Research Points

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Essential Information for Education Policy

Do the Math:
Cognitive Demand Makes a Difference

Extending high expectations to *all* students in mathematics is a relatively new idea. Even the 1960s movement to improve U.S. mathematics education, which was based on the argument that an excellent scientific education was necessary for a strong economy and national defense,[1] largely was limited to "college-capable" students.[2, 3]

Today, mathematics education faces two major challenges: raising the floor by expanding achievement for all, and lifting the ceiling of achievement to better prepare future leaders in mathematics, as well as in science, engineering, and technology. Although these goals are not mutually exclusive, this *Research Points* tackles the challenge of ensuring that whole groups of students are not excluded from higher mathematics learning.

In our global economy and democratic society, limiting math education to select students is unacceptable. A recent ACT study provides evidence that college and the workforce require the same levels of readiness in mathematics. One implication: All students require a greater level of "cognitive demand" in mathematics than once was considered appropriate. In other words, high school students need learning experiences in algebra, geometry, data representation, and statistics whether they are planning to enter college or workforce training programs.[4]

The term "cognitive demand" is used in two ways to describe learning opportunities. The first way is linked with curriculum policy and students' course-taking options — how much math and which courses. The second way relates to how much thinking is called for in the classroom. Routine memorization involves low cognitive demand, no matter how advanced the content. Understanding mathematical concepts involves high cognitive demand, even for basic content. Both types of cognitive demand are associated with student performance on achievement tests, but they are not substitutes for each other.

Course-Taking

Large-scale assessments have found that mathematics achievement can be predicted by the number of mathematics courses taken and the amount of time spent studying advanced mathematics. Generally, these predictors are inter-related.[5, 6, 7]

Course-taking options in the United States are organized according to curricular and ability tracks. Most students are sorted into tracks involving specific course

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sequences and, ultimately, different opportunities to learn mathematics. Traditionally, high schools have had three curricular tracks — college preparation, vocational, and general education. The college-preparation track has top status and provides greater opportunity to learn more demanding mathematics.

Although many schools have done away with such three-track sorting, hidden forms of tracking persist. In one common situation, students are divided by perceived ability under the same course label. For example, an algebra course might sort students into fast and slow speeds of learning, so that by the end of the year students in the same class have not had the same opportunity to learn. Another sorting strategy offers different entry points into college-preparatory coursework (e.g., freshman versus junior year). For students who enter the college-preparatory track late in high school, it might be too late to learn enough mathematics to pursue higher-level college courses.

Signs of Progress

Despite continued overt or concealed tracking, there has been progress — students who in the past might have been left out of high-demand courses increasingly are being placed in higher-level mathematics. For example, the 1980s saw striking increases in the percentage of African American students earning credits in college-preparatory courses.[8] These increases largely reflect many states' new standards and graduation requirements for more mathematics credits. Such policies, and their encouraging results, have overlapped with steady upward movement in the percentage of African American students earning undergraduate and master's degrees in science and engineering.[9]

In theory, tracking helps all students by providing instruction suited to their ability and learning styles. However, research strongly suggests that not all students are benefiting.[10] Instead, the positive effects of tracking on overall achievement are associated most with a small minority of students assigned to high-status tracks.[11, 12] We still need to prepare many more students in elementary and middle school to handle high-demand courses in high school, and we need to figure out how to keep the positive trends moving forward.

Quality of Mathematical Thinking

In a review of school impact on the test score gap between African American and white students, Ronald Ferguson concluded that the basic problem is not tracking *per se* but the expected quality of instruction — the second form of cognitive demand.[13]

Traditionally, American mathematics teaching has emphasized whole-class lectures with teachers explaining a problem-solving strategy and students passively listening. The lecture usually is followed by students working alone on a large set of problems that reflect the lecture topic.[14, 15, 16] In contrast, high cognitive demand mathematics programs generally deviate in important ways from the "normal" approaches to mathematics instruction and classroom practice.

The 1999 Trends in International Mathematics and Science Study looked at the ways that mathematics instruction differs among seven countries.[17] It found that although effective teaching varies from culture to culture, the key difference between instruction in the United States (the lowest performer in the study) and the other countries was the way teachers and students work on problems as a lesson unfolds.[18]

While higher achieving countries did not use a larger percentage of high cognitive demand tasks compared to the United States, tasks here rarely were *enacted* at a high level of cognitive demand. High-performing countries avoided reducing mathematics tasks to mere procedural exercises involving basic computational skills, and they placed greater cognitive demands on students by encouraging them to focus on concepts and connections among those concepts in their problem-solving.

Other research found that in classrooms in which instructional tasks were set up and enacted at high levels of cognitive demand, students did better on measures of reasoning and problem-solving than did students in classrooms in which such tasks were set up at a high level but declined into merely "following the rules," usually with little understanding.[19, 20] In successful classrooms, task rigor was maintained when teachers or capable students modeled high-level performance or when teachers pressed for justifications, explanations and meaning through questioning or other feedback.

International comparisons also have shown that some top countries teach fewer concepts in greater depth, while U.S. math curriculum is "a mile wide and an inch deep."[21] To focus the wide scope of topics presented to U.S. students, new curriculum guidelines from the National Council of Teachers of Mathematics

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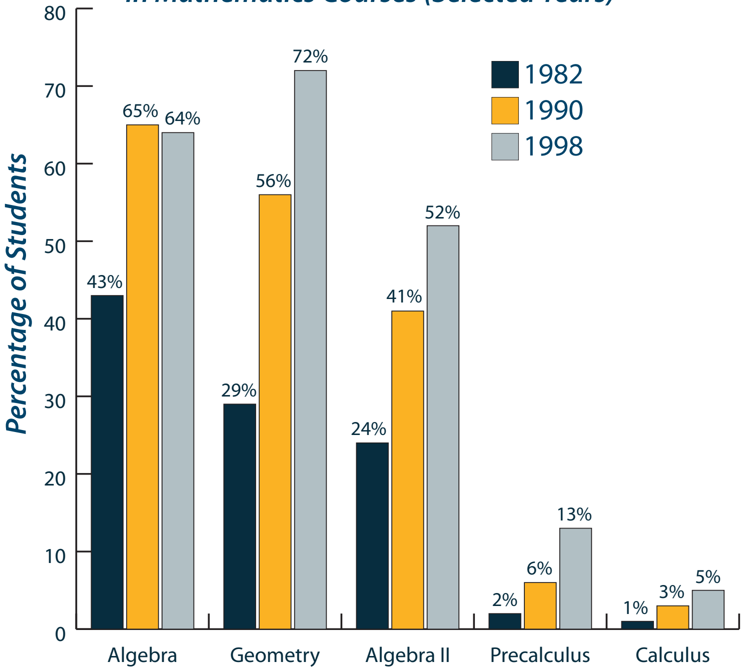
High-Level Mathematics Course-Taking

The percentage of African American students earning credits in college-preparatory mathematics courses increased dramatically between 1982 and 1990. These increases reflected state policy changes involving new standards and graduation requirements calling for more mathematics credits.

Despite the welcome progress, a word of caution: Merely mandating a narrow curriculum consisting of traditional college-prep mathematics courses will not undo problems endemic to the preK–8 mathematics program. Cognitive demand and instructional quality must be raised both in the lower grades and in high school.

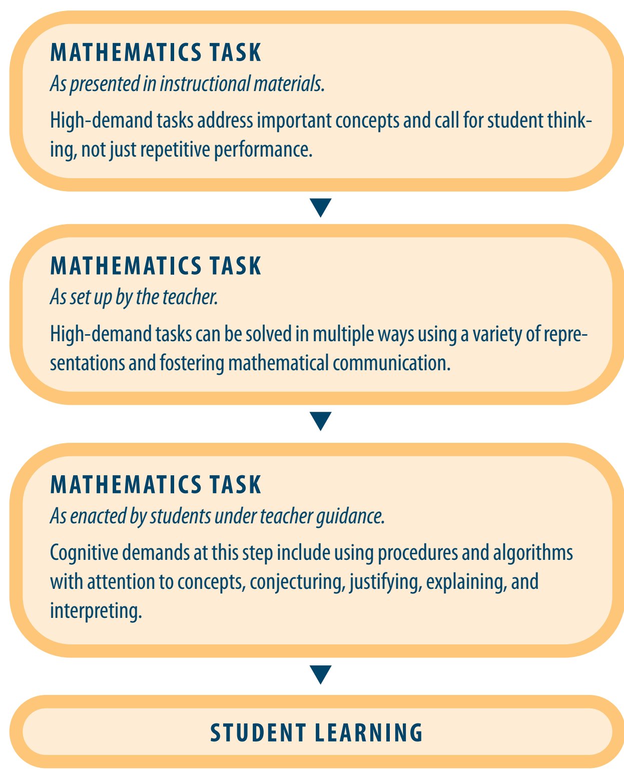
African American Graduates Earning Credits in Mathematics Courses (Selected Years)



Source: Roey, S., et al. (2001). **The High School Transcript Study Tabulations: Comparative Data on Credits Earned and Demographics for 1998, 1994, 1990, 1987, and 1982 High School Graduates.** *National Center for Education Statistics. (NCES 2001-498). Washington, DC: U.S. Government Printing Office.*

Mathematics Tasks in a Classroom

Mathematical tasks convey messages about what mathematics is and what doing mathematics entails. A typical task passes through three phases. High-demand tasks are the starting point. As these tasks are carried out, teachers must keep students engaged in high-level thinking and reasoning, avoiding the urge to do the hard thinking for students when they struggle with a problem. Teachers should encourage students to use more than one problem-solving strategy, represent the problem in multiple ways, and explain and justify their work. High cognitive demands or thinking processes involved in solving a task can include the use of general procedures connected to underlying concepts and meaning, complex thinking, and reasoning strategies.



Source: Adapted from Stein, M.K., Grover, B.W., Henningsen, M. (1996). "Building Student Capacity for Mathematical Thinking and Reasoning: An Analysis of Mathematical Tasks Used in Reform Classrooms." **American Educational Research Journal,** *Vol. 33, pp. 455–488.*

emphasize key mathematical ideas on which to build deep understanding and connections.[22]

Conclusion

Learning math can be tough. Not learning it is tougher. Many students lack access to higher-level mathematics courses and teaching at all levels of precollege schooling. This is unacceptable in the face of the ever-expanding technical demands posed by higher education and the 21st-century job market. Research reveals that strong academic experience is needed for both college and the workforce. Raising the cognitive demand in the curriculum is necessary for enhancing students' career prospects.

Recent trends show progress, such as growth in the number of minority students taking higher-level mathematics classes and earning degrees in mathematics. Still, there is much work to be done. Curriculum policies that limit course options restrict opportunities to learn for traditionally underserved students. This problem is compounded by the sorting of students according to ability within the same mathematics classes and the low quality of some mathematics instruction in elementary and middle schools.

Bringing less advantaged students into higher mathematics study and preparing our future leaders in mathematics and science are not mutually exclusive ends. If we teach math at a higher level of cognitive demand, even in the early grades, we can look forward to a future in which high mathematics achievers better reflect the country's diverse population. To accomplish this, schools need to be staffed by well-prepared teachers, and high curriculum standards should be a priority. Teaching in high-performing schools requires a learning environment that supports sustained student engagement on both basic skills and cognitively demanding conceptual mathematics tasks.

Research Points

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What Should Policymakers Do?

First, embrace high expectations for all students in mathematics. Informed civic engagement and a competitive, global economy demand higher levels of technical skill.

Second, institute curriculum policies that broaden course-taking options for traditionally underserved students. This includes avoiding systems of tracking students that limit their opportunities to learn and delay their exposure to college-preparatory mathematics coursework.

Third, raise cognitive demand in mathematics teaching and learning in both elementary and secondary schools. Elevated thinking processes come into play when students focus on mathematical concepts and connections among those concepts. High cognitive demand is reinforced when teachers maintain the rigor of mathematical tasks, for example, by encouraging students to explain their problem-solving.

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