The Center
FOR COMPREHENSIVE SCHOOL REFORM AND IMPROVEMENT

LEARNING POINT Associates®

NEWSLETTER

www.centerforcsri.org

April | 2008

IT PAYS TO COMPARE!

Using Comparison to Help Build Students' Flexibility in Mathematics

by Jon R. Star, Ph.D., *Assistant Professor, Harvard Graduate School of Education*

A key instructional outcome in mathematics is the development of flexible knowledge (National Research Council, 2001). Being flexible means knowing a variety of ways to solve problems and being able to apply these methods adaptively on a wide range of problems. Both international and national assessments indicate that U.S. students may learn to execute rote procedures, but they often fail to gain robust, flexible knowledge. In algebra, flexibility is especially important, in that students need to know a range of strategies for solving problems that are represented using graphs, tables, and symbols. In addition, flexible knowledge of important prerequisite mathematical topics, such as rational numbers, also appears to be linked to later success in algebra (National Mathematics Advisory Panel, 2008).

One important component of flexibility in mathematics is knowing more than one way to solve a given problem. Knowing multiple approaches for solving mathematics problems is a hallmark of expertise in mathematics and thus is an outcome worthy of our best efforts in our mathematics classrooms. Too often, students memorize only one method of solving a certain kind of problem, without understanding what they are doing, why a given strategy works, and whether there are (perhaps better) alternative solution methods. In other words, flexible knowledge—knowing more than one way to solve problems—supports transfer. Thus flexibility is not only an instructional goal in later courses such as algebra but also is critically important in the teaching of important prerequisite-for-algebra topics, such as working with fractions and proportions.

In fact, some teachers might believe that drilling students in the use of a single strategy is optimal. Some teachers might ask, "I have a hard enough time getting my students to know even one strategy—wouldn't it be even harder to focus on multiple strategies?" Certainly there are challenges associated with teaching multiple strategies. But the payoff is worth the extra effort: Students who know more than one method for solving a particular class of problems are likely to be *more* successful when faced with unfamiliar problems; if they forget one method, they have alternative strategies that they can fall back on.

What instructional strategies have been shown to be effective in helping students develop flexibility—particularly knowledge of multiple strategies? One important tool that teachers have to help students learn multiple approaches is comparison. For at least the past 20 years, a central tenet of effective instruction in mathematics has been that students benefit from sharing and comparing solution methods (Silver, Ghousseini, Gosen, Charalambous, & Strawhun, 2005). Case studies of expert mathematics teachers emphasize the importance of students actively comparing solution methods. Furthermore, teachers in high-performing countries, such as Japan, often have students compare multiple solution methods (Stigler & Hiebert, 1999). This emphasis on sharing and comparing solution methods was formalized in the National Council of Teachers of Mathematics (NCTM) standards (1989, 2000).

Comparison helps focus learners' attention on critical features of examples. It is easy to see the importance of comparison intuitively. Suppose we were interested in purchasing a digital camera from an online electronics store. Before going online, from many possible cameras in our price range, let's say we narrowed our choice down to two cameras. How do we make the choice between these two? One way to make this choice is to first learn about camera 1 and then learn about camera 2. So we first look at the long list of dozens of features of camera 1, and then we look at the long list of features of camera 2. When looking at these feature lists one at a time, it is very difficult to notice which features are the same between cameras and which differ—there is too much information! But now imagine that we could compare the specifications of both cameras by viewing them at the *same time*, side by side. With a glance, we could tell whether the cameras were the same or different for a given feature, making our decision-making process much easier. By comparing the cameras by looking at their features side by side, we could easily identify important similarities and differences.

Similar to the camera example, the power of comparison can be easily realized in a mathematics classroom to help students learn multiple strategies. However, some common instructional strategies used by many mathematics teachers may not enable their students to realize the benefits of comparison. For example, imagine that Mr. S, a high school algebra teacher, wants his students to become more flexible—particularly that they should know multiple strategies for solving a given problem. So he decides to show students several examples of the problem type, as well as two different solution methods. Mr. S writes a problem on the board and demonstrates its solution, step by step, asking a lot of questions as he works to see if students are getting it. "Any questions?" Mr. S asks. "OK, let's do another example." Mr. S *erases the board* and writes another similar problem on the board and, with students' help, solves it, using a very similar

Students who know more than one method for solving a particular class of problems are likely to be more *successful when faced with unfamiliar problems; if they forget one method, they have alternative strategies that they can fall back on.*

There is evidence that students who see worked examples side by side, with prompts to compare and contrast the examples, become better problem solvers and develop greater flexibility than students who see the same examples listed one at a time.

method to the one used in the first problem. "Any questions?" Mr. S asks. Now, Mr. S thinks, I'm going to do a third problem for my students—a problem that is a bit different than the other two, and I'm going to solve it in a slightly different way. "Here is a third example—one that is a bit different from the other two," he says. Mr. S *erases the board,* writes a third problem, and solves it, using a different method than what was used before.

Despite his good intentions, Mr. S may not realize his goal of having students become flexible with and knowledgeable about multiple solution strategies because he has made it very difficult for students to compare and contrast multiple approaches. Mr. S has assumed that his students notice that the third problem is different than the first two, even though only one problem was on the board at a time. Mr. S has also assumed that his students notice that the method used to solve the third problem is different than what he used with the other two, even though only one method was on the board at a time. And finally Mr. S has assumed that the students see the features of the third problem that led him to decide that a different method would be a better way to approach the problem—a critical observation that is very difficult for students to see when only one worked example is visible on the board at a time. In essence, Mr. S is not giving students the opportunity to compare because he has erased the board after solving each problem. As in the camera example, there is so much to notice and remember about each individual example that it is very challenging to compare across multiple examples when viewing them one at a time.

But what if Mr. S had put two or even all three of the problems on the board at the same time? There is evidence that students who see worked examples side by side, with prompts to compare and contrast the examples, become better problem solvers and develop greater flexibility than students who see the same examples listed one at a time (Rittle-Johnson & Star, 2007). Although it may seem to be a relatively minor change to make, Mr. S would be more likely to impact students' flexibility if he manages board space so that multiple problems and methods can be visible at the same time. Doing so would then enable Mr. S to help his students more productively engage in conversations and reflections on the similarities and differences between problems and methods.

In order to achieve the critical instructional goal of flexible knowledge in mathematics—where students know a variety of ways to solve problems and are able to apply these methods adaptively on a wide range of problems—it pays to compare. Comparison is a powerful tool that mathematics teachers can use to introduce students to multiple strategies. Comparison can and should play a key role in the teaching of algebra as well as in the teaching of prerequisite-for-algebra topics, such as rational numbers.

The Center
FOR COMPREHENSIVE SCHOOL REFORM AND IMPROVEMENT

NEWSLETTER

References

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.), Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel.* Washington, DC: U.S. Department of Education.

Rittle-Johnson, B., & Star, J. R. (2007). Does comparing solution methods facilitate conceptual and procedural knowledge? An experimental study on learning to solve equations. *Journal of Educational Psychology, 99*(3), 561–574.

Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Strawhun, B. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *Journal of Mathematical Behavior, 24,* 287–301.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom.* New York: Free Press.

Administered by Learning Point Associates under contract with the Office of Elementary and Secondary Education of the U.S. Department of Education.

LEARNING POINT
Associates®

The Center
FOR COMPREHENSIVE SCHOOL
REFORM AND IMPROVEMENT

1100 17th Street NW, Suite 500
Washington, DC 20036

P: 877-277-2744 > **W**: www.centerforcsri.org
P: 202-223-6690 > **F**: 202-223-6728