THE ROLE OF TOOL AND TEACHER MEDIATIONS IN THE CONSTRUCTION OF MEANINGS FOR REFLECTION

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ABSTRACT

This article reports on a study aiming to design learning systems in which students' knowledge of reflection is brought closer to institutional knowledge of this isometry and to compare how their activities shape and are shaped by different forms of mediation. It presents descriptions of interactions of groups of students (aged 12-13 years) with two computational microworlds, based on either dynamic geometry or multiple-turtle geometry, during attempts to construct and use a tool for reflections and considers how the tools of the microworlds along with the instructional approach adopted by the researcher were important in mediating the passage between meanings emphasizing reflection as property and those emphasizing reflection as function.

INTRODUCTION

Recent considerations of research into the role of technology in mathematics education have stressed the importance of considering the complex inter-relations between all the elements of technology-integrated learning systems, including: the mathematical domain and its epistemological structure; the resources brought by the learners to the system; the affordances and constraints of the technology itself; and the pedagogical structuring of the learning systems by the teacher in the institutionalized setting (see, for example, Herschowitz & Kieran, 2001; Lagrange, Artigue, Laborde & Trouche, 2001). Generally speaking, research into the pedagogical structuring of technology integrated learning systems have been less extensively documented in the research literature than the learning potentials and pitfalls that characterize learners' interactions with technological resources. This paper reports upon a study which analyzed both these aspects of mediation.

The aim of the study was to design learning systems in which learners' knowledge about reflection becomes connected to the institutional knowledge about geometrical transformations that they are intended to learn in school mathematics. Previous research related to geometrical transformations suggests that the majority of school aged students have some knowledge of the properties associated with the isometry reflection, but do not tend to characterize it as function (for a review of this research see Healy, 2002). In

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short, research suggests that students' analyses during work on reflections are characterized by what Piaget and Garcia (1989) describe as an *intrafigural* perspective generally involving associations with properties of symmetrical designs and that *interfigural* perspectives which might favor emphasis on the functional aspects of the transformation are less evident.

THE STUDY

To address the dual concern of investigating the processes by which mathematical knowledge is mediated while building learning systems which would support students in building from views of reflection based on intrafigural relationships to views of reflection as a function, the study was divided into two phases, the design phase and the comparison phase.

During the first phase, four learning systems were iteratively designed, through a series of successive steps during which tools, tasks and teaching interventions were developed as students' activities with them were observed and analyzed. To focus on tool mediation, two computational microworlds were designed: dynamic-Euclidean Geometry (DEG) and multiple-turtle geometry (MTG). The microworlds presented learners with different models of geometry along with different means for interacting with them: DEG interactions involved direct manipulation of a model of the theoretical field of Euclidean geometry; MTG interaction involved the programming of multiple turtles, whose movements around a two-dimensional surface were controlled by symbolic code. A set of five tasks was developed for use in the learning systems with the condition that the mathematical demands remained consistent regardless of which microworld was in use.

To examine teacher mediations, two instructional approaches were developed on the basis on a major difference between constructivist-rooted approaches on mathematical teaching (didactical engineering and the emergent approach of Cobb *et al.,* 1997, for example) and those guided by sociocultural ideologies (and particularly the work of Davydov as described in Renshaw, 1996). This difference related to the primacy assigned to the individual or the cultural in the learning process. Constructivist approaches emphasize a filling-outwards (FO) flow in which personal understandings are moved gradually towards institutionalized knowledge. A reverse filling-inwards (FI) flow of instruction described in sociocultural accounts stresses moving from institutionalized knowledge to connect with learners' understandings. Teaching interventions in this study were hence designed to allow investigation of these two different instructional approaches: the FO approach aimed to develop general mathematical models from learners' activities; and the FI approach intended to support learners in appropriating general mathematical models previously introduced.

In the comparison phase, an in-depth analysis of the evolution of the four systems was conducted, as a group of six 12-13 year old girls interacted together with the researcher within each system. The students were selected on the basis of their responses to a paper-and-pencil questionnaire, so that response profiles of students were similar across groups and each group consisted of students across the achievement range representative of the inner-city school in which they studied. Data, in the form of audio transcriptions, researcher notes, computer constructions and written work produced by any of the participants, were collected as students worked in pairs during five ninety-minute session of microworld activities and one forty-five minute teaching episode.

During data analysis, the various data sources were synthesized into group profiles, telling the story of the development of each system. To aid in the comparison between systems, data were the organized along the following dimensions: main strategies used in systems incorporating the same microworld; between-pair variations around the main strategies; variation associated with the use of FI or FO approaches; and microworld evolutions. Finally, these dimensions were further analyzed to identify the ways in which the tools, tasks and teaching interventions appeared to constrain and afford the abstraction and concretion of mathematical meanings for reflection in terms of its properties and functional aspects, movements between intrafigural and interfigural perspectives and students' considerations of figures and planes.

RESULTS

In general, the results suggested that students working in all four systems developed new meanings by coordinating intra and interfigural analyses as they built computational models of reflection. The microworld tools had a central role in mediating all aspects of the students' activities: with DEG tools, reflection tended to be represented as a correspondence relationship, usually based on perpendicular distances; MTG tools, in contrast, afforded the expression of reflection as a mapping of one set of turtles onto another and emphasized equal turns and distances. Mathematical meaning-making in all four systems involved the forging of connections between general models of reflection and physical movements on screen, with the support built into the microworlds helped students to see and investigate the generality behind the geometrical figures they were producing. However, results also indicate that the same support system allowed them to find ways of expressing this generality not through formalisation as intended, but through action. The impact of instructional approach on students' meaning-making activities was less marked and also mediated by microworld tools.

To illustrate the overall finding and concentrate on the particular effects associated with the instructional approaches, the remainder of the paper will consider the relationships between students' interactions on two of the five microworld tasks and the mathematical issues discussed in the teaching episodes.

One of these tasks – the third in the sequence – involved students in attempting to build a tool for constructing images of points (in the DEG systems) or turtles (in the MTG systems) under reflection. The second (representing the final of the five tasks involved students in operating with a set of elements to be reflected (see Figure 1).

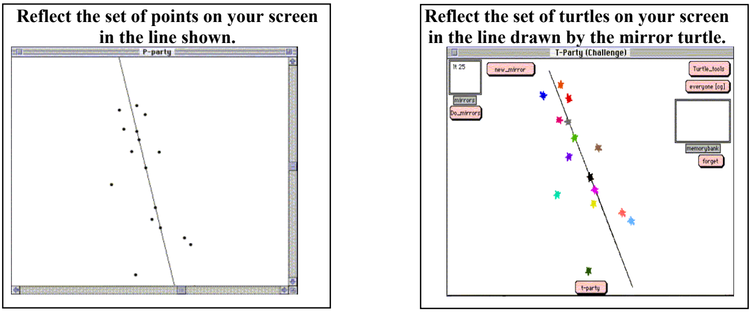

Figure 1: The DEG and MTG versions of the final microworld task

Characterizing the differences in the instructional approaches

The FI and FO instructional approaches differed both in terms of global structuring of the teaching episode and local structuring which related to teaching interventions made during microworld activity.

The teaching episode, which had the aim of emphasizing knowledge of reflection in terms of institutional mathematical practices, was an important aspect of all learning systems. The challenge in designing the teaching episodes was to discuss functional views on reflection in ways that would be meaningful to students and that stressed its connection with what they already knew about reflection. Three foci for discussion were planned: co-ordination of interfigural and intrafigural properties; function as relationship (static view) and as transformation (dynamic view); and meanings for planes and their elements. All discussion took place away from the computer, and was expressed using paper/transparency-and-pencil media. In the two FI systems, this teaching episode occurred before students started on the five microworld tasks and in the FO systems after the tasks had been completed.

In terms of the local structuring, regardless of instructional approach, the intention was that students would be in control of their own solution processes, making decisions and following directions of exploration that they chose for themselves. The differences between instructional approach related to the introduction of new tools at the beginning of each microworld session. In FI systems, the tools were introduced in ways that attempted to emphasis their connection to aspects of the intended knowledge and in particular stressed geometrical objects as sets of points or turtles. In contrast, in the FO systems, emphasis was on encouraging students to connect the empirical effects of a tool with their own knowledge and students were asked to come up with the own descriptions of their output.

Interactions in the FI learning systems

For the FI instructional approach, the teaching episode involved the presentation of models for reflection, in particular, planes were described using a flatland metaphor as two dimensional worlds, with no up or down, consisting of points and the reflection transformation enacted in ways that emphasized the axis of reflection as a perpendicular and angle bisector. To encourage the active interaction of students with this models, four teaching strategies types were adopted, based on those described in Renshaw (1996) *exposition* (presentation of models using mathematical voice), *leading questions* (encouraging students to use general and precise terms), *staging mistakes* (drawing attention to inconsistencies or errors) and *clashing* (provision of different valid representations of the same relations).

Despite participating in identically structured teaching episodes before microworld interaction, the interactions around the task of constructing a reflection tool varied considerable between the two FI systems. In the DEG-FI system all three pairs made use of the equal perpendicular distances to position the image point, although none of them managed to formalize the relationships involved to produce a robust construction. Instead they first constructed a line perpendicular to the axis and passing through the original point, two pairs using the appropriate DEG tool and the third constructing this line by eye and then manually adjusted the image point along the perpendicular line. When the image point was moved, the same procedure was adopted, hence students expressed – at least some elements of – generality through dragging and not through the construction tools.

In the MTG-FI system, one pair managed to formalize a variable Logo procedure for constructing image turtles. The method they used involved hatching a new turtle on top of the turtle to be reflected, sending this turtle so that it had the same location and heading as the mirror turtle (that is, to a special state on the axis of reflection) and then reproducing the remembered distances and angles on the opposite side of the mirror turtle's line. The other two pairs used a second method, which involved the use of an MTG primitive tool which constructed a new turtle at the meeting point of any two turtles (a Logo equivalent of an intersection point). The complete method entailed sending a turtle from the position of the original turtle until it met with the mirror turtle, turning this turtle to have the same heading as the mirror turtle, then repeating the turn and the distance traversed so that the image turtle was in the correct position. The two pairs using this method did not produce variable procedures but expressed its generality method by reusing the same Logo command sequence and manually altering the distances and angles following a change in the position of the original turtle.

When it came to the task of reflecting a set of elements, all the students in both FI systems attempted to transform all the element on both sides of the axis consistently using the same method for each element. However, whereas in the DEG-FI system none of the students operated upon the points located along the axis or discussed the point-set as anything other then the set of specific turtles on screen, those working with the MTG microworld were motivated to begin to connect the idea of reflection to acting upon a more abstract conceptualization of a turtle-set – evident in discussions referring to *"the world of turtles"* and to consider elements invariant under the reflection transformation. One student, for example, described her view thus:

> *"Every turtle has its own reflection turtle with the same distance away from the mirror and the same angle, except for left and rights. This one* (pointing to the mirror turtle) *has no distance away and no angle, but it still has its own reflection."*

Interactions in the FO learning systems

The global structuring of the FO systems did not include the presentation of “ready-made” models for reasoning about the intended knowledge. Instead the aim was that students would construct their own models during computer interactions which would serve as the basis from which they could *(re)invent* models for reasoning about objects of reflection during the teaching episode. The teaching strategies adopted in FO systems involved using the students’ *voices* to re-express the intended knowledge from the researcher’s *perspective*, along with *matching* (identifying and evaluating identical or overlapping solution approaches) and *contrasting* (identifying and evaluating different approaches to task solution). Although the same three areas for discussion identified above were planned for the two FO teaching episodes, in practice only two areas were covered as students reflected on the intra and interfigural aspects of their constructions and presented their models for reflection. No student descriptions of planes or figures as point (or turtle) sets emerged during the episode of either DEG system, hence this aspect was not discussed.

The differences in the models of reflection discussed in the two FO systems related to differences in the methods used during the task of constructing images. In the DEG-FO group a variety of different methods were constructed – one pair in particular came up with a total of four different construction methods formalized into macro tools (see Figure 2). The other two pairs, like the students in the DEG-FI system, were able to enact but not formalize properties sufficient to define a reflection tool.

a: Using two `angle carry` constructions | b: Using `angle carry` and `compass` | c:Using two `compass` constructions | d: Using `perpendicular line` and `compass`

Figure 2: The four robust image point constructions defined a FO-DEG pair

Students interacting in the MTG-FO system constructed image points using the same two methods as the MTG-FI students, all though none of the three FO pairs formalized the method in a variable procedure, again choosing instead to express generality in the action of reusing a set of commands, altering only the specific values as necessary.

On the final task, the strategies of the DEG-FO students matched those of their DEG-FI counterparts, with none of the students choosing to operate on the points located along the axis for reflection. This proved to be more of an issue for MTG-FO students who did tend to discuss the effect of the transformation on turtles along the axis. However, there was no evidence that the students were thinking beyond the specific set of turtles on screen or conceptualizing figures, let alone the screen, as representable as a turtle-set.

FINAL REMARKS

In summary, the evidence presented in this paper suggests that the efficacy of combinations of microworld interations and instructional approaches is likely also to depend on the specific learning objectives associated with the learning systems. Students in all four systems appeared to extend their knowledge of reflection, but each system had its own particular characteristics.

The DEG-FI system was the only one in which all students referred to the reflection construction traditionally emphasized in school texts, suggesting this system-type might be the most efficient in steering of students to some predetermined set of responses. The greatest variety of reflection constructions were built by a pair in the DEG-FO system, it seemed to offer to them opportunities for exploring equivalent expressions of the same geometry construction. In the two MTG systems, all students invented and explored their own models of reflection. These models were rather different than the traditional school model, especially in that the perpendicular relationship of reflection was not featured. The MTG-FI was the only system in which students connected to the notion of geometrical objects as point-sets, suggesting that connection to this particular abstraction may be facilitated by a system in which students are encouraged to connect the behaviors of geometrical agents in mathematical systems to with those of more animate agents in social systems.

REFERENCES

Cobb, P., Gravemeijer, K., Yackel, E., McClain, K. & Whitenack, J. (1997). Mathematizing and symbolizing: The emergence of chains of signification in one first-grade classroom. In D. Kirshner & J.A. Whitson, (eds*.) Situated cognition: Social, semiotic, and psychological perspectives*. NJ: Erlbaum, pp. 151-235.

Healy, L.(S.). *Iterative design and comparison of learning systems for reflection in two dimensions*. Unpublishlised PhD thesis, University of London.

Hershowitz, R. & Kieran, C. (2001). Algorithmic and meaningful ways of joining together representatives within the same mathematical activity. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of PME 25*. (pp. 111- 122). Utrecht University, The Netherlands

Hoyles, C. & Healy, L. (1997). Unfolding meanings for reflective symmetry, *International journal of computers in mathematical learning* , 2 (1) pp. 27-59.

Lagrange, J.-B., Artigue, M., Laborde, C., & Trouche, L. (2001). A meta study on IC technologies in education. Towards a multidimensjonal framework to tackle their integration. In M. van den Heuvel-Panhuizen (Ed.), *Proceedings of PME 25*. (pp. 111- 122). Utrecht University, The Netherlands

Piaget, J. & Garcia, R. (1989). *Pschogenesis and the history of science*. New York: Columbia University Press.

Renshaw, P.D. (1996). A sociocultural view of the mathematics education of young children. In H. Mansfield, N. Paternan & N. Bednarz (Eds.) *Mathematics for tomorrow's young children: International perspectives on curriculum*, Dordrecht, The Netherlands: Kluwer Academic Publishers, pp.59-78.