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AUTHOR	Brover, Charles; Deagan, Denise; Farina, Solange
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ABSTRACT

This paper explains the investigative attempts of The New York City Math Exchange Group (MEG) on elementary mathematics teachers' content knowledge in Adult Basic Education (ABE). The study is comparative in nature and took place in a workshop at the Adults Learning Maths Conference in Boston. The new members of the MEG professional development group were compared to the veteran members. It is observed that among the experienced MEG members, the ability to compute a division problem, create story problems, and reason mathematically and abstractly were higher than in other studies of U.S. teachers and the sample of new MEG members. It is concluded that it is only through comprehensive and ongoing staff development that all teachers can better understand, apply, and teach mathematics to their students. (SOE)

Why Understanding 1 ¾ ÷ ½ Matters to Math Reform: ABE Teachers Learn the Math They Teach

Charles Brover, Denise Deagan, and Solange Farina
The New York City Math Exchange Group, USA

It is widely recognized that making significant change in mathematics instruction requires profound change in teaching and teachers. The National Council of Teachers of Mathematics' *Professional Standards for Teaching Mathematics* (1991) declared that its guidelines for professional standards rest on two assumptions:

- Teachers are key figures in changing the ways in which mathematics is taught and learned in schools.
- Such changes require that teachers have long term support and adequate resources (NCTM, 1991, p. 2).

The first of these assumptions is self-evident. The second, however, remains a distant goal for most schools. The lack of long term support and adequate resources for teachers of mathematics is particularly evident in Adult Basic Education (ABE). Still, even in this environment of scarcity we should consider what models of professional development actually work best, deserve support, and will lead to the changes we seek. The seven-year experience of The New York City Math Exchange Group (MEG) and some recently published cross-cultural research suggest that teacher-researcher collaboratives may provide the best model for changing both teachers and teaching in ABE programs.

Despite all that has been written about the need for reform in mathematics education, the culture of teaching in the U.S. remains stubbornly unchanged (Hiebert, 1999; National Research Council, 1989). If teachers are key figures in changing the way mathematics is taught, the prospect for reform in ABE is particularly fraught with difficulties. Adult Basic Education teachers share the wider teaching culture of mathematics in the U.S. that emphasizes isolated procedural skills and repeated practice. The marginal status of the study of mathematics in many literacy programs and the non-math background of many ABE teachers called upon to provide math instruction compound the problem of professional development. The limited mathematical content and knowledge of ABE teachers constitutes a decisive barrier to meaningful reform because without it they lack the confidence and agility to make significant changes. Instead, this deficiency leads to restricted and reductionist views of mathematics and math teaching.

In this problematic context, MEG developed in the 1990s as a volunteer, ongoing teacher collaborative committed to improving mathematics instruction in the ABE classroom. MEG's mission is to help teachers learn more math, as "It seemed self-evident that if teachers didn't know and do math, they could not teach it effectively." (See Brover, Deagan, & Farina, 2000.) The acquisition of math content is generally viewed as a sequential and cumulative accomplishment most often measured by college coursework. As Thomas J. Cooney observes, the level of difficulty is conflated with the level of understanding (Cooney, 1994, p. 11). The question most often asked of teachers' math knowledge is: How far? But we should also ask: How deep? Does the successful completion of advanced coursework necessarily correlate to a profound understanding of fundamental mathematics and problem solving?

For MEG, developing a deeper understanding of the math we teach is organically connected to our mission of reforming classroom mathematics instruction. MEG seeks to effect change in the culturally embedded activities of teaching mathematics, so its model for professional development goes beyond "telling" and "showing" teachers what to do. "Changing the beliefs about mathematics teaching and learning that teachers possess requires giving them powerful experiences in mathematical thinking and conceptual understanding." "Professional development programs should actively engage teachers in doing mathematics, with the leaders facilitating experiences that model what teachers should do with their own pupils" (Hyde, Ormiston, & Hyde, 1994, p. 50). As an ongoing mathematical community, MEG bases its work on the idea that teachers can learn the way we expect our students to learn—by constructing mathematical knowledge and understanding socially. Although this model of learning is sometimes suggested for students, it is rarely recognized as a professional development model for teachers.

Two books published in 1999 engaged this question of the relationship of mathematics content and knowledge to pedagogy and change by stressing the need for teachers to have a deeper understanding of fundamental mathematics: *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*, by Liping Ma, and *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*, by James W. Stigler and James Hiebert. These books created controversy and interest in the wider mathematics education community not only because they closely examined the content-pedagogy connection, but also because they relied on cross-cultural data.

Liping Ma based her study on a small sample of interviews with U.S. elementary school teachers conducted as part of the 1980s study by the National Center for Research on Teacher Education (NCRTE); for the rest of her data she returned to interview teachers in China where she had been an elementary school teacher before becoming a professor at Berkeley. Stigler and Hiebert based their book on carefully observed and coded videotapes of classroom teaching that were part of the TIMSS research. At the heart of both of these books is the intuitive idea that knowing math deeply is critical to teaching math well. Content knowledge and confidence support flexibility and pedagogical effectiveness.

These books strongly suggest that U.S. teachers don't prepare and present lessons that are as rich, textured, coherent, and challenging as Asian teachers' lessons. Stigler and Hiebert emphasize the culture of teaching; they see in their video study "a distinctly American way of teaching, which differs markedly from the German way and from the Japanese way":

> What we can see clearly is that American mathematics teaching is extremely limited, focused for the most part on a very narrow band of procedural skills. Whether students are in rows working individually or sitting in groups, whether they have access to the latest technology or are working only with paper and pencil, they spend most of their time acquiring isolated skills through repeated practice. (pp. 10-11)

What's the problem in the U.S.? Ma says that "in the United States, it is widely accepted that elementary mathematics is 'basic,' superficial, and commonly understood. The data in this book explode this myth. Elementary mathematics is not superficial at all…" (p. 146). Ma says that Chinese teachers are more able to see the connectedness of math concepts and to draw from complex "knowledge packages" to inform their pedagogy. Unlike their U.S. counterparts, experienced Chinese elementary school teachers, she says, are more likely to possess "PUFM."

> Profound understanding of fundamental mathematics (PUFM) is more than a sound conceptual understanding of elementary mathematics—it is the awareness of the conceptual structure and basic attitudes of mathematics inherent in elementary mathematics and the ability to provide a foundation for that conceptual understanding and instill those basic attitudes in students. (p. 124)

Ma provides extensive reports from experienced Chinese teachers who see many different ways to understand a problem and present it effectively to students. When U.S. elementary teachers were asked to solve a subtraction problem by regrouping, for instance, they simply applied a rule about "borrowing" whereas Chinese teachers entertained different approaches and aimed for the understanding that the standard algorithm is rooted in number system and place value.

When U.S. teachers were asked to compute 1 ¾ ÷ ½ and to create a word problem they might use in class for that numerical representation, only 43 percent could compute correctly, and *none* could invent appropriate word problems. Their problems represented division by 2 or multiplication by ½; some teachers just gave up. It was clear that even those U.S. elementary school teachers who could do the computation did not understand the process of division of fractions. All of the Chinese teachers interviewed by Ma, on the other hand, were able to compute the problem and provide appropriate word problems representing the computation. Moreover, most of the Chinese teachers were able to demonstrate alternative computational approaches and invent problems that showed a deep understanding of different models of division.

MEG decided to research this question among our colleagues in ABE in New York this summer, and we brought a model of this mini-research project to our workshop at the Adults Learning Maths Conference 2000 in Boston in July. We posed to our colleagues Ma's problem about division of fractions.

In our mini-research we gave the problem models to two groups of MEG members. The first group was comprised of "new" MEG members (<1 year in MEG) who were relatively new to teaching math and/or participating in professional development in mathematics, as represented by MEG participation. The second group was comprised of more "veteran" MEG members (>5 years), who had much experience participating in and presenting staff development in mathematics.

Of the first group, 87.5% of the teachers were able to compute the problem, but only 25% could create an appropriate story problem, and only 12.5% could create more than one story problem illustrating more than one model of solving the problem. Of the 87.5% who could compute the problem, all but one teacher used the traditional "invert and multiply" algorithm, with little or no understanding of why this method worked. Of the second, more experienced MEG group, 100% could do the problem and create an appropriate story problem.

At the ALM Conference we presented the same two tasks ("Divide 1 ¾ by ½" and "Write an appropriate story problem") to 4 groups of teachers, researchers and practitioner-researchers in our workshop.

Each group worked collaboratively (as opposed to participants in Ma's research and in MEG's mini-research). Each group was able to compute the problem; some examples of story problems from each group are as follows:

Group 1
We have 1 ¾ pizzas left over from last night. If each person in our family likes to eat ½ a pizza, how many people can we serve?
Our challenge: Can we think of a situation where the remainder (of ½) matters?

Group 2
1. I want to put 1 ¾ liters into ½ liter bottles and I want to know how many bottles we need. Answer: 4
2. Mary has a piece of wood that is 1 ¾ ft. long. She wants to make signs that are ½ ft. long. How many signs can she make? Answer: 3
3. Sheila has 1 ¾ cans of cat food. If she gives her (fat) cat ½ can per meal, does she have enough food to give her cat 4 meals? Explain your answer. Answer: No

Group 3
1. I have 1 ¾ meters of ribbon. How many ½ meter pieces can be cut off? How many "full" pieces?
2. I have 1 ¾ acres of land to sell in plots for cash but zoning requires ½ acre plots.
   How many plots can I make? How many can I sell?

Group 4

1. A movie is $1\frac{3}{4}$ hours long, with $\frac{1}{2}$ hour parking time [per quarter]. How many quarters do I need?
2. Ribbon: $1\frac{3}{4}$ yard ribbon. How many $\frac{1}{2}$ yard pieces can I cut?

The measurement or quotitive model, represented by finding how many $\frac{1}{2}$ lengths there are in something $1\frac{3}{4}$ lengths long, was used in 89% of the problems. The partitive model, represented by finding how long the whole is if half a length is $1\frac{3}{4}$, was used in 11% of the problems. The product and factors method, represented by finding the length of a side of a $1\frac{3}{4}$ square foot rectangle if one side is $\frac{1}{2}$ foot long, was not used at all.

We found that among both the experienced MEG members and the self-selected ALM attendees, the abilities to compute the division problem, create story problems, and reason mathematically and abstractly were higher than in Ma's sample of U.S. teachers and our sample of new MEG members.

This suggests that it is only through comprehensive and ongoing staff development that *all* teachers can better understand, apply, and teach mathematics to their students. The Chinese teachers understand what many American teachers do not—that fractions and other "elementary" mathematics are *not* "basic" and "elementary" but represent rigorous and connecting strands of an integrated mathematics curriculum. By applying comparable intellectual rigor to these strands as to later "advanced" mathematics, students are able to build strong foundations and make connections in mathematics throughout their lives. This works not only in support of professional development for teachers, but in support of adult and elementary school learners, who too often get the message that they are incapable of understanding even "easy" mathematics, and who are not provided with the appropriately trained teachers necessary to explore and construct knowledge of complex subjects such as fractions and other "elementary" mathematics.

The books by Ma and by Stigler and Hiebert underscore the proposition that mathematics reform in ABE will require teachers to have a deeper understanding of fundamental mathematics. They also offer strong support for teacher-researcher collaboratives as an effective model to develop that understanding. Both books insist that for substantial change to reach the classroom, teachers must drive the engine of change. And they recognize substantial obstacles in the U.S. system: Stigler and Hiebert stipulate that "A requirement for beginning the change process is finding the time during the workweek for teachers to collaborate" (p. 144). "We must empower teachers to be leaders in this process" (p. 127). Ma notes that Asian teachers have much more opportunity than U.S. teachers do to collaborate with peers. Further, Chinese and Japanese teachers have much more non-teaching time. Chinese teachers spend most of the day working collaboratively to develop a deeper understanding of their specific subject matter and engage in "materials study." Japanese teachers work together on what they call "Lesson Study" (Stigler and Hiebert, 1999, pp. 110-127), in which they develop, edit, and polish particular lessons. According to Stigler and Hiebert, a Japanese teacher-researcher collaborative may spend an entire year developing a single lesson.

If the development of teachers' "PUFM" is critical for math reform in ABE, the prognosis for meaningful reform is grim but not altogether hopeless. Although many ABE teachers do not have sophisticated academic math experience, they may become better math teachers while teaching. For Asian teachers, significantly, profound understanding of fundamental mathematics develops mainly *after* they start their professional careers. "...[T]he key period during which Chinese teachers develop a *teacher's* subject matter knowledge of school mathematics is when they teach it—given that they have the motivation to improve their teaching and the opportunity to do so" (Ma, 1999, p.147). The same may well be true for ABE teachers *if teacher-researcher collaborative models of professional development gain long-term institutional support.*

Deepening teachers' understanding of fundamental mathematics is key to bringing reform into the classroom. This should be a central goal of mathematics professional development in ABE. Though teachers' efforts to pursue coursework in higher math education should be supported, it may not be the most productive path to meaningful improvement in ABE math teaching. The best opportunity to realize professional growth in ABE may be to put teacher-researcher collaboratives at the heart of professional development.

This implies bridging the gap between researcher and practitioner and raises a number of interesting research questions: What mathematics do ABE teachers know, and how does this knowledge affect their instructional flexibility and effectiveness? What mathematics do ABE teachers need to know and *understand* to be effective math instructors? There are many barriers to professional development in ABE, and the overarching question in the U.S. is political: Will support and resources be provided for the professional development of teachers whose students have been socially and economically marginalized?

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