

Appalachian Collaborative Center for Learning, Assessment and Instruction in
Mathematics

**Mathematics, Mathematics Education, and Views of Reform:
Perspectives from Two Mathematicians Involved with Mathematics Education for
the Appalachian Collaborative Center for Learning, Assessment, and Instruction
in Mathematics**

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**Foreword written by Craig Howley
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ACCLAIM's mission is the cultivation of *indigenous leadership capacity* for the improvement of school mathematics in rural places.

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Foreword

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This Occasional Paper is a dialog about mathematics education between two mathematicians, Reid Davis, formerly of the University of Tennessee and Jeff Connor, of Ohio University. Their views differ and they speak with distinct voices because the reform of mathematics education is not a settled matter. This paper was literally occasioned by conversations among members of the Center's Management Team.

Many of the differences of perspective articulated here are endemic to the operation of this Center, and in fact, to the operation of all Centers for Learning and Teaching commissioned by the National Science Foundation where mathematics is a concern. The reason for this tension is that the Foundation requires the collaboration of mathematicians and mathematics educators, and prominent representatives of the two groups often take differing positions on the desiderata of mathematics education.

As a dialog, this Occasional Paper comes to no conclusions about this debate. Rather, it is, I believe, an interesting contrast of views well articulated by the two major contributors, Dr. Davis and Dr. Connor. The interest in such a dialog may actually be higher among rural educators and the general public than among mathematicians and mathematics educators, for whom it is familiar territory. Although the public has not yet paid sufficient attention to the dynamics of mathematics education reform, the public interest is clearly in play in mathematics education reform and the issue of what might constitute the public interest is a highly contested part of the debate. Rural education researchers, too, should be paying attention.

Mathematics has been taught for a very long time in a very familiar way in the United States. Indeed, the discipline of *mathematics proper* remains the province of a rather small elite. Under this scheme, one might claim, the rest of us are needed mostly as a sort of test-bed in which the mathematically talented can learn the ropes—but on this view our mathematical knowledge is less important than theirs. We might be said to need exposure to mathematics principally to appreciate the important work to be done by the elite, supposedly on our behalf.

Mathematics reform, as conceived by the National Council of Teachers of Mathematics has taken a very different view of what the system of mathematics instruction should aim for and how it should conduct itself.

I am personally a skeptic of reforms of all stripes, and I have seen many of them come and go, making at least as much mischief as noise. My view of mathematics education reform, however, is somewhat different. The 1989 *Standards* (ca-authored by my colleague Jim Schultz) resonated strongly with my experience and instincts about mathematics teaching and learning. When I first read them, I had just finished two years of teaching mathematics to undergraduate students who had seemed to me (with few exceptions) thoroughly dismayed by their previous instruction. Whatever my subsequent contribution to their growth, there was no denying that they arrived in my classroom fearing and loathing something that they should have understood as breathtakingly beautiful and irresistible intriguing. Not to mention useful. In fact, many of them had plucked up the courage to butt heads with mathematics again because their job training, and future job performance, required it.

The 1989 *Standards* were very much concerned with taking mathematics out of the province of the elite and also with developing capacity in K-12 instruction for teaching that widely promoted mathematical thinking (and not merely the rituals of exact calculation that occupy so

much classroom time). It struck me as a grand and wise vision, and it struck me as particularly wise in aiming to communicate the big picture and being much less concerned with developing a myopic or dogmatic view of the requirements for implementing the big picture. Was that a fault? To my mind it was a strength. Reforms founder permanently when astute vision collapses into dogmas, mantras, and oaths of allegiance.

Acting locally on the big picture, nonetheless, is where reform gets tough. The vision remains, but the struggle for the math curriculum is a mighty one, with many dangers. These dialogs articulate perspectives that bear on those issues. Reid and Jeff are speaking civilly, articulating some of the difficult issues that confront all of us, whether we know it or not.

Reaching Out to Colleagues and the Public:
A University Mathematician Engages School Mathematics Reform

Reid Davis
University of Tennessee

Introduction

The Appalachian Collaborative Center for Learning, Assessment, and Instruction in Mathematics (ACCLAIM) braids strands of mathematics, mathematics education, and rural education into a single Center for Learning and Teaching. Mathematics Education dominates, but mathematics and rural education are essential. ACCLAIM depends on the participation of mathematicians, rural educators, and rural sociologists to accomplish its ambitions in mathematics education.

In August 2000 the University of Tennessee Department of Mathematics took the novel step of hiring two outreach mathematicians, faculty members pursuing service to K-12 mathematics in place of a research program. I have the privilege to be one of those two assistant professors. As a liaison between mathematics and mathematics education, I mostly make the mathematics department's resources available to others, but the department expects some reciprocation. I provide this in part by infrequent departmental e-mails educating the mathematics faculty on crucial issues and hot topics in mathematics education.

Since these messages addressed people who are neither mathematics education reformers nor mathematics education researchers, they may have a wider audience than originally imagined. Indeed, part of my work is to seek a wider audience. For this reason, I am pleased to offer this essay as an ACCLAIM Occasional Paper. It unites and revises three of those messages for a broader audience in the hope that it will give mathematicians, rural educators, and other interested scholars a taste of the public issues and activism in mathematics education. *Taste* is the key word:

the point is to give flavor, not sustenance. This is a narrow selection of facts, thoughts, and opinions designed to whet the reader's appetite, perhaps enticing him to a full meal.

Defining Reform: The NCTM Standards

Debate over reforming the teaching of mathematics generally starts with standards drawn up by the National Council of Teachers of Mathematics (NCTM). In 1989 the NCTM produced the book-length *Curriculum and Evaluation Standards for School Mathematics*. Until recently, people mentioning the "NCTM Standards" meant this book. In 1991 the NCTM published the *Professional Standards for Teaching Mathematics* and in 1995 the *Assessment Standards for School Mathematics*. In 2000 the NCTM published a revision of the 1989 Standards entitled *Principles and Standards for School Mathematics*. This book, which also incorporates material from the 1991 and 1995 documents, is the new "NCTM Standards."

The *Standards* – available online at <http://www.nctm.org> – are influential and controversial. In developing statewide mathematics curriculum requirements, many states keep the *Standards* in clear view, either to implement them or to oppose them. Many textbook publishers promote their texts as conforming to the *Standards*; a few advertise theirs as being untainted by the *Standards*. The NCTM heavily promotes implementation of the *Standards*, as do "Mathematically Sane" (<http://www.mathematicallysane.com>) and the National Science Foundation through the ShowMe project (<http://showmecenter.missouri.edu/>). Critics such as "Mathematically Correct" (<http://www.mathematicallycorrect.com/>) decry the *Standards* as "fuzzy math" or "feel-good math": mathematics at which everyone succeeds because essential content and rigor are missing.

The *Standards* suffer from excessive length and obtuse writing. The text drowns in vague generalities. "Teachers should help students recognize that all mathematics can and should be understood" (sidebar on p. 125). "Instructional programs from pre-kindergarten through grade 12

should enable all students to recognize and use connections among mathematical ideas” (from the connections standard on p. 64). Some precepts are obvious to the point of silliness, on the order of “Water is wet.” The sidebar on p. 126 of the new standards declares, “Teachers can understand students’ thinking when they listen carefully to students’ explanations.” On p. 197 the reader learns: “Teachers may need to explicitly discuss students’ effective and ineffective communication strategies.” Others are unnecessarily obscure. On p. 148 the Number and Operations Standard for grades 3-5 says students should “...develop fluency with basic number combinations for multiplication and division...” In context this appears to mean students should learn the multiplication tables. Much of the remaining text consists of examples “illustrating” the general statements.

As a mathematician, I understand the notion of stating a general theorem and illustrating it by examples. In the *Standards*, however, the precepts strike me as so broad and the examples so narrow that the illustrations often fail. It is like saying, “High school students should learn algebra. For example, they should learn that the line $y = 2x + 4$ has a slope of 2.”

The writers of the *Standards* are mathematics educators, and their writing seems to assume that their readers have an insider’s knowledge of the issues they address. This is an added source of confusion for lay readers, including most mathematicians. For instance, the Number and Operations Standard for grades 3-5 downplays memorizing the multiplication tables and learning the standard pencil-and-paper algorithms for multiplication and long division, saying instead that students should “focus on the meanings of, and relationship between, multiplication and division.” As a mathematician I see this as bad advice that weakens the curriculum: I know something of the important mathematics, both practical and theoretical, that grows from these algorithms.

Respectable math educators offer a different view. In their experience many schools teach multiplication and division in a mindless fashion. Students do rote drill after rote drill with never a word said about *why* the multiplication table is true, *how* one might derive a forgotten fact (e.g., $7 \times 6 = 7 \times 5 + 7$) or what multiplication *means* in the real world. In light of this insider knowledge, the *Standards'* emphasis on understanding becomes a reasonable rebuke to dreadful teaching rather than a poorly informed abandonment of basic skills.

Again, the *Standards* insist that all students must learn “important mathematics.” A current application of this principle is that all students can and should learn algebra. In conversation with a mathematics education professor, I took issue with this notion, suggesting that algebra is difficult and is beyond the ability of some students. The professor responded that the word *all* should not be taken literally, that the real point of the word *all* is that girls and minority students should not be excluded from learning mathematics. His knowledge as an insider in the field provoked him to read between the lines in a way that I could not. Of course this professor is not an authoritative arbiter of the meaning of the *Standards* (indeed there is no arbiter), but his claim is plausible: The first topic in the new *Standards* is the “Equity Principle.”

In short, the *Standards* make for difficult reading, and a lay audience cannot always take them at face value. This being the case, it is hardly surprising if open-minded readers look at the *Standards* and find radically different meanings. Some readers may welcome such ambiguity, but personally and professionally I find it a serious problem that greatly diminishes the *Standards'* usefulness.

Teaching Everyone: Technological Accommodation

It is easy, and probably correct, to see in the *Standards* an expectation that students of varying ability learn “important mathematics” together, suitable accommodations allowing weaker

students to work in fruitful, and perhaps equal, partnership with stronger ones. Commenting on this explanation, a mathematical colleague asked me, “Does this mean we will always teach calculus to students with poor algebra skills?” How indeed can such accommodation work?

The *Standards* appear to take a view something like the following: *Computational skills* (e.g., pencil-and-paper arithmetic, algebraic manipulation, calculation of derivatives and indefinite integrals by hand) differs from *conceptual understanding* (e.g., the meaning of arithmetic operations, proportion, graphs, differentiation, and integration in the real-world). Students learn best when they learn computational skills and conceptual understanding together. Some students, however, learn computational skills poorly or not at all. These students can still gain conceptual understanding if we aid them in computation, usually through calculators, computers, or other products of modern technology.

A student finishes elementary school unable to calculate with fractions and decimals. How should his mathematical education continue? Traditionally he spends the sixth through eighth grades fruitlessly repeating the same arithmetic drills that taught him nothing in elementary school. The *Standards* call for a change. Schools must teach this student the same mathematical concepts that more successful students see: ratio, proportionality, percentage, descriptive statistics, basic probability, linear and quadratic functions, algebraic representation of real-world problems, solution of equations, maximization and minimization. He must learn to solve problems dependent on these concepts, using calculator and computer support to carry out computations he cannot do by hand. The same principle applies to teaching calculus concepts to a student weak in algebra. He should learn, for instance, the meaning of the derivative (i.e., rate of change) in a wide variety of settings and be able to work with this concept, even if he needs a graphing calculator to compute particular derivatives.

Conceptual understanding itself will vary in depth. The 1989 *Standards* literally speak of understanding at level one, level two, etc. For instance, consider the probability of rolling seven on a pair of dice (a real-world concern in Nevada, Atlantic City, and Cherokee). At level one students find the “experimental probability” of this event by repeatedly rolling a pair of dice and recording the fraction of rolls yielding sevens. At level two they do the same using a random-number generator in place of dice. At level three they write a calculator program to simulate rolling 100 pairs of dice and calculate the fraction of sevens. At level four they list the sample space of rolls and compute the “theoretical probability” of rolling seven. Interested and capable students might study counting rules and independence in relation to this problem. Some students will not progress beyond level one; others will go much further. In any case all will, by this approach, gain some understanding of the concepts involved.

Two axioms underlie this approach to teaching. First, good mathematics instruction grows from real-world problems. A good teacher does not introduce quadratic equations and parabolas as abstractions. Rather he introduces his students to problems naturally modeled by quadratic polynomials, say, setting sales price to maximize the revenue or describing the path of a thrown javelin. Thus successful mathematics instruction prepares students to handle real mathematics in adult life using all available tools.

Second, mathematical skill is crucial to participating fully in our increasingly quantitative society. The society needs mathematically astute leaders and workers. Further, students competent in mathematics enjoy open doors of economic, professional, and political opportunity. Students lacking such skill find the doors shut. Our schools must, therefore, give all students, especially girls and children from racial minorities, “mathematical power” – NCTM’s term – to meet the needs of the society and the demands of justice.

I should note that I dispute the *Standards'* positions on these matters, seeing grave problems in them. In this text I am presenting those positions as clearly and attractively as my limited understanding allows, but my disagreement could easily lead me to misrepresentation.

To understand and evaluate this notion of accommodating weaker students, I suggest we apply the idea to other subjects. How would we apply the same principle to the teaching of English, for instance? A student finishes elementary school unable to read and write. Should his further education consist exclusively of remedial lessons in reading and writing? Or will we accept his deficiency and, with oral instruction and current technology, train him to work around it? With speech recognition software he can still produce written work. To do this well, he must learn proper English grammar and style. We will teach him these topics orally. Indeed we will even teach him punctuation (at least until software can punctuate accurately). Using widely available books on tape, we will teach him prose and poetry. The forms, devices, and techniques of good literature – simile, metaphor, foreshadowing, careful and economical choice of words, strong imagery, compelling description, onomatopoeia, meter – are not confined to the written page. We can teach them orally as well – indeed we may find classical inspiration in Homer, the blind poet. With speech software, our student can read anything on the Internet, from e-mail to public domain literature. With scanners and increasingly sophisticated optical character recognition, he will eventually have access to most printed works as well.

Other examples readily suggest themselves. In Elizabethan England basic musicianship was an expected part of social intercourse, but today lack of musical skill hinders no one. Boom boxes, compact discs, and MP3 players produce music at the touch of a button. In a different field, C.M. Kornbluth's chilling, classic short story, "The Little Black Bag," depicts a future world in which

medicine is the work of the slow-witted, advanced technology having made diagnosis and surgery easier than following a recipe.

Attractive as these pictures may be, I judge them unworkable. In Kornbluth's story, a better scalpel leads the ignorant to accidental suicide. Ready access has turned music into a commodity rather than promoting its practice and appreciation. The illiterate will not flock to websites offering spoken classic literature. Those who cannot calculate with fractions will find rates of change a mystery, regardless of what software tries to fill the gap. Without a foundation the tower will not stand.

Assessing Results: TIMMS and NAEP

Evaluating the need for and the success of reform requires knowledge of the state of mathematics learning in America. The Third International Mathematics and Science Study (TIMMS) and the National Assessment of Educational Progress (NAEP) are widely cited studies measuring the performance of schoolchildren in mathematics. Both are projects of the National Center for Educational Statistics (NCES at <http://www.nces.gov>), an office of the U.S. Department of Education. Both provide ample sound bites for the media and for promoters and critics of mathematics reform.

Conducted in 1995, TIMMS is a massive study comparing curriculum, teacher preparation, classroom practice, and student performance at fourth, eighth, and twelfth (or final) grades in 41 countries. The NCES repeated the study in 1999, producing TIMMS-r. The sites <http://nces.ed.gov/timss/> and <http://ustimss.msu.edu/> have extensive information on both. The studies defy simple summary, but some findings have received much attention. For instance U.S. fourth graders performed above the international average (12th out of 26 countries) while eighth and twelfth graders performed below it (28th out of 41 countries and 19th out of 21 countries,

respectively). In all three grades, the differences from average were statistically insignificant. Also in grades K-8, U.S. schools introduce more topics in earlier years and repeat the topics for more years than do schools in higher-performing nations. This observation has provoked the oft-quoted comment that the U.S. curriculum is “a mile wide and an inch deep.” Further, U.S. teachers meet students for 30 sessions each week on average. In Germany and Japan the averages are slightly above and slightly below 20 sessions respectively.

In contrast, the NAEP studies only students in the U.S. There are two versions: the main NAEP and the long-term trend NAEP. The main version changes over time to reflect changes in curriculum and approach in the schools; recent versions reflect the precepts of the NCTM *Standards*. The long-term trend version remains essentially unchanged at each administration, allowing comparison of student performance over time. It tests students primarily on basic facts, pencil-and-paper algorithms, measurement formulas, and practical daily mathematics involving time and money. The NCES has administered both versions of the math NAEP to fourth-, eighth-, and twelfth-grade students on an irregular schedule since 1972 (the main version eight times, and the long-term trend ten times).

The main NAEP shows statistically significant gains in student performance from 1990 to 1992 and again from 1992 to 1996. The changing content of the test precludes comparisons to earlier years. Roughly speaking, the long-term trend NAEP shows stable or declining performance from the 1970's to the early 1980's, improvement from the mid-1980's to the early 1990's, and stable performance through the remainder of the 1990's. Scores from 1999 are higher than those from 1973 by a small but statistically significant amount. The NCES offers more information through the mathematics link at <http://NCES.ed.gov/nationsreportcard/site/home.asp>.

The Main Course

Recently a mathematician asked me to name the biggest misconception mathematicians have about mathematics education. Without hesitation I replied, “It is that mathematics education is a branch of mathematics, when it is, in fact, a social science.” Little in the tidbits above indicates how great that gulf is. A mathematician making the acquaintance of mathematics education confronts a bewildering array of only vaguely defined (by mathematical standards) concepts: theories of cognition, constructivist (and radical constructivist) theories of learning, the sociology of the classroom, and the notion that mathematics possesses value because it is a “human activity” rather than because it is true. Worse, the research supporting these ideas is at best pseudo-experimental, lacking even the experimental controls expected in the natural sciences, to say nothing of the deductive rigor of mathematics. This is indeed foreign territory for mathematicians! I suspect that rural educators and rural sociologists will find mathematics education less foreign, though I wonder whether they may struggle to put a spotlight on mathematics in their teaching and research in rural settings. Of course if they want to understand how mathematicians view and promote mathematics, they will have to make some leaps. For scholars anywhere who want to go from this appetizer to a main course, I recommend (in addition to the links above and references they provide) the bibliography in the *Standards* and the NCTM’s *Journal for Research in Mathematics Education*.

A Response to “Reaching Out to Colleagues and the Public”

Jeff Connor
Ohio University

Before responding to the article, I wish to introduce myself and describe some of my connections to mathematics and mathematics education. I am a member of the Department of Mathematics at Ohio University, which is also a member institution of ACCLAIM. The department currently has 25 tenure and tenure-track faculty, two of whom are mathematics educators, and is a Ph.D.-granting department. While my training and the bulk of my publications are in mathematics, during the past six or seven years I have developed an interest in Mathematics Education, especially the mathematical education of prospective secondary school teachers. (In practical terms, this means that I teach the geometry sequence.) I have taught Masters level courses to in-service teachers, attended and presented at regional and national NCTM meetings, and make it a point to attend the research pre-session of the national NCTM meeting.

Confusing Mathematicians with Mathematics Education

The conclusion of “Reaching Out” suggests that the biggest misconception that mathematicians have about mathematics education is that it is a branch of mathematics. This insight certainly explains my own initial confusion (and occasional frustration) when working with mathematics educators. I was often dismayed at the flat response I received after giving a presentation or – even worse – an entire course on what I felt was absolutely compelling mathematics to a group of teachers. I had the same experience when I started to attend NCTM meetings, only this time I was the one who found most of the talks to be uninformative and irrelevant. I don’t think that I suffered from the confusion that mathematics educators would be presenting and proving theorems. I was perplexed, though, that, at least from my perspective, the

talks did not seem to be driven by mathematics.

Reaching Out to the Standards

I, too, find some aspects of the NCTM *Standards* document unsatisfying. It is a bit murky in the area between general discussion and classroom practice. In the geometry section, grades 9 – 12, I would have appreciated more on the place of formal proofs and axiom systems. My personal taste runs against the use of vignettes to illustrate general themes; teaching rarely works out so nicely in a real class.

However, I do think that “Reaching Out” is unfair in its presentation of the *Standards*. For instance, it cites the bullet “. . . develop fluency with basic number combinations for multiplication and division . . .” in the Number and Operations Standards (p. 148) as an example of “unnecessarily obscure” writing and goes on to interpret this as meaning that “students should learn the multiplication tables.” Reading further into the section on Numbers and Operations, one finds, on page 152:

By the end of this grade band, students should be computing fluently with whole numbers. *Computational fluency* refers to having efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. The computational methods that a student uses should be based on mathematical ideas that the student understands well, including the structure of the base-ten number system, properties of multiplication and division, and number relationships.

Fluency with whole-number computation depends, in large part, on fluency with basic number combinations—the single-digit addition and multiplication pairs and their counterparts for subtraction and division. Fluency with the basic number combinations develops from well-understood meanings for the four operations and from a focus on thinking strategies (Thornton 1990; Isaacs and Carroll 1999). By working on many multiplication problems with a variety of models for multiplication, students should initially learn and become fluent with some of the “easier” combinations. For example, many students will readily learn basic number combinations such as 3×2 or 4×5 or the squares of numbers, such as 4×4 or 5×5 . Through skip-counting, using area models, and relating unknown combinations to known ones, students will learn and become fluent with unfamiliar combinations.

For example, 3×4 is the same as 4×3 ; 6×5 is 5 more than 5×5 ; 6×8 is double 3×8

This passage indicates that a student should develop a good understanding of multiplication of single digit numbers in terms of the structure of the natural numbers. The phrase “learn the multiplication tables” does not convey the same message.

“Reaching Out” also gives short shrift to the *Standards* in its discussion of how to interpret *all* in the context of the *Standards*. The author reports learning in conversation with a professor of mathematics education that *all* should not be taken literally, but rather that “girls and minority students should not be excluded from learning mathematics.” He adds, as a disclaimer, that this professor is not “an authoritative arbiter of the meaning of the *Standards*” and goes on to state that “His knowledge as an insider in the field provoked him to read between the lines in a way I could not.”

As noted in the article, the Equity Principle is indeed the first topic in the *Standards*. Not only is it the first topic of the document, its authors also state that “Educational equity is a core element . . .” of their vision and that “it is interwoven with the other Principles.” The principle is

Excellence in mathematics education requires equity—high expectations and strong support for all students.

This principle is discussed in pages 11 through 14 of the *Standards*. Part of the discussion clarifying the principle includes the following:

The vision of equity in mathematics education challenges a pervasive societal belief in North America that only some students are capable of learning mathematics. This belief, in contrast to the equally pervasive view that all students can and should learn to read and write in English, leads to low expectations for too many students. Low expectations are especially problematic because students who live in poverty, students who are not native speakers of English, students with disabilities, females, and many nonwhite students have traditionally been far more likely than their counterparts in other demographic groups to be the victims of low expectations. Expectations must be raised—mathematics can and must be learned by all students. (p. 12)

It is also worth noting that *all* also includes talented and gifted students, as evidenced by the following passage:

Likewise, students with special interests or exceptional talent in mathematics may need enrichment programs or additional resources to challenge and engage them. The talent and interest of these students must be nurtured and supported so that they have the opportunity and guidance to excel. (p.13)

“Reaching Out” also identifies “good mathematics instruction grows from real-world problems” as a key “axiom” of the *Standards*. I think this is, at least in part, a reformulation of the ‘Learning Principle’:

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge. (p. 15)

The *Standards* makes frequent reference to problem solving. While one can interpret “experience and prior knowledge” as *real-world*, one should make sure that a “real-world” problem is in fact based on a student’s experience and prior knowledge. If the student doesn’t have the background to understand a model for throwing a javelin or tracking sales revenue, their use won’t help the student understand quadratic polynomials. It is also important to keep in mind that part of a student’s experience and prior understanding includes mathematical understanding. These last points are addressed in the Teaching Principle, which states that “Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well” (NCTM, 2000, p. 16).

While I am disappointed that the *Standards* was not represented more fairly, I also don’t think that a surface level, point-by-point discussion of phrasing is capable of developing a productive discussion, much less leading to collaboration among people with somewhat differing views. I would like to turn to what I believe to be more fundamental issues.

Fundamental Issues

There are some major distinctions between how mathematicians and the proponents of the *Standards* perceive mathematics. These different views have significant implications in terms of the educational mission, the nature of scholarship and the range of creative activity of the two disciplines. This article is a good illustration of the far-reaching implications of these two different views of the nature of mathematics.

The next few paragraphs are paraphrased from Paul Ernest's book *The Philosophy of Mathematics Education*.¹ I highly recommend this book; much of what Ernest says rings true and provides a useful way to understand the differences between the disciplines of mathematics education and mathematics.

Ernest identifies five ideologies of mathematics education and discusses these in depth, addressing topics ranging from their view of mathematics; their theories of society of teaching and of learning mathematics; to their theory of social diversity. The two schools that seem most relevant to this article are the "old humanist" and the "public educator".

Many of the mathematicians that I know fall into the "old humanist" camp. Members of the group tend to view mathematics as a self-subsistent "body of pure objective knowledge, based on reason and logic . ." Mathematics is viewed as being value free and having both a logical and hierarchical structure. The mathematics teacher's role is to demonstrate the pure conceptual structure of mathematics, and a successful student is one who can internalize this structure. The aim of mathematics education is to expose students to mathematics. Students are encouraged to move up as far along the mathematical hierarchy as possible. The farther they ascend, the closer

¹ This book may be hard to find. Paul Ernest has recently published another book, [Social Constructivism as a Philosophy of Mathematics](#) (1998), which is more readily available and which builds on themes of this text. Ernest indicates that there are some conceptual differences between his earlier and later books (i.e., between [The Philosophy](#)

they get to “real” mathematics. A main goal of assessment is to identify excellence. One consequence of this view is the development of a mathematics curriculum that prepares a small group of students to become mathematicians “...whilst leaving the rest to stand in awe of the subject” (Ernest, 1991, pp. 168-180).

The “public educators” have a very different view of mathematics and a different set of goals for mathematics education. This group views mathematics as a cultural product, and a major goal of mathematics education is to have students learn how to pose and solve problems within a social context. There is also a desire to have mathematics education be a means of promoting social justice. School mathematics is not to be externally imposed, but rather is to be developed within student culture, allowing the students to “acquire ownership” of mathematics. A curriculum designed by the public educator is friendly to females and minorities and has been designed to remove obstacles of success for *all* (Ernest, 1991, pp. 197-213).

According to Ernest, the roots of these differences go back to basic beliefs about the nature of mathematical knowledge. If mathematical knowledge is viewed as an idealization of an absolute truth, akin to Plato’s forms, it is natural to teach mathematics as a logical hierarchy of concepts. A student progresses by working with higher and higher abstractions; it is presumed the student can apply them when necessary. If, on the other hand, mathematics is viewed as being constructed from human experience, it is viewed as developing in a social context, and the goals of mathematics education change to those of the public educator.

The views expressed in “Reaching Out” are consistent with the beliefs of the “old humanist” camp. The discussion of “accommodating weaker students” appears to pivot on the student’s ability to acquire high culture. It objects to having ready access to music in that it “turns

and Social Constructivism). Although it is on my reading list, I have not read his more recent book, so I cannot report

music into a commodity rather than promoting its practice and appreciation.” I sense some of the elitism inherent in the old humanist school in the remark that “in Elizabethan England basic musicianship was an expected part of social intercourse.” For those who *had* horses, I imagine basic horsemanship was also expected.

For the old humanist school, the aim of mathematical education is to teach mathematics “for its intrinsic value, as a central part of the human heritage, culture and intellectual achievement. This entails getting students to appreciate and value the beauty and aesthetic dimension of pure mathematics” (Ernest, 1991, p. 176). I suspect, however, that members of this school would also agree with the unsupported assertion of the second axiom identified in “Reaching Out”:
 “Mathematical skill is crucial to participating in our ‘increasingly quantitative’ society.”

On the other hand, the Learning Principle and the Equity Principle place the *Standards* squarely in the public educator camp. I believe that these ideological differences were the source of my own frustration and confusion when I started working with in-service teachers and attending NCTM meetings; we just weren’t coming from the same place.

Dr. Davis senses the difference between the two camps as well. In his conclusion he states that educators will have to make some leaps “if they want to understand how mathematicians view and promote mathematics.” I agree.² I would add that mathematicians will also have to make some leaps if they want to understand how mathematics educators promote and view mathematics.

What about rural context? For collaborations such as ACCLAIM to succeed, I think that leaps need to be made in both directions. The public educator and old humanist schools are bound to have different approaches to rural mathematical education. On one hand, given the educational

on it.

² I suggest reading G. Hardy’s *A Mathematician’s Apology*. This classic, written by a top-notch mathematician, discusses beauty and elegance in mathematics using elementary mathematics.

aims of the old humanist, it is unlikely that, beyond considering the general mathematical background of the students, *where*, city or country, mathematics is taught would not affect how the material is presented.

On the other hand, I would anticipate that a rural public educator would spend some time considering the general culture, background and experience of rural students; it may not make sense to use materials designed for urban students in a rural setting. While members of both groups may be genuinely enthusiastic about joining a project to foster a “solid knowledge of mathematics,” the project will be short-lived if the participants do not take into account their different views of mathematical knowledge and different goals for mathematical education.

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