BRIDGING MATHEMATICAL KNOWLEDGE FROM DIFFERENT CULTURES: PROPOSALS FOR AN INTERCULTURAL AND INTERDISCIPLINARY CURRICULUM

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Teaching mathematics in a multicultural scholastic context is a current research theme in countries where the phenomenon of immigration is becoming important. There is a project being carried out in some southern European countries which aims to identify the needs of maths teachers in the lower secondary schools. The theoretical framework of the project is ethnomathematics and mathematics education in multicultural contexts. Bearing in mind the needs expressed by the maths teachers and the problems pointed out by them, three extensive didactic proposals have been prepared based on the activities of craftsmen in those countries with the highest percentage of emigration towards the South of Europe. In the final part of this article some of the initial indications following the introduction of one of these proposals into some Italian schools are briefly described.

THE SOCIAL AND SCHOLASTIC CONTEXT

The phenomenon of emigration towards countries in the south of Europe, such as Italy, Portugal and Spain, is a recent if somewhat significant one. The socio-cultural context is changing slowly but clearly. This change also affects the school context, especially compulsory schooling. In terms of percentage the figures are not yet at the levels of many northern European countries but, for instance, in Italy immigrant pupils represent 2.3% of the school population at present and are expected to reach 8% by 2011 with a higher percentage in primary and lower secondary schools (CARITAS, 2002).

While studying the phenomenon of immigration it is important to note the great diversity and variability which accompanies it. A diversity in the areas and countries of origin (in Italy more than 180 countries are represented), a diversity of distribution over the country, a variable in time and an inequality in the numbers present in a specific area of certain communities. All this makes it difficult to put into action a program both in the social and educational sphere which allows the transition from a state of emergency to the following of a project.

Nevertheless, when we talk about education and school and what has been done to give the teachers a basic knowledge which can be of use to them in facing the new professional reality of classes with pupils of different cultures, we can say that up to now the main aim has been to create a culture of cordiality which helped the immigrant pupil to settle down better in the class.

Little or nothing has been done to give the teachers (with the exception of L1 teachers) the tools to help them to use didactic methodology and to prepare curriculum proposals which take into account the new cultural context in the classroom. This lack of attention to the didactics in multicultural contexts is strongly felt today even in those subjects which were once considered to be culture-free and universal, like mathematics, especially by those teachers looking for educational strategies which respect the identity of each single pupil while being didactically effective for the whole class.

THE IDMAMIM PROJECT – ITS THEORETICAL FRAMEWORK

The European project IDMAMIM[1] - *Innovazione Didattica Matematica e sussidi tecnologici in contesti Multiculturali, con alunni Immigrati e Minoranze* is placed in the above socio-educational context. It is a three-year project in the final stages which is being carried out in Italy, Portugal and Spain.

The project, targeted for maths teachers in the lower secondary schools, is organised as follows:

- appropriation of information on the state-of the-art as far as maths teaching is concerned in multicultural contexts in the project partner countries;
- analysis of the opinions and experiences described by the teachers in their answers to a questionnaire (Favilli, César, M. and Oliveras, in press) and a semi-structured interview (César, Oliveras and Favilli, in press) as well as identification of their didactic needs;
- a course of seminars, aimed at teachers, on the principal indications which emerge from the international research on ethnomathematics (Powell and Frankenstein, 1997) and mathematics education in a cultural context (Bishop, 1988), which form the theoretical framework of the IDMAMIM project (Oliveras, Favilli and César, 2002);
- projecting, elaborating, experimenting, consolidating, evaluating and finalising (on paper and CD) formative material for teachers and didactic proposals for mathematics, especially destined for use in the multicultural context.

As just said, the theoretical framework of the IDMAMIM project is ethnomathematics and didactics situated in a cultural context. In their article Vithal and Skovmose (1997) underline how between the four strands which can be seen in research into ethnomathematics - the challenge to the traditional history of mathematics, the analysis of the mathematics of traditional culture groups and indigenous people, the exploration of the mathematics of different groups in everyday settings, the relationship between ethnomathematics and mathematics education - the last one is *the unifying strand as it pulls together the other strands:* even when it is *an under-researched area compared to the other strands* (p. 135). We agree with them and add that it is important that, in the programming of didactic activities in maths classes, the findings of such research would be taken into consideration not only in traditionally multicultural countries or in those countries where the culture of the colonising country has taken over and replaced local culture, but also in countries where the socio-educational context is undergoing a rapid and continuous transformation as described above for our countries. It is not at all easy to modify the maths teacher's approach to the curriculum in the different western school systems, but we believe that the presence of pupils from different cultures in the same class makes it indispensable for maths teachers to be made aware of the fact that

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mathematics, as a product of human intellect, is by its own nature under the influence of the cultural context in which it is produced both in its creation and development (Bishop, 1998). Instead of talking about mathematics we should be talking about mathematical activities which can grouped under the same categories but are not the same on a practical level; the existence of these mathematical activities, which have been developed in radically different ways, must be known to the teacher who must also be able of bearing in mind the different contributions that other cultures have given and continue to give to mathematical knowledge when carrying out class activities (Zaslavsky, 1991)

Members of these *minority* cultures (macro- or micro-cultures as they may be) are not always aware of this contribution and of their mathematical knowledge, which are often used implicitly; intercultural mathematical education (Oliveras, Favilli and César, 2002) can give teachers the opportunity to help their pupils, even those belonging to this same culture, discovering these contributions and knowledge. In this way pupils coming from minority cultures see their cultural heritage enhanced, while the others can be made to realise that knowledge, even mathematical knowledge, does not stem from one culture alone (Joseph, 1992).

THE MICRO-PROJECTS

It is in this light that the projects for the didactic proposals, which are being tested with the help of some teachers in lower secondary schools, in the IDMAMIM project were chosen. These teachers meet periodically with the project national steering groups.

The proposals were made taking as an example the activities in class developed around micro-projects, as shown in (Oliveras, 1996). Ideas were taken from some craft activities which were significant and representative of cultures from geographical areas with the highest number of emigrants towards southern Europe. The entire process of making a zampoña (pan pipes) in the Andes, African craftsmen from the sub-Sahara area making Batik cloth and north African rug makers were video-taped. After studying these videos, the various stages of the craftsmanship were identified for each activity and for each stage the mathematical knowledge and notions used by the craftsmen were identified both implicitly and explicitly. The mathematical knowledge and notions were handed down by older more expert craftsmen or were gained from their own direct experience: in fact, these notions were rarely gained in a scholastic context as these craftsmen did not have the possibility of attending school, at least no further than the first classes. In addition to these mathematical notions that the craftsmen use, we tried to identify others that we could generically term the mathematics that a researcher can *see* and/or associate with the various phases of the activity.

As it is only natural, the sum of knowledge and mathematical notions identified in the development of the working process of the craftsmen are difficult to place in sequence in a standard school curriculum for one class; in some cases some notions are better introduced at different times or even in different classes with respect to other notions used immediately before or after by the craftsmen. For this reason the didactic proposals programmed and elaborated were presented to the teachers in an open-ended fashion, not as a set pack with indications on how and when they should be followed.

The teacher-researchers, who are taking part in the empirical work, were advised to follow the viewing of the video showing the craftsman at work with a practical session in class where the students actually make the object either individually or in small groups. This activity which appears to be of an entirely technical nature and only mimics the activity of others, actually obliges the pupils to think, often in an unexpected manner, along mathematical lines.

These reflections, both collective and individual – but always shared with the other pupils (César, 1998) – are brought about by the necessity to face and solve small practical problems or by abstract considerations that the pupils make during the elaboration.

This first stage, dedicated to the construction of the object and present in each of the three didactic proposals is designed to be carried out by various teachers, not only the maths teacher, in a co-ordinated and collaborative fashion:

- the technical education teacher has to provide help in the construction activity, not only from a practical point of view, but also by providing information on the choice of materials and tools needed;
- the geography teacher can give the pupils a lot of information about the geo-physical aspects of the area in which these objects are made;
- the history teacher can provide news and comments on the principal events which have occurred in that area from the past to the present;
- the human science teacher can help to define the cultural context of the area by providing information on the roots and principle characteristics of the local language, its use and development, on the method used to transmit the culture, on the principal representatives of the culture (writers, poets, artists, scientists, etc.).

Depending on which project is being carried out it would also be possible to involve the music teacher and the science teacher (for the zampoñas) or the art teacher (for the batik material and the rug). We believe that this interdisciplinary didactical methodology should also be followed systematically in the teaching of mathematics, especially during compulsory schooling. The transmission of mathematical concepts should not be seen to be an end in itself but rather to stem from the necessity to solve problems in real life, following the essence itself of mathematics, because as Galileo Galilei said in his "Saggiatore": *chi non la conosce non può leggere il grandissimo libro dell'universo (*those who don't know it [mathematics] can not read the great book of the universe).

This type of didactic proposal means to instil in the pupil the knowledge that mathematics is a cultural product give them an example of a mathematical activity which is not scholastic and not linked to a western culture help them work in an intercultural manner interest and motivate the entire class enhance other cultures through an exchange of knowledge promote the academic performance in mathematics of the foreign students and those from a minority culture.

THE MICRO-PROJECT OF THE ZAMPOÑA (PAN PIPES)

The micro-project elaborated and tested in Italy is the one which makes use of the construction of the zampoña (pan pipes from the Andes), a wind instrument usually made of two series of seven and six pipes placed side by side. This instrument is part of the traditions found in the population of Ecuador, Peru, Bolivia and Chile. The micro-project is based on the construction of a zampoña done by a craftsman from Cuzco in Peru. The micro-project follows three steps:

1. introduction and construction (discovering the zampoña!)
2. qualitative analysis (getting to know it better!)
3. quantitative analysis (let's make one bigger or smaller!)

By watching the video and the direct construction of the zampoña we move straight to step 2. Numerous references to mathematical concepts are used even unknowingly by the craftsman or may be linked to various phases of his activity by the researcher.
Here are some of these concepts and mathematical activities:

Relation - function - sequence - ordering - classifying - measuring - average, mode and median - cylinder - circle - ratio - proportionality.

In the following part of this article we will present some comments which came out of the development in the classroom of the micro-project with reference to step 3 - *a quantitive analysis,* and in particular to ratio and proportionality.

The problems linked to the introduction by the teacher and the appropriation by the pupils of notions of ratio and proportionality have been the object of great study and research, due to the intrinsic difficulty of such concepts usually introduced to pupils aged 12 -13, during their seventh year of schooling. Following the studies carried out by Piaget and his collaborators on proportionality (Piaget, Grize, Szeminska and Bang, 1968), which led to believe that once pupils had understood linear functions they would be able to solve problems of proportionality whatever the problem situation, Vergnaud's studies (Vergnaud, (1983), however, suggest that in order to understand the concept of proportionality the nature of the problem situation plays an important role. Nunes, Carraher and Schliemann (1993) in the chapter on ‘Understanding proportions’ complain that:

> Little attention is given in math textbooks to connecting the mathematics with the problem situation, and the initial phases of teaching involve mostly formal demonstrations. The formal demonstrations are followed by exercises in application of the procedure. In the application it is assumed that the procedure just learned is appropriate; therefore students do not concentrate on a discussion of what connections there may be between mathematical models and empirical situations. (p. 86)

We can confirm that, at least as far as Italian textbooks are concerned, the situation is identical!

The micro-project on the zampoña goes exactly in the direction wished for by Nunes, Carraher and Schliemann (1993): to create the desire in class to solve a specific problem, to stimulate debate and an exchange of ideas, reflections, observations and proposals by the pupils and finally to create the need in the class for the introduction of new mathematical concepts which may be essential to solve the problem assigned to the pupils, as well as the opportunity to discuss at home about out-of-school mathematics (Hughes, 2001).

In order to build the zampoña the pupils have two tables of figures which refer to the measures, taken by the craftsman at the end of his construction, of the length and diameter of each of the two series of tubes cut by him and used to make the musical instrument. In actual fact the craftsman *measures* the length of the tubes using a plank of wood which is in proportion to the length and breadth of the zampoña he wishes to make and with markings which correspond to musical notes. This way of measuring, which relies on a sort of graduated scale, bases on the craftsman's experience and is in itself a point for reflection and comment in the class because of the implicit knowledge and mathematical activities put into play by the craftsman. It was only after a specific request from the researcher that the measurements were taken using a ruler on completion of the construction. This confrontation between mathematics used in and out of a school context and the various mathematical abilities used by both illiterate workers and students has already been noted by Nunes, Carraher and Schliemann (1993) and by Masingila, Davidenko and Prus-Wisniowska (1996).

	Ray	Fah#	Lah	Doh	Mi	Soh	Ti
Length	18.1	14.5	12	10.2	8.0	6.6	5.5
Diameter	1.2	1.1	1.1	1.0	0.9	0.8	0.7

Figures for the series with 7 tubes

	Mi	Soh	Ti	Ray	Fah#	Lah
Length	16.2	13.9	10.8	8.7	7.3	6.2
Diameter	1.2	1.15	1.0	0.9	0.8	0.75

Figures for the series with 6 tubes

These measurements are obviously only approximate, especially as far as the diameter is concerned: for this reason the teachers are asked to make their pupils reflect on the significance of measurement, error, average, etc. and to take little notice of the diameter of the tubes (considering the modest values) but rather to choose them in order of decreasing diameter as the tubes get shorter or even of similar diameters. The pupils are then asked to build a zampoña of a different size for example bigger. They are given a table - with incomplete numbers - showing the lengths of the six tubes which make up part of the instrument.

Notes	Mi	Soh	Ti	Ray	Fah#	Lah
Length	28		18.2	14.8		

As the students use this table they realise, with a little help from their teacher, that there is a constant relationship between the length of two tubes representing the same notes in both the instruments of different lengths; in fact

Mi: 28 / 16.2 = 1.73 - Ti: 18.2 / 10.8 = 1.69 - Ray: 14.8 / 8.7 = 1.70

The values obtained are slightly different and this is due both to inaccurate craftsmanship and to the difficulty in obtaining precise measurements; the calculations of the relationship between the values is an excellent way of introducing some simple statistical notions. In particular, as far as the figures recorded are concerned, it is possible to calculate the average (1.71) and the median (1.70).

In this way the pupils are given a simple tool which allows them to find a more accurate value in the relationship between the two musical instruments.

The presence of an almost constant relationship is also useful in calculating the length of the other tubes; following the example above, if 1.71 is taken as being the value of the ratio between the two zampoñas (the one built by the pupils and the one we have some figures for) it is possible to deduce the length of the tubes of the other notes; for example, Soh: 13.9 x 1.71 = 23.77.

Therefore the construction of the zampoña appears to be a concrete way of treating the concepts of ratio and proportion!

In particular, the teacher can focus attention on the fundamental properties of the ratio (multiplying both members by the same number does not alter the ratio). Further, finding the ratio between the two classes of quantities (the tubes of the two zampoñas) is the key to using the measurements of the first elements to find the length of the second and viceversa.

SOME FIRST INDICATIONS

The micro-project of the zampoña has already been used in some classes of lower secondary schools in Italy during the past school year (involving four teachers and around a hundred pupils, some of whom were immigrants) and is being more widely used during this school year.

The first indications referred back by the teachers in their reports and by the pupils themselves in their comments in class seem to be very positive.

The pupils seem to be highly motivated by the will to construct something either individually or in small groups: *it's the first time I've built anything!*

Interdisciplinary teaching is a surprising novelty for many pupils (and teachers…): *What has mathematics to do with the zampoña?… I think it's useful to bring together two such nice subjects, maths and music, in one task!*

Mathematics is seen under a new light and appears less boring: It was interesting because it is nice to do maths like this! *The lessons were useful and helped us find out more about the zampoña and they were also fun…*

Mathematics turns out to be less of an ordeal than it usually is: *The lessons were much nicer also because it was a more fun way to reason, to think up original solutions… The mathematical notions were very easy to understand…*

At the end of the second cycle of using of this didactic proposal a more detailed analysis of the impact it has had on the pupils will be done, in particular using a comparison with students of other classes where the concept of the ratio and proportionality will have been introduced in a traditional manner. Furthermore, we will try to assess not only the greater interest shown in learning mathematics but also the real didactic effectiveness of the proposal itself.

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