THE USE OF GRAPHING CALCULATORS WITH SYMBOLIC CALCULATIONS ON PERFORMANCE GAINS IN A COLLEGE ALGEBRA CLASS[1]

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[1] This article is based on the first author's doctoral dissertation, completed at the University of Texas at Arlington.

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More sophisticated graphing calculators available for college students can be incorporated into college algebra classes to result a higher student achievement. For this purpose, in this study 117 college students were randomly assigned into either experimental groups where students were allowed to the use of graphing calculators or control groups where the traditional lecture format was given. Pre-and-post measurements were made on students' algebra knowledge in two levels: action and process levels. Action questions were routine, basic, and conceptual questions, whereas process questions were high level, procedural questions. Result indicated that regardless of initial differences, the average and process pre- to post-treatment gain of students in the experimental group was significantly higher than in the control group. There was no significant difference detected between the groups in terms of action gain. Suggestions for future research were discussed.

Mathematics is one of the most important and challenging subjects that students have to learn in schools. In addition to this innate difficulty, the needs and expectations of students from schools, in general, and from mathematics, in particular, are changing as our society rapidly changes. Therefore, school mathematics should be adapted to meet the changing needs of students and society (NCTM, 1989;1991). *The* S*tandards* (1989) gives guidelines that can be used to make changes in curriculum, instruction, and assessment for different grade levels. One recommendation was shifting emphasis from a curriculum dominated by memorization of isolated facts and procedures to conceptual understanding, multiple representations, mathematical modeling, and mathematical problem solving (NCTM, 1986). Constructivist theories that emphasize active learning might be useful in this shifting process (von Glasersfeld, 1989). It is suggested that a learning environment should give opportunity for students to investigate, to make sense of, to construct meanings from new situations; to make and provide arguments for conjecture; and to use a flexible set of strategies to solve problems both from within and outside mathematics. A classroom environment should be created in which teachers and students are natural partners in developing mathematical ideas and solving mathematical problems. There is a close parallelism between what was suggested and the assumptions of constructivist theories. Simply, constructivist learning theory assumes that the learning environment will promote and encourage the development of each individual with different alternatives to powerful mathematical constructions for posing, exploring, constructing, solving, and justifying mathematical problems and concepts (Brooks and Brooks, 1993; Confrey, 1990).

Constructivist learning theory requires active student involvement. Because graphing calculators are just tools for teaching and learning, there is a need to investigate different aspects of them that may facilitate constructivist learning. The use of graphing calculators with symbolic calculations may create a learning environment in which the attention is shifted from time-consuming calculation to conceptual understanding of the subject being discussed. Students can explore by pondering on different aspects of a given problem--numerical, graphical, symbolic, which can lead to a higher-level understanding of the subject. As Quesada (1996) stated each student can approach problems using different representations. Given the opportunity to choose a method that fits best for the student causes the student to actively involve and become more confident.

Research findings are in favor of using graphing calculators in different content areas of mathematics (Rodgers, 1995; Tobias, 1993). Graphing calculators with symbolic capabilities are becoming part of mathematics education. Several researchers have investigated the relationships between the use of graphing calculators and different learning environments, which promote different aspects of constructivist theory. Emese (1993) examined students who were taught using guided discovery style of instruction with the aid of graphing calculators and found that students were in favor of this style of learning. In another study, Coston (1994) investigated the effects of graphics calculator-enhanced instruction, and cooperative learning on college algebra students' understanding of the function concept, achievement of algebraic skills, and attitude towards mathematics and found that there were significant gains in achievement and attitudes of students. Because graphing calculators with symbolic capabilities are becoming more widely used, students have greater access to information from numerical, graphical, and symbolic representations (Dick, 1992). This might help multiple representations of topics, which leads higher level constructions on students' mental structures.

The spreading use graphing calculators in mathematics education raises the question of their best use in mathematics classes. There is less research on what kind of teaching style and learning environments will take advantage of graphing calculators with symbolic calculation features. Graphing calculators have potential to facilitate constructivist learning environment. Therefore, the goal of this study was to investigate how graphing calculators with symbolic calculations could be used to facilitate a constructivist learning environment in college algebra classes.

New technologies such as calculators and computers have made calculations and graphing easier. There is growing interest in their potential to facilitate and enrich the teaching and learning of mathematics. Sophisticated computer software and symbolic-graphing calculators are becoming available at low cost (Dick, 1992). Depending on the availability of resources and subject matter, administrators with the coordination of instructors might decide on choosing one technology over another but the following three main reasons may favor using graphing calculators at this time: (a) Establishing computer labs are still a costly investment. (b) Dependency on desktop computers and expensive software used in these computer labs is a barrier to implementing serious technology-based curriculum reform in mathematics. (c) There are also some limitations on accessing to those computers such as number of computers and time restrictions. On the other hand, the cost of getting powerful graphing calculators for schools is not the same as the cost of buying computer systems. The user-friendly aspect of the graphing calculator is another advantage over computers. Although the speed of technological progress is closing the gap between the use of computers and graphing calculators, the graphing calculators seem to have more advantages at this time. Therefore, the main discussion should be how to take advantage of available technologies for mathematics classes.

Students using graphing calculators have the opportunity to study algebra as a meaningful and related representation of functions, variables, and relations. Functions using graphing calculators can be represented by a graph, a verbal statement of the real-world context, and a table value other than algebraic expressions. Flexibility in translating among different representations of functions is an important aspect of constructivist learning environment because students have different experiences and goals, and the goals or context should determine which representation is most appropriate. Students can construct conceptual links among these representations (Wilson and Krapf, 1994). Students should be able to see the importance of moving from one representation to another. Students, with necessary practice, can reach to a level at which they can recognize and understand the equivalence of different representations that describe the particular functions (Dyke and Craine, 1997). The graphing calculators allow students to draw accurate graphs of standard and complicated functions. Students can analyze different aspects of the functions such as symmetry, minimum (maximum) values, and zeros of functions. Although a graphing calculator can graph functions, students should still have necessary skills to get the complete graph of a given function by finding appropriate window(s). A complete graph gives all the important points and features of a given function. Students cannot answer many questions without getting the complete graphs. This would be a new skill that students should acquire, which requires basic knowledge of graphing such as x and y intercept and symmetry.

Graphing functions can be helpful to confirm algebraic solution for a given equation. For example, solutions of $x^3 + 6x^2 + 3x - 10 = 0$, in addition to symbolic calculation feature of the calculator, can be obtained by finding zeros of $f(x) = x^3 + 6x^2 + 3x - 10$. Students should know how to derive solution using some kind of algebraic technique. In this example, writing this expression in factor form using rational zeros theorem and division algorithms (synthetic or long division) leads to three different solutions, $x = 1, -2, -5$, but there is no need to spend time for expressions similar to the one in this example. Graphs or symbolic algebra features of the calculators can be used to find solutions for a given equation. This process can also be reversed (i.e. students can get their solution(s) using some paper-and pencil techniques and then confirm their results using graphing and/or symbolic features of the calculators). Different features of calculators give students a way to confirm their results.

Consequently, knowing a way to confirm the result may lead to positive attitude. There are some cases that the graph of function corresponding to the given equation may not show all zeros. For example, assume the question is to solve $x^3 - 8 = 0$. Real solutions can be answered by finding x-intercepts of the graph of $f(x) = x^3 - 8$. Although there are three solutions, $x = 2, -1 \mp \sqrt{3}i$, the graph gives one real solution, $x = 2$. Therefore, the graphing is not enough to find all solutions. It can be used as a first step to analyze the problem. Although symbolic algebra capability of the calculator can give all solutions, students should still need to know basic algebraic skills to get all solutions. The role of the calculator is to give students opportunities to investigate the question from different perspectives. After developing different techniques for solving different equations, the graphing calculators can be used for more complicated problem situations. Time spent on using different algebraic methods to solve equations can be used for real-world problems that may lead to some kinds of a model, and these kinds of models may not have easy solutions using algebraic manipulations.

As a summary, the graphing calculators have potential for students to:

1. explore and experiment with mathematical ideas such as patterns, numerical and algebraic properties, and functions,
2. develop and reinforce skills such as estimation, computation, graphing, and analyzing data,
3. focus on problem-solving processes rather than the computations associated with problems,
4. perform the tedious computations that often develop when working with real data in problem situations, and
5. gain access to mathematical ideas and experiences that go beyond those levels limited by traditional paper-and-pencil computation (NCTM, 1991).

The effects of graphing calculators on students’ achievement have been investigated. As Dunham (1994) stated, "comparing common test scores of students receiving graphing calculator-based instruction to those of students receiving traditional instruction gives some information, but this process is much like comparing apples and oranges if the course goals are different". Penglase and Arnold (1996) emphasized the same point that effects of graphing calculators on students' achievement and learning in many studies equated with performance of students on some assessment tasks. Therefore, the effects of graphing calculators must be carefully distinguished between the tool and the context in which it is used. Better-designed research on the topic is a great necessity.

Milou investigated secondary school mathematics teachers' attitudes and their use of graphing calculators (1998). A majority (85%) of algebra teachers indicated that a graphing calculator was a motivational tool and could make students try harder. More than 61% of teachers responded that graphing calculators allowed for algebra classes to cover additional material and modifications of the algebra curriculum were beginning to appear.

Rich (1990) examined the ways in which the use of a graphing calculator as a teaching tool affected pre-calculus students' learning function and related concepts, teachers' methods, and beliefs. Learning graphing concepts was positively affected. Although there was no significant difference in overall achievement, the learning of graphing concepts was positively affected. Students using a graphing calculator learned to solve algebra problems graphically and algebraically. Students in the control group understood the connection between algebraic equations and their graphs and viewed graphs more globally (domain, range, intervals where the function increases and decreases, asymptotic behavior, and end behavior). Similar results were also supported by other researches (Army, 1991; Slavit, 1994). Another finding of Rich's study was that the teacher who used graphing spent more time on explorations, asked more higher level questions, used examples differently, and emphasized the importance of graphs and approximation in problem solving.

Giamati (1990) investigated the effects of graphing calculator use on advanced graphing techniques that focus on stretches, shrinks, reflections (horizontal/vertical) translations, and forming reciprocals of functions. The graphing calculator was used to allow students to observe and analyze the effects of various parameter changes on graph of functions and relations. Results showed that the students in the control group outscored the students in the experimental group at sketching functions, understanding translations and stretches and shrinks, and describing parameter variations. The graphing calculator did not aid students on understanding stretches and shrinks and translations. Based on classroom observations and students interviews, Glimati suggested that these results might be due to unfamiliarity with certain characteristics of the graphing calculators.

Vazquez (1990) and Steele (1993) found that the use of graphing calculators may not lead to higher level understandings of different aspects of graphs (Penglase and Arnold, 1996).

In a meta-analysis, Hembree investigated studies that focused on the effects of graphing calculators for different subjects. Seventy-nine studies investigated the effects of calculator use in pre-college mathematics. The study supported the claim that calculator use for instruction and testing enhances learning and performance of arithmetical concepts and skills, problem solving, and attitudes of students. The study also concluded that students were in favor of the presence of calculators during most mathematical activities. Students in these studies seemed to display more reserve regarding calculators on tests and on basic computational skills (Hembree, 1992).

Thomasson (1992) investigated the effects of various treatments using a calculator on achievement and attitude of college students enrolled in elementary algebra. The study was experimental pretest-posttest control design. Two instructors each taught three classes: class with total calculator use, including instructor as demonstration and students use at all times including tests, class with partial calculator use, including calculator use as demonstrations and student use in class only but not on tests, and class with no calculator use by instructor or students. Students in the total calculator use group performed higher on posttests of achievement scores but it was not statistically significant.

Dunham (1993) also summarized the research findings on impact of graphing calculators/technology in teaching and learning of mathematics. Students who used graphing calculators/technology, along with others, had more flexible approaches to problem solving, and were more successful on problem solving tests. Later, he concluded that students who participate in technology-enhanced curricula had more positive attitudes toward mathematics and were more confident about their ability to do mathematics (1996).

Asp, Dowsey, and Stacey (1993) compared the use of graphing calculators and computer software package on linear and quadratic graphing with six year-10 level classes. Students in the treatment group had significant gains in interpreting graphs with maximum, minimum values, and intersections. The treatment group also had significant improvement on plotting and reading points and drawing graphs.

Norris (1994) investigated effects of graphing calculator on algebraic skills and functions concepts of students by using quantitative and qualitative instruments. Although there was no significant difference between treatment and control group on algebraic skills, the performance of the treatment group on a posttest of basic function concepts and graphing was significantly higher than that of the control group. Student and faculty questionnaires revealed that there was a strong support for the graphing calculator as a visual aid for teaching and learning of pre-calculus.

Rodgers (1995) compared the effectiveness of graphing calculator activities for traditional algebra II curriculum of quadratic equations and related problem-solving situations with control groups that were taught the same subject without graphing calculator activities. Overall achievement scores from pretest to posttest and from posttest to retention test were compared. Attitude scores were also compared. Treatment group had significant gain in overall achievement scores from pretest to post. The gain in achievement scores for problem-solving situations was significantly higher for the treatment group. The control group had significant gain in paper-and-pencil achievement scores.

Hollar (1996) examined the effects of a graphing approach with a TI-82 graphing calculator to college algebra curriculum on students' understanding of the function concept. Four intermediate college algebra classes totaling 90 students were chosen at a large state university. Two teachers each taught one experimental and one control class in parallel time periods. Same topics were discussed in both groups, but less emphasis was given to the traditional paper-and-pencil calculations and manipulations in the experimental group. Students in the experimental group had a better understanding of functions as a group than students in the control group. They had significantly higher scores on the function test and on all four sub-tests--modeling, interpreting, translating, and reifying. There were no significant differences between the two groups on the departmental final exam that measured traditional algebraic skills without graphing calculator and on mathematics attitude scores.

Smith (1996) synthesized the findings from existing research on the effects of calculators on students in mathematics education. Twenty-four reports were integrated by the process of meta-analysis to determine the effects of calculators on students' achievement and attitudes. Research findings in meta-analysis indicated significant differences in the achievement of students using calculators for problem solving, computation, and conceptual understanding compared to students who do not use calculators. There were significant differences on attitude scores between students using calculators and without using calculators. Smith concluded that the use of calculators in mathematics classes should improve students' attitude toward mathematics and improve a student's ability to learn mathematical concepts, solve mathematical world problems, and perform mathematical computations.

Mustafa (1997) investigated the extent to which college algebra students' understanding of the numerical experience associated with the global behavior of polynomial functions was or was not influenced by the availability of the graphing calculator. This understanding was determined by students' use of a table of values to find the x- and y-intercepts, the increasing and decreasing regions, and the end behavior of polynomial functions. The students were divided into two groups: (1) the graphing calculator group where teachers used the calculator in conjunction with teacher explanation and (2) the non-graphing calculator group where teachers used explanation with no graphing calculators. There were three sections for the graphing calculator group and three sections for the non-graphing calculator group. Students in both groups took pre- and posttests and sixteen students (eight from each) were interviewed. The pre- and posttest questions focused on students' use of tabular representation of polynomial functions along with algebraic and graphical representations. Interviewed students were asked to explain their reasoning aloud while solving three problems similar to problems in the posttest. The results indicated that the graphing calculator group achieved higher posttest scores than the non-graphing calculator group. In terms of students' understanding of the numerical representations associated with the global behavior of polynomial functions, the results of quantitative and qualitative data have shown that the graphing calculator group performed better than the non-graphing calculator group.

Drijvers and Doorman (1996) developed a project called The Graphics Calculator in Mathematics Education from August 1991 to September 1994 in the Netherlands. Their goal was to develop a learning environment in which the graphing calculators can stimulate the use of realistic contexts, the exploratory and dynamic approach to mathematics, a more integrated view of mathematics, and a more flexible behavior in problem solving. Their results consisted of experimental instructional material, observations of the lessons, and reflections based on these observations. Students were able to get a good picture of the problem using graphing calculators. Although realistic data led to complex formulas with unattractive coefficients, the graphing calculators overcame the difficulties. The graphing calculators engaged students in exploratory activities. Students connected links between algebraic and geometric aspects of the same problem. This developed positive attitudes. Graphing calculators removed students' inhibitions to making a quick sketch. Trace feature offered the opportunity to follow the graph point-by-point and to read the changing coordinates on the screen. This direct feedback encouraged the students to reflect on what they had done.

Most studies investigated indicated a positive effect of using calculators in mathematics classes and mathematics achievement, positive attitudes toward mathematics, and positive attitudes toward computers. Present study investigated the use of graphing calculators with symbolic calculations in a constructivist learning environment and achievement in a college algebra class.

Method

Sample

110 college students enrolled in an algebra course participated in this study. These students passed the Mathematics Placement Test prior to enrolling for this algebra class. The researcher taught the experimental group and the other instructor taught the control group. There were 47 students in the experimental group and 64 in the control section.

Material

TI-92 graphing calculators loaned from Texas Instruments were used in the experimental group. Students in the experimental group were provided with TI-92 graphing calculators during the class meetings. Calculators were made available to use after class in the computer lab for homework and lab assignments. TI -ViewScreen for TI-92 was used in the experimental group. TI-ViewScreens enlarge and project the image of a calculator display so that it can be viewed by an entire class. The TI-92 graphing calculator was chosen due to its graphing and symbolic calculation capabilities and being user-friendly. Students in both sections used the same textbook, *College Algebra: Graphs and Models* **(**Bittinger et al., 1997). Students in the control section replaced the calculator exercises with similar activities not using the calculator.

Procedures

This study was conducted in two sections of a college algebra class. The researcher taught the experimental group and another graduate teaching assistant taught the control group. The researcher and the other instructor taught the same sections from the same textbook. The method of instruction, in the control group, was a traditional lecture format with the instructor as information giver. For the experimental group, the method of instruction was an investigative approach with the researcher as a facilitator. Solution for a given equation can be found algebraically by using the TI-92 calculator. In the experimental group, students were required to show their work on how they got the solution. The graphing feature of the calculator was used to demonstrate mathematical concepts or show another way of solving a given problem. Because the graphing calculators were available, concepts, theorems, and relations were presented using a graphical approach in addition to an algebraic approach. The graphing calculators were used to guess, see patterns, and develop a model that best described the given information. The graphing calculators saved time in graphing and calculating expressions so that extra time was used to investigate patterns, find models that describe the data best, and interpret the solutions. The instructor always asked the students to investigate the posed problem using a graphical approach so that they might see another dimension of the problem. Since the graphing calculator was an integral part of the lecture, homework and lab assignments included the graphing calculator and non-graphing calculator exercises. In some cases, detailed instruction indicated the approach that the student was expected to use. In others, the students were left to choose the way that seemed best, thereby encouraging a critical reasoning approach.

The researcher and the other instructor met during the semester to make sure that the same topics in each section were covered with different emphasis. The researcher emphasized graphical, tabular, and algebraic methods, but the other instructor emphasized algebraic paper-and pencil manipulations and numerical approach. Topics in the textbook were presented in three ways: graphically, algebraically, and/or by a numerical approach, so the same textbook was used with different emphasis in presenting topics.

Analyses

Pre-and post-tests designed by the researcher were used to measure students' ability scores using the graphing calculator with a constructivist learning setting in the study. Both tests included twenty multiple choice and five free response questions. The objective of the tests was to measure the student's algebra knowledge in two levels: action and process levels. Action level questions were routine, basic, conceptual questions in pre- and post tests. Process level questions were high level, procedural questions in pre-and posttests. The students in both sections were allowed two hours and thirty minutes timeframe to complete these two tests.

Dependent variables were defined as the difference between pre-and post-treatment scores of the students for different modes of responses. Pre-test ability scores were used as a covariate for overall gain in achievement scores, a composite of action and process level questions; because it was suspected that the gain in dependent variable might depend on initial ability. The covariate adjustment strategy did not compare the unadjusted means, but instead compared mean scores $\left(\mu_{y.x}\right)$ within a subpopulation with fixed confounder.

Results

Students in the experimental group expected to have significantly greater gain on achievement scores than students in the control group, after controlling for initial ability differences. Table 4.1 reports pre-test overall achievement scores of both groups in percentage out of 100 points.

Table 4.1. Summary of Pre-test Achievement Scores of Two Groups (in Percentage out of 100 points)

Group	Number of students	Mean of pretest score (%)	Standard deviation
Control	63	27.56	9.09
Experimental	46	25.93	7.76

Table 4.2 reports the parameter estimates of $\beta_o, \beta_1,$ and β_2.

Table 4.2. Parameter Estimates

Variable	DF	Names	Parameter estimate	Standard error	p values
Intercept	1	$\hat{\beta}_o$	38.09	5.35	.0001
Pretest	1	$\hat{\beta}_1$	–0.21	0.18	.2405
Group	1	$\hat{\beta}_2$	12.66	3.10	.0001

Results showed that regardless of initial overall achievement score, the average pre- to post-treatment gain of students in experimental group is 12.66% higher) than in the control group (P<.0001). Since both the experimental and the control groups began at a level of about 26 %, this 12.66 percentage excess translates to roughly a 50 percentage greater relative improvement. Therefore, students in the experimental group had significantly greater gain on achievement scores than the students in the control group. Students in the experimental group also expected to have significantly greater gain on action scores than students in the control group, after controlling for initial action ability scores. Table 4.3 reports pre-test action scores of both group in percentage out of 28 points.

Table 4.3. Summary of Pre-test Action Scores of Two Groups (in Percentage out of 28 Points)

Group	Number of students	Mean of preact score (%)	Standard deviation
Control	63	41.04	18.21
Experimental	47	39.82	18.11

Table 4.4 reports the parameter estimates of $\beta_o, \beta_1,$ and β_2.

Table 4.4. Parameter Estimates

Variable	DF	Names	Parameter estimate	Standard error	p values
Intercept	1	$\hat{\beta}$	35.36	0.04	0.0001
Preact	1	$\hat{\beta}$	-0.89	0.08	0.0001
Group	1	$\hat{\beta}$	3.61	0.03	0.2032

Thus, regardless of initial action score in $[4,24]$, the average pre-to post-treatment gain of students in the experimental group is 3.61 % higher than students in the control. This relative improvement was not statistically significant (p>05).

In terms of ability, students in the experimental group were also expected to have significantly greater gain on process scores than students in the control group, after controlling for initial ability. Table 4.5 reports pre-test process scores of both groups in percentage out of 72 points.

Table 4.5. Summary of Pre-test Process Scores of Two Groups (in percentage out of 72 Points)

Group	Number of students	Mean of preproc score (%)	Standard deviation
Control	63	22.31	9.46
Experimental	47	20.60	7.75

Table 4.6. Parameter Estimates

Variable	DF	Names	Parameter estimate	Standard error	p values
Intercept	1	$\hat{\beta}$	53.06	0.06	0.0001
Preproc	1	$\hat{\beta}$	-0.35	0.22	0.1233
Group	1	$\hat{\beta}$	15.63	0.04	0.0001

Therefore, regardless of initial process level score in $[1,32]$, the average pre- to post-treatment gain of students in the experimental group was approximately 16 % higher ($p<.0001$) than the students in the control group. Since both the control and the experimental group began at a level of 21 percentage, this 16 percentage excess translates to approximately a 76 percentage greater relative improvement. Indeed, students in the experimental group had significantly greater gain on process scores than the students in the control group.

Discussions

The main purposes of this study were to implement constructivist learning theory at college algebra level and to investigate different aspects of symbolic graphing calculators that may facilitate a constructivist learning setting. The researcher presented research findings on effects of graphing calculators and some classroom examples that could lead to higher level constructions of concepts using the calculators. Pre- to post-treatment gains of students in experimental group was 12.66 % higher (statistically significant) than students in the control. This result is supported by findings of other studies (Coston, 1994; Flores and McLeod, 1990; Rich, 1990; Thomasson, 1992). The average pre- to post-treatment gains of the students in the experimental group was approximately 16% higher than the students in the control group ($p < 0.05$) on process level questions, which measure students' ability to go beyond what is presented/discussed in the classroom. It seems that graphing calculators combined with a constructivist learning environment have the potential to increase problem-solving ability of students. The findings of Tobias (1993), Hart (1992), and Mustafa (1997) also support this result.

One of the more important limitations of this study was that constructivist learning environment required detailed discussion of the subject. Students' active involvement in discussions and explorations required a lot of time to get to the point. This is the most important aspect of constructivism. Some topics were not discussed in detail in the classroom; therefore, students, in the experimental group, were given homework assignments to learn other aspects of the topic.

For the future research, first of all, this study should be repeated in other mathematics courses to determine how symbolic calculators with a constructivist learning setting will facilitate different subjects of mathematics. More research is needed to investigate different aspects of the symbolic calculators that may facilitate teaching and learning of mathematics.

Additionally, studies with larger samples are needed. Studies focusing on a limited number of topics are also needed as to have more time to discuss the subject in detail. Lastly, long-term effects of the graphing calculators with a constructivist setting need to be investigated to see how much of the information will be retained and how students will adapt the approach they learned to different subjects of mathematics.

As a conclusion, the positive results of the study encourage the use of the graphing calculator with a constructivist setting. Students using graphing calculators attained higher algebraic skills. This study indicates that the symbolic graphing calculators have the potential to facilitate constructivist learning. Therefore, the challenge is to find the best combination of different aspects of the calculators with constructivist tenets in order to prepare students for the next century.

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