The Components of Numeracy

Lynda Ginsburg, Myrna Manly, and Mary Jane Schmitt

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Harvard Graduate School of Education
101 Nichols House, Appian Way
Cambridge, MA 02138

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In 1994, we and close to 100 practitioners, researchers and policymakers gathered together to discuss issues of adult numeracy. From that meeting, the Adult Numeracy Network (ANN) was created. This organization has taken the leading role in promoting high-quality adult numeracy instruction and professional development, supporting research efforts, and informing policy in the United States. ANN has also provided collegial support to its members as they, individually and collaboratively, seek to improve adult numeracy provision in all educational settings. We gratefully acknowledge the ongoing support of the board and membership of ANN.

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INTRODUCTION

The term “numeracy” is used in the adult education community to include an array of mathematically related proficiencies that are evident in adults’ lives and worthy of attention in adult education settings. There are various definitions of the term “numeracy” (see, for example: Coben, 2000; Cockcroft, 1982; Crowther, 1959; Gal, van Groenestijn, Manly, Schmitt, & Tout, 2003; Johnston, 1994; Lindenskov & Wedege, 2001; and Steen, 2001). While differing in phrasing and emphasis, the definitions recognize that mathematics and numeracy are related but are not synonymous.

Pure mathematics is abstract and context-free, yet “unlike mathematics, numeracy does not so much lead upward in an ascending pursuit of abstraction as it moves outward toward an ever richer engagement with life’s diverse contexts and situations” (Orrill, 2001, p. xviii). Most definitions of numeracy refer to this richer engagement by including a connection to *context, purpose,* or *use*. In some cases, the emphasis is on critical numeracy needed for active participation in the democratic process (Johnston, 1994), and in others the emphasis is more utilitarian—the needs of the workplace or competition in the global economy (Wedege, 2001). Numeracy connotes mathematical topics woven into the context of work, community, and personal life. Moreover, numeracy requires the ability and inclination to explore this situational mathematical content, thus is owned differently by each person. Unlike pure mathematics, numeracy has a distinctive personal element.

In Volume 3 of the *Annual Review of Adult Learning and Literacy,* Tout and Schmitt (2002) reviewed the current practices and policies with regard to numeracy instruction in adult basic education and urged that the field pay more attention to the subject. Their recommendation continues to gain validity. Since its publication, the economic impact of having low numeracy skills has been documented by the Adult Literacy and Lifeskills Survey (ALL). U.S. adults performing at numeracy levels 1 and 2 (the lowest of five levels) are about three times more likely to receive social assistance payments from the state (after adjusting for gender, age, education, and income) than those who score in levels 3, 4, or 5 (Statistics Canada and OECD, 2005, p.171).

As quantitative and technical aspects of life become more important, adults need higher levels of numeracy to function effectively in their roles as workers, parents, and citizens. The increased need for numeracy skills is amplified by results from recent large-scale surveys of the adult population that indicate that a strikingly large proportion have inadequate skills for the numeracy demands of the twenty-first century. These studies found that the numeracy proficiency of 58.6% of U.S. adults was below level 3, the minimum level for coping with today’s skill demands (Statistics Canada and OECD, 2005), and that the quantitative literacy skills of 55% of U.S. adults are at a Basic or Below Basic level (NCES, 2006). Moreover, in both studies, the percentage of the population scoring at the lowest level, “below basic” or “level 1,” in quantitative literacy and numeracy was significantly higher than it was for prose or document literacy. (See Note 1 for a discussion of the terminology and definitions used by these assessments.)

To address the need to improve and expand numeracy instruction in adult basic education programs, this paper attempts to describe the complex nature of numeracy as it exists today. All stakeholders—including policymakers, program directors, educators, professional developers and curriculum designers—need a full understanding of numeracy to know how to provide adults with effective numeracy instruction.

While there are large-scale assessments, standards documents, and position papers, thus far, there has not been a field- and research-based synthesis of the components required for adults to *be* numerate, to *act* numerately, and to *acquire* numeracy skills. By components of adult numeracy, we mean those fundamental elements that are inherent in proficient numeracy practice. This paper will attempt to identify and clarify the nature of the components that are specific to adult numeracy with the hope that such identification and clarification will provide a vision that will guide instruction, contribute to the design of assessments, frame research, and inform policy.

METHODOLOGY

To inform our analysis of the components of adult numeracy, we looked at two sources of information. First, we gathered as many adult numeracy and mathematics frameworks as we could find from the United States, as well as international frameworks that were available in English. We included documents identified by the authors as curriculum frameworks, assessment frameworks, or standards documents, many of which were national in scope. In addition, some states have developed their own adult education standards and/or curriculum frameworks. We included all eight of the state standards documents that were available electronically (as of May 2006) on the Adult Education Content Standards Warehouse Web site (http://www.adultedcontentstandards.org/howto.asp).

Second, we examined K–12 and community college mathematics frameworks, which are well accepted, of high quality, and were based on extensive research. These documents were among the first national frameworks and were designed to guide the educational system that most adult learners have experienced or the educational system that many aspire to enter. In addition, we included frameworks from large-scale mathematics assessments that target teenagers.

Our goal in examining these two groups of frameworks was to seek the implicit or explicit theoretical bases underlying these documents and look for commonalities across them. A complete list of documents examined is provided in Appendix A. As we identified numeracy components that emerged from our examination of the documents, we also looked at the existing adult numeracy research base and the rich K–12 research base in mathematics education to augment our understanding and inform our descriptions of the components.

In total, we found 29 appropriate or informative frameworks applicable to adult numeracy. From these documents and from our understanding of the existing body of related research, we propose three major components that form and construct adult numeracy:

1. **Context —** the use and purpose for which an adult takes on a task with mathematical demands
2. **Content —** the mathematical knowledge that is necessary for the tasks confronted
3. **Cognitive and Affective —** the processes that enable an individual to solve problems, and thereby, link the content and context

While each component can be described separately and is different in nature, in actuality they interact, are intertwined, and have little meaning in isolation. Furthermore, each of the components has subcomponents as described below.

In order to illustrate the components and to establish a shared frame of reference for the readers of this paper, we have inserted situations or tasks to demonstrate the breadth and variety encompassed by real-life numeracy. The tasks range from the relatively simple (multiplying two numbers) to the more complex (comparing telephone plans).

THE CONTEXT COMPONENT

Context is *the use or purpose for which an adult takes on a task with mathematical demands*. In most definitions of numeracy, the notion of a decontextualized, entirely abstract mathematics is laid to rest by such phrases as “real-world” and “real contexts.” Attention to context is evident in many of the adult numeracy frameworks we examined,

However, there are noticeable differences in the frameworks’ treatment of use or purpose. The adult-focused frameworks use three different approaches as to how they position context: (1) context as the primary organizing principle; (2) math skills as the organizing principle, while paying attention to context throughout; and (3) math skills as the organizing principle, yet paying little explicit attention to context.

An example of the first approach—context as the primary organizing principle—is found in the Australian *Certificates in General Education for Adults.* The authors state that the framework is based on the idea that “skills development occurs best when it is within social contexts and for social purposes” (http://www.aris.com.au/cgea/). Learning outcomes are organized into four different “numeracies” depending on their purpose:

- **Numeracy for Practical Purposes** … addresses aspects of the physical world to do with designing, making, and measuring.
- **Numeracy for Interpreting Society** … relates to interpreting and reflecting on numerical and graphical information of relevance to self, work or community.
- **Numeracy for Personal Organization** …focus is on the numeracy requirements for the personal organizational matters involving money, time and travel.
- **Numeracy for Knowledge** …deals with mathematical skills needed for further study in mathematics, or other subjects with mathematical underpinnings and/or assumptions (Butcher et. al., 2002, p. 215).

Also leading with context, the United States’ *Equipped for the Future* (EFF) content standards identify three roles within which adults use mathematics: as worker, family member, and citizen. In the EFF framework, instruction and assessment are embedded in meaningful contexts that support learners in enacting their adult roles. The mathematics standard (1 of their 16 standards) states that the purpose of adults acquiring mathematics proficiency is to “use math to solve problems and communicate” (National Institute for Literacy, 1996, p. 35). The assessment framework of the Comprehensive Adult Student Assessment System (CASAS) is organized around nine competencies, six of which are contextual (health, government and law, community resources, employment, independent living skills, and consumer economics). Two competencies are skill based (basic communication and computation) and one is cognitive (learning to learn). While numeracy-related tasks occur within the contextual competencies, the CASAS framework treats computation as a separate competency.

An example of the second approach—math content as the organizing principle, while paying attention to context throughout—is the Adult Numeracy Network's (ANN) framework, which categorizes numeracy by mathematical content and processes consistent with the National Council of Teachers of Mathematics approach. However, the ANN framework adds a category: relevance. The inclusion of this extra category was motivated by an analysis of stakeholder focus group discussions examining the important mathematics adults do in their lives.

The curriculum framework used in Massachusetts and the numeracy core curriculum required in the United Kingdom offer contextual examples for each mathematical benchmark or outcome. For example, in the Massachusetts framework, a mathematical benchmark such as "Read and understand positive and negative numbers as showing direction and change" has corresponding "examples of where adults use it"; in this case, "Reading thermometers, riding an elevator below ground level, staying 'in the black' or going 'into the red' on bill paying" (Massachusetts Department of Education, Adult and Community Learning Services, October, 2005, p. 8). Similarly, the United Kingdom's core curriculum is organized by skills (e.g., "count reliably up to 20 items") and a contextual example is offered ("count the items in a delivery"). Other state frameworks (Arizona and Nevada, and other countries such as Scotland and Ireland) also lead with math content, but are similar to the ANN and Massachusetts frameworks in that context or use is ever-present, even while the primary organizer is mathematical content.

An example of the third category—math skills as the organizing principle, while paying little attention to context—is the United States' National Reporting System (NRS), in which the description of outcome measures focuses only on mathematics computational skills, even though the category is labeled "numeracy" rather than "mathematics." Some states that organize their frameworks based only on math skills are Florida, Washington, and West Virginia.

One adult-focused document did not fit into these three categories. New York's math standards are organized into four areas: Analysis, Inquiry, and Design; Information Systems; Mathematics; and Interconnectedness: Common Themes. This categorization combines context, content, and cognition as the organizers.

Context Categories

While most adult numeracy frameworks include use and purpose, there is some variation in how they identify categories of societal contexts. Categories more or less correspond to one or more of four adult roles and responsibilities:

- **Family or Personal** is related to an adult's role as a parent, head-of-household, or family member. The demands include consumer and personal finance, household management, family and personal health care, and personal interests and hobbies.
- **Workplace** deals with the ability to perform tasks on the job and to adapt to new employment demands.
- **Community** includes issues around citizenship, and other issues concerning the society as a whole, such as the environment, crime, or politics.
- **Further Learning** is connected to the knowledge needed to pursue further education and training, or to understand other academic subjects.

A Swedish position paper (Gustafsson & Mouwitz, 2004) puts forth a more general humanistic view than any others we reviewed, emphasizing the democratic aspect, and the concept of "Bildung"—the shaping of a person to be prepared to handle life. The ways that context is included in each adult-focused document are found in the table in Appendix B.

The inclusion of societal contexts in adult-focused frameworks stands in marked contrast to the exclusion of such contexts in school-based frameworks. While most of the school-based documents include in their introductions a reference to the importance of mathematical literacy to the individual's and society's future, usage or context is not included other than as realistic applications that appeal to the particular age group. In practice, the addition of context is almost always in service to a mathematical content knowledge goal (e.g., sharing a pile of cookies is a way to understand better the partitive model of division). For adults, the context may well be the impetus for learning the mathematical content and will frame the application of that learning (e.g., "How many packages of cookies will I need to purchase so that each child at the party gets at least two?"). The focus on applying mathematics in a context or having a social purpose to the use and application of the mathematics provides motivation for learners to engage with and learn about mathematics. This leads us to conclude that it is the focus on, and prioritization of, context that differentiates an adult numeracy framework from a formal school mathematics framework.

One Dilemma Within the Context Component: "Realistic" Is Not "Real"

The contrast between decontextualized, abstracted mathematics (e.g., "What is 23 x 13") and highly contextualized mathematics (e.g., "When can you retire and how do you know?") might be best described as a continuum from abstract to real, with "realistic" somewhere in the middle. Word problems and standardized test items are designed to approximate real situations, but when they are used in educational settings, they generally are structured so that they have only one correct answer. This is especially evident when items are presented in a multiple-choice format (see Figure 1). These tasks might be considered “realistic” but are hardly “real.”

Figure 1: Calculating Batting Averages

A baseball player’s batting average is the number of hits he gets divided by the number of “at bats.” Joe had 75 “at bats” and made 22 hits. Find his batting average to the nearest thousandth.

1. 0.356
2. 0.333
3. 0.320
4. 0.299
5. 0.293

When considering a real-life problem such as finding the best telephone plan (see Figure 2), people generally have a number of variables that must be taken into account before the problem can be solved: How much money is available in the family budget to pay for telephone service? What telephone calling patterns do family members use? Which “free,” portable telephone will be included?

Figure 2: Which Telephone Plan Is Best?

MONTHLY CHARGE	PLAN FEATURES	
$60	900 Anytime minutes	$.40 per additional minute
		Unlimited night and weekend minutes
$80	1400 Anytime minutes	$.35 per additional minute
		Unlimited night and weekend minutes
$200	Unlimited Anytime minutes	

The appreciation of the differences along the continuum from decontextualized math, to realistic math, to the math embedded in real life has implications for instruction and assessment. Contrast the following two instructional strategies, and how they differ in the ways the lesson is “contextualized.”

Strategy 1. A teacher launches a lesson by teaching the mechanics of multiplying decimal numbers (e.g., 20.5 x 15.75). After students have practiced the method, they attempt some word problems in which they are asked to apply the new skill by, for example, determining the area of a 12.5 foot by 18.25 foot room. The word problem serves as a “realistic” application and a reason for using and practicing the skill.

> However, many students know that since they had just practiced multiplication of decimals, there is a signal to do that operation with the numbers in the problem.
>
> Strategy 2. A teacher launches a lesson from a context by posing the challenge: "How much would it cost to carpet your bedroom?"

The first instructional strategy for contextualizing instruction is frequently found in traditional textbooks. However, while word problems are an attempt at realism, they do not provide the same experience as engaging students in tasks that have the features of real problems. In real life, the task is less well-defined and requires consideration of several aspects, in this case, the range of carpet prices, the need to measure, estimate and compute with rather messy numbers (a decimal or fraction will likely have to be dealt with, motivating the need to learn to multiply decimals), to take seams into consideration, and the labor to remove the old carpet or prepare an underlay. The "non-mathematical" aspects of the problem brought in by students will depend on their familiarity with the context. An adult in the class who has worked as a carpet layer will bring much to the discussion, sharing new knowledge with those who have little experience. A study comparing the performance of adolescents who experienced learning in each of these ways, concluded that students who learned with the problem-solving approach (such as Strategy 2) were better able to remember and apply knowledge to new situations (Boaler, 1998).

Assessing knowledge through performance-based tasks similar to Strategy 2 yields different information about what a student has learned and is able to do than a test on computational skills. The dilemma faced by educators are the trade-offs such as cost, time, generalizability, and ease of scoring. If one accepts the premise that "realistic" is not the same as "real," a serious question is raised about the extent to which "efficient," short-response standardized test items are valid measures of a person's numeracy when the items are not structured to elicit the practices an adult actually employs in a real situation.

There is yet another distinction between "real" and "realistic" when contrasting any school experience with adults' actual mathematical practices. Researchers who conduct ethnographic studies of adults managing real mathematical demands in the workplace or marketplace point to how little out-of-school math resembles school math. Calculations tend to be less error-prone, people focus on the meaning more often, and the resources that people turn to are more varied. For example, in a study of nurses' thinking, Noss, Hoyles, and Pozzi (2000) found that when the nurses considered adjustments in medication, they used not only mathematical procedures they learned in nursing courses, but also used self-invented procedures, and took into account time management, the specific characteristics of the particular drug, the authority of doctors, and past experience.

There are different judgments as to which contexts are important, the extent to which context is incorporated, and the pedagogical approaches for teaching in or with context. Nonetheless, the overwhelming consensus across the documents we reviewed is this: c*ontext matters.*

THE CONTENT COMPONENT

The content component of numeracy consists of *the mathematical knowledge that is necessary for the tasks confronted*. There are two elements in that description:

1. The depth of mathematical knowledge that is necessary
2. The kinds of tasks that one faces

The first element is a deep and coherent understanding of the mathematics that is being used. The essential concepts are those that provide critical structural elements for a flexible form of knowledge that can be used in context (Boaler, 1998).

The second is the nature of the tasks that presently face adults. Recognizing that these tasks change as technological advances are made and the goals of society are adjusted to them, the content component of numeracy will shift over time to meet the demands. For example, while accuracy with arithmetic operations involving large numbers was demanded of bookkeepers in the mid-twentieth century, today's bookkeepers must be able to program their requirements into spreadsheet software and estimate as they check to see if the results calculated by that program are reasonable. Full participation in careers and citizenship in today's technological society requires a different set of skills than was required 60 years ago (Murnane and Levy, 1997).

Numeracy content will also vary from context to context within the same time period. A carpenter may need a high level of practical understanding of measurement and geometry to ensure accurate fits and structural integrity; an office worker may need an understanding of the algebraic concepts of variables and equations to use spreadsheets effectively; and a factory worker may use statistical process control measures that require an understanding of what constitutes abnormal deviation in the quality of the output of a certain machine.

Acknowledging that the content varies with time and context, we focus on the general numeracy content used by adults now, at the beginning of the twenty-first century. This paper organizes numeracy content around four mathematical strands:

1. Number and Operation Sense
2. Patterns, Functions, and Algebra
3. Measurement and Shape
4. Data, Statistics, and Probability

The word *strand* is significant because it carries the idea of concepts from all areas being interwoven into a cohesive instructional path, distinguishing it from content that exists in *layers*, where, for example, algebra content is not considered until after the number content is mastered. Thus, the organizational scheme is not intended to divide the content neatly into self-contained, strictly sequential packages nor to set limits on the content to be included.

In our literature review, we found examples of many curriculum frameworks in which content is organized into similar strands of mathematics. The first entry in the table in Appendix C is the standards document from the National Council of Teachers of Mathematics (NCTM) that represents school mathematics as it is widely accepted in the United States today. It is notable that in this seminal framework all the strands are intended to span the grades, so are included at all levels of instruction in varying proportions. For example, early elementary school mathematics instruction is not devoted solely to numbers and operations. Measurement, algebraic reasoning, and data are introduced in grades K–2 (NCTM, 2000, p. 30). Similarly, the National Assessment of Educational Progress (NAEP) includes items from each of the strands at all the different levels that it assesses (Grades 4, 8, and 12). Contrary to this trend, some adult curriculum frameworks omit the concepts of algebra completely and others do not address them until the higher levels.

It is clear from Table 2 that the prevailing conception of mathematical content is one that agrees in principle with NCTM's framework, but there are subtle differences that hint at different priorities. For example, The Organization for Economic and Community Development (OECD) Programme for International School Assessment (PISA) that evaluates how well 15-year-old students can apply what they have learned to real-world contexts (OECD, 2003) uses the overarching ideas of Quantity, Space and Shape, Change and Relationships, and Uncertainty as titles for its strands. The titles themselves reveal a shift of emphasis to the concepts and ideas that facilitate the problem-solving aspects of mathematics rather than a purely academic focus on skills and mental practices like inquiry and reflection that prepare students for further study in mathematics.

We note a similar shift in emphasis in the article by Forman and Steen (1999) that describes Functional Mathematics, a core curriculum for high schools that combines the historical focus on the abstract with the utilitarian focus of vocational studies. Their strands (Numbers and Data, Measurement and Space, Growth and Variation, Chance and Probability, Reasoning and Inference, Variables and Equations, and Modeling and Decisions) name mathematical topics that are most likely to be used in the contexts of real life. They describe a narrower curriculum than the broad NCTM strands (Numbers and Operations, Algebra, Measurement, Geometry, and Data Analysis and Probability) by focusing on utility yet containing the essential concepts that provide a foundation for subsequent courses in mathematics.

The content strands in adult numeracy documents also emphasize the utility or functionality of the concepts. The titles of the strands often imply competencies along with content, perhaps intending to show where emphasis should be placed within the areas of content. For example, many use the term "Number Sense" for the quantitative strand and include the concept of "patterns" for the algebraic strand. Both modify the NCTM titles so that they reflect the abilities that are important to adults functioning in today's world. While this does not set limits, it does point to a tacit recognition that a focus on the skills and understandings needed for proficient performance in real situations is more appropriate than an academic focus for preparing adults for a variety of roles.

Number and Operation Sense

This first content strand, considered by most to be *arithmetic*, has gradually moved from rote, skill-based study to one in which the understanding of concepts is the goal.

In its Foundation-level standards, which are meant for underprepared students, the American Mathematical Association of Two-Year Colleges (AMATYC, 1995) goes beyond the realm of academics and explicitly recognizes that students should develop a sense of how numbers work, for the purpose of being able to use mathematics effectively in their future roles in society, as well as for establishing a foundation for further study:

> Number sense involves the intuitive understanding of the properties of numbers and the ability to solve realistic arithmetic problems using appropriate mathematical tools. … Number sense is developed through concrete experiences. It includes knowledge of basic arithmetic facts and equivalent numerical representations and the ability to estimate answers. … Such intuition is based not on being able to perform an algorithm, but rather on meaningful experiences with numbers.
>
> Number sense includes a conceptual understanding of numerical relationships and operations. Students should be able to use numbers to express mathematical relationships that occur in everyday situations. In particular, they should know how to use percent and proportionality relationships. They should also understand concepts on which arithmetic algorithms are based and be comfortable devising their own methods for performing mental arithmetic. Students with a well-developed number sense will have a basis for building an understanding of algebra and the properties of real numbers. (p. 26)

Some of the documents reviewed emphasized "operation sense" as well. Operation sense has been described as understanding the meanings and models of operations, the real-world situations with which they connect, and the symbols that represent them.

Operation sense includes understanding the relationships among the operations, and the effect an operation will have on a pair of numbers (Huinker, 2002, p. 73).

In agreement with the goals expressed above, the Number and Operation Sense strand consists of those concepts that have applicability in solving everyday problems and those that are important in building the intuition and reasoning necessary for flexible thinking and for understanding concepts in other strands.

Examples of these concepts are:

1. **Relative size and multiple representations of numbers**; e.g., 1/4, 4/16, 0.25, and 25% are equivalent and they are less than 1/3
2. **Place value, computation, estimation**; e.g., the base-ten structure of our number system explains many steps in computational procedures and estimation strategies
3. **Meanings of operations**; e.g., real-world actions such as joining, separating, comparing, and growing underlie the mathematical operations
4. **Relationships between numbers**; e.g., numbers can be compared using addition (8 is 5 more than 3) or using multiplication (12 is 4 times as large as 3); rates, ratios, proportions and percents represent a multiplicative relationship

Patterns, Functions, and Algebra

In academic settings, the study of algebra has traditionally concentrated on the rules of symbol manipulation that govern tasks such as simplifying expressions, solving equations, and applying them to a few types of word problems. These skills are seen as the entry-level competencies for the next course in mathematics. With this focus on the abstract, successful students generalize many arithmetic principles and acquire disciplined thinking skills, but they often ask, “When are we going to use this?”

The reform mathematics algebra curriculum recommended for all grade bands (pre-K–2, 3–5, 6–8, and 9–12) by the NCTM (2000, p. 90) includes the following four student outcomes:

1. Understand patterns, relations, and functions
2. Represent and analyze mathematical situations and structures using algebraic symbols
3. Use mathematical models to represent and understand quantitative relationships
4. Analyze change in various contexts

These outcomes suggest an instructional strategy that begins with the real world (patterns, relationships, mathematical situations) and moves toward the abstract by modeling the situations using algebraic symbols. For example, a comparison of a constant or linear rate of change (observed when a dollar is added to a piggy bank each day) to an exponential rate of change (observed when a population of cells increases by doubling each day) deepens the students' understanding of real-world phenomena because they analyze the mathematical structures of each. In turn, the students' own experience with change lends meaning to the abstract mathematical equations. The utility of mathematics works hand in hand with the abstractions of mathematics.

In numeracy, context is the critical factor, so the focus of algebraic content for numeracy are those concepts that will help to build the reasoning, skills, and strategies that enable a student to interpret the mathematical demands of situations of the real world. Concepts clustered under the following topics (which are consistent with the organizing themes offered by the National Research Council after a symposium on the "Nature and Role of Algebra in the K-14 Curriculum" [NRC, 1998]) interact with each other to form the basis for using algebra:

1. **Language and representation**; e.g., tables, diagrams, and algebraic expressions with variables and constants can be used to capture the mathematical aspects of a real situation
2. **Structures and properties**; e.g., properties of numbers and equations govern the systematic manipulation of algebraic symbols to simplify expressions and solve equations
3. **Mathematical modeling**; e.g., a real situation is analyzed for its mathematical relationships and represented using algebraic terms
4. **Functions**; e.g., a specific kind of relation between quantities can be represented with words, tables, graphs, and equations, each of which can be used to analyze change

The concepts from the algebra strand are illustrated in the sample numeracy task, "Which telephone plan is best?" (Figure 2), in which the total cost of a plan is a function of how many minutes are used per month. An equation for each plan can be found by analyzing the information, but a clearer picture of the situation might come from graphing the three options on the same axis. It is clear that there is not one "best" plan for everyone; the choice will be determined by the calling habits of each individual.

Measurement and Shape

Geometric ideas are pervasive in everyday life. A statement from the Massachusetts Adult Basic Education Curriculum for Mathematics and Numeracy says "Adult learners who attend basic mathematics classes at any level share a wealth of pragmatic experience surrounding geometric and spatial concepts. They have probably built a bookcase, laid out a garden, applied wallpaper or tiled a floor, all the while discovering informally the rules that formally govern the study of geometry itself" (Massachusetts Department of Education, Adult and Community Learning Services, 2005, p. 19).

The practical value of studying the elements of measurement and shape is also recognized in the numeracy module of the curriculum of the Australian Certificates in General Education for Adults (CGEA) in which one category of numeracy, Numeracy for Practical Purposes, is wholly devoted to two learning outcomes related to aspects of geometry: Designing and Measuring (see http://www.aris.com.au/cgea).

In addition to its enormous everyday practical value, geometry provides opportunities for developing various types of reasoning skill. These opportunities can be as simple as devising an informal unit for measuring, as when a tile setter notices that the length of the sole of his work boot is about the same as the length of the 12-inch tiles that he is laying and decides to use his boot when approximating length in the future. More extensive reasoning is required in a classroom challenge to explain *why* carpenters test for equal diagonals in a rectangular floor bed when they want to ensure that the corners are right angles.

As with all the numeracy content strands, the concepts from geometry that an individual needs in order to function depend on lifestyle and career. The following concept clusters are examples of the knowledge that the general population needs to participate fully in today's society:

1. **Direct measurement**; e.g., using a ruler or tape measure with standard units, converting between common units, and estimating length by using some personal reference points
2. **Indirect measurement**; e.g., using the proportionality of similar figures, the Pythagorean theorem, or trigonometric ratios
3. **Angles and lines**; e.g., using the properties of parallel or perpendicular lines and the relationships between pairs of angles (vertical or complementary)
4. **Attributes of shapes**; e.g., categorizing shapes by the number of sides or angles
5. **Perimeter, area, and volume**; e.g., understanding the basis for the formulas that provide a method to determine these attributes from simpler measures such as length, width, height, and radius
6. **Coordinate plane**; e.g., using ordered pairs (x,y) to define the location of points in the plane

The Measurement and Shape strand is interwoven with the other strands of numeracy content. The Cartesian coordinate system, with its x and y axes, serves as the basis for graphing the functions of algebra. Mathematical modeling from algebra, which captures the essence of a situation in a mathematical expression, is used to analyze the patterns that occur with shapes and their angles. Measurement concepts provide a visual representation of the basic ideas of number, as when a ruler depicts fractions on a number line and the number of square units in the area of a rectangle illustrates the answer to a multiplication fact.

The sample numeracy task in Figure 3 below is an example of problem solving using the interaction between number sense and measurement. "Is it a cup?" requires some familiarity with the measures of capacity, as well as number sense about fractions.

Figure 3: Is It a Cup?

To make some pancake batter while on a camping trip, Julia needs to add 1 cup of water to the prepacked amount of dry pancake mix. She poured some water from a liter bottle that had been full.

How can she know that she has poured about the right amount?

Knowing that there are 4 cups in a quart and that a quart is nearly equal to a liter, it becomes evident that pouring a cup of water would use up about 1/4 of the bottle of water, and would leave the bottle 3/4 full.

Data, Statistics, and Probability

"The median home price in our city has increased by 40% in the past year."

"This anti-wrinkle cream reduces 68% of deep wrinkles in six months."

"20% of all U.S. women are iron-deficient."

"The average person consumes 2,500 calories and 86 grams of fat during Thanksgiving dinner."

The examples above show that sets of data and their interpretation confront us in all the aspects of our lives and may make this content strand the most commonly encountered and relevant of them all. "Some understanding of the meaning and sense of data, including their acquisition and manipulation, is required for intelligent participation in a society in which decisions increasingly rely on interpretations of data" (Manaster, 2001, p. 71). Data literacy can be defined as the ability of adults to describe populations, deal with uncertainty, assess claims, and make decisions thoughtfully.

Technology may well be the critical factor in this explosion of data in our lives. With the push of a button, using inexpensive computers and software, researchers can create and analyze data sets and publishers can design appealing graphs and diagrams that highlight the results. When exposure to data was less widespread, there was less need for the general population to be aware of how statistics can be manipulated to sway an audience. Today, people need a more critical stance, looking for manipulations in the data that could be a source of bias. They need to ask who sponsored the study, how representative was the sample, or if the results are displayed in a way that leads to improper interpretation.

The number of career choices in which statistics play an important role has also increased in recent years. Social sciences are dominated by research studies and their results; businesses operate under data-driven models; marketing is driven by public opinion survey data; political campaigns are dominated by the polls. Workplaces that once valued workers with sufficient brawn to handle manual labor are now looking for brains that can understand the basics of statistical process control like sampling and variation. In many cases, robots do the repetitive, non-thinking tasks.

School curricula have evolved so that students are exposed to elementary ideas of statistics in early grades. The NCTM standards list these four student outcomes for data analysis:

1. Formulate questions that can be addressed with data, and collect, organize, and display relevant data to answer them
2. Select and use appropriate statistical methods to analyze data
3. Develop and evaluate inferences and predictions that are based on data
4. Understand and apply basic concepts of probability (2000, p. 248)

In adult education in the United States, both the EFF Performance Continuum and the Massachusetts Adult Basic Education (ABE) Curriculum Framework address the need for adults to understand the "ways of data" so that they can intelligently respond to its growing presence. As would be expected, the lists of student objectives from EFF and Massachusetts include more references to context than the NCTM listing. For example, this statement, "Make and evaluate arguments or statements by applying knowledge of data analysis, bias factors, graph distortions, and context" (Massachusetts Department of Education, Adult and Community Learning Services, 2005, p.19) mentions many of the common techniques that are used by publishers of data to mislead the public.

The concepts that can contribute to a critical stance when interpreting the barrage of data that confronts adults today can be clustered under:

1. **Collection, organization, and display of data**; e.g., the type of data and the story that it is meant to tell determine the type of chart or graph that is most appropriate for its display
2. **Analysis and interpretation of data**; e.g., changes in the data set can affect the mean and median in different ways
3. **Chance**; e.g., in terms of probability, zero represents an impossible event and one represents an event that is certain to occur

Using statistics well is as much an art as it is a science. An open-ended question such as "What is a typical adult education student?" requires choosing, collecting, organizing, and possibly displaying data, as well as sampling. It also requires interpreting and decision making when determining which measure of central tendency is more informative (e.g., would we include the most frequent age, the middle point of the range of ages, or the arithmetic mean of all students' ages to best describe a "typical" adult education student?).

Numeracy skills do not stop at "being good with numbers." Numeracy for the twenty-first century is a much richer construct, grounded in the content of these four strands. The mathematical demands of today's technological society are different from those of earlier decades. Some concepts have become more important for coping with the demands while others are not as critical as they once were.

THE COGNITIVE AND AFFECTIVE COMPONENT

The cognitive and affective component of numeracy includes *the processes that enable an individual to solve problems, and thereby links the context and the content*. To solve a numeracy problem, the numerate adult must: (1) have available a rich understanding of the mathematical ideas or concepts involved so as to be able to make sense of the problem. To begin to solve the problem, the person needs to (2) reason or think logically about the relationships within the situation and the concepts that might be related to it. Then, the person needs to (3) formulate the mathematical problem and strategize ways to look at the information, represent it in meaningful ways, and decide, if necessary, how to manipulate numbers to come to a useful solution. Only then, does the person (4) perform any needed precise calculations or make estimates, using computational procedures that may require pencil and paper or that may be done mentally or with a calculator. This is an iterative process, by which each step must be monitored and reevaluated to see if the process is working as it should, if what is being done continues to seem reasonable, and if changes in direction should be made. Along the way, it is likely that the person will need or want to communicate with others regarding assumptions, strategies, or solutions. This entire process is only possible if the person (5) is emotionally able and willing to engage with the task, and persevere in the process, dealing with possible confusion, frustration or ambiguity as it arises.

These five processes have been formally identified as: **Conceptual Understanding, Adaptive Reasoning, Strategic Competence, Procedural Fluency, and Productive Disposition.** We have chosen to use the terminology of The National Research Council's volume, *Adding It Up: Helping Children Learn Mathematics* (Kilpatrick, Swafford, & Findell, 2001) as a way of relating the rich research base of K–12 education to the numeracy activities of adults. Each of these subcomponents of the cognitive and affective component is described more fully below, with a particular emphasis on how they relate to adult numeracy learning and behavior.

While these five subcomponents have been identified in the K-12 literature, we also examined the existing U.S. and international adult numeracy frameworks to see if and how these processes were addressed from an adult perspective. We found that many of the documents address both cognitive and affective aspects of numerate behavior, as well as the mathematical content and context. In Appendix D, we list the frameworks and examples of the language used within each framework to address each of the cognitive and dispositional areas or subcomponents of numeracy. While it is difficult to pigeonhole phrasing from the different framework documents, most of the frameworks recognize the centrality of the cognitive and affective processes as crucial to what it means to be numerate and to act in a numerate fashion. These processes are not subsumed under either the Content or the Context but rather *are the mechanisms that enable the linkage between them*. That said, different frameworks mention or focus on particular cognitive and affective subcomponents to different extents, sometimes reflecting the particular function of the framework. For example, many of the assessment frameworks give minimal space to descriptions of adaptive reasoning or conceptual understanding because their assessments do not attempt to evaluate these subcomponents. On the other hand, some frameworks make mention of the various cognitive and affective subcomponents but primarily focus on identifying particular arithmetic content areas throughout their documents. This is particularly true of the state standards documents (with the notable exception of the New York State Standards).

Conceptual Understanding

Conceptual understanding is defined as *an integrated and functional grasp of mathematical ideas* (Kilpatrick, Swafford & Findell, 2001, p. 118). These two aspects of conceptual understanding—integrated and functional—frame the ability to think and act numerately and effectively. Across the frameworks, this idea is referenced through words such as “meaning making,” “relationships,” “model,” and “understanding.” Some of the assessment frameworks mention and attempt to assess the development of conceptual understanding. (See, for example: GED, TIMMS, and NAEP.)

Frequently, as people learn math, both in K–12 and in adult education settings, they and their teachers focus on mathematical activity that can be seen and easily assessed; i.e., becoming facile with computational procedures and producing correct answers to specific computational tasks. But, to be able to use those well-learned procedures flexibly and appropriately when needed in the world outside the classroom, people need to understand the meanings behind the operations and procedures, behind other important mathematical concepts, and the interconnections among these various mathematical ideas.

For example, the question “What is 23 x 13?” can be answered using a traditional computational procedure that can be memorized and, with practice, can be employed efficiently. In this particular problem, the problem solver does not have to decide which operation to use; multiplication is specified. However, what happens when there is no paper handy, and the solver needs to find a solution mentally? With a conceptual understanding of how the multiplication algorithm (procedure) works, as well as an understanding of place value and the distributive property, a person might say, “This is the same as 23 times 10 (which is 230), added to 23 times 3 (which is 69). Then I need to add 230 and 69, or add 230 and 70 and then subtract 1.”

Conceptual understanding helps learners produce reasonable estimates that can help them catch computational errors. Alternatively, it may be that an exact product is not necessary, but an estimate is enough for the purpose. In this case, a person might suggest that an estimate of 15 times 20 or 10 times 25, both easily computed mentally, might be sufficient. In this case, the person recognizes that the numbers 20 and 15 or 10 and 25 are easy numbers to work with mentally because one number is 10 or a multiple of 10. The person also chooses the particular pair of numbers because one is slightly less than the original number and the other is slightly greater, leading to a better estimate than if both were less than or greater than the original numbers.

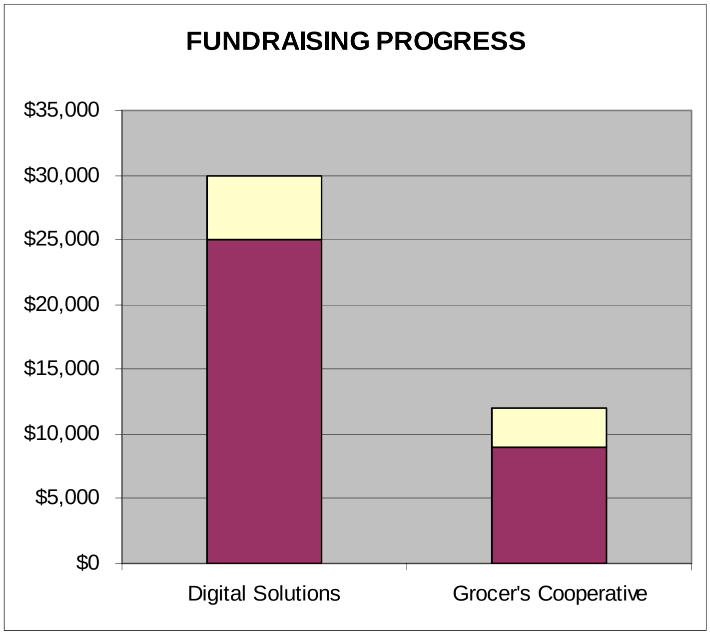
Both of these scenarios presuppose a level of conceptual understanding that undergirds a learned procedure and that serves to increase the usefulness and power of the procedure. Conceptual understanding also permits one to be free from relying on memory for all methods and procedures. One can think about the meaning of the task and "construct or reconstruct" a representation that both illustrates what it means and suggests a method for solution. For example, fundamental conceptual understandings include interpreting and visualizing 23 x 13 as the repeated addition of 13 objects, 23 times (one could arrive at an accurate answer by adding groups), or as a 23 by 13 rectangular array (one could count the elements in the array).

Most numeracy tasks are less clearly defined and directive than the example used above. They require one to understand a situation; define the problem; generate possible solution plans; and make decisions about appropriate strategies, suitable representations, and levels of accuracy. To make such plans and decisions, a solver needs to be able to see beyond surface characteristics of the situation to the underlying issues or patterns that structure the problem of the situation, and then to match those with appropriate strategies and procedures that will afford a meaningful solution. This ability to see beyond irrelevant surface characteristics and identify underlying structure requires rich conceptual understanding.

In the problem, "Who is ahead?" (in Figure 4 below), there are at least two ways of considering the information provided to determine which business is closer to meeting its goal. In order to make sense of the information and also of the solution generated, the solver needs to understand the benefits and limitations of alternative strategies as well as the meaning of the resulting solutions. One could use an additive strategy to compare the two, finding that Digital Solutions and Grocer's Cooperative need to raise an additional $5,000 and $3,000, respectively, to meet their goals, thus leading to the conclusion that Grocer's is closer to its goal. On the other hand, one could use proportional reasoning and compare how each is doing relative to its goal, leading to the conclusion that Digital Solutions has accomplished five-sixths of its goal while Grocer's has met only three-fourths of its goal. Either of these two solutions could be considered correct and reasonable. It is only with a conceptual understanding of both additive and proportional reasoning that the differences between the two solutions make sense, the implications of each solution can be considered, and a decision made about which solution best meets the situation within a particular context.

Figure 4: Who Is Ahead?

In a community, fundraising campaign, local businesses set goals for the amount of money they expected their employees to raise.



Which of these businesses is closer to its goal? How do you know?

It would also help the numerate solver to understand that fractions, decimals, or percents are meaningful alternate representations of such information, and while they might be equivalent to each other, each representation affords different benefits for communicating information. For our fundraising example, a solver might decide that using percentages (83.3% vs. 75%) to describe the situations of the two groups might more meaningfully communicate the comparison (than would fractions) because percents always use a common standard (100%) and are powerful ways of comparing portions of quantities of unequal size.

In this case, the integrated and functional understanding of fractions, decimals, and percents informs a meaningful solution and goes far beyond the ability to manipulate the numbers within each system or even to know the equivalents across the representations (such as that 1/4 is equivalent to .25 and to 25%). Similarly, the fundraising problem calls upon conceptual knowledge that is more purposeful than understanding why 3/4 of an inch is the same as 12/16 of an inch on a ruler in that it demanded an appropriate and practical response to the situation.

Knowledge that is learned with understanding is more likely to be remembered and available when needed. Yet so often, the rush to use a procedure—sometimes *any* procedure—is the mistaken goal of the mathematics classroom. Drill and practice on a procedure that makes no sense to the learner, whether it be using a formula or an arithmetic algorithm, will give rise to pleas such as, "I know how to do it; just tell me if I need to multiply or divide." This indicates that understanding is incomplete, and consequently there is little chance that a procedure can be used or applied appropriately. Indeed, the development of conceptual understanding can be hindered by an early focus on formalized procedures and formulas (Pesek & Kirshner, 2000).

Adaptive Reasoning

Adaptive reasoning is defined as the *capacity to think logically about the relationships among concepts and situations* (Kilpatrick, Swafford, & Findell, 2001, p. 129). When described as "ongoing sense making" (Donovan & Bransford, 2005), the metacognitive aspect of learning is emphasized. Many of the frameworks we examined mentioned "reasoning" in some way, often within the context of communicating and/or justifying results after problem solving, rather than as an ongoing mathematical process.

There are several aspects of adaptive reasoning to consider. In some instances, it involves an ability to recognize logical (mathematical) connections between individual elements in situations and to make generalizations about their relationship. It often leads to choosing a particular solution method and justifying that an answer is reasonable. This ability is often nurtured in the early stages of learning arithmetic by practice in recognizing recurring patterns and completing analogies.

Another aspect of adaptive reasoning is the ability to follow a logical path of reasoning that is based on basic ideas and principles that underpin that concept. The Massachusetts ABE Standards use "reasoning to support solutions and ideas" as an example of "respect for evidence," a habit of mind crucial to the development of numeracy (p. 15). Asking and answering the question, "Why does that work?" is a typical instance of this kind of reasoning. For example, a well-known mental shortcut when multiplying a number by 25 (as in 12 x 25 = 300) is to divide the multiplier (12) by 4 and then to multiply the result (3) by 100. Evidence that supports and legitimizes that particular solution method is provided by reasoning that 100/4 is equivalent to 25 and can be used in its place (12 x 100/4 = 300).

Elements of logic also play a role in the communication of mathematical ideas, an important activity when bringing the worlds of mathematical content and real-world context together. Forman and Steen (1999, p. 13) state that "being able to speak clearly about mathematical ideas" is an important aspect of presenting arguments, and van Groenestijn (2002, p. 41) stresses that communication skills are critical in acquiring and sharing numeracy-related knowledge. Using mathematical terms precisely and recognizing the significance of mathematical vocabulary when it is confronted are hallmarks of a numerate person. A mathematical term carries with it a precise description of its nature and properties. For example, a specification for a “circular” window suggests particular characteristics; that is, that all points on its circumference are equally distant from the center.

Ongoing sense-making, a metacognitive process of reflection, connects early learning with more sophisticated concepts and procedures that follow. Concepts learned in whole-number multiplication are critical to understanding later concepts of rates and functions. (Donovan & Bransford, 2005). Reflection and synthesis elevate learning beyond a study of isolated facts to that of a coherent whole. Making the connections between basic understanding of measures and fractions in a question such as, “Is it a cup?” (see Figure 3) is sufficient to conclude correctly that approximately a cup of water has been poured.

Kilpatrick, Swafford, and Findell describe adaptive reasoning as the “glue that holds everything together” (2001, p. 129). They detail instances in which reason is interwoven with the other strands of the proficiency rope in critical ways: a problem-solving strategy is legitimized by reason, a procedure is deemed to be appropriate for a situation by adaptive reasoning, a representation of a concept requires reason to recognize its limitations, and reason works hand in hand with a productive disposition to find an alternate solution plan for a complex problem when the first one runs into a dead end. Adaptive reasoning is, in effect, the management system that makes sense of the task and then monitors and evaluates progress.

Strategic Competence

Successful mathematical problem solving requires *the ability to formulate mathematical problems, represent them, and solve them* (Kilpatrick, Swafford & Findell, 2001, p. 124). Problem solving represents the heart of numeracy—using mathematical content appropriately in a real situation—and has also moved to a more prominent place in the study of mathematics itself in recent decades. Being able to use mathematical knowledge strategically serves both as an aid to understanding and as a tool for empowerment.

All of the framework documents promote the development of problem-solving skills, with descriptions that capture various aspects of problem solving such as: “skills in exercising judgment” (Sweden); “cognitive and metacognitive strategies” (Scotland); “systems thinking” (Functional Mathematics); “the ability to recognize inappropriate assumptions and intellectual risk-taking” (AMATYC); “apply knowledge of mathematical concepts and procedures to figure out how to answer a question, solve a problem, make predictions, or carry out a task” (EFF); “appropriate selection and use of strategies and tools and by distinguishing between relevant and irrelevant information" (Arizona AE Standards).

By "problem solving," we do not include completion of computation "exercises" (such as multiplying 23 times 13). The goal of exercises is to provide practice in computation skills or to assess the accuracy or speed of such skills. Problem solving, on the other hand, suggests dealing with a complex situation in which a solution path is not explicit but must be developed to meet the needs of the situation. The problem may be clearly defined such as in the calculating batting averages task (see Figure 1), commonly found in text- or workbooks, or it may be ill defined, such as "Which telephone plan is best?" (see Figure 2). In either case, the problem can only be solved once an appropriate "strategy" can be identified.

Many strategies help "mathematize" a situation or organize the information from either a routine or nonroutine problem into a mathematical form or model that enables the solver to "see" the underlying mathematical structure of the problem. Such problem-solving strategies include: drawing a diagram, making an organized list or table, looking for a pattern, and modeling the activity of the problem with objects. It is sometimes helpful to simplify the problem by substituting smaller numbers or rounded numbers so that the patterns can be seen more easily. Sometimes, working backwards is an effective strategy; other times it seems most efficient to guess and check, not generating guesses randomly but keeping track of guesses (and results) to hone in on an answer or pattern.

In school, students traditionally are encouraged to use various strategies to solve "word problems." Sometimes, they are advised to look for "key words" as clues to which operation applies to the problem situation. While this strategy is sometimes effective, it responds only to surface features of a word problem and does not reflect any understanding of the mathematical structure of the problem. Further, the strategy is of no use if a problem does not contain the proper key words or if the information is presented in any form other than a terse word problem. A more coherent approach is to look for the underlying relationships among the quantities in a problem.

Other problem-solving strategies are more domain-specific in that they are aligned with particular types of problems. For example, learners who are familiar with algebra would recognize that one productive strategy for deciding "Which telephone plan is best?" (Figure 2) might be to graph the cost of each plan for 0 through 2000 minutes, putting all three graphs on the same axis. This strategy would be unlikely to occur to someone who does not see the underlying algebraic structure of the problem.

Genuine problem solving is rarely a linear process—one generally does not proceed directly from the problem to the choice of strategy, the enacting of the strategy (with whatever calculations or procedures are needed), and on to the production of a solution. Instead, the process is dynamic and cyclic in that there are frequent moments of reflection and questioning, “Is this working?” “Is this the most reasonable approach?” “Why is this coming out differently from what I expected?” This constant monitoring of the problem-solving process often leads back to the original problem, altering or changing the strategy, and adjusting the implementation of the strategy (Wilson, Fernandez, & Hadaway, 1993).

Numeracy tasks, because they are embedded in real situations, are not as well defined as the situations used to promote learning in school or as word problems. They are often complicated by nonmathematical constraints, contextual issues that complicate the choice of methods, or the certainty of a solution. Forman and Steen (1999) say that *systems thinking* is required when adults are confronted with all the factors that need to be considered when making a decision in a real situation. One needs an insider’s contextual knowledge in addition to mathematical knowledge to make a wise decision. In a question such as, “When can you retire and how do you know?” the decision may be influenced by considerations other than the financial repercussions of remaining on the job.

Gal (2000) says that adults *manage* numeracy situations rather than merely solve problems that have right and wrong answers. They have to consider factors such as time and resources available, cost, and accuracy requirements before planning a course of action. He categorizes the kinds of numeracy situations that confront adults as being *generative* (such as computational tasks that have a correct answer), *interpretive* (where an opinion may be required), or *decision* (where a course of action is determined). Often in interpretive and decision situations, the underlying mathematics may be nearly invisible on the surface. For example, in “Is it a cup?” the situation must be mathematized first to produce mathematical information that can be interpreted, analyzed, and considered to make a decision.

Procedural Fluency

Procedural fluency is an essential component in completing many numeracy tasks that require efficient and accurate calculation. In addition to paper-and-pencil procedures, it includes *using mental mathematics to find certain answers, estimation techniques to find approximate answers, and methods that use technological aids like calculators and computers* (Kilpatrick, Swafford and Findell, 2001, p.121). Every framework we examined included this subcomponent; in some frameworks, it was the most salient one. However, beyond knowing (or being able to figure out) a procedure, it is also essential to understand what the operations do, how they are related to each other, and when a situation requires their use.

By definition, numerate behavior is triggered by a purpose. Thus, as a component of numeracy, computation techniques are not learned for their own sake, but are learned so that they can be used as tools in problem solving. A real-life situation in which the computation is embedded often suggests the most appropriate computational technique to find a good solution. An estimate may be good enough to answer the question or make the decision. The numbers involved may lend themselves to finding the answer mentally. Written procedures may be necessary for an accurate answer in a simple situation but using a calculator may be better for more complex problems. The goal is to be comfortable with many methods so that a person can choose an efficient method and (perhaps) use another one to check to see if the answer is reasonable.

With respect to written procedures, fluency does not demand that one knows *the* textbook algorithm for finding answers. Alternative strategies that are based on sound mathematics are often used flexibly and fluently to find answers in special circumstances or even for general use. For example, when adding 215 to 128, the traditional algorithm starts on the right, focuses on the digits themselves, and does not consider their value in the number as a whole. (5 + 8 = 13, write the 3 and carry the 1, 1 + 1 + 2 = 4, 2 + 1 = 3 and the answer is 343.) An alternate algorithm starts at the left side and considers the digits along with their place values. (200 + 100 = 300, 10 + 20 = 30, 5 + 8 = 13, 300 + 30 + 13 = 343.) Use of such alternative procedures is especially evident in adult education where students come to programs knowing computation methods that are easier for them to understand than the textbook algorithms. Some are unique procedures that they use at home or at work (Lave, 1988; Scribner & Stevens, 1989), while others are techniques that they learned in different countries (Schmitt, 2006; Zaslavsky, 1973). The goal for numerate behavior is to flexibly use procedures that lead to error-free results (are accurate) and take the least time and effort (are efficient).

A survey of how adults do mathematics in everyday life concluded that “it’s mostly estimation, mostly mental” (Northcote & McIntosh, 1999). Another study of how mathematicians compute found a similar result (Dowker, 1992). The flexible ability to find exact answers without using paper and pencil represents the epitome of adult numeracy. Often it is the features of the numbers and their relationship to one another that determine if mental techniques are more efficient. For example, if the situation requires finding 36% of 50, a person can mentally find half of 36 (50% of 36) to quickly arrive at the answer to the original question. The same mental strategy would be more cumbersome with other numbers, such as 36% of 60. In most cases, mental math procedures involve insight beyond that required by traditional paper-and-pencil procedures.

Estimation, a version of mental math that uses rounded numbers to construct simpler situations, is used to find approximate answers. The left-handed (or front-end) approach, shown in the addition procedure above, leads naturally to the estimation technique of rounding numbers before adding because it starts with the digits that have the most consequence to the size of the answer. Like mental math, the techniques involved with estimation require the integration of elements of the strands of conceptual understanding, adaptive reasoning, and problem solving.

Procedural fluency also involves being able to use technological aids to complete computations that are more complex. Technology surrounds us in modern society, and facility in its use is especially critical in the many workplace tasks. Calculators and computers eliminate the burden of tedious computations and ensure accuracy when the data and procedures are entered correctly, but do not remove the need for understanding what the operations do and whether they are appropriate for a particular situation.

Productive Disposition

The affective component of numeracy includes the beliefs, attitudes, and emotions that contribute to a person's ability and willingness to engage, use, and persevere in mathematical thinking and learning, or in activities with numeracy aspects. The literature on affective issues suggests that beliefs, attitudes, and emotions toward mathematics may differ from those directed toward other aspects of people's lives (Tobias, 1978), are frequently correlated with mathematics achievement (Dossey, Mullis, Lindquist, & Chambers, 1988; McLeod, 1992), and are likely to affect adults' motivation to engage in numeracy activity and persistence during that activity.

Kilpatrick, Swafford, and Findell (2001) identify a "productive disposition" as a necessary component of mathematical proficiency and argue that it should be developed during the course of K–12 mathematics education. The implication is that if a person leaves school without having developed such a disposition, one is unlikely to be able to be numerately effective or effectively numerate. Unlike children, adults cannot develop productive attitudes, beliefs, and emotions on a relatively clear slate and may have to counter existing, entrenched negative beliefs, attitudes, and emotions.

A number of the adult numeracy frameworks acknowledge the importance of the affective component of numeracy and recognize its impact on numeracy acquisition and use. Their terminologies include:

- "Enabling beliefs and attitudes" *(ALL)*
- "Confidence" (Australian Holistic Adult Numeracy Assessment, Marr, Helme & Tout, 2003)
- "Emotional dimension" (Scotland *Curriculum Framework*)
- "Habits of mind" (Massachusetts ABE Framework for Mathematics)

The Swedish report, *Adults and Mathematics—A Vital Subject* (2004), specifically identifies "affective factors and adults' mathematics learning" as a "critical area in which research and development work are a matter of particular urgency" (p. 13).

While the affective component of numeracy is recognized as significant, none of the frameworks position this component at a level worthy of extended description or of instructional or assessment imperative. Perhaps this is because research has only begun to address and seek to understand this component and/or because it is seen as a psychological construct rather than a cognitive one.

Affective responses emerge from an individual's life events, experiences, and perceptions; may be culturally grounded; and may build up over time to vary in their stability and power. Over the last 15 years, the affective domain has generally been divided into three subcomponents: beliefs, attitudes, and emotions (McLeod, 1992; see also DeBellis & Goldin, 1997; Evans, 2000, 2002).

Beliefs include *a set of ideas* that ground expectations and provide a lens through which new experiences are understood. People often have developed beliefs related to math use, including beliefs about:

- Their own ability or capacity to learn math (some seem to believe it's fixed at birth)
- The nature of math (set of procedural rules vs. a body of conceptual ideas and meaningful procedures)
- The usefulness of math in their lives (in fact, adults often underreport mathematical use [Coben, 2000])
- The mathematical problem-solving process itself (appropriate solution paths are clear and obvious to those who are able)
- How one goes about learning math (memorizing, teacher tells vs. constructed by student—passive/active).

These beliefs may develop in response to classroom experiences, a cultural environment, and/or messages from family members that contribute to feelings of confidence.

Attitudes include *feelings and preferences* about math, particular content, or instructional practices. Some common negative attitudes frequently expressed include a dislike of word problems, feelings of anxiety about algebra, and discomfort in asking questions of a teacher. Evans (2000) found that among his adult research subjects, "every single student expressed some emotion related to the doing of mathematics, or the use of numbers. Not only was anxiety expressed by many, as expected, but also confidence, pleasure, and sometimes dislike or anger" (p. 179). McLeod (1992) theorizes that such attitudes may develop as a result of repeated, intense emotional reactions.

Emotions are *intense, short-lived feelings*, such as panic, joy, or frustration. Emotions are embedded in the contexts in which they occur in that they are immediate and involuntary responses. Such emotions are to be expected as individuals engage in numeracy activity that is challenging to them. We have seen how panic over not knowing how to solve a word problem can cause an adult learner to simply grab whatever numbers are available (regardless of their meaning) and perform some calculation with them just to have something to put down on paper (Ginsburg, Gal, & Shuh, 1995). In our own teaching experience, we have also seen the unbridled joy expressed as a resounding "Yes!" when a learner solves a problem that had initially been perceived as difficult or complex.

How adults respond to inevitable feelings of frustration during mathematical activity may vary. For example, frustration may result in stopping work on a problem after the first unproductive attempt, or it can ignite a burst of resolve and energy to attack the problem until a solution can be found (see Goldin, 2006, for a rich discussion of the representational and communicative nature of frustration). Recognizing and acknowledging the emotion and examining the response it engenders can lead to an understanding that such emotions are common to all, but that productive responses *can* be developed.

Adult learners may have already established counterproductive attitudes, beliefs, and emotions as a byproduct of their earlier school and other experiences. Avoidance of mathematics courses has been linked to math anxiety, self-perceptions of incompetence, and feelings of lack of control (Tobias, 1978). Indeed, failure to successfully complete secondary education may be partially attributable to patterns of interfering and counterproductive beliefs, attitudes, and emotions. Further, these affective responses may continue to negatively influence cognition and performance, limit empowerment, and inhibit confidence-building unless they are addressed and dissipated.

The consequences of positive attitudes, beliefs, and emotions toward numeracy allow for a productive engagement, the expectation that mathematics should and will make sense, and a willingness to commit to a problem-solving stance and persistence when encountering false starts or other frustrations.

SUMMARY AND DISCUSSION

Together and individually, we (Ginsburg, Manly, and Schmitt) have enjoyed the privilege of spending many years as classroom teachers, researchers, teacher trainers, and curriculum and assessment developers in the midst of and around the edges of the emerging field of adult numeracy. We took on the task of writing this paper with impact in mind, always returning to the question: What might this paper contribute to the people in the field—our colleagues and our students?

We were commissioned to write a paper covering the components of numeracy and, while that work was intellectually stimulating in and of itself, for us writing this paper was more than an intellectual exercise. Always central to our discussions were the adults who return to study in adult learning programs, their teachers, and the organizations that set policy and guide practice. It is with this perspective that we now reflect on several issues that emerge directly from the document review and subsequent identification and explication of the components of numeracy.

All three essential components of adult numeracy—**Context, Content, Cognitive and Affective**—are necessary to *be* numerate, to *act* numerately, and to *acquire* numeracy skills, and any one without the others is insufficient. These components are not independent of one another, and should remain interwoven during instruction and assessment. Since the components are all always in play to some extent during any numeracy activity, they must each be part of meaningful adult numeracy learning and development. Therein lays the challenge.

The following table is a summary of the components of numeracy, along with their subcomponents, that we have discussed in this paper.

Components and Subcomponents of Numeracy

CONTEXT – the use and purpose for which an adult takes on a task with mathematical demands

Family or Personal—as a parent, household manager, consumer, financial and health-care decision maker, and hobbyist

Workplace—as a worker able to perform tasks on the job and to be prepared to adapt to new employment demands

Further Learning—as one interested in the more formal aspects of mathematics necessary for further education or training

Community—as a citizen making interpretations of social situations with mathematical aspects such as the environment, crime and politics

CONTENT – the mathematical knowledge that is necessary for the tasks confronted

Number and Operation Sense—a sense of how numbers and operations work and how they relate to the world situations that they represent

Patterns, Functions and Algebra—an ability to analyze relationships and change among quantities, generalize and represent them in different ways, and develop solution methods based on the properties of numbers, operations and equations

Measurement and Shape—knowledge of the attributes of shapes, how to estimate and/or determine the measure of these attributes directly or indirectly, and how to reason spatially

Data, Statistics and Probability—the ability to describe populations, deal with uncertainty, assess claims, and make decisions thoughtfully

COGNITIVE AND AFFECTIVE—the processes that enable an individual to solve problems and, thereby, link the content and the context

Conceptual Understanding—an integrated and functional grasp of mathematical ideas

Adaptive Reasoning—the capacity to think logically about the relationships among concepts and situations

Strategic Competence—the ability to formulate mathematical problems, represent them, and solve them

Procedural Fluency—the ability to perform calculations efficiently and accurately by using paper and pencil procedures, mental mathematics, estimation techniques, and technological aids

Productive Disposition—the beliefs, attitudes, and emotions that contribute to a person's ability and willingness to engage, use, and persevere in mathematical thinking and learning or in activities with numeracy aspects

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Implications for Practice

We turn now to consider how these three components of numeracy and their subcomponents might provide insight and support improvements to the field of adult numeracy education. In the following section, we begin to list how teaching and learning might be influenced when considering each aspect of practice—curriculum and instruction, assessment, and teacher professional development—through the prism of the three-component definition of numeracy. These lists are not complete, but we offer them to begin a dialogue in the field. It is up to teachers, professional developers, curriculum and assessment developers, and policymakers to work together to enact a numeracy educational program that serves well the adults who return to education.

Re-examining Curriculum and Instruction

Good teachers always attempt to tailor instruction to the particular needs of their students and strive to build on their learners' existing knowledge and experience. However, for diagnostic purposes, adult education teachers typically use standardized arithmetic tests that at the minimum provide a grade-level score and at most identify errors in computational procedures. On the basis of these, teachers often hand learners a workbook (generally one beginning with whole numbers or fractions) for further computational practice. The numeracy components and their subcomponents indicate that these common practices, while well meaning, are limiting and perhaps even counterproductive for empowering and enabling learners and as a model of numeracy instruction. Therefore, adult educators would benefit from a critical examination of their curriculum and instructional practices through the lens of each of the numeracy components.

1. Include context in curriculum and instruction.

Learning and teaching "math in context" has often been interpreted as first practicing computational rules, and then applying those skills to word problems that are instantiations of that particular computational process. As a result, the contexts are often stripped of the messiness of reality, include no opportunities to consider and decide on alternatives, and thus carry little instructional value. Referring back to the earlier discussion of "real" versus "realistic," teachers would do well to devise a curriculum that strives to (1) begin with context and teach problem solving and procedures in service of solving real or realistic problems; (2) draw upon contexts that are important to adults and that are part of their experience and, at the same time, provide a variety of numeracy tasks that emerge from contexts that are less familiar to the learners, but are worthwhile to know. Instructors must become familiar with the mathematics needed to manage the demands of family, workplace, community, and further education.

2. Restructure the scope and sequence of mathematical content.

The notion that mastery of whole numbers, fractions, decimals, and percents must precede algebra, geometry, and data analysis and statistics has been challenged by research and several of the frameworks. (See for example, the Equipped for the Future Performance Continuum, the Adult Literacy and Lifeskills Survey Framework, the Massachusetts ABE Math and Numeracy Curriculum Framework, and the NCTM Principles and Standards.) Therefore, teachers are challenged to devise a scope and sequence that integrates all four content strands at all levels, from Beginning ABE through transition to college, paying attention to how students' thinking develops within and across each content strand.

3. Address all aspects of cognition and affect.

Practicing arithmetic computational procedures is a part of the process of being mathematically literate, but only one part. Take, for example, the topic of addition of fractions. At present, much class time is allotted to the procedural (e.g., finding common denominators when adding fractions). However, time and attention must be also paid to developing learners' conceptual understanding of the meaning of rational numbers, what the operation of addition means, what is a sensible answer, and how the numbers "look" with manipulatives, number lines, or diagrams.

Other subcomponents of cognition require a pedagogical approach that helps adult students develop a collection of problem-solving strategies and the ability to apply them effectively, and mathematical reasoning so learners can make judgments about procedures and strategies based on understanding mathematical relationships.

With regards to affective factors, adult educators are challenged to support learners' existing and developing sense of themselves as numeracy learners and doers, helping them to manage frustration with false starts during problem solving as well as to develop the confidence and agency to better manage real-life numeracy situations.

4. Create learning environments focused on problem solving.

Restructuring the classroom environment from lecture or individualized learning modes to small, collaborative, problem-solving groups can support the development of mathematical problem solving and communication skills. Learning to be numerate and to function numerately outside the classroom is best done as a social activity, in which people brainstorm strategies, propose alternatives, articulate and defend their reasoning, and learn from one another.

Boaler (1998, 2000) showed that adolescents who studied mathematics in an environment that used only open-ended, project-based activities developed conceptual understanding, reasoning, and problem-solving skills. On computational assessments, these students performed as well as similar students whose learning was described as traditional, textbook-based. However, these student far outperformed the others on applied, realistic tasks that used the same mathematics content as was assessed on the computational assessments. This outcome was attributed in part to the students' attunement to the constraints and affordances represented in multiple situations, their engagement in practices that were similar to those of the real world, and their own experience of appropriate behaviors in math class. As one student described the difference between project-based math learning and practice with formal, procedural methods:

> Well, when we used to do projects, it was like that—looking at things and working them out, solving them—so it was similar to that, but it's not similar to this stuff now. It's, you don't know what this stuff is for really, except the exam (Boaler, 2000, p. 117).

Many adult education numeracy classes have more closely resembled the traditional textbook-oriented classrooms in which students become proficient at finding and interpreting cues that help them proceed through exercises (such as using all of the numbers given in a word problem, using the same procedure for all of the exercises on a page, focusing on "target words" that suggest specific operations, and not bringing in any information or ideas from the real world). Such cues are specific to a math classroom, but are of no use at all in solving problems outside the classroom.

Assessment

The field should undertake efforts to develop and adopt assessments that are aligned to standards that address the components of adult numeracy. Although we have seen "pockets of good practice" in numeracy instruction in the United States, in many jurisdictions there is a severe misalignment between the elements of the system—official policy, what is assessed, and teacher beliefs and practice. Classroom experiences should include requirements such as explaining *why* a procedure will or won't always work, describing a reasoning process that leads to an estimate of the answer, and making a sketch of the elements of a problem from the real world. Likewise, assessments should evaluate the full spectrum of cognitive abilities and reflect the emphasis on a broader curriculum at all levels.

Assessments such as the Tests of Adult Basic Education (TABE), CASAS, and the General Educational Development (GED) are influential for documenting learner progress and for accountability purposes. Teachers often find themselves in the position of "teaching to the test," allowing the assessments to dictate the content of instruction. However, none of these assessments fully address all three numeracy components. An ideal assessment should evaluate (and thereby promote) complete performance; that is, it would consider all three components in the structure of items. Complexity would vary in all the content strands (number and operation sense; patterns, functions, and algebra; measurement and shape; data, statistics, and geometry), cognitive strands (conceptual understanding, adaptive reasoning, strategic competence, procedural fluency), and contexts. Such assessment tools would reveal a more complete profile of learner proficiencies than the "skill gaps" that are exposed by current measures. Because we tend to value what is tested, it is imperative that programs be held accountable by showing learner progress toward goals that are meaningful. Practically speaking, no one test can "do it all" so a range of tools and procedures are needed to assess the various components of the construct of numeracy, including performance-based assessments, teacher observations, portfolios, self assessments, etc.

Professional Development

Support teachers who seek to address all the components of numeracy. Helping teachers rethink their instructional practices in response to a vision of numeracy that goes beyond shallow computational procedures is not an insignificant task. We expect that many existing professional developers will need to further their own learning significantly in order to provide effective professional development in numeracy to others.

With regard to context, teachers would benefit from the ability to enmesh instruction into contexts as well as to draw the math out from the contexts with which learners are familiar. Teachers should have mastery of all of the content strands, including but not limited to the computational aspects. In addition, they should have a vision of how understanding develops within each of the strands and across them. Finally, it is important that teachers have a deep understanding of the cognitive and affective processes involved in numeracy learning and performance.

To address these needs, professional developers will have to create workshops and experiences that enable teachers to enrich their own mathematical understanding, provide opportunities for teachers to identify or develop activities that are numeracy-rich, facilitate teachers' attempts to implement such activities, and then provide a structure in which teachers can reflect on the process and impact of changes in their practice.

Implications for Further Research

Because the field of adult numeracy is young, the research base is thin. The field now requires a strategic research program to learn how numeracy develops throughout adulthood and how to foster that development. The components and subcomponents identified in this paper together describe the terrain of numeracy proficiency and suggest broad-based research questions, such as:

- How do learners' prior encounters with particular mathematics ideas impact their new learning?
- In what ways is adult numeracy learning the same or different from children's learning?
- To what extent and how do current instructional practices aid in the development the full range of numeracy components and subcomponents?
- How does teacher knowledge of mathematics, numeracy, and mathematical pedagogy impact student learning?
- Does the nature of teachers' perceptions of themselves as numeracy learners and instructors impact their instructional practices and learners' progress?
- How is effective numeracy professional development similar to or different from effective literacy professional development?

Separately, each component presents a locus for further research and suggests different kinds of questions related to numeracy learning and teaching. For example, the **context component** gives meaning and purpose to numerate activity, and also provides entry into the everyday mathematical activity of adults. Starting from familiar contexts and the embedded mathematical practices, instruction should help adult learners expand their understanding of the mathematical aspects of these already familiar contexts. But,

- To what extent does adult "school math" align with adults' out-of-school numeracy practice?
- How do learners develop competence within new contexts?
- What mechanisms help adults learn to see the underlying similarities across contexts and then to transfer and apply their skills and knowledge appropriately?

From the perspective of the **content component**,

- What kinds of prior learning do adults bring within each content strand, and to what extent do earlier learnings serve as affordances or constraints to new learning?
- How do learning trajectories differ within and across different content strands?
- Which kinds of representations (e.g., graphs, tables, equations, or manipulatives) seem to be particularly salient for different topics? For different learners?
- Should teachers employ different teaching strategies for different content strands?
- What effect does including and integrating the content strands at all levels have on adult learning?

The **cognitive and affective component** focuses on the processes that must be in place for proficient numeracy performance:

- How does earlier math learning in school constrain or support subsequent development of conceptual understanding, adaptive reasoning, strategic competence, and computational fluency in adulthood?
- How do teachers negotiate a shift from focusing *only* on procedural fluency to beginning to address all the cognitive subcomponents? What instructional strategies do they incorporate? How do they feel about teaching differently?
- How do age, gender, culture, and prior experiences inform models of differentiated instruction, and how are these enacted in classrooms?
- How do teachers recognize, attend to, and engage with learners' positive and negative emotions (including fear, anger, frustration, elation, satisfaction, etc.) in the context of classroom numeracy activities? Under what circumstances do these emotions change?

There is also a need for research that **examines the instructional impact of the current guidelines and frameworks as well as other system elements for adult numeracy education with respect to the components:**

- To what extent do existing standards documents address all components and subcomponents of numeracy?
- How do teachers use and interpret the standards documents?
- Are there differences in classroom practice due to the adoption at the state or program level of the EFF, CASAS, or other frameworks?
- How do the various curriculum materials influence instructional practice?
- To what extent do current standardized assessments align with and measure the components of numeracy?

These are all examples of research questions that would further our understanding of how we can best help adults build functional numeracy skills. There are, of course, many additional questions and topics worthy of research. The methodologies appropriate to begin to address such questions are also varied and could include quantitative, qualitative, and mixed method studies. Exploratory as well as impact studies are needed on issues of student learning, instructional practice, and teacher preparation. Research programs that include collaborations among teachers and researchers have the added benefits of bringing together theoretical perspectives as well as the wisdom gained from practice.

Conclusion

When numeracy is considered as the interaction among the three components—context, content, and cognitive and affective—there can be no debate as to its value, both for an individual's full participation in today's society and for a nation's development of its democratic potential. Recognizing the critical value of numeracy carries a challenge for adult education practitioners and policymakers to take action at all levels—to expand the existing practices, frameworks, assessments, and research agenda to include the broader construct that is discussed in this paper. Incorporating context as a necessary component of numeracy challenges the field of adult education to understand where and when adults use mathematics. Acknowledging that numeracy content is more than arithmetic challenges the field to include elements from all content strands at all levels. Envisioning a confident, numerate adult challenges adult educators to develop learners' productive disposition, understanding of concepts, and ability to reason, solve problems, and carry out procedures. This vision of numeracy should revitalize instruction, making learning a more meaningful and lasting experience for adults.

Note 1: In the National Assessment of Adult Literacy (NAAL), Quantitative Literacy is defined as the knowledge and skills required to perform quantitative tasks (i.e., to identify and perform computations, either alone or sequentially, using numbers embedded in printed materials). Thus, all quantitative items involve using arithmetic operations. In the Adult Literacy and Lifeskills survey (ALL), numeracy is defined as the knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations. In addition to computing, the numeracy items from the ALL required identifying, interpreting, communicating about, and acting upon by ordering and sorting, counting, estimating, measuring, and modeling.

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APPENDIX A: REVIEWED FRAMEWORKS AND STANDARDS DOCUMENTS

TITLE	VERSION	ORGANIZATION/AUTHOR	COUNTRY OR STATE	TYPE	ABBREVIATED TITLE (USED IN TABLES)
Adult-focused Documents (United States or international)					
Adult Numeracy and Its Assessment in the ALL (Adult Literacy and Lifeskills) Survey: A Conceptual Framework and Pilot Results	2003	Statistics Canada	International	Assessment	ALL
Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus	1995	American Mathematical Association of Two-Year Colleges (AMATYC)	US	Grades 13 and 14 standards	AMATYC Crossroads
A Framework for Adult Numeracy Standards: The Mathematical Skills and Abilities Adults Need to Be Equipped for the Future	1996	Adult Numeracy Network (ANN)	US	Standards framework	ANN Framework
Comprehensive Adult Student Assessment System (CASAS) Competencies	19xx	CASAS	US	Assessment	CASAS
Equipped for the Future (EFF) Performance Continuum for Use Math to Solve Problems and Communicate Standard	2004 electronic	National Institute for Literacy	US	Content standards and assessment framework	EFF

TITLE	VERSION	ORGANIZATION/AUTHOR	COUNTRY OR STATE	TYPE	ABBREVIATED TITLE (USED IN TABLES)
Official GED Practice Tests Administrator's Manual	2002	GED Testing Service, American Council on Education	US and Canada	Assessment	GED
The National Reporting System for Adult Education: Implementation Guidelines	2001	Division of Adult Education and Literacy, Office of Vocational and Adult Education, U.S. Department of Education	US	Assessment guidelines	NRS (US)
Tests of Adult Basic Education (TABE) Teacher's Guide for Mathematics: Linking Assessment in to Learning.	2005	CTB/McGraw-Hill	US	Assessment	TABE
State Adult Basic Education Standards Documents					
Arizona Adult Education Standards	electronic	Arizona Department of Education	Arizona	Content Standards and Performance Standards	Arizona AE Standards
Florida Department of Education Curriculum Framework—Mathematics	2005	Florida Department of Education	Florida	Curriculum framework	Florida AE Framework
Massachusetts Adult Basic Education Curriculum Framework for Mathematics and Numeracy	2005	Massachusetts Department of Education	Massachusetts	Curriculum framework	Massachusetts ABE Framework

TITLE	VERSION	ORGANIZATION/AUTHOR	COUNTRY OR STATE	TYPE	ABBREVIATED TITLE (USED IN TABLES)
Nevada's Content Standards for Adult Basic Education: Mathematics	2006	Nevada Department of Education	Nevada	Content standards	Nevada ABE Standards
New York Adult Education Resource Guide and Learning Standards—Mathematics	1997		New York	Learning standards	New York AE Standards
Ohio Adult Basic and Literacy Education (ABLE) Ohio Mathematics Benchmarks, Revised	2003	Ohio Department of Education, Adult Basic and Literacy Education Unit	Ohio	Content benchmarks	Ohio ABLE Benchmarks
Washington Adult and Family Literacy Competencies–Adult Basic Education (ABE) Mathematics Competencies	2000	Washington State Bureau of Community and Technical Colleges, Adult Basic Education	Washington	Standards	Washington ABE Competencies
West Virginia Instructional Goals and Objectives (IGOs)—Mathematics	2004/2005	West Virginia Adult Education and Literacy Information Network	West Virginia	Standards	West Virginia IGOs
Adult-focused Documents (Non US)					
Certificates in General Education for Adults	2002 electronic	Community and Further Education Board: Melbourne, Victoria	Australia (Victoria)	Curriculum framework	Australia: CGEA
National Reporting System	1994/5	Commonwealth of Australia and the Australian National Training Authority.	Australia	Standards	Australia: NRS

TITLE	VERSION	ORGANIZATION/AUTHOR	COUNTRY OR STATE	TYPE	ABBREVIATED TITLE (USED IN TABLES)
The Level Descriptions Manual: A Learning Outcomes Approach to Describing Levels Of Skill in Communications & Numeracy	2000	Ontario Literacy Coalition	Canada (Ontario)	Learning outcomes	Ontario, Canada
Mapping the Learning Journey: NALA Assessment Framework for Literacy and Numeracy	2000	National Adult Literacy Agency (NALA), Ireland	Ireland	Assessment framework	Ireland: Assessment Framework
An Adult Literacy and Numeracy Curriculum Framework	2005	Communities Scotland	Scotland	Curriculum framework	Scotland: Curriculum Framework
Adults and Mathematics—A Vital Subject	2004	National Center for Mathematics Education (NCM), Goteborg University.	Sweden	Policy paper	Sweden: Adults and Mathematics
Adult Numeracy Core Curriculum	2001	Basic Skills Agency, London	United Kingdom	Curriculum	UK Curriculum
The National Standards for Adult Literacy, Numeracy and ICT	2000	Qualifications and Curriculum Authority, London	United Kingdom	Curriculum standards	UK Standards
Children- or Adolescent-focused Documents (United States or International)					
Beyond Eighth Grade: Functional Mathematics for Life and Work	Print 1999	National Center for Research in Vocational Education/ Steen. L. & Forman, S.	US	Secondary curriculum framework	Functional Mathematics

TITLE	VERSION	ORGANIZATION/AUTHOR	COUNTRY OR STATE	TYPE	ABBREVIATED TITLE (USED IN TABLES)
The Nation's Report Card: Mathematics, 2005. National Assessment of Education Progress (NAEP)	2005	National Center for Education Statistics, U.S. Department of Education.	US	Grades 4, 8, 12 Assessment framework	NAEP
Principles and Standards for School Mathematics (2000)	Print 2000	National Council of Teachers of Mathematics (NCTM)	US	K-12 curriculum standards	NCTM Standards
The OECD Programme for International School Assessment (PISA)	Electronic	Organization of Economic and Community Development (OECD)	International	Assessment framework	PISA
Third International Mathematics and Science Study (TIMSS) 2007 Assessment Frameworks	Electronic 2005		International	15 year-olds assessment	TIMSS

APPENDIX B: CONTEXT IN ADULT-FOCUSED DOCUMENTS

	FAMILY OR PERSONAL	**WORKPLACE**	**FURTHER LEARNING**	**COMMUNITY**
Adult-focused Documents (United States or International)				
ALL Survey	Everyday life	Work	Further learning	Societal
AMATYC Crossroads			Preparation for four types of programs: technical, mathematics-intensive, prospective teachers, and liberal arts	
ANN Framework**	Family member	Worker		Community member
CASAS	Health Consumer economics	Employment		Community resources Government and law
EFF	Family member	Worker		Community member
GED	While test problems are often contextualized, the assessment framework refers to mathematical content only.			
NRS (US)	The framework for numeracy is organized by math skills only.			
TABE	The mathematics assessment framework is organized by math skills, not by contexts.			
State Adult Basic Education Standards Documents				
Arizona AE Standards	Family	Workplace		Community
Florida AE Framework	The framework is organized by mathematical skills only.			

	FAMILY OR PERSONAL	WORKPLACE	FURTHER LEARNING	COMMUNITY
Massachusetts ABE Framework*	Everyday life	Work	Further learning	Societal
Nevada ABE Standards***	Health Consumer economics Independent living	Employment		Community resources Government and law
New York AE Standards	Organized by content and cognitive areas. No explicit mention of contexts.			
Ohio ABLE Benchmarks	Organized by content. No explicit mention of contexts, but connected to the EFF standards.			
Washington ABE Competencies	The framework is organized by mathematical skills only, even though they mention the EFF standards.			
West Virginia IGOs	The framework is organized by mathematical skills only.			
Adult-focused Documents (Non US)				
Australia: CGEA	Family and social life	Workplace and institutional settings	Education and training	Community and civic life
Australia: NRS	Personal settings	Workplace settings		Community settings
Ontario, Canada: Performance Indicators	Organized by math skills, but skills have examples of everyday use.			

	FAMILY OR PERSONAL	WORKPLACE	FURTHER LEARNING	COMMUNITY
Ireland: Assessment Framework	Organized by math skills, but every skill has an example of everyday use.			
Scotland: Curriculum Framework	Family life Private life	Working life		Community life
Sweden: Adults and Mathematics	"Bildung"—the shaping of a person to be prepared to handle life			
UK Curriculum Framework	Organized by math skills, knowledge, and understanding, but there is an example of how each skill can be used in an adult context.			
UK Standards	Domestic and everyday life Leisure	Economic activity	Education and training	Citizen and community Using ICT in social roles

* adopts the ALL contexts ** adopts the EFF roles *** correlated to the CASAS competencies

APPENDIX C: MATHEMATICAL CONTENT STRANDS IN SELECTED FRAMEWORKS

STRANDS / FRAMEWORK	NUMBER AND OPERATION SENSE	PATTERNS, FUNCTIONS AND ALGEBRA		MEASUREMENT AND SHAPE		DATA, STATISTICS AND PROBABILITY	OTHER
Children- or Adolescent-focused Documents (United States or International)							
NCTM	Numbers and operations	Algebra		Measurement	Geometry	Data analysis and probability	
Functional Mathematics	Numbers and data	Variables and equations	Growth and variation	Measurement and space		Chance and probability	Reasoning and inference, modeling and decisions
NAEP	Number sense, properties, and operations	Algebra and functions		Measurement	Geometry and spatial sense	Data analysis, statistics, and probability	
PISA	Quantity	Change and relationships		Space and shape		Uncertainty	
TIMSS	Number	Algebra		Geometry		Data and chance	
Adult-focused Documents (United States or International)							
AMATYC **Foundation level**	Number sense	Symbolism and algebra	Function	Deductive proof	Geometry	Probability and statistics	
ANN Framework	Number and number sense	Algebra		Geometry		Data	Relevance

<table>
<tr><th>Strands / Framework</th><th>Number and Operation Sense</th><th colspan="2">Patterns, Functions and Algebra</th><th colspan="2">Measurement and Shape</th><th>Data, Statistics and Probability</th><th>Other</th></tr>
<tr><td>EFF</td><td>Numbers and number sense</td><td colspan="2">Patterns, functions, and relationships</td><td colspan="2">Space, shape, measurement</td><td>Data, statistics</td><td></td></tr>
<tr><td>GED</td><td>Number operations and number sense</td><td colspan="2">Algebra, functions, and patterns</td><td colspan="2">Measurement and geometry</td><td>Data analysis, statistics, and probability</td><td></td></tr>
<tr><td>ALL</td><td>Quantity and number</td><td>Pattern and relationships</td><td>Change</td><td colspan="2">Dimension and shape</td><td>Data and chance</td><td></td></tr>
<tr><td>TABE (Applied Mathematics)</td><td>Number and number operations, computation in context</td><td colspan="2">Patterns, functions, and algebra</td><td>Measurement</td><td>Geometry and spatial sense</td><td>Data analysis, statistics, and probability</td><td>Problem solving and reasoning, estimation</td></tr>
<tr><td>NRS (US)</td><td>Number skills (Levels 1–5)</td><td colspan="2">Solve simple algebraic equations (Level 5)</td><td colspan="2">Measurement and geometry (Level 6)</td><td>Tables and graphs (Level 5)</td><td></td></tr>
<tr><td colspan="8">State Adult Basic Education Standards Documents</td></tr>
<tr><td>Arizona AE Standards</td><td>Number sense</td><td colspan="2">Algebra</td><td>Measurement</td><td>Geometry</td><td>Data analysis</td><td></td></tr>
<tr><td>Massachusetts ABE framework</td><td>Number sense</td><td colspan="2">Patterns, functions, and algebra</td><td colspan="2">Geometry and measurement</td><td>Statistics and probability</td><td></td></tr>
<tr><td>Ohio ABLE Benchmarks</td><td colspan="7">Same as EFF (above)</td></tr>
</table>

<table>
<tr><th>STRANDS / FRAMEWORK</th><th colspan="2">NUMBER AND OPERATION SENSE</th><th colspan="2">PATTERNS, FUNCTIONS AND ALGEBRA</th><th colspan="2">MEASUREMENT AND SHAPE</th><th>DATA, STATISTICS AND PROBABILITY</th><th>OTHER</th></tr>
<tr><td rowspan="2">New York AE Standards</td><td colspan="3">Number sense</td><td rowspan="2">Algebra</td><td colspan="2" rowspan="2">Spatial sense and measurement</td><td rowspan="2">Data analysis, probabilities, and statistics</td><td rowspan="2"></td></tr>
<tr><td>Whole numbers and integers</td><td colspan="2">Fractions (common/ decimal/percent) and ratio/proportions</td></tr>
<tr><td>*Florida AE Framework</td><td colspan="2">Number</td><td colspan="2">Patterns, functions, and algebra (from Level 2)</td><td colspan="2">Measurement and geometry</td><td>Data (Level 3 and GED)</td><td></td></tr>
<tr><td>*Nevada ABE</td><td colspan="2">Number</td><td colspan="2">Patterns, relations, functions, and algebra (from Level 2)</td><td colspan="2">Measurement and geometry</td><td>Diagrams, charts, and maps (from Level 3)</td><td>Money

Consumer skills</td></tr>
<tr><td>*Washington ABE Competencies</td><td colspan="2">Number (at all 4 levels)</td><td colspan="2">No algebra at any level</td><td colspan="2">Measurements and formulas (at Levels 3 and 4)</td><td>Tables, graphs and schedules (at Level 4)</td><td></td></tr>
<tr><td>*West Virginia IGO's</td><td colspan="2">Number</td><td colspan="2">Algebra (at Levels 4, 5, 6 and GED)</td><td colspan="2">Measurement and geometry</td><td>Tables, charts, graphs and maps (at Levels 2, 3, 4, 5, 6 and GED)</td><td>Money

Trigonometry</td></tr>
<tr><td colspan="9">Adult-focused Documents (Non US)</td></tr>
<tr><td>Australia: CGEA</td><td colspan="2">Numerical information</td><td colspan="2">Algebraic and graphical techniques (only at the higher levels)</td><td>Design and measuring</td><td>Location and direction</td><td>Data</td><td></td></tr>
<tr><td>Sweden: Adults and Mathematics</td><td colspan="2">Numbers and operations</td><td>Representations of relationships</td><td>Familiarity with symbols</td><td>Geometry and visualization</td><td>Measure -ment and units</td><td>Statistics and probability</td><td></td></tr>
</table>

STRANDS / FRAMEWORK	NUMBER AND OPERATION SENSE	PATTERNS, FUNCTIONS AND ALGEBRA	MEASUREMENT AND SHAPE	DATA, STATISTICS AND PROBABILITY	OTHER
Canada (Ontario)	Numbers	Patterning and algebra	Measurement and geometry	Data and probability	
***Ireland: Assessment Framework**	Number	No algebra	Measures	Graphs and charts	
Scotland: Curriculum Framework	Number	No algebra	No geometry	Graphical information	
UK Curriculum	Number	No algebra	Measures and shape and space	Data	

Note: The content strands are to be addressed at all levels unless specifically stated.

* Objectives were scanned to ascertain whether a content strand was represented.

APPENDIX D: COGNITIVE AND AFFECTIVE REFERENCES IN SELECTED FRAMEWORKS

FRAMEWORK	CONCEPTUAL UNDERSTANDING	ADAPTIVE REASONING	STRATEGIC COMPETENCE IN PROBLEM SOLVING	PROCEDURAL FLUENCY	PRODUCTIVE DISPOSITION
Children- or Adolescent-focused Documents (United States or International)					
NCTM	Conceptual understanding, conceptually grounded ideas, connect knowledge, understand how mathematical ideas interconnect and build on one another	Various types of mathematical reasoning, make and investigate mathematical conjectures, mathematical justification	Apply and adapt a variety of appropriate strategies to solve problems, monitor and reflect on the process of mathematical problem solving	Procedural facility and fluency, procedural proficiency	Autonomous learners, confident, eager, rewarding, feeling of accomplishment, willingness to continue
Functional Mathematics	Modeling	Speak clearly about mathematical ideas, and write summary reports Reasoning and inference	Systems thinking	Fluency in the language of mathematics	
NAEP	Conceptual understanding	Reasoning, connections, communication	Problem solving	Procedural knowledge	
PISA		Thinking and reasoning, argumentation, communication	Problem posing and solving	Using symbolic, formal, and technical language and operations, use of aids and tools	

Framework	Conceptual Understanding	Adaptive Reasoning	Strategic Competence in Problem Solving	Procedural Fluency	Productive Disposition
TIMSS	Knowing and understanding concepts	Reasoning, analyze, generalize, synthesize, justify	Solve routine and non-routine problems	Know facts and procedures, recall, recognize, compute	
Adult-focused Documents (United States or International)					
AMATYC Foundation Level	Modeling, meaning and use of mathematical ideas	Mathematical reasoning, test conjectures, judge validity of mathematical arguments	Problem solving	Number sense, mental arithmetic, estimation	Self-confidence, persistence, tenacity
ANN Framework	Conceptual understanding, develop and connect mathematical ideas	Ongoing sense-making	Problem solving	Procedural fluency	Positive attitude about learning mathematics
EFF	Knowledge of mathematical concepts, variety of mathematical representations, including graphs, charts, tables and algebraic models	Communication, verify results are reasonable	Use math to solve problems; select and apply the knowledge, skills, and strategies	Fluency; procedures, including estimating, tools, informal strategies	Independence
GED	Conceptual		Application, modeling, problem solving	Procedural	
ALL		Interpret, manage, and respond	Enabling problem solving skills, locating and modeling	Count, estimate, compute	Beliefs and attitudes as enabling factors

FRAMEWORK	CONCEPTUAL UNDERSTANDING	ADAPTIVE REASONING	STRATEGIC COMPETENCE IN PROBLEM SOLVING	PROCEDURAL FLUENCY	PRODUCTIVE DISPOSITION
TABE **(Applied Mathematics)**		Reasoning, analyze and synthesize information, evaluate outcomes	Problem solving	Operations	Reduce test-taking anxiety, enhance self-esteem
CASAS			Utilize problem solving strategies	Computation	
NRS (US)				Perform operations with accuracy	
State Adult Basic Education Standards Documents					
Arizona AE Standards	Meaning and relationships, equivalent forms	Determine if results are reasonable	Applies math strategies, analyze and solve real life problems	Computation	
Massachusetts ABE Framework		Reasoning to support solutions and ideas, reflection, connecting, communicating	Problem solving, decision-making	Mathematical fluency	Habits of mind, curiosity, persistence, ownership
Ohio ABLE Benchmarks	Model meanings, demonstrate the meaning of operations and their interrelationships	Communicate results	Use problem solving strategies	Quantitative procedures	
New York AE Standards	Extending understanding through exploration, concepts of operations	Reasoning skills, relationships, connections, communicating mathematical ideas	Problem solving	Computation, manually and using calculator	

FRAMEWORK	CONCEPTUAL UNDERSTANDING	ADAPTIVE REASONING	STRATEGIC COMPETENCE IN PROBLEM SOLVING	PROCEDURAL FLUENCY	PRODUCTIVE DISPOSITION
Florida AE Framework	Describe a variety of patterns and relationships through models	Explain reasoning steps, use and justify different strategies, draw inferences	Solve problems	Proficiency with operations	
Nevada ABE			Solve real world problems	Computation	
Washington ABE Competencies		Creative thinking skills	Problem solve and think critically, use a variety of methods	Master facts	
West Virginia IGO's				(computation)	
Adult-focused Documents (Non US)					
Australia: CGEA	Mathematical knowledge and techniques	Interpretation, language	Problem solving, different "numeracies"	Numerical information	
Australia: NRS	Meaning making, Mathematical representation	Reflect, interpret results, judge their reasonableness in the context, comment on the appropriateness of the math for the circumstances	Problem solving strategies, identify the embedded mathematical information and relationships	Perform procedures	
Sweden: Adults and Mathematics	Understanding concepts	Communicating, presenting arguments	Skills in exercising judgment, solving problems	Mastering procedures, using aids	

Framework	Conceptual Understanding	Adaptive Reasoning	Strategic Competence in Problem Solving	Procedural Fluency	Productive Disposition
Ontario, Canada: Performance Indicators	Develop an understanding	Judging the reasonableness of results, evaluates arguments		Perform computations, develop number sense	
Scotland: Curriculum Framework	Understanding "how" numbers work	Cognitive and metacognitive strategies, knowledge that sometimes getting the exact answer and using a particular method matters and sometimes it doesn't	Solve problems	Fluency	Independence
UK: National Standards	Select and compare relevant information from a variety of graphical, numerical, and written materials	Interpret results, present findings	Identify suitable calculations, procedures appropriate to the specified purpose	Calculate and manipulate	

NCSALL

National Center for the Study of Adult Learning and Literacy

NCSALL's Mission

NCSALL's purpose is to improve practice in educational programs that serve adults with limited literacy and English language skills, and those without a high school diploma. NCSALL is meeting this purpose through basic and applied research, dissemination of research findings, and leadership within the field of adult learning and literacy.

NCSALL is a collaborative effort between the Harvard Graduate School of Education, World Education, The Center for Literacy Studies at The University of Tennessee, Rutgers University, and Portland State University. NCSALL is funded by the U.S. Department of Education through its Institute of Education Sciences (formerly Office of Educational Research and Improvement).

NCSALL's Research Projects

The goal of NCSALL's research is to provide information that is used to improve practice in programs that offer adult basic education, English for speakers of other languages, and adult secondary education services. In pursuit of this goal, NCSALL has undertaken research projects in four areas: (1) learner persistence, (2) instructional practice and the teaching/learning interaction, (3) professional development, and (4) assessment.

NCSALL's Dissemination Initiative

NCSALL's dissemination initiative focuses on ensuring that practitioners, administrators, policymakers, and scholars of adult education can access, understand, judge and use research findings. NCSALL publishes *Focus on Basics*, a quarterly magazine for practitioners; *Focus on Policy,* a twice-yearly magazine for policymakers; *Review of Adult Learning and Literacy,* a scholarly review of major issues, current research, and best practices; and *NCSALL Reports* and *NCSALL Occasional Papers*, periodic publications of research reports and articles. In addition, NCSALL sponsors the Connecting Practice, Policy, and Research Initiative, designed to help practitioners and policymakers apply findings from research in their instructional settings and programs.

For more about NCSALL, to download free copies of our publications, or to purchase bound copies, please visit our Web site at:

www.ncsall.net