MATHEMATICAL AND PEDAGOGICAL UNDERSTANDING AS SITUATED COGNITION

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One mathematics lesson was planned by two Grade 2 teachers together. Their separate teaching of it was videotaped, and each teacher was interviewed before and after her lesson. The "same" lesson resulted in different sets of worthwhile learning outcomes. In this research report, the notion of situated cognition is used as a tool for analysis of how this divergence seemed to happen. It is agued that the teachers' development and uses of mathematical concepts were mediated by the social situations.

CONTEXTS OF ACTIVITY

Vygotsky's seminal work focused on how social activity mediates cognitive development. In socio-cultural theories that draw on this work, understanding is portrayed as developing through interpersonal activity. One of these approaches, situated cognition, focuses attention on the influence of specific contexts (see, for example, Lave, 1988, 1993, 1996; Lave and Wenger, 1991; and Wertsch, 1991, 1995). "Contexts", here, include unique personal interests, perspectives, interpretations and purposes of participants as much of the general nature of social locations and interactions (Cobb, 1990). They incorporate social settings, which are "repeatedly experienced, personally ordered and edited" (Lave, 1988, p. 151). Any social interaction is shaped and constrained by the features and norms of the particular context in which it evolves; so action takes place not merely *in* or *on* an environment, but *with* it. This is not a one-way influence because participants in any social activity bring to it their own sets of socially mediated attitudes, beliefs, experiences, and goals. The distribution of cognition thus depends on situated affordances (Salomon, 1993).

In this paper, I analyse how this complexity seemed to be played out when the "same" mathematics lesson was taught by two Year 2 teachers. My focus is on teachers' cognition, rather than that of the children, although these are clearly interrelated.

THE RESEARCH CONTEXT AND METHODS

The data are drawn from a case study of four experienced primary teachers who planned their lessons for Grade 6 and Grade 2 in year-level pairs. The general aim of the project was to analyse what teachers do to develop children's mathematical understanding. Case study, an epistemology of the particular (Stake, 1994), was considered appropriate for this research because it allows us to look beyond human behaviour in order to focus on meaning as it is developed and used in individual contexts. The focus of the project was mainly on the teachers' understandings of their work. The fact that the "same content" was taught in two classrooms provided an opportunity to compare and contrast the finer elements of school contexts in which mathematics cognition develops.

The data were collected in one primary school over a period of four months. Mathematics lessons taught by each teacher (or each pair when they team taught) were videotaped for about one month. The teachers were interviewed before and after their lessons. The audiotapes of the interviews and the videotapes were digitized and compressed. This footage was reviewed for episodes that illustrated specific foci of the project: activity, children's thinking, discourse, and assessment. Parts of the interaction were transcribed.

Some of the lessons were then selected for a closer analysis using the theory of situated cognition. It is one of these lessons, taught by "Ruth" and "Trina" that I report on here. These two Grade 2 teachers planned many of their lessons together.

RUTH'S LESSON

On the day previous to this particular lesson, Ruth and Trina (together) had asked me about possible meanings of a topic that was included in the school curriculum outline, because, as Ruth claimed, they had "never known what it means". The topic listed was "Rotational symmetry", and I could not think of any traditional Year 2 curriculum content that would be given that name. Ruth had already asked a Grade 6 teacher, Rob, and reported to Trina and myself that, "Rob said it's order by rotation or something". That comment was helpful for me, and I demonstrated what order of rotation means, saying that the word "symmetry" would be appropriate because images are repeated as some shapes are turned. Ruth thanked me and said, "We'll work something out anyway". Later that day, after I had left the school, the teachers planned their lesson together.

In the pre-lesson interview the next day, Ruth said that they had decided to have the children paste colored sectors inside circles drawn on art paper "to make a rotating pattern". They had found this activity in a teachers' resource book. They had decided "not to worry about the symmetry bit", said Ruth.

Ruth's lesson started with the children sitting on the floor. The meanings of the words "pattern" and "tessellating" were recalled from previous lessons. Ruth then said that they were going to make a tessellating pattern "to go around in a circle". She produced cut out colored sectors of circles, and there was some discussion about what they might be called but no specific name was given to the shape. Ruth illustrated how the sectors were to be pasted onto a piece of white paper to make a circle, and stressed the need to "put them in order to make a pattern". She then turned her pattern around and around, saying "It's round, so we can rotate it, like a wheel. It's a rotational pattern. We are going to make *rotational* patterns". The children were then directed to "Choose either two or four colors", with no explanation of why, to move to their tables, to make their "rotational patterns", and then to cut around their "circles". (This latter direction was repeated later.)

As their patterns were completed, Ruth introduced individual children to the notion of "quarter turn" and "half turn". She realised that the degree of turn was hard for them to see, given that the cut-out patterns were circular and each had more than one sector of the same color, so she encouraged them to write a figure 1 on their "first bit" and to think of that as the "top of the circle". Towards the end of the lesson, Ruth started to lead the children to see that use of two colors would mean that the pattern would look the same at each quarter turn; but use of four colors would require a half turn for this to happen. While videotaping the lesson I thought that this latter perceptual development had been planned, but after reviewing the videotape I realised that Ruth had come to understand this only as she was teaching the lesson. (I later asked her, and she agreed that this was the case.) One section of the videotape shows an unsuccessful attempt to help a child solve a problem. The girl had used two colors, but in the order BGGBGG. She could not continue the pattern to complete the circle. Over the next few minutes Ruth looked at other children's patterns, listened reflectively to their discussions, physically turned and read out some of the patterns herself, and gradually started to talk with children about whether and when their images "look the same". She then went back to the girl to suggest making "a pattern … like two lots of four or four lots of two. The girl understood what she meant and changed her pattern to BBGGBBGG, then showed Ruth and commented that it was "twos as well as fours". Ruth agreed and then asked the girl to turn her circle half way round to see if it looked the same "upside down", and then to use quarter turns.

By the time that the children had been called back to the floor to show and describe their work, Ruth seemed to have a clear understanding of the notion of order of rotation. This information was offered to them, employing a subtle shift from the idea of patterns forming a circle to the idea of the image being repeated several times as a circle is turned.

Ruth:	Put up your hand if you think Lewis' is a rotational pattern. (Many hands were raised.) How do you know, Nick?
Nick	It fits in the circle.
Ruth:	(Nodded.) What's rotational mean? What does rotate mean? (Some children's hands were raised. Ruth drew a circle in the air with her hand.) If something rotates, what can it do? If something can rotate … (Drew another circle in the air. More hands raised.) Can you make your picture rotate, please Lewis? (Lewis hesitated.)
Jamie Lee	I can. (Turned her circle with four distinct quarter turns.)
Ruth:	Oh, you can! (Nodded, smiled. Lewis copied the action, turning his own picture around.) Good, Lewis. (Other children also turned theirs.) Is it the same pattern, then, as he turns it? … Can you see the same pattern repeating? (Lots of nods.) Good.

As I interviewed Ruth after the first lesson, Trina listened to her describe what had happened. Ruth illustrated the order of rotation principle using two examples of children's work. In her description, there was a lot of emphasis on "turning the circles" and observing when they "looked the same". (At this stage I was still under the impression that this had been the intended learning outcome.) Ruth stressed the need for children to put a one on the first piece "so they can tell where the pattern starts". Ruth said that she was going to "do more on rotational patterns" in her next lesson.

For the following lesson, Ruth had the children color in four circles (marked into eights on photocopied sheets) to record what the patterns that they had made the previous day looked like after each of four quarter turns. The last part of the lesson was whole-class discussion about when two-color patterns looked the same (after each quarter turn) and when four-color pattern did (after each half turn). Some children spontaneously noted that the first and third circles always looked the same and the second and fourth did too, and a child that Ruth asked was able to talk about why this was the case. One girl said her record of the four turns was "a special one because all of the pictures [are] the same". She was also able to explain why: "There's four lots of black and white, and black is always on the top because it starts each quarter". Not all of the children would have understood these explanations, but from their fiddling with their own patterns while waiting for their turns and their responses to Ruth's questions it seemed that many had grasped the idea..

TRINA'S LESSON

Trina had planned the first lesson with Ruth, and she used the same materials to teach it to her own class. There was much more discussion than in Ruth's lesson about what the shapes are called, and specifically about why they are called "eighths". The idea of turning the "circle pattern" was introduced quickly, and the children were told to "put a number one where the pattern starts"—a direction that was repeated several times later. The children then lined up to chose "eight eighths, and only two or four colors".

Trina	(Apparently filling in time usefully for the children waiting in line to collect their sectors.) I wonder how many pieces you all need to make the circle.
Jay	It depends on how many colors you're getting.
Jessica	Eight.
Trina	(Nodded, smiled.) You'll need eight pieces. Good girl. Eight pieces to make your pattern. Eight eighths.
Jessica	If you have four colors you need two halves, two lots; and if you use two pieces you need four.
Trina	Mm, right. (Paused.) What did you say? You want two pieces …
Jessica	If you have four colors you need two lots; and two colors need four lots.
Trina	Ah, right. Very good. You need eight all together.
(and later)	
Trina	(Filling in more time because children were still waiting for others to collect their pieces.) I want you to have a think about why they need two or four.
Jessica	I know.
Trina	Ahha! Jessica?
Jessica	If you use four it repeats twice, half, and if you use two it repeats four times, quarters. (…) If you had three you'd fill up three and then six and then you could not fit in three more.
Trina	Right. So you wouldn't really have a repeating pattern, would you?

As they finished the pasting activity, Trina asked each child to read out the pattern. She asked a few to "turn the circle around and read it again", but this seemed to be essentially a describing exercise with a focus on repetition of the pattern of colors around the circle.

Trina	What's your pattern?
Child 1	Red, blue.
Trina	Good boy. Red blue. (Pointed to shapes.) Red, blue, red, blue, red, blue, red, blue. Easy, yes. (Attended to next child in the queue) What's your pattern?
Child 2	Orange, yellow, red, blue, orange, yellow, red, blue.
Trina	You're going this way? Orange, yellow, red, blue. Excellent! (To next child) You tell me your pattern.
Child 3	Red, green, red, green, red, green, red, green.
Trina	Okay. (Rotated child's circle several times as she spoke.) So if I turn it around any old way, it should still be red, green, red, green, red, green? (Child nodded and smiled.) Okay! Good!

Trina did not ask any questions about when quarter or half turns were needed to make the pattern look the same. However, there was a lot of talk about their "eights". In the post-lesson interview, Trina later told Ruth and I that it was "a good lesson for fractions". She commented that the idea of “eighths” was not in the Grade 2 curriculum, but since the children had “understood it very well” she had thought that she “may as well press on with it”.

COGNITION AS SITUATED IN SOCIAL ACTIVITY

A key principle developed by Vygotsky is one of unity between mental functioning and activity, with the development of the mind resulting from goal-oriented and socially determined interaction between human beings and their environments. It is interesting to use this idea as a tool to analyse how implementation of ostensibly the same lesson plan led to different mathematical outcomes (and potentiality) for the two sets of pupils.

The teachers seemed to start with similar expectations for the lesson, and their knowledge about rotational symmetry seemed similar before the lesson. Many objects of the context—including the school curriculum, a resource book and the teaching aids it suggested, specific words (“pattern”, “rotational”, “order”, etc.)—seemed to be used in common ways. Interactions with Rob (the colleague) and with me may have too, but again these were shared. Apparently it was the enacted curriculum in their classrooms that led to a divergence. It was this social activity that seemed to have the strongest bearing on what mathematical ideas were made available to the children.

Understandings and interpretations gradually emerge through interaction, distributed among the participants’ interactions rather than individually constructed or possessed; and because cognition is both created and distributed in a specific activity context, it is necessarily situated (Salomon, 1993). Activity contexts are necessarily complex. For example, part of Ruth’s classroom interaction was with a girl who had followed her instructions but could not make a pattern, leading to reflective activity and growth of understanding by the teacher. Her development here was dialogic, involving social and mental activity. Note, though, that Trina also had this prompt, in the form of a girl’s impromptu explanation of why six sectors would not work, but that she did not seem to engage with it. Perhaps because it was not a problem for the child Trina was not put into a position where it became an “epistemological obstacle” (Sierpinska, 1994) that she had to engage with.

A further part of Ruth’s cognitive development was engagement with the idea of symmetry, even though she did not think of it this way. As Wertsch (1985) pointed out, the inner word extends the boundaries of its own meaning. Ruth employed her new understanding of order of rotation in ways that set up a context where the children were expected to grasp the same idea. Clearly some did, and so this cognition became somewhat distributed.

> A concept … will continually evolve with each new occasion of use, because new situations, negotiations, and activities inevitably recast it in new, more densely textured forms. So a concept, like the meaning of a word, is always under construction. (Brown, Collins & Duguid, 1989p.33)

It is useful to reflect on why neither of the teachers saw repeated patterns as anything to do with symmetry. Even a child’s comment to Ruth that “the same colors are opposite each other in all the patterns” did not provoke perturbation here. Sierpinska (1994) describes understanding as overcoming conceptual “epistemological obstacles”.

> Students' thinking appeared to suffer from certain "epistemological obstacles" that had to be overcome if a new concept was to be developed. These "epistemological obstacles"—ways of understanding based on some unconscious, culturally acquired schemes of thought and unquestioned beliefs … marked the development of a concept in history, and remained somehow 'implicated' … in its meaning. (p. xi)

It seems that the teachers' conceptualisation of "symmetry" was bound by their experience. The common notion of symmetry being reflection around an axis probably constrained their understanding of Rob's and my brief explanations of another form of symmetry. Higher-level aspects of symmetry such as "correspondence" and "congruity" are rarely articulated outside of secondary and tertiary mathematics classrooms, especially in relation to "symmetry". Perhaps a textbook exposition, or an immediate and confident explanation from me would have helped give the term new meaning for the teachers, but in actuality these were not elements of the context. Rob's explanation was drawn on in part, though, with the words "order" and "rotation" being used as a basis for their finding an appropriate activity in a textbook. In fact, it is not hard to see how the mathematical understandings of each of the participants in the unfolding scenario above (including my own) could be viewed as parts of a mutually constructed whole. It is also possible to see this whole as being made up of diverging, converging and interdependent learning trajectories.

Activity in the classroom situations served to raise some knowledge to a level of consciousness. For example, I would not have thought about "order of rotation" unless Ruth had misquoted Rob's interpretation of this term. Then Ruth's interpretation of my resulting explanation seemed not to be taken up until social activity stimulated her to reflect on the patterns that her pupils were making. Did Rob's and my "mentoring" roles on the previous day move Ruth's understanding enough to set up a zone of proximal development? This development certainly evokes Newman, Griffin and Cole (1989) discussion of a process whereby meaning is negotiated through groups of participants seeking "common ground of comprehension and understanding" (p. xi), and then progressing from that point. This involves trying to discover what the other has in mind and adapting the direction of interaction accordingly, operating in what Newman, Griffin and Cole called "the construction zone" (p. xi).

The social and person were inextricably interwoven. Without the contributions of others, the struggle to understand others, reflection on what had been said and observed, then translation of this into further activity and the resulting understandings of the teachers, students and researcher would not have eventuated. Lerman (1996), discussing Vygotsky's work, pointed out that development of knowledge is something that takes place between people that is then internalised secondarily by individuals (p. 137). Clearly, though, internalization sets up zones for contributions that people can make to further social leaning. Ruth's realization about quarter and half turns was communicated after the lesson to Trina, but Trina focused on other aspects of the potential of the activity for teaching and learning. Thus the social distribution of knowledge here was not a "blanket" one, but a networking of activities, tools (including words and concepts), and that was mediated by individuals. The community members participating in the learning context developed as a group, but not as one mind. Their learning was distributed among co-participants rather than over it.

I was interesting to me to see how individual engagement, or not, with an idea led to essentially different learning situations. However, the activity itself had still set up in Trina's room an opportunity for children to notice the phenomenon of the patterns being related to quarter and half turns, and in fact Jessica had reasoned about it even before undertaking the activity. The video of Trina's lesson shows a few instances of children in this classroom saying to each other, "It's the same, it's the same", as they turned their circles and paused after quarter or half turns—actions that were not suggested or modeled by their teacher. These children were commenting on the same observation that Ruth had made. They had achieved this understanding as a result of interactions with peers and interactions with physical (and perhaps linguistic) objects. Mentoring for some of the children, here, resulted from their observation of peer activity. There was a feeling (as I wrote in my filed diary) that "we are making some kind of sense together". Thus the differences in the teachers' developments of the same lesson seemed to lead to different learning foci for their students—but not necessarily to different learning opportunities and outcomes for those children who were ready to learn from the activity itself.

CONCLUSION

The research project reported on here draws on case study methods and theories of situated cognition. Case study allows researchers to capture evidence of and synthesise dimensions of teaching theory as well as practice. For me, it provided a methodological approach for describing components of classroom interaction; but also allowed inquiry into the origins of these elements, the meanings that they seem to hold for the subjects of the research, and the ways teachers and children interact with tools and traditions of mathematics education. Similarly, literature on notions of situated cognition has provided powerful tools for analysis of the case data.

Lave (1988, 1993, 1996), and Lave and Wenger (1991) proposed that the development of cognition is an integral part of generative social practice in the lived-in world, so the development of knowledge and social interactivity are interdependent and indivisible (Lave and Wenger, 1991). A further key principle of situated cognition theories that this paper has utilised is that dynamic mutual activity between actors and their environments lead to changes in participants and contexts over time. It is clear that the teachers' and children's mathematical understandings (as well as potentialities for further development of these) were mediated by the social environments in which they were developing.

My description of this "one" lesson, taught by two teachers, and of the subsequent lessons has been necessarily brief here, but is sufficient to demonstrate how mathematical understanding seemed to exist not only amongst individuals acting together in a social context, but across the tools (including language, beliefs and customs) and artifacts that were used. There's an element of voluntary control over what was learned by the various participants, but this too, was seen as "a product of the instructional process itself" (Vygotsky, 1934/1987, p. 169).

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