

ROUTINES IN NON-ROUTINE PROBLEM SOLVING PROCESSES

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Nonroutine problem-solving, too, involves elements of routine. Often, elementary routines such as solving for a variable are not carried out without error. In this paper, we use new findings from neuroscientific research to explain why even excellent students make mistakes in elementary routines.

1. INTRODUCTION

Mogens Niss, in his paper, "Aspects of the nature and state of research in mathematics education", wrote:

"It is important to realise a peculiar but essential aspect of the didactics of mathematics: *its dual nature*. As is the case with any academic field the didactics of mathematics addresses, not surprisingly, what we call *descriptive/explanatory* issues in which the generic questions are 'what is (the case)?' (aiming at description) and 'why is this so?' (aiming at explanation)." (Niss, 1999; 5)

In this paper, I begin with descriptive issues, in Niss's sense. While correcting maths exams at university, I noticed something very surprising. To pass an exam, students had to complete complex nonroutine tasks that required sophisticated mathematical ideas in the solution process. The student often solved the nonroutine part of a task excellently, while in the routine parts made simple mistakes such as

$$\frac{ax+b}{cx+d} = \frac{a+b}{c+d}$$

or, for instance,

$$\sqrt{a^2 + b^2} = a + b.$$

When the students saw their papers after they'd been marked, they couldn't understand how they could make such simple mistakes. In a first analysis of the problem, I found that all nonroutine problem-solving processes utilise routines as important constituents; therefore I had to understand the role of routines in nonroutine problem-solving processes. Many such routines — often called skills, they include tasks such as solving for a variable in an equation — are essential in any problem-solving process. One ought therefore understand the aspects specific to such skills or routines.

2. SKILLS

In the course of their education in mathematics, students learn algorithms and rules to calculate numbers, and later, to solve for variables. These algorithms and rules are so

important and are used in so many cases that the student should implement them automatically, meaning that when a student sees certain signs or expressions (i.e. stimuli), the steps of the algorithm are engaged automatically without further thought as to which step comes next or which rule is necessary now. This automatic implementation of algorithms and rules in mathematics is a consequence of a special acquisition process. With practice, knowledge of, for instance, a problem type is converted into procedural form, and is then applied directly without interference from other interpretative procedures. The gradual process by which knowledge is converted from declarative to procedural form is called *knowledge compilation* (Anderson, 1982; 370).

Anderson stresses that the distinction between procedural and declarative knowledge is fundamental. "Procedural knowledge is represented as production, whereas declarative knowledge is represented as a proposition network." (Anderson, 1982; 370)

The distinction between two kinds of knowledge suggests different systems at work in the long-term memory. Most memory researchers classify memory into two basic types: (1) the declarative or explicit memory; and, (2) the procedural or implicit memory (Roth, 2001; 151). Each basic memory type can, in turn, be classified into a number of subcategories. For example, the explicit, or declarative, memory can be viewed as being composed of the episodic memory, which stores remembrances from our personal past, and the semantic memory, which contains associations and concepts, i.e. our general world knowledge.

Skills memory is part of implicit or procedural memory. This type of memory contains the many functions that help us handle everyday problems in life (Schacter, 1999; 222). Skill memory is situated in the cerebellum. Although neuroscientific researchers had long held the view that skills were a motor function, like playing piano or cycling, in recent years, the cerebellum, too, has come to be seen as being important for cognitive skills:

"Research over the last ten years has shown that the cerebellum is active not only during purely motor tasks, but also [when the brain is performing] tasks that are unambiguously cognitive." (Roth, 2001; 396, translating by W.S.)

Nevertheless, researchers are not implying that the cerebellum is itself responsible for the whole cognitive process:

"As Ivory and Fiesz write, as with the purely motor tasks, it is rather difficult to find cognitive tasks in which the cerebellum is not active. However, neuroscience has at present no consistent idea of the extent to which the cerebellum participates in the cognitive activity." (Roth, 2001; 396)

For the problem we are considering here, it is important to know aspects that are specific to automated processes, particularly the ways in which they contrast with non-automated thinking and acting. Roth described the situation in the following

way:

"Automated, or implicit, processes:

- 1) are independent of the limitations of cognitive resources;
- 2) are not – or are only weakly – voluntary acts;
- 3) do not require attentiveness and awareness – indeed, these can be an interference;
- 4) proceed quickly and effortlessly;
- 5) are usually unimodal;
- 6) have a low susceptibility to error;
- 7) can be improved through training, and are difficult to change after having been acquired or consolidated;
- 8) cannot be described in detail in any natural language."

(Roth, 2001;229.)

To understand the function of skills – and automated – processes for our acting and thinking, we must consider the implications of our use of conscious – and controlled – processes as opposed to skills:

"In contrast, controlled, or explicit, processes

- 1) depend strongly on the provision of cognitive resources (a good example being the notoriously limited working memory);
- 2) require attentiveness and conscious awareness;
- 3) proceed slowly – i.e. in seconds or minutes – and are frequently arduous;
- 4) require intensive access to long-term memory;
- 5) are very susceptible to interference;
- 6) show limited response to training; they are, nevertheless,
- 7) quickly alterable; and
- 8) can be described in detail in a natural language."

(Roth, 2001; 229–230.)

Putting the contrast in a nutshell, we may say that skills and automated processes are very important in coping on an intuitive level with tasks encountered in everyday life.

Automation of functions is achieved through training processes (Anderson, 1982; Ciompi, 1999; Roth, 2001; Schacter, 1999). To understand how skills are learned, it is important to note that people with amnesia of the explicit memory are also able to learn skills (Schacter, 1999; 291-292). In most cases, skill learning begins with learning the effects of an action – the result of doing something is stored in the explicit or declarative memory – and the process of training transforms this knowledge into procedural form (Anderson, 1982; 370; Roth, 2001; 151-154). During the training process, we lose conscious awareness of this knowledge, together with the ability to describe the skill. Roth writes that

"In the light of this theory, the explicit, declarative system of consciousness is a *special tool of the brain*. This tool is implemented by the brain when faced with a new problem, of either motor or cognitive type, that is difficult or significant." (Roth, 2001; 231).

For the brain, conscious awareness is energetically expensive, and automation reduces energy consumption. Use of skills and routines in problem-solving significantly extends one's capabilities. Focussing on mathematics in particular, we may assert that solution of complex problems is only possible if we are able to store complex knowledge as a compact unit, and this is achieved through making effective use of the imagination, routines and skills.

While learning mathematics, students acquire basic "building blocks" that can be used in complex situations. In my view, all cognitive processes dealing with new, complex situations place a great demand on cognitive skills, routines and the imagination. These reduce the perceived complexity of a problem and allow the brain to overcome its limitations (e.g. a limited working memory) in solving it. However, cognitive routines used in mathematics – such as algorithms, rules etc. – do not work perfectly in all cases. This fact warrants further investigation.

3. ATTENTIVENESS AND ROUTINE PROCESSES

We have seen that one goal of the creation of routines is the reduction of conscious awareness of the routine. Also, implementation of routines is necessary to reduce attentiveness. Indeed,

"...nothing is more closely tied to conscious awareness than attentiveness."
(Roth, 2001, 224.)

Implementation of routines is also accompanied by a lowering of emotionality. Ciompi describes this reduction of emotionality (that occurs as a result of routinisation of a thinking process), which results from frequent repetition, as being a prerequisite for all routinisation processes:

"...in this way, the affective components that were originally present during rational thought – the 'affective components of logic' – enter ever more smoothly that giant pool of apparently affect-neutral 'self-evident things'...". (Compi, 1999; 111.)

We may assert that the importance of routinisation lies in the low level of attentiveness and conscious awareness that is necessary to run a routine.

We may define "normal cases" to be those in which routine processes usually run smoothly with a low level of error. Thus the level of error depends on the circumstances. Let us consider a simple example. We are all able to walk. We acquired this skill at an early stage. We are usually able to walk without thinking much about it (i.e. without being attentive to the details of the process). Even when the ground we are walking on is somewhat uneven, we are able to think about any number of other things while we're walking (a maths problem, for instance). But if the ground is very uneven, we need to concentrate on the act of walking, otherwise we'll stumble.

This foregoing example illustrates the existence of "normal cases". Circumstances strongly influence the level of attentiveness necessary to guarantee error-free implementation of routines.

A set of applications exists where a routine runs smoothly and error-free, but outside this set, we must increase our level of attentiveness, otherwise the number of mistakes we commit increases. In situations that demand higher cognitive skills, it seems that a certain level of attentiveness is always necessary if errors are to be avoided. (At work or in everyday life, it is often difficult to hold the required level of attentiveness to routine processes. One therefore speaks of a "duty of care" to indicate that attentiveness is necessary to avoid errors)

All researchers in the field of attentiveness emphasize the importance of the emotions. For example, Matthews and Wells write,

"Emotion and attentiveness are intimately linked."
(Matthews and Wells, 1999; 171.)

and LeDoux writes,

"Arousal is important in all mental functions. It contributes significantly to attentiveness, perception, memory, emotion and problem-solving. Without arousal, we fail to notice what is going on – we don't attend to the details. But too much arousal is not good either. If you are overaroused you become tense and anxious and unproductive. You need to have just the right level of activation to perform optimally. Emotional reactions are typically accompanied by intense cortical arousal." (LeDoux, 1998; 289.)

If emotion, arousal and attentiveness – the key elements we are considering in this paper – are so strongly linked, we have to consider their role in the relationship between affect and cognition.

On the neuronal level we must distinguish between two processes: namely,

"the affective-emotional, relatively details-poor registration of a situation at hand, together with comparison with the emotional memory; [and] the more or less unemotional, detailed registration of the situation with the help of the cortico-hippocampale system, which leads to a 'rational' weighing up of the particular situation and all the consequences stemming from it." (Roth, 2001; 322.)

The central function of this process is to appraise the meaning of a situation, person or thing:

"Both these systems have assigned to them the attentiveness faculty, which directs our gaze – consciously or not – onto whatever seems conspicuous or important to the brain. This control of attentiveness is an important part of our appraisal system." (Roth, 2001; 322.)

An important point is that the appraisal process is self-regulated:

"Self-regulation focuses attentiveness onto the self, and [enables] appraisal of the personal significance of external stimuli, somatic stimuli and internal cognitions." (Matthews and Wells, 1999; 183.)

This sheds light onto the question we are considering in this paper – namely, understanding why errors occur when routines are invoked in non-routine problem-solving processes. The appraisal system operates on stimuli that are noticeable, new or challenging. But problems requiring non-routine strategies have new, challenging as well as routine parts. Goldin (2000) analysed the affective pathways, together with their representation, in mathematical problem-solving by characterizing the crucial stages of the problem-solving process. The non-routine parts are the ones that are challenging and new, and these are identified by the appraisal system. Therefore, the problem-solver's attentiveness is focused very strongly on this part of a task.

Conversely, when faced with the routine parts of a task, the emotional appraisal system identifies routines as well known, and appraises them as being connected to a well-known, complete solution to the problem. The emotional system is not activated, consequently no special attentiveness is directed towards this part of the task.

We therefore have a situation in which full attention is directed towards some parts of a task but not others: the cognition system is activated for the challenging part, the entire working memory is utilised to handle the non-routine situation. We may suppose that all systems are activated in order to solve the crucial parts of the task, but hardly any resources are available to handle the routine parts of the task.

We know that in "normal circumstances", routines only need a low level of attentiveness to operate error-free. However, it seems that cognitive routines such as mathematical algorithms require more attentiveness to operate error-free. Particularly when manipulating unknowns in an equation, "super-rules" exist that can lead to over-interpretation of the required rule, and hence to an unconscious tendency towards error (Malle, 1993). Lack of attentiveness in non-routine problem-solving processes that are free of routine processes can also lead to error, even if the student could have dealt with the problem had he been more attentive.

The entire picture is made much more difficult if a student has had negative experiences in mathematics learning. Non-routine problems are then often linked to fear of failure. Researchers speak of "impairment effects associated with a reduced quality of efficiency of performance" (Matthews and Wells, 1999; 171). Some ideas for understanding this may be found in Schloeglmann (2002, 2003).

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