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ABSTRACT

This study investigates student understanding of the part-whole concept in mathematics at an independent two-year post-secondary institution in New Mexico. The mathematics portion of the Tests of Adult Basic Education (TABE) Summary Form was used as the evaluative instrument. Evidence for a lack of part-whole concept understanding as a hidden cause of innumeracy is reported. Data shows a trend of a higher final grade corresponding to better part-whole understanding at the start of the course. In light of this evidence, it is concluded that direct instruction of the part-whole concept should be part of adult basic math and algebra classes. (SOE)

Does "Part-Whole Concept" Understanding Correlate With Success in Basic Math Classes?

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Introduction

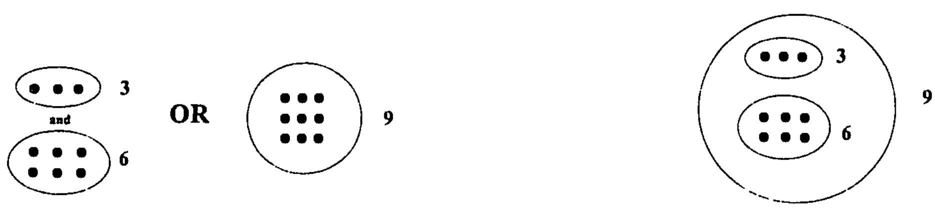
Adults with difficulty in math may lack skills in number manipulation or may fail to grasp fundamental concepts of number relationships or both. It is important to uncover students' weakness in conceptual understanding so that math "skills" can be applied appropriately.

One major transition concept from concrete to abstract thinking in math is the part-whole concept. Students who grasp this concept have the sense that the parts or partitions of a quantity and the whole quantity exist together at the same instant, rather than only the whole or only the parts existing at any given point in time (Figure 1).

Figure 1

NO PART-WHOLE
Either the parts or the whole exist

PART-WHOLE UNDERSTANDING
Both the parts and the whole exist at the same time



Pilot studies (Steinke, 1999) indicated that many adults lack the part-whole concept. A later study (Steinke, 2000) reported a high percentage of errors on problems requiring part-whole understanding on a standardized test (Test of Adult Basic Education Summary) among a large sample population already possessing a high school diploma or GED. The current report relates part-whole understanding on the same standardized test to final grade in a Basic Math or Algebra I course in portions of the large-sample population.

Background

In studies with children, Steffe and Cobb (1988) proposed a 3 Stage model of number understanding, the third stage being grasp of the part-whole concept. Using the physical signs and oral responses described by Steffe and Cobb as differentiating the 3 Stages, Steinke (1998) developed a short non-pencil-and-paper assessment for part-whole understanding. In three different pilot groups of adult volunteers, Steinke (1999) found unexpectedly high percentages of adults who lacked the part-whole concept: 4 of 11 students from a general population at a two-year college (36%); 8 of 12 students in a pre-GED class (67%); and 2 of 15 pre-service teachers in a "mathematics methods" class (13%). Even with visual cues (circles to represent the given part; the written numeral for the whole), these adults were unable correctly to answer, or showed lack of part-whole understanding in the way they answered, a question of the form: If I have 7 cookies on the plate and 23 cookies altogether, how many cookies are still in the box?

Attempting to support or refute these unsettling pilot results, a *post-hoc* data analysis was undertaken (Steinke, 2000) of a large sample of responses from the mathematics portion of the Tests of Adult Basic Education (TABE), Summary Form (CTB, 1990). The entire Summary test is used as a placement exam for entering students at an independent two-year post-secondary institution in New Mexico. A TABE "locator" test determines the appropriate reading level for a person, and the correspondingly difficult form of the TABE Summary is then administered (Form A, D, M, or E). Prior to data collection and analysis, the questions in the Math Application portion of the TABE were reviewed to determine which required part-whole understanding.

Single-operation math problems fall into two types:

1) given the parts, find the whole (P + P = ?) (7 + 9 = ? or ? – 9 = 7); or
2) given the whole and a part, find the missing part (P + ? = W) (9 + ? = 16 or 16 – 9 = ?),

The second type requires part-whole understanding to find the "missing part."

This segregation of problems into two basic types is supported by comments from the literature. Steffe and Cobb (1988) speak repeatedly of "part-whole operations." Kamii (Kamii with others: 1985, 1989, 1994) and Ross (1986, 1989) imply these two categories of problems in their writings. Carpenter et al. (1999) use the terms "part unknown" and "whole unknown" to describe word problems.

Analysis for Type 2 "missing part" problems in the TABE Summary found 5 problems in Form A, 7 in Form D, 4 in Form M, and 4 in Form E. Scores on the "missing part" problems in the large sample (2,909 individuals) showed a high level of errors (Steinke, 2000). For all such problems, the percent of wrong answers (% errors) on "missing part" problems were: 6.6% to 50.0% in Form A (424 tests), 12.3% to 65.1% in Form D (1,066 tests), 1.2% to 42.7% in Form M (1,118 tests), 7.6% to 48.5% in Form E (301 tests).

Procedure

Three of the Type 2 problems from each TABE Form, those most straightforward and most similar to each other across the Forms, were used as the basis for determining part-whole understanding in the present analysis. The % errors for these three problems were: Form A: 6.6%, 24.3%, 37.7%; Form D: 12.3%, 24.9%, 24.2%; Form M: 1.3%, 25.7%, 42.8%; Form E: 48.2%, 26.6%, 48.5%.

For the present analysis, the database was organized to show TABE scores, by-item TABE errors, math classes attempted, and final grades for students who took the TABE Summary Math Applications Section and enrolled in a math course from the Fall 1996 term through the Summer 1999 term. "Final Grade" is recorded as the standard A through F plus W for "withdraw." "W" students left the course between four weeks from the start of the term and four weeks before the end of the term with no penalty. In other words, many of those who are recorded as W may have earned a poor grade had they not withdrawn.

Fewer students took math classes than the total who took the TABE. Of particular interest were individuals who had taken either Math 100 ("basic" math) or Math 102 (high school Algebra I) as their initial math course. Also of interest were students who took Math 100 and continued to Math 102.

The data from all four TABE Forms were combined with all results sorted by math class and final grade (Table 1). This was done to provide more data for each grade/errors pairing. For example, all the errors made by students who earned an A grade in Math 100 were summed and divided by the number of such students (N = 159) to give All Problems Average Errors of 3.1447. To provide clearer independence of data, the Average Errors for the three "part-whole" problems (3 P/W) and for the remaining 12 problems (12 Prob.) excluding the three "part-whole" problems were also found by final grade in each course.

Table 1: Average Errors by Letter Grade in Math 100 and Math 102 for All Forms of the TABE Summary Combined

Math 100 (Basic Math) N = 500				Math 102 (Algebra I) N = 291			
Grade/N	All Prob.	12 Prob.	3 P/W	Grade/ N	All Prob.	12 Prob.	3 P/W
A 159	3.1447	2.6038	0.5409	A 56	2.6964	2.4107	0.2857
B 107	3.1121	2.3551	0.7570	B 54	2.6852	2.2407	0.4444
C 87	3.4367	2.6207	0.8161	C 59	3.2034	2.6271	0.5763
D 33	3.0303	2.1212	0.9091	D 23	3.2174	2.0870	0.6957
F 64	2.9688	2.2188	0.7500	F 25	3.1600	2.4800	0.6800
W 50	3.3600	2.6000	0.7600	W 74	3.3378	2.7887	0.6622

This combining of Forms was felt to be justified by the TABE publisher's assertion of the equivalence of the Forms when the Form on which a subject is tested is based on taking a "Locator" test for reading level, as was done for all these subjects. Furthermore, separate analysis of scores for Forms M and D showed a pattern of results within each Form similar to that of the scores for the combined forms.

Because of interest in success in Algebra I (102) of those who had taken Basic Math (100) compared to those whose initial math class was Algebra, another analysis of the sample was done (Table 2). In this smaller group, Forms A and E had too few samples to be fairly distributed across letter-grade categories. Only the Forms D and M pairings were used. Students' math registration was known from Fall 1996 forward through Summer 1999. The "Initial Math 102" sample was limited to those Math 102 students who took the TABE no earlier than July 1996 and whose first registered math course after that date was Math 102. There were 156 students in this group. Another 130 students took Math 100 followed by Math 102 in this time period.

Table 2: Math 102 Final Grade and 3 P/W Average Errors for Prior Math 100 Students and for Initial Math 102 Students (TABE Forms D & M)

	100 Prior	% Tot. N	3 P/W	**102 Initial**	% Tot. N	3 P/W
Grade	N (Tot. 130)		Av. Error	N (Tot. 156)		Av. Error
A	16	12.31	0.3750	36	23.08	0.2500
B	18	13.85	0.8333	28	17.95	0.2857
C	34	26.15	0.5588	37	23.72	0.4595
D	31	23.85	0.8065	11	7.05	0.4545
F	11	8.46	0.4545	15	9.62	0.5333
W	20	15.38	0.8500	29	18.59	0.3793

Analysis

What is striking about the letter grade/TABE All Problems and 12 Problems Average Errors relationships in Math 100 students (Table 1) is that the people who received D's or F's in the course did better on the TABE than those who got A's, B's, or C's. Also, those who withdrew (W) did better than the people who earned C's.

Students in Math 102 who earned D's and F's had fewer average errors on the 12 problems (those other than the three selected part-whole problems) than did students who earned A's. Granted there are other factors affecting classroom success, but at face value it seems that this version of the TABE does a poor job of predicting success in basic math and first year algebra for this sample.

A student's grasp of the part-whole concept (3 P/W average errors) prior to starting Basic Math (Math 100) does seem to relate to final grade in the course. The same trend appears for those who successfully complete Algebra I (Math 102) (see Chart 1). This group of Math 102 students probably includes some who took Math 100 prior to 1996; this may have the effect of showing slightly higher 3 P/W average errors in Math 102. The reason for the 3 P/W average errors for F and W students in Math 100 being close to that for B and C students has any number of explanations. One factor in academic success is self-motivation. Perhaps the F and W students could earn a B or C but give up. One reason for giving up is that they are aware that they aren't "getting it." They are unaware that "it" is an understanding of the part-whole concept.

Chart 1:
3 Part/Whole Problems
Average Errors on TABE and
Students' Final Grades
in Math 100 and Math 102

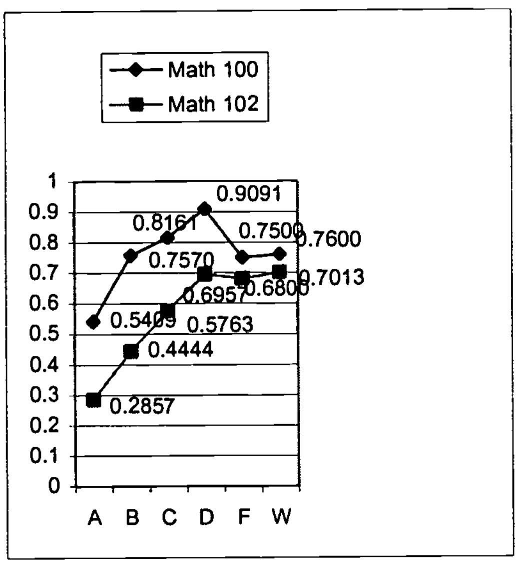
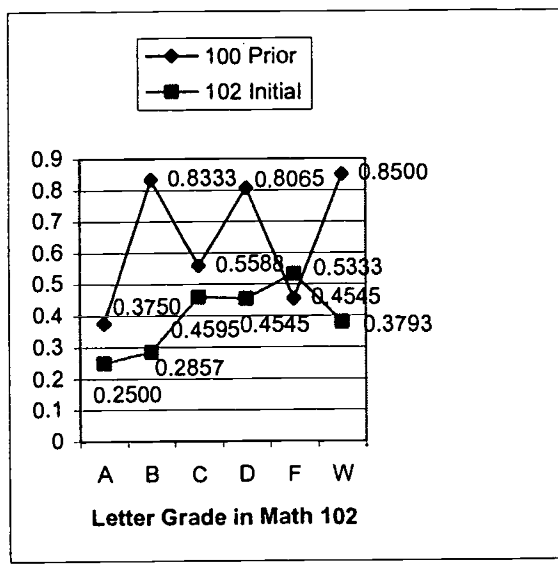


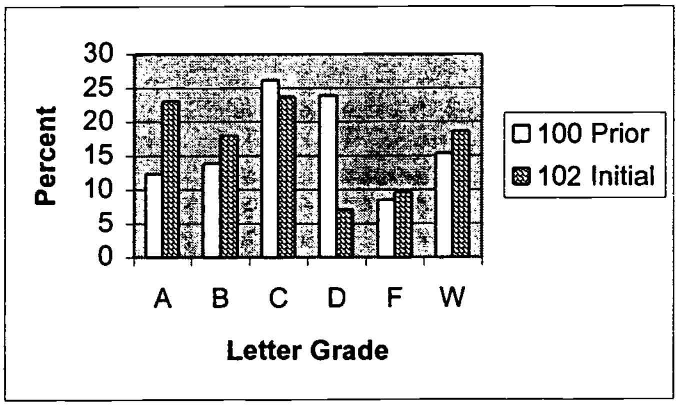
Chart 2:
Math 102 Final Grade and 3 P/W Problems
Average Errors on TABE for Students Taking
Math <u>100 PRIOR</u> to Math 102 or
Taking Math <u>102</u> as <u>INITIAL</u> Math Course



The better question is: do B and C students in Basic Math (Math 100) get better grades by improving their part-whole understanding? Perhaps they are able to earn an acceptable grade by using rote skills on tests, turning in homework, and doing "extra credit" work. The success rate of Math 100 students going on to Math 102 provides further insight.

A comparison of 3 P/W average errors and final grade was made between students who took Math 102 after taking Math 100 (100 Prior) and students who it is certain took Math 102 as their first math course at the college (102 Initial). The comparison was limited to Forms D and M (for a fair distribution of Form/Grade pairings) and "Math 100 Prior" students with grades of A, B, or C in Math 100 (Table 2). The "102 Initial" group shows a 3 P/W-to-grade trend; the "100 Prior" group does not (Chart 2). Again the question is whether the "100 Prior" students improved their part-whole understanding in Math 100 or worked harder in Math 102.

Chart 3:
Percentage of Students at Each Final Grade in Math 102 With Math 100 as Prior Course or Math 102 as Initial Course



Another interesting comparison between the Math 100 Prior/Math 102 Initial groups is the percentage of students in each group at each final grade in Math 102 (Table 2; Chart 3). The percentage of F and W are close. In the Math 102 Initial group the remaining grades are skewed toward A. In the Math 100 Prior group the remaining grades are skewed toward D. If the importance of the part-whole concept to success in math were accepted, then it would seem that students in Math 100 (Basic Math) who lack the concept might not be picking it up from current instruction methods. If they were, the Math 100 Prior success rate, measured by percentage of students at each letter grade, should match that of the Math 102 Initial group.

Another source provides insight into the extent of the lack of the part-whole concept. After providing in-depth instruction on the fundamental principles of arithmetic to pre-service teachers, Bloom and Zimmerman (2000) reported a statistically significant increase in the number who could correctly answer these questions without calculating them:

1. Without calculating an exact answer, select the best estimate for $\frac{12}{13} + \frac{7}{8}$.
   - A. 1
   - B. 2
   - C. 19
   - D. 21
   - E. I don't know

2. John had two pizzas. He gave one-third of one pizza to his sister and one-half to his brother. How much pizza remains?
   - A. More than 1 pizza
   - B. Less than 1 pizza
   - C. Exactly 1 pizza
   - D. No pizza

However, even after a semester of instruction, 20% of the pre-service teachers in the course still were unable to answer these problems correctly. Solving these questions requires part-whole understanding, a sense of the whole coexisting with all the parts.

Concluding Remarks

Evidence for lack of part-whole concept understanding as a hidden cause of innumeracy is reported here. The evidence is based on "data mining" and as such is subject to hidden traps. Therefore no confidence values (*p* values) have been attached to the numbers reported. Still, the data do show a trend of a higher final grade corresponding to better part-whole understanding at the start of the course.

In light of this evidence, it would seem that direct instruction of the part-whole concept should be part of adult basic math and algebra classes. This is not the case with any of the many adult math texts reviewed by this author. Part-whole understanding is assumed. Therefore, many adult math students continue to struggle and continue to fail. They do not know that it is this concept, rather than skills, that they are lacking; neither do the curriculum developers.

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