilgard (1986) these approaches to instruction had important consequences for educational psychology.

- the individualized instruction based on the idea that best learning outcomes can only be obtained with one to one tutoring approach. The claim related to those behaviorist models of instruction was that they could provide learning environments closer to one-to-one tutoring (Bloom, 1976).
- the diagnostic teaching which is based on the immediate feedback obtained from the responses of students to the questions being asked during the instructional process.
- the step-by-step approach they presented in the instruction of a certain learning task. Those programs advance in a way such that one sub-task followed the other, in other words one frame at a time as in teaching machines. Therefore, in each step, what is expected from the user should be specified in terms of observable behaviors.

According to Mager (1962) a behavioral objective has four basic components. The first component is the actor or the learner who is supposed to act in the prescribed manner. Second component is the behavior itself. The condition(s) under which that action would occur and the criteria to judge if the behavior is applicable are the third and fourth components respectively.

1.2. The types of CAI programs

CAI means, in broader terms, the use of a computer to provide the course content in the form of drill, practice, tutorial, and simulations. Demonstration, testing, information, and communication are the main facilities provided by CAI. Hatfield (1985) counts eight basic types of CAI. The first type of CAI includes “drill and
practice” programs. Here, the students rehearses different elements of teaching and develop related skills. They are presented to students in the educational softwares produced for school and home computers. That type of program relies heavily on positive reinforcement. That means, a reward follows a correct response, negative reinforcement is also used but not frequently. A good example for drill and practice programs are the ones designed to help children learn multiplication tables. The drills might be presented to students with a car race game for instance. The rule is simple: the refuelling of the car depends on the correct answer the student gives to a multiplication question.

Besides drill and practice facilities the use of “computers as tutors” are as widespread as the former. The tutorial programs are the one designed to teach basic concepts or methods as well as certain subject in mathematics for instance. Those type of programs try to behave like a good stimulating teacher. It involves explanations, questions, as well as feedback and correctives. For example, when the user asks for help in a certain step he/she may need help for handling the problem.

The third type, “simulation” is really a very useful advantage that CAI provides for the users. It is the facility to set up the reality in the classroom, which makes CAI very attractive for the users. This feature of CAI presents great advantages for the science curriculum. With this way, experiments that are difficult, time consuming and costly could be realized in the classroom (Watson, 1984). It is also useful in the mathematics curriculum. Many of the three dimensional objects that are very difficult to be visualized by the students. These objects can, then be presented to the learner through the screen of a computer.

“Gaming”, the fourth type of CAI is the most stimulating use of computers. These programs are a kind of simulation offering competitive situations in which one or more persons can play and win. That does not mean that all games are educationally useful. The stress here is upon the ones that produce worthwhile learning situations related to the objectives contents and processes in the mathematics curriculum (Hatfield, 1984).

They are other types of games which are not necessarily based upon a winning and loosing theme. Such programs give opportunity to the user to work as team and explore the environment with other team members. For instance the game DEFCIT asks to form two themes, both having the goal of balancing the budget (Watson, 1984). The programs really offer great opportunities for the mathematics students to work within a co-operative environment to learn difficult mathematical concepts.

The idea that games can be used to enhance the learning of mathematical concepts stems from two different views (Dugdale 1985)

- "Making mathematics fun": mathematics is uninteresting and difficult. It can be made easier and enjoyable by playing computer games.
- By providing the structured environments with which students manipulates things in mathematics: mathematics is useful and interesting.

1.3. Effects of CAI on mathematics teaching and learning

The view that complex learning behaviors comprise of a network of stimuli response associations is the fundamental idea underlying the application of CAI programs. This idea has been the natural continuation of the behavioristic mode of teaching.

Hartley (1981) stated that the stimuli response bonds are established by providing positive reinforcements such as knowledge of results. Then, it was the teacher (or the organiser of the instruction) who was responsible for the selection and arrangement of content to help the desirable responses to be elicited. Then, this process leads to the discovery of feedback, the message which follows the response made by the learner. This, in fact acts as a positive reinforcement for the learner. It is also the information which shows the error and informs the student to correct this error. This is called feedback-corrective cycle. Learning takes place by rewarding the correct associations by questions and answers (Howe and du Boulay, 1981) which is known as the reinforcement learning.

CAI also brings the possibility that student interaction with the computers may result in less interaction with the teacher and the classmates. This, in fact is very harmful for the process of socialization provided by the school environment. Most of the applications of CAI have been based on the individualized learning, one student working with one computer. The model proposed by Johnson et al (1978) was a group based model. They found that the co-operative CAI was far more effective compared to the individualistic CAI approach in mathematics and concluded that the assumption being “all CAI should be individualistically oriented” is not a valid argument.
Hartley (1977) was the first person who meta-analysed the findings of CAI on mathematics achievement (in Kulik et al, 1983). He found that CAI had a significant effect (effect size 0.41) on the achievement levels of primary and secondary school mathematics students. Burns and Baseman (1981) also reviewed the findings on the effect of CAI on mathematics achievement. The effect size he found was almost the same (0.45 for tutorial, 0.34 for drill and practice) verifying the results of previous analyses.

In brief, this review of literature revealed that most of the CAI studies and reviews of those studies, as expected, were done in the USA, most of which suggested the effectiveness of CAI. It must be noted, however, that main focus of those studies are generally easily observable measures such as achievement, not more complex phenomena such as cognition.

2. Student programming and its effects on mathematics curriculum

2.1. Its origins and philosophy

Student programming movement started from the realization that there are strong connections between thinking processes of learners during writing their own computer programs and many aspects of mathematical thought (Hatfield, 1985). Its main assumption is that, computers can help students in learning certain mathematical topics by programming the computer which can be considered as an anti thesis to CAI. Most CAI programs were criticized because they were mainly drill and practice based which could unlikely 'relational understanding' (Skemp, 19??) of the content presented.

Seymour Papert, an influential figure in this movement, proposes that, the use of computers as teaching machines gives nothing to students (Papert, 1980). “Programming the computer, not being programmed by it" was his motto. He also argued that the students can develop their thinking skills through writing their own programs rather than using programs that were developed for them previously. Underwood and Underwood (1990) note that the open ended programs like LOGO for instance are like pencils and bicycles, in the sense that they are tools to make students reach the ultimate goal: to develop thinking. CAI programs are criticized in the sense that they have known and well defined goals which inhibits discovery.

Students sometimes give right answers for wrong reasons or wrong answers may be the result of rational thinking (Dubinsky and Tall 1993). This statement is a brief summary of the new paradigm of learning. Learning can occur by giving the student the opportunity to construct their own knowledge. This could be provided by a rich computer environment with which the students can develop new mathematical ideas and play with them. Another important feature of computer environments is their power in making abstract ideas more concrete. The computer promotes the minds of the children, causing a shift from concrete to abstract, iconic to symbolic (Kelly, 1984). Computers also assist the child to develop abstract modes of learning to direct them to higher levels of conceptual understanding. Through design, coding and revision, and debugging of a new computer program students can have an opportunity to develop higher mental skills such as deductive reasoning and problem solving. Therefore, it becomes more crucial now to incorporate computer programming into existing mathematics curricula.

Children should be actively engaged into the activities which they are exposed to (Kelly, 1984). In that way the learner can use a computer in the positive sense. As opposed to passive learning which is enhanced by the ready-made packages, the student can have better learning experiences. Kelly labels CAI programs as “second-hand programs which have been prepared for them by others."

Underwood and Underwood (1990) criticises the use of computers as teaching machines in that it encourages passive learning which result in undesirable outcomes. On the other hand students can understand conceptually rather than merely acquiring the facts through programming the computer themselves. The prepositional knowledge (the facts), and the procedural knowledge (the algorithms) are less important than the relational knowledge (the process and the insights of the subject) (Kelly, 1984). The computers is a very suitable tool to provide opportunities for the learner who asks “why” rather than “what” or “how”. Papert (1980) pointed out that education has to change its attitude towards the learning process. A shift is seriously needed from quantitative knowledge towards qualitative one. The important thing, then, is not to have more knowledge but doing something with the existing knowledge.

Papert (1980) also noted that the traditional curriculum which he calls “the worksheet curriculum should be abandoned as quickly as possible and allow minds of the children develop through the exploration of computer stimulated microworlds. This necessitates a revolution in the educational practice all over the world.
Salamon (1988) draw attention to the major misuse of computers, that is, the distinction between machines that work for us those that work with us. In the first category there is drill and practice programs while in the second the use of computers as programming devices (in Underwood and Underwood, 1990).

2.2. The effects of student programming on mathematics teaching and learning:
There are two basic traditions in programming. They have their own philosophy and teaching styles (Cope and Walsh, 1990). The first one is the BASIC-PASCAL tradition. The other tradition, LOGO is rooted from the artificial intelligence and supported by Piaget’s cognitive development theory (Papert, 1980).

2.2.1. BASIC
Although it is not widely used any more, BASIC had a strong impact on mathematics education in the previous years. It is indeed an easy language. Students could write their own programs to explore mathematical ideas. They could be used for the purposes of problem solving, investigation and practical work. Although its applications were very limited, it was and (may be) is being used because of its ease of use.

2.2.2. LOGO
Seymour Papert in 1970 wanted to design a computer language that is suitable for children, not able to use complicated programming languages such as BASIC and PASCAL. It should be easy to manipulate and have the power of a structured programming language.

LOGO, the outcome of this effort, is a general purpose programming language. The programs in LOGO can be written to perform different tasks. It can be used to write programs across many different subjects in the curriculum, mostly in mathematics. The turtle geometry, a part of LOGO helps young children program the computer. It is used to draw whatever the learner likes, such as geometrical figures.

Programming in LOGO combines with use of microworlds. Then through programming certain problem area can be explored. With LOGO children are assumed to develop useful practices as a result of programming in microworlds. Contrary to the other programming languages, LOGO is often taught in an open ended and child centred fashion. The learner himself/herself gives directions to his/her learning. Children using LOGO develop a knowledge of how programming works through testing hypothesis.

The Mathematical Association (1992) lists different uses of LOGO across mathematics curriculum.
- the concept of ratio, for instance, is easier with LOGO graphics (p 47).
- the idea of measurement: distance on a floor can be described in terms of units of
- moment of a robot toy (p 45).
- the perception of a function as a procedure taking a number as a input and after an algorithm giving the output (p 154).
- natural number concepts can be developed through guessing the number. Computer generates a random number student makes prediction the program gives a feedback for the accuracy of that prediction. With that way, student gets closer to the number and finally finds it (p 45).
- the program comments “turn” and “corner” changes the direction of moving object helps the understanding of the concept of the angle (p 29).
- It is very difficult to visualise 3-D objects and draw it on a sheet of paper. It is an easy test in LOGO. Indeed LOGO can produce 2-D representations of 3-D objects. For instance the turtle can move in the directions as if it is moving in 3-D space (p 34)

The early findings of Papert (1980) indicated that LOGO had an effect in improving cognitive skills. Clements (1983) also found that LOGO was also effective in problem solving skills. Clements and Gullo (1984) compared the effects of CAI and computer programming on young children’s cognition, and found that programming was more efficient than CAI in various measures of cognition, such as reflectivity and divergent thinking. Roblyer (1989) reviewed 82 studies to provide information about the effect of computer use in schools on student achievement, attitudes, dropout rate, and learning time. The effects of Logo applications on problem-solving and general thinking skills were found to be significant.

3. Mathematical tools in mathematics education
By tools what is meant is the computer packages by which the learner can develop his/her thinking skills. In this context the spreadsheets such as excel, computer algebra systems (CAS), the databases, communication facilities, word processing will be analysed. Those are the tools that are used in the educational computing.

There were very optimistic expectations from student programming moment. But the realization of the fact that it failed to satisfy these expectations caused a shift towards general purpose software tools such as databases and spreadsheets (Case & Walsh, 1990). As stated in the Mathematical Association report (1992) these general purpose packages present an alternative vehicle for programming. The skill in the handling of the algorithms which is the essential parts of programming can be developed by the use of these spreadsheets and databases.

The Mathematical Association (1992) summarises different uses of spreadsheets in the mathematical curriculum.

- drawing graphs (p 59) comparing the graphs of \( y=x^2 \) and \( y=x^3 \)
- finding the second and thirds roots of numbers through iteration. Finding for instance, length of the inside edge of cubicle box given the volume numerically (p 62).
- calculating the area under a curve ( p 87).
- introducing the function concept (p 57). For example, a set of numbers in one column and another set of numbers in another column can be represented by the symbols \( x \) and \( y \). The function can be defined as relation between two sets of data.
- solving differential equations. (p 90)

With the use of spreadsheets data handling can be done in a minimum amount of time and effort, allowing the students interact with the data focus on the interpretation of it. Hence, children can understand data more easily, so they can get rid of many unnecessary calculations which inhibit the learning of important mathematical concepts which are the main objectives of a mathematics lesson.

Computer spreadsheets which are readily adaptable for problem solving, can also enhance the user's insight into the development and use of algorithms and models, free students from being hampered by laborious manipulation of numbers, and allow students to see the progression of calculations on the screen as they are generated (Masalsky, 1990).

Malara et al. (1992) in a review about the use of spreadsheets in teaching typical topics in high schools such as algebra, calculus, and statistics and concluded that there are many advantages as well as problems (in Dettoori et al., 1995). One particular problem; according to Dettori et al. (1995) is with the use the sign “=” differently in algebra and in computer languages. In the former it represents an equality while in the former it is a relation. This problem may cause misunderstandings in the learning of algebra.

In their analysis about the use of spreadsheets in algebra Dettori et al. (1995) conclude that spreadsheets are very useful tools in the introduction of many algebraic concepts. With the use of these tools they can understand meaning of “solving an equation” and can learn the concept of “approximations”.

Sutherland (1993) investigated the effects of participation in computer spreadsheet sessions on learners’ understanding of mathematical symbols, and found that participation improved students' attitudes toward problem solving. According to Nevile (1995) the spreadsheets are useful tools for the students to understand processes in the problem solving situation, and that they can recognise that codification and symbolization in word problems is not arbitrary.

Databases are organizational structures into which information is placed and from which the information can be retrieved. These programs can provide cognitive experiences for the user. Databases are useful in stimulating a process oriented curriculum (Underwood and Underwood, 1990). Brown and Howlett (1994, in Underwood et al., 1994) list the arguments for the educational uses of databases which include stimulating the ability to classify objects and symbols, facilitating multiple representation of data (e.g. a numerical data can be represented by a straight line), developing the ability to compare data, encouraging skills in the selection of the data and modes of representing it, developing questioning skills and the understanding of scientific method.

Computer Algebra Systems (CAS) are digital devices used to manipulate symbols. Monaghan (1995) lists five basic things that a CAS can do, which include simplifying algebraic expressions, doing calculus (calculate limit, integral etc.), evaluating functions in several areas such as statistics, physics, and engineering,, doing matrix algebra, and analysing two or three dimensional objects (Cartesian or polar or parametric representations). The computer packages like Derive, Mathematica, Mathcad, Mathlab, and Maple are all included in this category.
Their difference from the spreadsheets lies on the fact that these programs are specifically designed for solutions of mathematical problems with which students can enhance their learning.

All computing technologies in general and CAS in particular offer many opportunities for the mathematics curriculum in designing new ways of teaching mathematics topics, making advanced topics more easily understandable, providing a different approach to thinking about a particular topic and working more effectively in modelling and the applications (Rothery, 1995).

With the use of such packages many aspects of teaching and learning will be affected and that they have the power to change the nature and sequence of the mathematics curriculum. Traditional skills that were taught the students will not be taught any more in the near future, since the skills can now be performed by these packages. CAS can act as a bridge between teaching and learning by matching the teachers’ program of work to students’ learning. Maximising a cone (p 333), devising parametric equations for a family of curves (p 336), and graphical representation of derivatives (p 340) are some of the topics that can be taught by using computer algebra systems (Rothery, 1995).

Hunter et al. (1995) in a study in which used a computer algebra system in the topic “quadratic functions” with 14-15 year old students found out that graphical work could not be made easier by the use of CAS in graph sketching or drawing, or through computer generated graphs and that a CAS is advantageous in learning abstract algebra if the students are mathematically ready to use it.

**SUMMARY**

Computer science had separated from a mathematical logic, and gained independence. That means the improvements in computer science are changing mathematics. Hence mathematics education is changing. It caused the revision of the mathematics curricula in many of the leading countries in the world.

This survey of literature indicated that various computer applications in mathematics education contributed greatly to classroom practices. But the limitations of a computer should be kept in mind that it will not solve all deep-rooted problems of mathematics education.

**REFERENCES**