The 27th Undergraduate Mathematics Teaching Conference took place in September of 2001 at the University of Birmingham. Major topics of the conference included preparing a teaching portfolio, engineering mathematics should be taught by engineers, issues in teaching discrete mathematics, action research, study skills, and issues for web-delivered assessment. Papers and presentations given at the conference include: (1) "Why Do Non-Mathematicians Need More Maths?" (Alison Wolf); (2) "Mathematics, Statistics and OR: Subject Provision Reviews, Benchmarking and Learning and Teaching" (Neville Davies); (3) "Computer-Based Learning and Assessment" (Neil Pitcher and Mike Barry); (4) "The Latex/PDF Route to Support Student Learning of Mathematics" (Robin Horan and Martin Lavelle); and (5) "The Lights Are Going Out All Over the UK" (Walter Middleton). Also included are reports of the working groups. (MVL)
Preparing a Teaching Portfolio –
supporting professionalism in mathematics teaching in HE

Engineering Mathematics should be taught by Engineers!

Issues in Teaching Discrete Mathematics

Action Research: Diagnostic Testing & follow-up support

Study Skills for Mathematics

Issues for Web-delivered Assessment

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Published by
University of Birmingham Press

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For more details about the Undergraduate Mathematics Teaching Conference, and for current information, visit our website:  www.umtc.ac.uk
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INTRODUCTION

In 2001, the twenty-seventh Undergraduate Mathematics Teaching Conference accepted the hospitality of The University of Birmingham for our annual meeting. We have been based at Sheffield Hallam for four successful and comfortable years, so the move came as quite a wrench. But we found we were exchanging one green and leafy campus for another, this time with a lake and canal included! Our times in Sheffield hold happy memories and we are grateful to colleagues there who put so much time and effort into supporting UMTC, but all good things are liable to change. Our initial experiences at Birmingham have been very positive, and UMTC is likely to stay there for the next few years at least.

We are most grateful to Professor Robert Curtis, the Head of Mathematics and Statistics at Birmingham for all the facilities he has made available to us. While we have been very comfortable housed this year in Chamberlain Hall for ‘bed and breakfast’ purposes, we have worked by day in the Watson Building, using the lecture rooms, syndicate rooms and computing facilities of the Maths Department.

Our new location has been considerably influenced by the fact that the University of Birmingham is the home of the LTSN Maths, Stats and OR Network. We feel that UMTC will gain much from a continuing close association with this national network of contacts in our subject area; we look forward to the possibilities of more collaboration. We are delighted that Ms Pam Bishop (Assistant Director of the LTSN Maths, Stats and OR Network) has agreed to come onto the committee of UMTC as an ‘ex officio’ member.

The plenary sessions, as usual, gave us high standards to follow and much food for thought. We were delighted that Professor Alison Wolf accepted our invitation to come up from The Institute of Education in London and talk on “Why do non-mathematicians need more Maths?”. This topic is close to the hearts of many of us. Professor Wolf discussed evidence which has been gathered concerning levels of competence in Mathematics, and how the present educational system serves people in our subject area. We were also extremely pleased that Professor Neville Davies came from Nottingham to speak on “Mathematics, Statistics and Operational Research – Subject Provision Review, Benchmarking and Teaching and Learning”, - a meaty and important subject, delivered with musical sound effects! Summaries of both these talks appear in these Proceedings.

As is traditional at UMTC, the Working Groups took up most of our time. This year several of the Groups involved innovative features. It was decided to run one Group, on ‘Preparing a teaching portfolio’, with an invited Expert to lead it. Dr Peter Kahn, from Manchester, came in this capacity; he is an old friend of UMTC and the Group gained much from his expertise and leadership. Another Group, on ‘Diagnostic testing and follow-up support’ was run for the purpose of setting up an extended programme of Action Research. So this Group only began its work at UMTC, and will be continuing it throughout the year. They have already been awarded a grant of £5000 from LTSN-MSOR to pursue the project (well done!) and will report back to UMTC 2002. A third Group directed its attention to reviewing and upgrading the booklet ‘Study Skills in Mathematics’, designed for distribution to students on Mathematics courses. The learning environment has changed so much since the original edition of this booklet was written,
and now includes a great deal of computer mediated material; it has become essential to make the booklet’s contents more up-to-date. 'Engineering Mathematics should be taught by Engineers' was the deliberately contentious title given to one of the Working Groups; they chose to address this ‘from both sides’, as if preparing for a formal debate. Other Groups focused their activity on, ‘Issues in teaching Discrete Mathematics’ and ‘Issues for Web-delivered Assessment’. Reports from the Groups are included in these Proceedings.

Other innovations at UMTC 2001 included deliberate links with activities immediately before and after the Conference. We were pleased that the Induction Meeting for new Mathematics Lecturers (run at Birmingham by LTSN-MSOR) occurred on the day we began, so that newly appointed colleagues could attend UMTC as a follow-on, and miss very little of our programme. In addition it was mutually beneficial that the MathsWise Users’ Group planned their Annual General Meeting to occur at Birmingham straight after UMTC 2001 finished; several colleagues were able to attend both events with maximum efficiency. Another new departure was the decision to invite an outside speaker to contribute to the Conference session for short presentations: Mr Selwyn van Zeller – with experience with ‘Maths Year 2000’, ‘Count on’ and SATRO - gave a brief talk entitled “Maths in a Suitcase” at which he displayed a fascinating selection of mathematical apparatus and activities.

Dr Dirk Hermans masterminded the smooth and efficient organisation of UMTC 2001. The transfer of the Conference to a fresh university venue made this a considerable task; I personally am most grateful to him, and I believe everyone else also recognises the superb job he has done! Lending administrative support throughout the planning period, as well as during the Conference, was Ms Sam McCauley. I thank Sam, too, for all her hard work and all the arrangements she made on our behalf, including (of course!) our trip to Cadbury World, and the Conference Dinner held in the historic surroundings of the Bonded Warehouse at Stourbridge. For several years Dr Neil Gordon has maintained the UMTC Website and the capability for on-line registration; this year these features were complemented by the collection and analysis of feedback on-line, in time for our final plenary session. This was put in place by Dr Joe Kyle (thank you!) and it certainly smartened-up our discussions and decision making.

UMTC 2001 saw a most encouraging rise in numbers to 46 participants, an increase of 52% on the figures for 2000 or 1999: may this trend continue! (We kept up our reputation for being an ‘international conference’ by welcoming this year three colleagues from Malta.) Many people have said how successful and enjoyable they found UMTC 2001; for this I thank everyone who contributed: Dirk and Sam especially, and all members of the Committee. I offer my good wishes to next year’s Chair, Dr David Pountney, and his organising Committee, hoping that UMTC will grow from strength to strength.

Patricia Egerton  
(Chair of UMTC 2001)  
University of Teesside  
March, 2002
The Plenary Talks
Why do non-mathematicians need more Maths?

Professor Alison Wolf
Institute of Education, University of London

Professor Alison Wolf is Professor of Education and Head of the Mathematical Sciences Group at the Institute of Education, University of London, and also Executive Director of the International Centre for Research in Education. Much of her current research deals with post-compulsory maths, and with education, training and the labour market. Her sharp focus, not only on Mathematics itself, but on the statistics of achievement levels and vocational needs, made her an especially welcome speaker at UMTC 2001.

In her talk Professor Wolf began by noting how rapidly the pyramid of students’ mathematical attainment narrows. In England at the present time there are around 600,000 16 year olds studying mathematics as part of their GCSE programme, and about 420,000 of them stay on in full-time education post-16. But only 55,000 students pass Maths at A level, which is 13% of those still in full-time education; and around 4000 achieve a degree in mathematics – which is a mere 1.5% of all graduates. From a broad base in compulsory education, achievement in mathematics is restricted to a small and specialised group of people at higher levels. This minimal involvement with ‘higher maths’ is despite the fact that in the 10 years from 1988 the percentage of students staying in the educational system to do A/AS levels increased by a third, from 27% to 36% of the age cohort.

We were shown detailed figures regarding mathematics results at GCSE, A level and degree level. In 1998, for example, approximately 670,000 candidates took Mathematics at GCSE (about 90% of the age cohort). However less than half of these passed with an A to C grade, and in fact only 1 in 6 had taken the ‘upper tier’ exam. This level of achievement is worrying. On the one hand, we in education continue to allow pupils’ morale to be knocked and the idea that ‘maths is hard’ to be perpetuated. On the other hand we see that a large proportion of those who attain this GCSE (itself a passport to many careers) have had very little exposure to mathematical thought processes beyond arithmetic. The figures for students offering Mathematics at A level have kept reasonably steady in absolute terms, but they show a depressing trend: in 1964 Mathematics accounted for 16.8% of all A level entries, but by 2000 this had dropped to 8.6%. In the five years up to 2000, the number of students graduating in England and Wales with a Mathematical Sciences degree rose from 3540 to 4090 (an increase of 15%), but as a percentage of all graduates this represented a fall from 1.8% to 1.5%. Over the years we are aware of the commitment of Governments to raise standards, and of many initiatives undertaken, including the National Curriculum, the National Numeracy Strategy, and variations of A/AS assessments. However all the figures show that current national attainment in mathematics, when compared with other subjects, is deteriorating.

Many of us suppose that the way to change these trends must lie with intervening in school mathematics, thus the supply of well qualified maths teachers is crucial. Professor Wolf showed us that there too there are problems. In the four years to 1998, the number of Mathematics graduates undertaking a PGCE course fell in absolute numbers from 514 to
308 (this represented a fall from 14.5% to 8.3% of all mathematics graduates). Where will the teachers come from to support mathematics learning in the next generation?

In other countries they have far greater pools of potential teachers, well qualified to pass on mathematical expertise. This is because the study of mathematics is accepted as a compulsory part of their post-16 educational systems. On the ‘academic track’ in continental Europe the integrated diplomas (such as Baccalaureate, Abitur) have mathematics as compulsory, with further maths as optional, and in North America the High School Diplomas have maths as a compulsory element. However in the UK the choice of A level subjects is completely unconstrained, and studying maths beyond GCSE is allowed to be a minority interest. In courses on the ‘vocational track’ in France, Italy and Spain, students still receive separate “abstract” mathematics teaching (especially in France there is a strong emphasis on general education as a universal entitlement). In vocational courses in Germany, Austria, Switzerland and the reformed Netherlands programmes the mathematics is taught ‘in context’ but separately. However for vocational courses in the UK there tends to be complete integration of the mathematics within the field of application: it is taught mainly by non-specialists, within other topics.

We must improve the quality and the quantity of the mathematics generally studied in this country post-16. The new AS level qualifications (including AS Use of Maths) may contribute to this, as may the free-standing Maths Units now available, but even these are perhaps not sufficiently serious approaches to change the national attitudes.

So why is it important to learn Mathematics? It is not for the sake of specific calculations or algorithms – in any context – but because ‘thinking mathematically’ is a transferable skill which promotes success in virtually all walks of life. We should all try to advertise widely the following Key Fact. In considering working adults with the same occupational status, who achieved the same GCSEs, the same number of A levels and the same class of degree, those who have Maths A level are paid, on average, 10% more than the rest. (The Maths A level has an insignificant effect on starting salaries: the positive returns cut in only some time later in the working career.) This is supported by NCDS sample data (people born in 1958): the ‘professional’ group are 18% of the total sample but include 30% of those having Maths A level. Professor Wolf emphasised that, holding other things equal, most people who report that their jobs require quantitative skills are paid at above average rates – but this is largely because they work in IT related areas. However in general most employees with Maths A level do not describe their work activities in mathematical terms - often they declare that they use very little maths in their workplace. Nonetheless the skills they have gained from the maths they have studied make them more effective at work, which is translated into greater personal remuneration.

Professor Wolf was very persuasive that non-mathematicians need – and deserve – more maths. In the UK we should be making the skills of mathematical thinking available more explicitly to more people: for the sake of their own personal achievement, and for the sake of the whole country, whose success depends on being competitive in the modern world.

*(Summarised from slides by PAE, with the approval of the Speaker. More details may be obtained from “The Maths We Need Now: Demands, deficits and remedies”, edited by Clare Tickly & Alison Wolf, published Institute of Education, University of London, 2000.)*
Mathematics, Statistics and OR: Subject Provision Reviews, Benchmarking and Learning & Teaching

Professor Neville Davies
RSS Centre for Statistical Education, Nottingham Trent

Professor Neville Davies is the Director of the Royal Statistical Society Centre for Statistical Education in Nottingham. He is also Deputy Director of the LTSN Maths, Stats and OR Network - so Birmingham was almost home ground for him! He brought a wealth of background experience to his talk, giving us insights into the working of national initiatives and also his concerns for the future. His presentation was detailed, thought provoking, and most enjoyable – because of his enthusiasm, and because of his unexpected use of music, including The Beatles and the Welsh National Anthem!

Professor Davies began by outlining aspects of his work at the RSS Centre. His plan for the talk was to look in detail at the QAA Review results, then consider Benchmarking, and finally the LTSN Maths, Stats and OR Network. (“Can we work it out….?”)

The Quality Assurance Agency’s mission is to ‘promote public confidence that quality of provision and standards of awards in HE are being safeguarded and enhanced’. In the Subject Provision Review for MSOR there were 87 reviewers involved, of whom 82 were male, 57 had PhDs and 21 were professors. Over half the reviewers came from ‘old’ universities, 41% from ‘new’, and about 4% each from Colleges of HE and industry. There were 71 institutions in England and Northern Ireland that had MSOR reviewed in 1998 – 2000: all but one of the HEFCE-funded old universities and 23 of the 35 new universities. The reviews concluded with 3% of departments scoring 24 (the maximum) and 56% of them deemed ‘excellent’ (scoring 22 – 24). Just how good was this?

The QAA reviewed six aspects of provision in each department: Curriculum design, content & organisation (CDCO), Teaching, learning & assessment (TLA), Student progression & achievement (SPA), Student support & guidance (SSG), and Learning resources (LR). Professor Davies analysed what we might learn from the overall results. He compared the old and new universities: the total scores appeared to have different distributions, but the chief (highly significant) difference was in the SPA aspect. Maybe this was due to the much lower A level average score of students in new universities? But new universities often give good ‘added value’. Professor Davies also compared MSOR reviews with other subjects. He supposed that Physics and Astronomy (P&A) might be fairly comparable: there were 41 P&A departments reviewed (34 old, 7 new), but 25% of them scored 24 and 80% of them were ‘excellent’ (22 – 24). Is P&A really ‘better’ than MSOR in universities? Professor Davies thought there might be true differences or maybe different criteria were being applied in the review process (or a combination). He looked further at other subjects, and concluded that there seem to be real differences between MSOR and P&A, Organismal Biosciences (OB), Medicine (M) and Psychology (Ps). Where only 3% of MSOR departments achieved the perfect 24, in P&A, OB, M and Ps the figure is 20% or more. For MSOR 56% were ‘excellent’ (22 – 24), but the figures for ‘excellent’ in P&A, OB and Ps were 85%, 71% and 75% respectively.
Was the QAA review in MSOR worth it? Departments said that the process took up much time (up to 2 years) and money (close to 1 person-year). Maybe there was some gain from the ‘bonding’, and tightening procedures, identifying good practice and reviewing courses. Some staff found the process character forming, and gained by thinking more deeply about course structure and appreciating quality assurance. But conversely, many departments thought the process was a waste of time: with teaching and research suffering before and during review, with problems from post-QAA exhaustion, and innovation being stifled. Individuals complained about stress, unsettling despondency and de-motivating cynicism. The modal score for MSOR was 22, so we may be seen as a failure overall; there are grave doubts whether MSOR teaching has been improved. Reviewers’ comments included criticism that MSOR staff engaged with the process less than others, that there was a low level of attention paid to ‘skills’ provision and that MSOR should interact more with the real world and advertise its value to society.

The overall aims of the QAA will only have been achieved if – at the end of the day – the student learning experience has been enhanced; at present this remains unclear. For MSOR there were many positive comments made: about good core provision, many options, generally good teaching, high rates of progression to employment, and strong academic and pastoral support. But Professor Davies reminded us of other issues we must continue to address: the decline in the mathematical preparedness of students, the range of competence within any group of students, and the fact that mathematical difficulties are often cited as reasons for failure.

The discussion then went on to Benchmarking. The QAA says this is “broad statements which represent general expectations about standards for the award of honours degrees” to “be developed by the academic community itself” with “subject associations and professional bodies playing a role in developing” them. The purpose is to inform the public about HE, guide intending students and employers and provide institutions with frameworks for developing programmes. Professor Davies stated that the academic community was widely cynical about the value of this exercise; the majority were against benchmarking including a prescriptive list of topics (as already developed in USA for Statistics). An MSOR document was published for formal consultation.

Whatever the outcomes, the original impulse for both QAA Review and Benchmarking was the improvement of learning and teaching. Another significant support for this is the LTSN Maths, Stats and OR Network. The LTSN’s aim is to promote high standards in the learning and teaching of Maths, Stats and OR by encouraging knowledge exchange, innovation and enterprise, leading to an enhancement of the learning experience for students. It encourages networking, provides a focal point for sharing innovation and good practice, disseminates information and supports and enhances academic practice. There are many ongoing activities (including publishing, workshops, conferences and web developments). LTSN MSOR is proactively addressing the issues raised in the QAA reviews and the matter of ‘standards’ (at the heart of Benchmarking); it also seeks a higher profile for educational research. Professor Davies recognised that ‘times are hard’ at present for our subject area (fewer students, reprefiling of departments, the lack of attraction in a career teaching mathematics), but thought that the LTSN does give us a gleam of hope. (“It’s been a hard day’s night.....”)

(Summarised from slides by PAE, with the approval of the Speaker.)
Presentations by Delegates
Computer-based Learning and Assessment: 
Sharing Experiences and Resources

Dr Neil Pitcher, University of Paisley 
and Dr Mike Barry, University of Bristol

Neil Pitcher began by introducing 'Mathwise', a computer based resource which can be shared by all mathematics lecturers in Higher Education. He emphasised that Mathwise is not a general solution for all teaching problems, but that it can be used for assessments with instant feedback and for a variety of directed and planned tasks.

There are several aspects to the value to be obtained from using Mathwise, with its interactive features: experimental tasks can be set, lessons can be given, reading can be set for students (eg for Maths in Society). Using Mathwise in a supported way, students can be trained in the use of technology, and also encouraged to develop ‘deep’ attitudes to learning, looking for connections and the underlying structure in their subject. This contrasts with those who have a merely ‘strategic’ approach (the priority is to pass the exam), or a ‘surface’ approach where facts appear unconnected (Entwistle's classification).

There is continuing research on the efficacy of Mathwise. Judy Goldfinch (Napier University) has shown that the ‘surface’ approach decreases most where students rely heavily on Computer-based learning, that strategic learners are challenged by the open-ended activities, and that the Computer-based learning experience is seen to be positive for students who are ‘growing’. Neil would be pleased to hear from any other colleagues who are evaluating the use their students make of Mathwise.

It is a fact that more students than ever now take mathematics as an integral part of their degree programmes. Sadly, teaching resources in universities do not always increase in proportion to the demand. Mike Barry suggested that some of the additional burden might be alleviated by sharing assessment material and resources, particularly with regard to first year Mathematics.

In putting the case firstly against, and then for resource sharing, Mike suggested that practical, usable, systematic and transferable Computer Aided Assessment (CAA) might tip the balance in favour. A large structured database of questions, with a full range of field descriptors, would be of considerable inter-institutional value in Higher Education provided it was robust enough.

A start in generating such a database has been made at Bristol University with the ‘Test and Learn’ (TAL) system of multiple-choice questions. Lecturers can set their own tests in accordance with a well-defined menu of topic, difficulty and execution time, all of which are objectively defined. A collective aim eventually must be to have a versatile WEB-based system which lecturers can contribute to, by adding material, and share. Once generated it would make sense for LTSN/MSOR to host this, but we need to work at getting it together first of all.
The LateX / PDF route to support student learning of Mathematics

Dr Robin Horan and Dr Martin Lavelle,
University of Plymouth

Robin Horan started this talk by stating aims for producing web-based support materials for students learning mathematics – that the tools should be easy to access, easy to use and affordable, and that the software should be portable and platform-independent giving professional quality presentation. Possible routes for this include HTML and LateX.

HTML has the advantage that it is free, but it is bad at Mathematics! Other software is needed, and the ‘maths’ is imported as graphics (since MathML is not yet ready). In addition, downloading (linking) can be slow, especially from home.

In contrast, LateX is a world standard for typesetting for technical documentation. TeX is a formatting programme which produces book quality text (it is not WYSIWYG!), and LateX simplifies the use of TeX, giving many powerful features for cross-referencing, equation numbering, citations, etc. Excellent editors (eg WinEdt) make it simpler to use.

For putting mathematics onto the Web, LateX output can be converted into Portable Document Format (PDF) files. (PDF files are electronic ‘documents’ – with hypertext properties - which maintain a fixed appearance.) These PDF files can contain professional mathematics and interactive features, such as links between problems and their solutions within the document, and the possibility of generating exercises and quizzes. PDF files can be read using the (free) Acrobat Reader software.

Robin showed some impressively successful results of using LateX for specialised mathematical notation. He considers that LateX is a very suitable tool for putting mathematical lecture notes, questions sheets and other handouts onto the Web. In addition it can be used to construct packages with which students can reinforce their understanding of material and answer quizzes to test themselves, obtaining immediate feedback.

His summary was that LateX gives high quality output, the PDF gives portability and interactive features, and all the software is free or inexpensive. Acrobat is free also (students can use it at home), and it downloads at once, with immediate response. This way of communicating Mathematics on the Web is ‘future-proof’, since the LateX route to MathML is under development. Moreover, students use it and students like it!
The Lights are going out all over the UK

Walter Middleton, University of Sunderland

Walter Middleton was prompted by the recent deletion of undergraduate mathematics programmes from Sunderland's portfolio to investigate the health and viability of programmes in 'maths' nationally. Anecdotal evidence suggested that problems exist and were increasing, so he carried out a survey by questionnaire. In his presentation, Walter summarised the results of his survey.

130 questionnaires were sent out, and 62 responses were received (24 from post- and 38 from pre-1992 universities). From the 'tick-boxes' Walter gained quantitative data under 8 headings. He also elicited many comments, whose overall 'flavour' is included below.

1. **Service teaching**
   It is clear that many maths departments are seeing a steady decline of their 'service' work. A few are gaining some hours in particular cases, and one reported that
   - "our increase in teaching came about via a deliberate attempt to do more research with engineering colleagues and by putting on courses that appealed directly to students over the heads of colleagues. It is very hard for teaching to be cut if it is the best that the students receive."
   However, overall there seems to be a
   - "gradual erosion of students opting for 'skill acquisition' courses such as Mathematics and Statistics
   - "steady decline in service teaching as other areas recruit staff to deliver courses in-house".

2. **New courses and course closures**
   Walter reported just 2 comments relating to new courses. One new module had been developed with 30% maths (ie ~12 hours). Actuarial Science was cited as a growth area.
   Several closures of 'maths' courses were reported. An HNC/D course closed because
   - "it was no longer seen as appropriate to a 'genuine university'".
   Some universities reported degree closures due to poor recruitment, and many others said that courses were under threat. Some Statistics courses had been abolished or suspended, but one university reported that with the closure of their Mathematics degree programme,
   - "The future, if any, is seen in Statistics".

3. **Institutional and faculty reorganisation**
   Walter received comments that many institutions are in the process of, or considering, reorganisation involving maths/stats/OR. Typically maths/stats/OR will combine with Computing or Engineering; expectations of the results vary.

4. **Staff age profile**
   Comments relating to the age profile of 'maths' staff included that the
   - "number of permanent staff has reduced from >20 to 10 over the last 10 years".
   However another university said it had recruited 3 'young' staff since January 1999.

5. **Resources and loss of autonomy**
   Different comments came from departments now combined with Schools of Computing; including increased computing resources, improvements in teaching accommodation and
library resources, but a loss of a Computing Officer. A loss of autonomy is felt because the Head is no longer 'of the subject area'. That budgets are now controlled by the School is mostly seen to be beneficial.

6. Quality and quantity of applicants
There were widely diverging comments about changes in quantity/quality of applicants. In one university the entry A level grades had risen recently but, even so, the students' maths background is less reliable and there are alarming gaps in their knowledge. Another university had their 'honours maths' FTEs doubled over the past 3 years. A third reported an increase in numbers for BSc/Mmath, and an increase in quality for 'joint honours'.

7. Career prospects
Walter received many comments on the likely effects that changes will have on the career prospects of young (< 30 years) staff. On the one hand:
- "Probably quite good, if mobile";
- "Career prospects have improved since 1995";
- "Employment prospects are good for top calibre people in a research driven department. Our problem is that salaries are too low to attract good PhDs into university teaching".

But on the other hand:
- "There is little chance of promotion for the younger members of staff."
- "Extremely bleak – no investment is planned in mathematics in the foreseeable future";
- "No real career prospects";
- "University is seeking to reduce staffing in Maths/Stats with an occasional RAE strategic appointment";
- "Not likely to have staff in this age range in the foreseeable future".

8. Additional comments
Responses indicate that there are still hopeful departments:
- "We may or may not be successful but we are trying to overcome perceived long-term threats to service teaching by developing a strong UG degree and using this to enter joint honours arrangements with other departments".

However the majority of comments indicate very low morale:
- "For my university the future without maths is extremely bleak. This will begin to hit engineering in 2 – 3 years time, and post-graduate research will feel the effects about the same time";
- "Within 6 weeks of two TQA visits (22 and 24 points) it was decided to close the Division of Maths/Stats. This was resisted for 12 months but the lack of undergraduates made closure inevitable. Mathematics is destined to become the preserve of a few dedicated specialists – in time very few people will be able to use algebra or the calculus";
- "The (maths) problem is the low esteem in which it is held by many 16-18 year olds. It is difficult and the purer aspects are seen as irrelevant."
- "Successive governments have failed to understand how changes in secondary school maths have adversely affected degree courses in maths, engineering and science."

Conclusions
Walter proposes that the health of 'maths' in UK universities is not satisfactory. Despite some bright spots, the majority of responses are not optimistic. His figures on staff ages indicate ~49% aged 50+. This, with the current rate of course closures and the negative comments about career prospects, implies that in ~5 years time the majority who currently teach 'maths' in HE will leave the profession. Given the time lag between action and effect it may now be too late to rectify the situation. The future does indeed appear bleak.

[For detailed results of the survey, including the figures he obtained, please contact Walter Middleton directly.]
Reports from the Working Groups
Working Group Brief: Preparing a Portfolio – supporting professionalism in mathematics teaching in HE

Introduction
The profile of teaching in higher education is growing in prominence. Professional development in support of teaching is now firmly on the agenda in higher education. This is particularly evident in the Teaching and Learning Support Network and the Institute for Learning and Teaching (ILT). Additionally, attention has been devoted to rewarding lecturers who display excellence in teaching, through the National Teaching Fellowship Scheme, institutional learning and teaching strategies and the current HEFCE initiative on performance-related pay.

Teaching is also a growing concern within the mathematics community in higher education. Mathematicians are only too aware of the challenges posed by such factors as the changing entry profile of new students, the need to attract sufficient numbers to the study of mathematics and the role of technology in teaching. For departments to respond to these challenges effectively, the staff who take the lead must have their professional development supported and be adequately rewarded for their leadership.

In recent years a new vehicle has been established within the higher education sector to assess expertise in teaching — the 'teaching portfolio'. A teaching portfolio comprises a body of evidence related to an individual's expertise in teaching and a commentary on that evidence. Portfolios serve a variety of purposes, including fostering and demonstrating professional development. They may also underpin claims by lecturers for excellence in their teaching.

Mathematics is a discipline where the gathering of the evidence required for a portfolio might be seen as an alien activity. The mindset required to evidence one's own expertise is clearly a step removed from mathematical activity. However, it is important that mathematics lecturers also are able to benefit from the rewards and opportunities on offer in the new climate in higher education. And many of these rewards and opportunities are made available on the basis of a teaching portfolio.

This workshop will take a dual focus. Firstly it will seek to help the members of the working group to develop a portfolio of their own. Secondly it will lead to a report which will assist other mathematics lecturers to develop portfolios. The chair of the workshop has already been nominated: he has considerable experience of supporting staff in the development of teaching portfolios.

The workshop will be relevant to lecturers who are:
- required to submit a portfolio as part of their initial professional development;
- constructing a case for promotion;
- either seeking to join or maintain membership of ILT;
- interested in facilitating their own continuing professional development;
- supporting other lecturers to develop portfolios;
- interested in helping to raise the profile of teaching within the mathematics community in higher education.

Remit for the construction of teaching portfolios
The workshop will begin with portfolio building and reflection on the process. Opportunities will be made available for participants to comment on each other's work as appropriate.

Participants are advised to bring with them materials that will aid the construction of a teaching portfolio of their own. (Contact Peter.Kahn@man.ac.uk for advice on what to bring.) Partially or fully constructed portfolios will be of particular value. Participants may wish to refer to advice on portfolio construction before attending the workshop: http://cel.net.ac.uk/se/portfolio.html or http://www.lgu.ac.uk/deliberations/portfolios/ICED_workshop/seldin_book.html.
Remit for the Report
For drafting the Report the following questions may be of assistance.
- How can mathematicians best be supported in the process of developing a portfolio?
- What fosters reflection on the practice of teaching mathematics?
- What kinds of systematic approaches might aid portfolio construction?
- What writing skills might be of assistance?
- How does the purpose to which a portfolio is put affect its construction?
- How should portfolios draw on the educational literature on undergraduate mathematics education?
- How does the nature of both mathematics and the teaching of mathematics affect portfolio building?
- Would it be appropriate to include extracts from portfolios created by the members of the workshop within the report?
- How can we help to ensure that portfolios provide evidence that is reliable and valid?

It is expected that the report will draw on the practical experience of building portfolios that members of the working group will engage in during the workshop itself.
Preparing a teaching portfolio –
supporting professionalism in mathematics teaching in HE

Members of the Working Group

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1. Introduction

Many of the staff of UK mathematics departments now wish for one reason or another to produce a ‘teaching portfolio’—a body of evidence related to their experience and expertise in teaching, and a reflective commentary on that evidence. The preparation and updating of such portfolios can allow individual lecturers to document and plan their professional development; to provide reasoned cases for appointment or promotion; or to gain and retain membership of professional bodies and institutes. In many cases, however, a portfolio will not merely serve the author themselves, but will also lead to developments in the support that is offered to learners.

In this paper we set out some of the characteristics of these different sorts of portfolio, and outline a process by which they might be prepared and enhanced, illustrating some of the steps of the process by means of examples. We further suggest some resources for those engaged in preparing a portfolio.

We are convinced of the value of developing a culture of ‘reflective practice’ in HE mathematics teaching, and believe that keeping current a personal portfolio can be helpful in achieving this. We have doubts, however, whether the external imposition of this task on younger staff without sufficient support and resources is likely to be useful. Many staff presently engaged in portfolio construction are doing so as a means of becoming members of the ILT; we take no view of the desirability or otherwise of such membership — but hope that our paper will be of use to colleagues who are so engaged.
The preparation of a teaching portfolio may seem initially to take mathematics teachers a long way from their central concerns; certainly we do not think that there is much mathematics involved — although perhaps a higher level of statistical sophistication than usually shown by Deans of Teaching might be thought appropriate! However, the work as described below involves many of the usual processes found in teaching: in particular, the need for clarity of thought about our intentions; for careful reflection (perhaps in cooperation with colleagues) on the outcomes; a willingness to modify and adapt; and an effort to communicate with the intended audience. We emphasise the need for reflective commentary. A portfolio is never merely a diary or a list of courses given or staff seminars attended; reflection on these from a variety of perspectives (especially the learners’ perspective), and realistic future plans, are integral to the enterprise. Amongst these perspectives is, of course, that of the professional educational world. A previous UMTC working group highlighted the communication gap between mathematics lecturers and educational researchers; but mathematics lecturers preparing portfolios which will be read (and assessed?) by those with an education background must bear this factor in mind.

2. Support for the writing of a teaching portfolio

The demands on university lecturers are increasing and the construction and maintenance of a teaching portfolio can become very time consuming. On this note, it should be recognised that time is an issue with lecturers and keeping a teaching portfolio should be a recognised administrative duty for which points are given. The amount of time and, in particular, the initial start up of a portfolio can certainly be reduced with existing materials and the introduction of possible new ones.

The ILT (see www.ilt.ac.uk ) provides information on portfolios, but this is limited and directed towards ILT application. Existing conferences such as the UMTC offer a good start. Fry and Ketteridge (1999) provide a good introduction with examples from two universities. The webpage www.Deliberations.ac.uk provides support for lecturers on most teaching matters. Nottingham Trent University provides many courses and online resources, see www.education.ntu.ac.uk.

Most universities provide staff development including, often, induction courses on portfolio construction. Mentors for new lecturers could provide help, although this requires the mentor to have such knowledge. This could easily be overcome with the introduction of a recognised portfolio aid within the department. This may be more useful than someone from outside the department, who may have little knowledge of your teaching field.

Possible useful introductions could include introductory documentation giving a basic outline of what a portfolio should consist of, with examples and basic practices that one should undertake when constructing and analysing teaching methods. This would make the portfolio construction an ongoing development, as it should be.

Finally, a regular induction course could be given. This could take the form of a teaching and research development day, where issues of learning and teaching are considered along with issues of grant and research proposals (both essential to university lecturers).
3. The process of preparing a portfolio

We outline here a six stage process of preparing a portfolio. While this process will clearly not fit all of the different types of portfolio that are possible, it highlights many of the features that are common to preparing a portfolio.

A. Clarify the nature of the portfolio

It is important when preparing a portfolio to be clear about its nature. A full portfolio will usually require a body of evidence and a commentary upon that evidence. Narrower portfolios may only require a commentary on your practice. Portfolios may be used for:

- initial professional development (making a claim for competence),
- continuing professional development (claiming or facilitating development),
- presenting a case for promotion (making a claim for excellence),
- applying for membership of the ILT (making a claim for competence).

The following case study provides an example of a more personal type of portfolio, intended for the development of the author.

Case study: A Teaching Portfolio as Continuing Professional Development

"I see teaching as a very personal activity embedded within numerous constraints that are capable of translating my thinking about teaching into ‘unrecognisable’ and dubious practices. These tensions between my thinking and practices are an undoubtable source of frustration - at times I just cannot fathom that someone who is doing a PhD in Education and who has lectured B.Ed students in Mathematics Education can feel so insecure about one’s own teaching. I see that a possible way to ease these tensions is to reflect upon my own practices in a systematic manner. Consequently, my interest in Teaching Portfolios does not emerge from any need to come up with a persuasive statement about my teaching competence (as some teachers seeking a career promotion are likely to do), but simply to record, understand better and hopefully improve my teaching. One may refer to such a portfolio as a means of continuing professional development. In other words, I am interested in a Teaching Portfolio that facilitates my professional development whilst acknowledging the constraints (e.g. top-down decisions, lack of resources, prescribed courses, large class sizes and student’s lack of motivation, just to mention a few!) with which I have to deal on a daily basis."

Sample Introduction to a Teaching Portfolio as Continuing Professional Development

"Teaching Portfolios (TP) are still a recent phenomenon in Malta and still do not have any ‘real’ currency for career promotion. However, I am aware that Student Portfolios are becoming a reality in some pilot schools and selected university departments at the University of Malta. My idea is to use this TP as a tool for continuing professional development. In particular, I am interested in reflecting upon and improving my teaching practices within my college context - the pre-university sixth form college of the University of Malta. This awareness arises from my current doctoral studies at the University of Nottingham in ‘classroom assessment within a context’, namely departmental, college and national policies and practices with regards to teaching, learning and assessment.”
“My research has involved observing my colleagues teach and follow-up interviews in relation to their thinking and teaching practices. I have focused on teaching, learning, the nature of mathematics and classroom assessment. Whilst observing my colleagues teach I distinctly remember taking up various roles, such as researcher (the most important one given the nature of my endeavour), student, teacher and even evaluator (even though I tried very hard to ignore this aspect!). Apart from providing my research data, the observations and interviews shed rich insights on my own teaching practices. It didn’t take me long to see that what had started as an external journey gradually developed into a very personal reflective experience. Seeing others teach and hear them air their views and practices has been a rather eye-opening experience for me - for the first time I could move back and see things from a distance. I definitely did not like all the things I was learning about myself. For instance, I became increasingly aware that my practices and thinking are frequently in direct contrast and that, in particular, my classroom practices are largely governed by the local highly summative high-stake external examination system. As I plodded further on in my research, I felt the pressing need to explore my own thinking and practices. I feel that a TP offers me an excellent opportunity to reflect in a systematic manner about my teaching - a holistic record that I hope would help me to understand and improve my teaching.”

B. Decide on specific criteria your portfolio needs to address

Once the nature of the portfolio has been clarified, it may be helpful to consider more specific criteria that the portfolio is seeking to address. This will both guide the preparation of the portfolio and provide a basis for subsequent analysis to indicate whether initial drafts of the portfolio are indeed fit for the stated purpose.

General criteria
- **Fitness for purpose.** The portfolio’s purpose determines its content; it must meet all of the criteria by which it will be judged. Appropriate claims are made for your practice.
- **Reliability.** Each claim for competence, development or excellence is substantiated with a sufficient range of examples (cited or evidenced as appropriate) or by reference to some kind of external verification (e.g. peer review, reference, student feedback, publications, action research, TQA, external examiner’s report, employers, award/prize).
- **Writing style.** The portfolio is appropriately structured in the light of its purpose. The writing is clear and easy to understand. Someone else would be able to follow your writing. The writing emphasises what is important in the portfolio.

Examples of specific criteria
- **Critical evaluation** in a ‘Portfolio for initial professional development’. The portfolio provides sufficient evidence of a critical approach to your existing practice. Appropriate suggestions are made to adapt your practice in the future.
- **Balance** in an ‘Application to join the ILT via the Initial Entry Route for Experienced Staff’. An appropriate balance exists between describing how you carry out your practice and justifying why you carry it out in the way that you do.
- **Development of teaching activity** in a ‘case for promotion’. The portfolio demonstrates a commitment to and expertise in the development of teaching activity.
- **Action planning** in a ‘Portfolio for continuing professional development’. Goals for continuing professional development are clearly stated and are specific, measurable, achievable, realistic and time-bound.
### C. Collect/decide on relevant information

Sources of information include material from oneself, material from others, outcomes of student learning. We provide here some suggestions for the sources of evidence on which lecturers might draw for their portfolio.

<table>
<thead>
<tr>
<th>Section</th>
<th>Examples of Sources and Evidence</th>
</tr>
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</table>
| Teaching Responsibilities | - Statement of the modules and level taught  
- Number of modules led  
- Number of students in each module |
| Curriculum Content | - Course documents (e.g., Program of study, Module plans)  
- Module description booklets  
- External Examiner’s comments (e.g., coverage of syllabus, etc) |
| Teaching Approaches | - Lecture and seminar/tutorial outlines  
- Strategy for building team work, whole classroom discussion, etc.  
- Strategy for using different learning resources such as: question sheets, the internet, distance learning materials, etc.  
- Student feedback forms  
- Peer review  
- Video/audio tapes of your own teaching  
- Staff appraisals forms  
- Progression rate and evidence of changes over time |
| Intended outcomes of teaching/learning | - Module description booklets (statement of teaching/learning aims)  
- Module review |
| Development and Support of Learning | - Module description booklets  
- Study guides  
- Handouts  
- ITC resources  
- Distance learning materials |
| Students’ assessment and feedback to learners | - Module description booklets  
- Comments from members of the course team (e.g., moderation)  
- Student feedback forms  
- Informal assessment (e.g., student performance in seminar/tutorial, presentation of a solution, etc)  
- Samples of examination papers/coursework/dissertation/essays  
- External examiner’s comments  
- Sample of students’ marked assignments |
D. Decide on a structure for the portfolio

The structure may be determined for you but, where there is choice, possible structures include a case study approach, commentary citing evidence in an appendix, commentary that is presented alongside the evidence and commentary that cites evidence but is not accompanied by evidence. In terms of structuring the commentary, or the evidence, various structures are again possible. For instance, one systematic approach to writing the portfolio could be to adopt the five headings used by the ILT (see www.ilt.ac.uk), namely:

- teaching and the support of learning,
- contribution to the design and planning of learning activities,
- assessment and giving feedback to students,
- development of effective learning environments and student learning support systems,
- reflective practice and personal development.

E. Draft individual sections of the portfolio

At this stage of the process the main issue to consider is drafting the reflective commentary. In many ways this text constitutes the heart of a portfolio. The short guide to drafting reflective text given below is based on the finding that a structured approach is invaluable. At the heart of the structure followed here is the distinction between description and reflection. The short guide is followed by a case study drawn from a portfolio presented as evidence of initial professional development. While the theoretical perspectives drawn on within the case study may go beyond what is required for many portfolios, it is valuable to realise that insights into one's own practice should be linked to the insights of others. This can occur not only through drawing on theoretical perspectives, but also through peer review, mentoring and reports of good practice.

Describing your practice

Clearly, there is a need for material upon which to reflect — you will need to provide a clear description of the relevant activity.

- Draw on a variety of sources of evidence — describe your own recollections of how the activity unfolded, the nature of any material given to the students, the notes you made in preparation for a tutorial and so on.
Describe the activity from a variety of perspectives, not just from your own perspective. Draw on evidence from peer review or student feedback. How did the students react?

Are there any critical incidents in your teaching or supporting learning on which to focus? It is valuable to look at notable successes or failures.

Stimulating reflection on your practice
Once you have generated a suitable description of the relevant activity, one of the most effective ways of stimulating reflection is to ask questions about the description:

- **Evaluation**
  - How effective was the activity?
  - What were its outcomes?
  - What were the positive and negative features of the activity?

- **Understanding**
  - Why did I carry out the activity the way that I did?
  - How did an understanding of student learning inform my practice?
  - Why was the activity appropriate to the educational context?

- **Future practice**
  To provide such guidance for future practice, your reflection will need to address questions such as: What will I change in my practice as a result of this reflection?

Grounding reflection in practice
It is important to retain a clear link with the original description of any activity. You may make a general statement about the way you undertake an activity - specific examples will be required for someone else to judge whether you have made a reasonable generalisation. Claims that you make about your activity need to be grounded in the description.

Case study: Developing problem-solving skills in applied maths & theoretical physics
(by Francesca O’Rourke, Queen’s University, Belfast)

Introduction

“The module that I taught in the first semester of the academic year 2000/1 was a course entitled: Vector Algebra and Dynamics. This module introduced the students to essential techniques used in applied mathematics: namely, differential equations and vectors. These techniques are applied to physics related problems later in the course.

This was my first time teaching this course, and I found that many students had great difficulty in relating and applying the mathematical techniques that they had learnt in the first part of the course, to the physics problems in the second part of the course.”

“Through discussions with the students, I have tried to identify whether their difficulties are associated with learning issues in mathematics, or physics, or both. Having obtained a greater insight and knowledge into the problems experienced by the students, I have reflected on some possible changes that could be made in the teaching of this module. In this case study I will attempt to show how I have tried to bridge the gap, in helping the students to make the ‘intuitive jump’ between linking the mathematical theory they have covered in lectures, to the representations of reality in problem solving.”
Problems that students encounter

"In the educational literature Schoenfield (1985) highlights many of the common difficulties that students experience in mathematical problem solving. These are that

- students do not extract all the relevant information given in the question,
- students are impulsive in that they embark on the first method that occurs to them,
- students often have difficulty in working out a methodical and logical approach to the problem, instead relying on poorly planned out approaches,
- whilst carrying out their approach they do not monitor their work, making it hard for them to see if they are on the right path.

Through interviewing the students, it became clear that many of them had difficulties with mathematical methods, many of them experienced 'physics related' cognitive difficulties in the way that they attempted to solve the problem. In order to try and ascertain where these difficulties arose, I broke down the process into a number of steps given below.

Process involved from problem to solution:
1. visualise the physical situation in the question,
2. draw a representative diagram of the situation,
3. organise a planned, well thought-out approach to the problem,
4. formulate the problem to be solved in mathematical language,
5. check each stage of the work for errors in reasoning and number crunching, before moving to the next stage,
6. reflect and generalise for how the approach could be applied in a new context."

Characteristics of surface and deep learning approaches

"It is extremely difficult to progress beyond steps 1, 2 and 3 above without a clear understanding of both the mathematical and physical principles involved in solving the problem. In solving problems in this module, I found that deep learners tend to 'work forwards' in that they decide what principle of physics is required and then use this to derive the equations from which they then calculate the unknown variables. On the other hand I found that surface learners tend to 'work backwards' looking for an equation that contained the unknown variable and variables that they had been given or could calculate.

The student adopting a surface approach is intent only on coping with the module requirements, content to memorise facts routinely for exams without deep understanding. Surface learning is characterised by this 'external' emphasis defined by Ramsden (1992) as 'Demands of assessments, knowledge cut off from everyday reality'. It is clear that the student adopting a surface approach will have difficulty progressing beyond steps 3 and 4.

"The student applying the deep approach to learning is motivated by a desire to understand the material in depth for himself. Deep learning is associated with this 'internal' emphasis, described by Entwistle and Marton (1984) as 'A window through which aspects of reality become visible, and more intelligible'. Deep approaches to learning are connected to a strong knowledge base, ability to apply one's own and other people's ideas to new situations and integration of knowledge (Ramsden 1992). These students will be less likely to experience difficulty with steps 3, 4, 5 and 6."
"There is quite a lot of evidence in the literature to show that students in mathematics and physics adopt the surface learning technique. For example Crawford et al (1994) reports that 82% of university mathematics undergraduate learn by rote memorisation. From my own student questionnaire I found that 80% of students reported that they learned by memorising the material. However surface learning is at best about quantity without quality as Biggs (1989) puts it:

'Rote learning scientific formulae may be one of the things that scientists do, but it is not the way that scientists think.'

Since student approaches to learning are heavily influenced by the teaching practices of their lecturers and their experiences in previous educational setting (Ramsden 1992), it is the responsibility of academic staff to ensure that teaching methods and assessment criteria foster a deep approach to learning that will enable students to acquire expertise in their subject area.”

Helping students to develop problem-solving strategies in Applied Mathematics and Theoretical Physics

“It is only possible to have a good understanding of applied mathematics and theoretical physics by spending a long time solving a wide variety of problems. The tutorial sessions are a vital mechanism in helping students develop the necessary problem-solving skills. The following approaches have been implemented in my tutorials to develop these problem-solving skills.

Algorithms and heuristic guides
Closed problems can be tackled using algorithms. Algorithms are set procedures that, if followed will inevitably lead to the right solution of the particular problem considered. Heuristics on the other hand are a better guide for novel problems and equip students for dealing with unfamiliar situations. Heuristic guides are usually combinations of hints, prompts, cues and general ideas for solving problems. They guide students without telling them what to do. Unlike algorithms, heuristic guides cannot guide one infallibly to a solution but they help reduce the complexity and ambiguity of solving the problem.

Reflection and abstraction
In solving problems in mathematical physics students need to remember the situations in which various problem solving techniques are applicable, and cues that will enable them to recognise other situations where they can apply their methods successfully to different problems. Reflective practice helps students develop their own strategies and adapt them later or invent new ones to solve problems (Brown et al 1997).”

Addressing the need to develop the role of intuition in problem solving

“Many students when interviewed felt that when given an algorithm for problem solving, they could apply it easily to the problem in hand and obtain the correct solution. They however found more difficulty with heuristic guides. This is because often students cannot link the mathematics to the representations of reality. I would relate this difficulty in transferring knowledge learnt to a new context due to a lack of ‘intuition’ or ‘insight’ as discussed by Burton (1999a,b).”
“Burton (1999b) describes intuition as ‘having a feeling for how things connect together’. The studies of Burton (1999a, b) on the role of intuition in mathematics education found that many mathematicians relate intuition to knowledge and experience. In my own opinion it is this hidden link that enables a student to abstract principles and apply them in a new context. It therefore follows that we need to build the role of intuition into teaching and assessment methods through reflective learning for transfer of knowledge to occur.

Concept maps
One way of developing an intuitive feel for the appropriate transfer of knowledge to problem-solving is to use concept maps. In particular students could be invited to prepare and justify a concept map of a particular problem. In particular they should be asked to explain the connections between various sections of their concept map. This method would enable them to identify linkages between other parts of the course and help them to develop their own ‘insight’ to acquire more proficient problem-solving skills.

Learning Logs
Learning logs for students could be a useful way of helping them to develop ‘insight’ into problem solving techniques. The log could be based on how they solved a set of problems, what false starts they made and what approaches they found helpful. The students could, as part of an assignment, be asked to submit a re-drafted part of the log for assessment.”

Conclusions

“In this case study I have focused on a learning difficulty which I observed within my own level 1 course on Vector Algebra and Dynamics. A common problem, students exhibited when problem-solving was a difficulty in linking mathematics to representations of reality. Problem solving involves the whole spectrum from logical thinking to creative thinking. It is concerned with perception, of seeing connections and making intuitive leaps. In order to help students to develop ‘insight’ or ‘intuition’, we should discourage teaching practices that foster surface approaches to learning, and adopt teaching methods that lead to deep learning. This would ensure that students had the right environment to develop ‘insight’ as proposed by Burton (1999 a, b) and would discourage the rote memorisation technique reported by Crawford et al (1994).”

F. Revise draft/collation in light of your analysis

Rather than simply finishing the process once an initial draft has been made, it is important to revisit the draft portfolio to consider whether it meets the criteria outlined at the start of the process. We now illustrate this with a case study focused on one specific type of portfolio. The following case study is an extract from an application for membership of the Institute for Learning and Teaching in Higher Education. The application essentially involves making a case for professional competence in teaching and the support of learning.
Case study: My reflective practice and personal development

"I receive direct feedback from students by questionnaire and adjust my teaching accordingly. I am pleased that I have received little or no criticism in these. I used to receive criticism concerning my handwriting – this is no longer the case. I now break up lectures with prepared transparencies which also serve to break up the monotony of reading handwriting!

My own personal development is characterised by my participation annually for the last 10 years in the UMTC as witnessed by the number of references above. Indeed, I was the chairman of the 1995 meeting which involved a confrontation between Roger Porkess and the late David Crighton on the declining mathematical skills needed by the A-level.

This year the UMTC has as one of its topics of current interest for investigation – 'Preparing a portfolio – supporting professionalism in mathematics teaching in Higher Education'. In the accompanying descriptor it states 'Mathematics is a discipline where the gathering of the evidence required for a portfolio might be seen as an alien activity.' I do not believe this. But I do believe that 'rewards and opportunities in the new climate in higher education be made available on the basis of a teaching portfolio.' The leader of this workshop has experience of supporting staff in the development of such portfolios. Thus I intend to participate fully in this workshop.

For many years I have participated in the assessment of Sixth Year Study projects in local schools for the Scottish Qualifications Authority. This has enabled me to keep abreast of changes in the knowledge which our entrants bring, especially recently.

Last week I attended a briefing meeting for the new replacement Advanced Higher where the whole thinking behind assessment has been overturned. Instead of candidates gaining marks for originality and completeness they are initially given the full mark, then marks are removed for each error (such as too many significant figures or lack of full references). Many of the assessors are teachers; they were up in arms about this assessment method. I have informed my colleagues about this and they too are concerned about this change."

Commentary on the above extract

- **Para 1** With feedback from students, bright students tend to reply genuinely; poor students do not participate; whereas students with a grievance against taking mathematics at all (not a personal one) will direct their ire at you, as an outlet. Handwriting problems can now be tackled by a variety of remedies.
- **Para 2** Attendance at UMTC is a strong indicator of the desire for continued personal development. This person’s long term involvement would obviously not apply to a younger person. Participation in UMTC is not always funded by a department seeing research as the investment major item.
- **Para 3** This indicates up-to-date participation in professional development.
- **Para 4/5** This indicates up-to-date knowledge of student skills and assessment procedures and acknowledges interest in what is going on the 'the outside world'.
- **Overall** Generally thought-provoking, in a 'personal' writing style.
4. Conclusions

Preparing a teaching portfolio takes a significant investment of time and effort. And for those new to the process support will usually be required. This report seeks to provide some pointers towards the kinds of the support that are needed. In part this has been achieved by breaking the process of preparing a teaching portfolio into a series of steps, but in practice the process may be less systematic. Yet the end point should be the same with the development of a portfolio that serves the purpose for which it is designed. Ideally this purpose will extend beyond any immediate concerns for the author's own development or advancement to ensure that students receive a more effective mathematical education.

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6. Acknowledgements

The short guide to reflective practice was adapted from materials provided for the Teaching and Learning Course for New Academic Staff at the University of Manchester.
Working Group Brief: Engineering Mathematics should be taught by Engineers!

Introduction
Over the years, there has frequently been tension within higher education institutions over the issue of who should teach mathematics to engineering students, and as to how it should be taught. In the current climate of falling numbers of students applying for places on traditional engineering courses, the temptation has increased for engineering departments to make their budgets go further by cutting service teaching in favour of in-house mathematics teaching.

The advocates of in-house mathematics teaching by engineering lecturers suggest that engineers are better equipped to convey to students the applications of the subject, and to integrate the mathematics with the engineering principles. This, they claim, makes the mathematics seem more relevant to the students. By contrast, mathematicians are often ignorant of and disinterested in the applications of the mathematics, and too apt to be side tracked by proofs and other technical details that are unnecessary for engineering students.

However, there is evidence that within some institutions this approach has met with problems. Opponents of mathematics teaching by engineering lecturers suggest that it is simply not true that mathematics can be taught by any numerate person with a degree in a scientific or technical discipline. Mathematics teaching is a specialist skill, which requires years of dedication and practice to excel in. Thus, they claim, mathematics teaching should be left to the specialists.

The Remit
It is hoped that a working party can be formed, comprising proponents of both points of view, to debate these issues in depth. The issues for discussion include:

- What are the advantages and disadvantages of mathematics being taught by engineers?
- What are the advantages and disadvantages of mathematics being taught by mathematicians?
- Is it the role of the persons teaching mathematics to describe the applications, or ought they confine themselves to conveying the required techniques?
Engineering Mathematics should be taught by Engineers!

Members of the Working Group

John Ahmad (Sunderland) - Chair
John Appleby (Newcastle)
Peter Edwards (Bournemouth) - Scribe

Experience and qualifications of the Working Group
The provocative title of this report has not been changed from the one presented to us by the UMTC committee, despite being offered for discussion at a mathematics conference. In order to clarify our standpoint on this issue at the outset, it was thought appropriate that some details should be given about the backgrounds of the members of this UMTC Working Group. All teach mathematics as a service subject to engineers and to business students. Currently, we do not teach specialist mathematics students, except on Open University courses. We all hold first degrees in mathematics, but also higher degrees in education, mathematics and engineering. Our principal concern is the effective teaching of mathematics to other disciplines, whoever does it, and we attempt to give a balanced argument below of what the main criteria should be and how they should be achieved.

1. Introduction

Engineering Mathematics used to be accepted as an important subject in its own right but, with a reduced mathematical requirement in some recent engineering degree syllabuses, this is changing. Traditionally, mathematics was taught in substantial quantities (of content and time) to all engineers and scientists by mathematicians. In the past twenty years or so, however, the traditional picture of mathematicians teaching engineering mathematics has been changing, influenced by changes in society and the workplace, in schools and in universities. The nature of engineering disciplines in the workplace and in research has changed, and indeed diversified, and with that the requirements for the training of engineering students. The tools available have increased, especially with the advent of powerful computer packages for engineering and mathematical analysis and, as a result, the problems that are routinely tackled have changed in type and extent. There is no longer a clear consensus about what should be taught to engineers, nor about how it should be taught. Some of the same questions and tensions exist within engineering disciplines, between theory and practice, hand and computer calculation, individual and team work, as between engineering and mathematics.

In schools there have been substantial changes in the curriculum, not just in mathematics and physics, aimed partly at addressing the needs of those who don't go on to science and engineering degree courses (whose needs arguably dominated earlier curriculum design). Consequently, those who do enter engineering will now necessarily enter with different qualifications due to the revised curriculum. Also, this change in style and content has taken place in a period when recruitment of qualified school teachers has been difficult, and schools have been under pressure from other changes and from shortage of funds.
At the same time there are increasing pressures on curriculum time and the resources of engineering and mathematics departments, and more rapid change. When many areas are changing in direct response to outside requirements, there can be expectations that all areas should change. Is this right, or should some patterns in education keep fairly stable? This rapid pace of change has led to impatience with lengthy consultation and with learning by experience, towards increased emphasis on rapid and measurable results in all areas. This can result in rather narrow calculations of benefits to students and to departments.

Changes in schools, in the actual or perceived graduate market, and in student finance have led to a decline in applications to traditional engineering and science courses. Now new entrants also have a rather different background, in knowledge and style, from previously; they tend to expect more help, and higher quality, better presented materials than formerly.

In engineering courses, these pressures have led to a more pragmatic style of teaching of engineering theory, and at the same time the idea that engineers will teach their own students mathematics more effectively than mathematicians would, - for reasons, it is claimed, of sympathy, pace and relevance. We acknowledge that financial pressures within universities also play a part, but restrict ourselves mainly to academic matters. In the two Cases discussed below (Sections 2 and 3) we explore following issues:

- what mathematics do engineering students really need?
- how should it be taught?
- who should teach it?

Many statements about engineering apply equally to the physical sciences. There are also many similar aspects to mathematics teaching in the social sciences and medicine, but in these areas the use of mathematics has been growing significantly in the past thirty years. At a time when a number of mathematics departments have recently closed and groups of mathematicians are being disbanded, it has not always been possible for the mathematical revision in social sciences and medicine, for example, to have been overseen by a mathematics team. At times such as these much of the mathematics teaching is absorbed into the host department. This is not always a problem since mathematicians from disbanded mathematics departments often find themselves in departments whose mathematics they may have been servicing for many years. Where this is not the case, most mathematics teaching is kept by the staff in those departments rather than given to mathematics staff. We examine some features special to these disciplines in Section 4.

Statistics teaching is a special case. Many engineering courses include more statistics than formerly (indeed, many courses contained no statistics until quite recently). In most cases, such modules continue to be taught by members of mathematics or statistics departments. Maybe this reflects the lack of statistical teaching experienced by the engineering staff themselves in the past. However, as regards content, many of our assertions about mathematics teaching apply equally to statistics.

In Section 6 we make some recommendations for best practice, and draw some overall conclusions for the following situations:

1. if mathematicians teach the engineering mathematics,
2. if engineers teach the engineering mathematics,
3. for effective teaching in either case.
Although it would be preferable to confine this paper to academic arguments, there is no doubt that financial considerations play a not insignificant part in deciding who teaches mathematics to engineers. Where departments or faculties are 'cost centres' in charge of their own budgets, they often feel it to be more prudent to keep all their teaching in-house, to ensure improved deployment (and employment) of their resources – and to cut costs. In extreme cases, some departments will use part-time staff from outside the institution, since they are less expensive compared with staff from servicing departments within. While we acknowledge that it is lamentable that financial factors should have such an influence on the outcome of pedagogic issues, we now proceed with academic arguments only.

2. Case A: Engineers teaching Engineering Mathematics

2.1 Knowledge of engineering and of applications of mathematics

Engineers are aware of the employment and the professional needs of their discipline and are familiar with realistic and up to date examples, applications and accompanying mathematical techniques. Consequently, engineers who teach mathematics are more likely to set the mathematics within the appropriate engineering context, introduced as needed. That a mathematical topic/technique is needed to help solve an engineering problem can motivate students, strengthen their perception of the subject and, at the same time, help avoid the traditional questions "Why are we doing this [mathematics]?" and "What's this [mathematics] used for?". Engineers have a knowledge of the appropriate units and magnitudes to use in examples; for instance, an electronics lecturer is unlikely to specify a capacitor rated at (a physically unreasonable) 100 farads in a mathematical problem set in context, despite the mathematical convenience of this numerical value!

The advent of powerful engineering computer packages for system modelling, analysis and forecasting has allowed engineers to tackle a wider range of realistic problems. They can concentrate on essential techniques, analysis and interpretation of the mathematics rather than being embroiled with repetitive computation and complicated analytical techniques.

Being an engineer, the lecturer is aware of the engineering curriculum and is able to select examples from a range of relevant situations. They can use appropriate language, notation and terminology and can maintain uniformity of delivery across the course as a whole. While some mathematicians may confuse (and amuse) engineering students by using "i" in complex numbers, the engineers require and will use "j". Engineers are unlikely to become entangled with providing unnecessary proofs and considering mathematically interesting "problem" cases just for their own sake. The engineer lecturer is unlikely to use \[ \int_{-1}^{1} \frac{1}{x^2} \, dx \]
since it has no application in the real world (although it may provide fertile ground in a specialist mathematics course). Only "realistic" integrals will be used, such as \[ \int_{1}^{e^\pi} \frac{1}{x^2} \, dx \].

This is the same integrand but, seemingly, a more mathematically interesting case with the infinite limit! However, this example arises in gravitational theory and elsewhere.
2.2 Knowledge of the students

There is a view that engineering lecturers relate better to their students. They are in the same department and, very likely, within the same building for most of the working day: this proximity can lead to a good rapport between staff and students. The rapport is strengthened by lecturers having followed similar engineering course themselves. On this last point, having been through the same experience can develop in lecturers a greater awareness of, and empathy with, the students' perspective, motivation, interest and likely difficulties. Mathematics taught by engineers is more likely to be seen by students as an integrated part of the course and not just an independent "add on" taught by outsiders.

3. Case B: Mathematicians teaching Engineering Mathematics

The dividing line between mathematics and some topics in engineering is often ill-defined, so that terms like "Applied Mathematics", "Engineering Mathematics" and "Mechanics" cover common ground. It goes without saying that mathematicians know the subject of mathematics well, but this does not in itself justify them teaching engineering students. So what can mathematicians do for engineers and for engineering that is of particular benefit?

3.1 Making mathematics work for engineers

Mathematics needs to be taken seriously if it is to be studied or used at all. Mathematicians are thoroughly familiar with the required mathematical topics, and will introduce them in a sequential manner with all prerequisites taught first. Teaching mathematics as a separate subject means that the mathematician will not have to interrupt, and possibly disrupt the flow of, an engineering topic just to introduce a new mathematical topic. However, the mathematician should justify the need for the mathematics by indicating some engineering context as a lead-in to each mathematical topic. Mathematicians appreciate that it is mathematics' overall structure that gives it its power, and its usefulness in the kind of generic skills needed in engineering problem-solving. Naturally this implies that if mathematicians do the teaching, they are able to communicate this structure.

Sufficient time must be provided for explanation, study, practice and application, and the importance of careful, correct and appropriate use of mathematical techniques needs to be emphasised. Mathematicians are more likely to make that time and space, to demand it even, and to aim to ensure that methods are used correctly, and therefore usefully and safely. One reason they do this is their own enthusiasm for their subject. This can also help to enthuse and motivate the students, especially those who go on later to advanced engineering theory. Links made in early courses may be reinforced in project work and options, in postgraduate work and subsequently in research as staff or in employment.

Mathematicians can be expected to understand why a technique works, and when it doesn't work. Although they should avoid mathematically introspective "problem" examples - see Case A - (often used to justify a rigorous approach) that will rarely, if ever, be met, they will pay attention to the exceptions, and not just teach methods that usually work.
It is important to make students aware of a range of methods, not just the simplest or the fastest, as these may apply at different times. At the same time, the mathematician can illustrate the use of the same technique in other areas of engineering or elsewhere, making links between disciplines and showing the power and generality of mathematics. For example, if resonance is introduced only in the context of damped spring systems, then not only are important applications in electronics overlooked, but also common theory that applies to quite different problems is not explored to advantage.

The increasing use of computer packages, both specifically engineering packages (such as finite elements) and mathematical packages (numerical solvers, spreadsheets, etc.), may have reduced the need for fluency in hand-calculation and analysis. However, opening up newly feasible areas for engineering analysis can expose the engineer to new uses of mathematics. To make effective use of these packages requires some understanding of the underlying theory, engineering and mathematics, or invalid results - and frustration - will arise. Mathematicians can use these packages to deepen student understanding and draw students' attention to the consequences of the theory using changes in parameters and visualisation (Edwards, 2001). They can also develop mathematical insights from this practice that feed back into other areas and courses.

Many mathematicians are involved in collaborative research with engineers and staff from other disciplines. As well as making valuable links with students through undergraduate teaching, these mathematician are also more aware of the needs of engineers if involved at this level, and therefore more likely to contribute to higher engineering work.

3.2 Recognising the needs of students

Nowadays, mathematicians often have more contact with first-year students than they used to, mainly due to an increased need for 'bridging mathematics'. Hence mathematicians are more likely to be empathetic and sympathetic to the mathematical requirements of engineering students - some of whom may consider mathematics as a 'necessary evil' and some of whose mathematics on entry is particularly weak. For this reason, first-year tutorial support from Ph.D. students, for example, whose learning and teaching skills may not have yet been finely honed, is not necessarily the best use of such a resource. In tutorials, students need to feel as though their contributions are valid and valued. A lecturer who is aware of students' learning difficulties in mathematics will know how to tread carefully with a student's feelings when the student is labouring under some mathematical misconception. Often mathematicians have very good contact with engineering students, partly because it is usually agreed that mathematics modules need significant numbers of tutorials. The lecturer should, whenever possible, attend and be involved in tutorials for their own modules. Many mathematics staff take an interest in school developments and are aware of current entry standards, and hence student needs, partly through their involvement in diagnostic testing and follow-up in recent years (Appleby, 1997 & 2000, Edwards, 1997). The mathematics lecturer also has more experience of mathematics teaching and a broader perspective, and may have tried a range of innovative methods for teaching mathematics.
Whatever mathematics is thought necessary for engineering students, it is important that all requisite logical steps are covered, with the vital pre-requisites in place. Teaching isolated techniques, even in an engineering application, may be unsuccessful if the terminology or logic is unclear or unfamiliar to the student.

Although it is important that engineering students appreciate the significance and usefulness of mathematical techniques in the context of their own discipline, sometimes they find a topic clearer if they concentrate on the mathematics for a while. Certainly they need to practice the technique in isolation as well as in context. Moreover, engineering students can actually enjoy mathematics on occasion!

4. Mathematics teaching in the Social Sciences and Medicine

So far we have discussed mathematics teaching in the context of engineering courses. However, similar issues are raised in other subjects that need a quantitative approach; for example, Business Studies, Economics, Biological Sciences, etc. Who is to teach the Mathematics and, more particularly, the Statistics required? Traditionally the Business Studies (or Economics, etc.) department undertakes the teaching of quantitative methods in the higher levels set in the context of the specialist field, often requiring a greater emphasis on interpretation rather than on concepts. Consequently students sometimes fail to appreciate the limitations of the methods/models used. Mathematicians have been involved mainly at the basic level covering topics such as elementary statistics and basic algebra and in introducing students to packages such as Excel or MiniTab. However, these mathematicians/statisticians who met, taught and helped students in their first year, are often called upon for help with the analysis necessary in final year student projects.

5. Recommendations

5.1 If Mathematicians teach Engineering Mathematics

It goes without saying (but needs to be mentioned here) that mathematicians chosen to service the mathematics of an engineering department should be sympathetic towards the needs of engineering students and interested in engineering itself. If the mathematician is employed within the serviced department, this may not be a problem. In particular, a lecturer of Engineering Mathematics who has a first degree in mathematics and postgraduate qualifications in engineering should be especially valued. Mathematicians should encourage understanding and thoughtful use, and may communicate enjoyment of mathematics and an appreciation of the subject that will be of particular value to those who pursue advance engineering theory. The generality and power of the subject, the range of methods available, and links to other subjects within and outside engineering can be emphasised. The ability to inculcate in a student the desire to interpret a mathematical solution in the real-world context is invaluable.
5.2 If Engineers teach Engineering Mathematics

If engineers teach the mathematics they must allow sufficient time for methods to be understood, and they must ensure that pre-requisites are covered and that all the logical steps are in place for adequate understanding. They should encourage an appreciation of alternative methods, of when and why methods work, and of links with other subject areas and applications. Effective use of engineering computer packages will require some understanding of the underlying theory if useful results are to be obtained consistently: the engineer should avoid the danger of using such packages as a 'quick fix'.

5.3 In either case

Mathematicians need to be aware of the needs of engineers and vice versa, and in regular contact with other staff and students. Engineers need to be aware of the range of mathematical techniques available, and may benefit from the experience and perspective of mathematics staff. The physical proximity of staff and students can contribute to collaboration, as can formal and informal involvement in curriculum development, course evaluation, joint research into engineering and/or mathematics and even pastoral oversight.

Flexible patterns of collaboration can usefully be explored. Mathematicians could contribute parts of engineering modules, as well as whole mathematics modules; engineers could contribute sessions on particular applications in mathematics modules. Engineers can also contribute case study material for use by mathematics staff in their classes.

5.4 Tutorials

The staffing of mathematics tutorials needs particular care, especially where these sessions are sometimes seen as optional extras. It is in tutorials that students ask most questions, and need immediate answers, so the tutors must be sympathetic, well-informed about the topic, and good at explaining. In addition, students' individual questions in a tutorial give opportunities to motivate the topic and to broaden the students' perspective by making links with other topics in mathematics and in their own engineering area.

The criteria for who should staff a tutorial are the same as those stated generally for classes, though the design of materials is not involved in this case. The important point is that the staffing of lectures should not be seen as more important than the staffing of tutorials, where the ability to explain and make relevant is vital. Tutorials are not just an afterthought!
6. Conclusions

Mathematicians used to have an accepted place in engineering teaching. Mathematics still does - although there are those who, for whatever reason, would like to see mathematical content reduced. In contributing to the debate about what should be taught, how it should be taught, and especially who should teach it, we have examined some arguments in favour of teaching by engineers and by mathematicians. These do not apply to all engineers or mathematicians; they serve to identify important criteria for appropriate teaching.

Mathematicians have much to offer engineering, in teaching and in research links. For effective mutual support they should be involved in curriculum development, there should be collaboration in teaching design and delivery, and they need to be in regular contact with engineering staff and students. Pursued by both sides, this partnership can be very fruitful.

7. References


Also available on: http://ltsn.mathstore.ac.uk/newsletter/feb2001/pdf/mathinstore.pdf

* MathinSite is available on http://mathinsite.bmth.ac.uk
Working Group Brief: Issues in teaching Discrete Mathematics

Introduction
Discrete mathematics modules are a feature of most undergraduate mathematics degree programmes, and have certainly featured in mathematics ‘service’ teaching for computer scientists and engineers. Such modules often start with such topics as sets, logic, Boolean algebra, functions and relations, etc. Many textbooks exist (see references for examples) that cover such topics. Some of these topics can be found on A-level syllabuses. One might also include such topics as graph theory, combinatorics, operational research, and other similar topics often described as ‘finite mathematics’ or perhaps ‘decision mathematics’. The applications of discrete mathematics to engineering and business seem to be increasing, with financial mathematics being one obvious growth area. Programme designers have an increasingly difficult task in ensuring that content is up-to-date, of the appropriate standard and relevant. The inclusion of elements of discrete mathematics, perhaps at the expense of more traditional continuous mathematics teaching, can cause much debate.

The use (or not) of technologies such as computer-based learning software (including that on the Web) or computer algebra systems is also a contentious issue in the teaching of (for example) calculus and the same issues can arise in the teaching of elements of discrete mathematics, although perhaps the impact in this area of mathematics is not so pronounced. Thus the inclusion of discrete mathematics can raise pedagogical issues as well as curriculum/programme development issues.

In summary, the questions posed are:
- What discrete mathematics topics should be taught?
- What innovative ways of teaching such topics have been or ought to be tried?
- How can technology be most appropriately used in the teaching of discrete mathematics?

The Remit
The working group is asked to consider innovative features that could figure in the design of a discrete mathematics module (or modules) either forming a component of an mathematics undergraduate degree programme or serving as a ‘service’ module(s). The following questions could form the basis for discussion.

Curriculum/Programme design issues
- What topics would a single module ‘discrete mathematics’ cover? Should all mathematics undergraduate degrees have such a module (or modules?) in year 1?
- What is a reasonable balance between ‘continuous’ and ‘discrete’ mathematics modules in a single honours mathematics undergraduate degree programme? Ought modules contain either discrete or continuous mathematics, or might both be tackled within the same module? For example, do we teach differential equations and difference equations in the same module? If the balance favours too much continuous work then what should be removed?
- Assuming a broad definition of the term ‘discrete mathematics’, should students be introduced to elements of topics such as graph theory, number theory, combinatorics, operational research, etc. as core activities or are these best developed as separate modules leading to final year option choices?

Teaching & learning issues
- To what extent should applications such as financial mathematics, computer network analysis, etc. be paramount in the teaching of discrete mathematics? What applications are important, both now and in the foreseeable future, for a working mathematician to be familiar with, and which are best left to dedicated ‘service’ modules? Does ‘fun maths’, such as the study of the mathematics of puzzles and games, have a place here?
- What impact are technologies such as computer-based learning materials (e.g. WWW, MATHWISE), computer algebra systems, etc. having now on the teaching, learning and assessment of discrete mathematics (including service teaching)? How are such technologies being used and with what result? What impact might or should they have in the future?
Indicative References


and many more........
Issues in Teaching Discrete Mathematics

Members of the Working Group

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Introduction

Modern industrial and commercial problems are increasingly complex and often require analytical tools in part for their solution. Discrete mathematics has considerable elegance and interest as a purely mathematical theory, but can also provide tools to solve problems in (amongst other areas) computer science, cryptography, DNA sequencing and management. Modern undergraduate mathematics courses need to reflect the growing importance of such applications and equip their students with the necessary understanding for the use of techniques in Discrete Mathematics. (One American aerospace company apparently now employs more ‘discrete’ mathematicians than ‘continuous’ mathematicians). The Group favours a Discrete Mathematics ‘core’ but with different emphases depending on the target audience. We explore ways in which discrete mathematics could be taught incorporating IT and innovative teaching methods.

We first consider two possible core modules in Discrete Mathematics: one for Mathematics students, one for Computer Science students, followed by a number of possible modules for subsequent years. The needs of Mathematics students, which (for example) include greater emphasis on the notion of proof, are likely to differ somewhat from those of Computer Science students; thus these two groups may well need to be taught in separate streams.

Core modules

We suggest two possible initial core modules: one for Computer Science students (with a good base in GCSE Mathematics) and one for Mathematics students. The Computer Science core is intended to be approximately 20 CATs points (one year is 120 CATs points). The Mathematics core is intended to be about 10-15 CATs points since we believe that a typical Mathematics programme already incorporates a large amount of essential material in the first year. Proofs play a very small part in the Computer Science module and a large part in the Mathematics module. Of course these possible syllabuses will need local modification in the light of the other courses in the curriculum.
Computing Science core (two semesters)

- Propositional logic: truth tables.
- Predicate Calculus: translation between English and logic; rules of inference.
- Set theory: language of sets; algebra of sets in terms of Venn diagrams.
- Boolean algebra; elementary minimisation techniques (e.g. Karnaugh charts)
- Functions and relations; composition of functions.
- Induction: the concept of proof by induction; elementary proof techniques in
  algebraic and other applications. (Induction is a necessary foundation for the idea
  of recursive programming, but students find it very hard to understand).
- Matrix algebra: addition, multiplication, inversion, use of computer algebra
  packages for matrix manipulation, applications of matrices in computing.
- Graph theory: basic definitions; path algorithms; (shortest path; maximum flow;
  critical path analysis).

Mathematics core (one semester)

- Propositional logic: truth tables.
- Predicate Calculus: translation between English and logic; rules of inference.
- Set theory: language of sets; algebra of sets in terms of formal proof
- Boolean algebra; algebraic and graphical minimisation techniques
- Methods of proof: induction; direct & indirect proof.
- Modular arithmetic: simple applications e.g. ISBN numbers.
- Combinatorics: binomial coefficients; recurrence relations.

Further modules

Some suggestions for modules to be studied in subsequent semesters are as follows.

For Mathematics students:
- Number theory and cryptography
- Graphs and networks
- Designs, Latin squares, error-correcting codes

For Computer Science students:
- Computer graphics and fractals
- Functional programming
- Formal specification languages

For the last three we envisage collaboration between the departments, as appropriate.
Innovative Teaching Methods

Using Applications
Discrete mathematics is inherently attractive to students because of the wide range of applications which are easy to formulate and comprehend. An applications-led approach to teaching discrete mathematics is a way to introduce novel ideas. We looked at separate topics in the core structure to see what applications could be used to motivate the subject.

Induction: Sequences could be generated iteratively and the closed formula given and checked by numerical investigation and then validated by induction.

For example, the Towers of Hanoi problem for \( n = 1, 2, 3, 4 \) requires 1, 3, 7, 15 moves respectively. The suggested formula for \( n \) discs is thus \((2^n - 1)\) moves. This can be proved by induction, as follows. When moving \( n \) discs first of all the top \((n - 1)\) discs have to be repositioned in the same order elsewhere, the bottom disc must then be moved to the free position, and finally the other \((n - 1)\) discs have to be repositioned on top of it. So, if \( a(n) \) denotes the smallest number of moves required for \( n \) discs then

\[
a(n) = 2a(n-1) + 1.
\]

If our induction hypothesis is \( a(k) = 2^k - 1 \),

then \( a(k + 1) = 2a(k) + 1 = 2(2^k - 1) + 1 = 2^{k+1} - 1 \).

So the result follows by induction. (see Anderson (1))

Similar problems using difference equations to model the complexity of algorithms, and problems using induction to prove the simple algorithms involving a loop, may be found in the course material for the Open University course M261.

Sets: Elements of the formal specification language $\mathbf{Z}$ could be introduced to demonstrate how sets can be used in computer science applications. Simple studies of this kind can be found in the First Year Mathematics course notes (author Dr D Cooper) for the BSc Computing course at Sheffield Hallam University.

Solving combinatorics problems using the inclusion/exclusion principle is another application whilst the mathematical ideas of union, intersection and relation can lead on to the study of databases.

Again in M261, Set Theory is used to simplify segments of code where decisions are made based on values taken by complex boolean functions.

Matrix algebra: There are applications here in computer graphics, as matrices are used to carry out transformations in two and three-dimensional space. In addition, adjacency matrices may be used in graph theory applications.
Group work
Discrete mathematics lends itself to group work more readily than other areas of mathematics because problems are often more accessible. We favour small-scale group work, which can help students to consolidate abstract ideas, rather than large-scale project work with presentations and reports. For example, students engage in workshop sessions in which they work on elementary applications from an investigative point of view. The application is described in a workshop pack in which the students are led through the problem solution via a series of leading questions.

Examples of applications tackled are the use of graphs to model the bracing of rectangular structures, and in solving recreational games such as 'Instant Insanity'. In another example, pairs of students use Maple to encode and send encrypted messages to each other, which are then decoded. (The basis of the coding was the RSA algorithm.)

Using IT
There is scope for using computer algebra packages such as Maple or Mathematica, which have specialised discrete Mathematics sections, and a number of texts have incorporated IT implementations of Discrete Mathematical techniques. For example, a recent book by Wilson and Aldous comes with a CD-ROM, which contains a database of small graphs and the facilities to manipulate and investigate these. The module 'Applications in Discrete Mathematics' in Mathwise presents a number of problems and applications to initiate the teaching of specific Discrete Mathematics topics. The Open University embeds IT usage in a number of its Discrete Mathematics courses - which gives the students a feel for the difficulty of the problem posed.

Andrew Rae of Brunel University has run a Discrete Mathematics Course for 140 students of varying background without giving any lectures. He devised a self-paced learning system based on laboratory sessions and tutorial help. This was backed up by video material, available for students to view in their own time in the library. The Course had ten units, each timed for two weeks. Each unit contained an initial handout, two half-hour videos and a diagnostic test, carried out under supervision. Students volunteered for the tests when ready, and the results were then discussed individually with the Course Tutor. The tests did not affect the students' results, as the Course was assessed by examination.

There are some web-based resources, such as
- ISETL, this is an interactive programming package for set language, downloadable from http://isetlw.muc.edu/isetlw
- http://www.earlham.edu/~peters/courses/logsys/glossary.htm This site is a glossary limited to basic set theory, basic recursive function theory, two branches of logic (truth-functional propositional logic and first-order predicate logic) and their metatheory. The material is good but it is non-interactive.
Other conferences have addressed the availability of web-based materials. See for example

- the Proceedings of UMTC 2000: http://www.umtc.ac.uk/
- CTI Mathematics Workshop 1990
  http://www.bham.ac.uk/ctimath/workshops/wdis.htm

As a future web-based development, we would like to see a database of appropriate problems for assessment purposes and developments in this area.

References

General

Open University courses
MT365: *Graphs, Networks and Designs*
M261: *Mathematics for Computing*

(1) Anderson I *A First Course in Discrete Mathematics*, Springer 2000
Biggs NL *Discrete Mathematics*, OUP
Garnier & Taylor *Discrete Mathematics for New Technology*
Gersting JL *Mathematical Structures for Computer Science*, Freeman

For suggested mathematics modules

Number theory and cryptography
Tattersall JJ *Elementary Number theory in Nine Chapters*, CUP 1999

Graphs and Networks
West DB *Introduction to Graph Theory, 2nd edition*, Prentice Hall 2001

Designs, Latin Squares and Codes
(2) Anderson I *Combinatorial designs and tournaments*, OUP, 1997
Hill R *A First Course in Coding Theory*, OUP 1986
Introduction
The activities outlined in this brief represent a radical departure from those usually carried out by a UMTC Group. Normally the bulk of the Group's work is completed by the end of the conference leaving only the writing up of a Report to be finished later. What is envisioned here, however, is a more extended piece of joint research activity to be planned during the conference but carried out over a period of a year or more following the conference. The research plan would be written up immediately after the conference for inclusion in the conference proceedings and the actual research itself would be expected to be published in a suitable journal.

Diagnostic testing of students entering mathematics-based degree courses is now widely practiced [1]. This is a response to a decline in the mathematical skills and level of preparation of these students in recent years [2]. The results of such tests are used to plan remedial action. Presently this testing and remedial action take a variety of forms [3]. Some institutions employ traditional paper-based tests and others use computer packages [4]. Some computer packages encompass both the testing and remediation [5].

The Remit
Whilst not wishing to be prescriptive the following are suggestions for areas of research which the Group might wish to address:
- A novel approach to testing and/or remediation to be piloted on a group of students and subsequently evaluated.
- A review of current method of testing and/or remediation with recommendations for "best practice".
- A follow up study of a group of students that have already been tested and received remediation to compare their subsequent progress with that of a group with a traditional entry profile.
- A comparative study of computer-based methods of testing and remediation with paper-based ones.

References

Acknowledgement
The authors of this brief wishes to record their thanks to Prof. Peter Saunders for the suggestion upon which this brief is based.
Action Research: Diagnostic Testing & follow-up support

Members of the Working Group

Richard Atkinson (Birmingham) – Scribe
Mike Barry (Bristol) – Chair
Bjoern Hassler (Cambridge)
Douglas Quinney (Keele)

Overview
This small Working Group was asked to consider the actions needed in Higher Education regarding the diagnostic testing of mathematics skills and follow up support. In a radical departure from the normal procedure at UMTC, where a final Report is written up by the Group at the end of the Conference, this group was invited to carry out a more extended piece of work over a period of a year and report back at UMTC 2002. So for the purpose of UMTC 2001 Proceedings, the following is an interim report on the work done to date.

The Working Group decided to apply for an LTSN-MSOR miniproject Grant of £5000 (see the edited Proposal below) to investigate the practices in mathematics diagnostic testing at a selected group of universities and to establish what follow-up actions and support were in place. This Grant was awarded in Nov 2001. A condition of the award was that the Working Group would cooperate closely with the Maths-TEAM at LTSN-MSOR (based at Birmingham), and would present a report that advised on best practice. The Maths-TEAM has partly begun some of the tasks that the Working Group plans to take on, in particular a survey of universities, but the Proposal is a summary of our discussions at UMTC and how these were focused into a successful grant application.

The Proposal for a LTSN Project Grant – Nov 2001

Introduction
For students entering Higher Education courses in Science and Engineering there is always some level of pre-requisite assumption and reliance on prior knowledge in a range of topic areas and mathematical skills. Such courses also tend to recruit large numbers of students with a rich diversity of intake qualifications and prior experiences. In order to determine individual student needs, many universities are now using multiple-choice computer marked diagnostic tests. Such testing of fresher students taking mathematical subjects is common in many universities and a considerable volume of experience of using them is now being accumulated [1,2,3]. We intend to concentrate on the principal aspects of the UMTC Brief. Lawson, Halpin & Croft in [1] list these strategies undertaken by institutions to address the question of what action to take following the diagnostic test:
- Curriculum changes to include more ‘revision’ material in the first year
- The provision of extra support units to be studied alongside the traditional syllabus
- The introduction of bridging units, studied intensively at the start of the course
- Staff voluntarily making themselves available to students outside scheduled classes
- The provision of some kind of mathematics support centre.
They then focus on the last three, notably the provision of a dedicated institutional support centre. The aim of this bid however is to concentrate on the second strategy of using the results of the diagnostic test to specify extra support units, to be studied alongside the main mathematics unit within the department teaching the unit. Recommendations for ‘best practice’ would then be made.

The research team, detailed above, is composed of Dr Richard Atkinson, Dr Mike Barry, Prof. Doug Quinney, and Mr Bjorn Hassler. The first three will investigate the provisioning of extra support units in their three universities using computer aided assessment methods. Moreover, diagnostic approaches and follow-up will be investigated at a further twelve universities which between them will be selected for:
(a) providing diagnostic testing and follow-up support,
(b) providing testing without follow-up support,
(c) providing neither testing nor follow-up support.

The Study

The members of the working party will undertake a systematic evaluation of diagnostic testing and student support in their own universities (Birmingham, Bristol and Keele - the focus group universities), looking at the effectiveness of different strategies of following up the results. They will also work in teams to administrate a questionnaire, conduct structured interviews and visit up to 12 other universities, which cover a full diversity of diagnostic delivery and follow-up action. To make the study effective it is believed that 4 universities in each of the categories (a), (b) and (c), will need to be surveyed and visited.

Part 1. An evaluation of existing diagnostic testing and student support

Diagnostic tests are intended to give new entry students, and their teachers, an overall profile of ability, knowledge and preparedness, after which students are invited to carry out a specific programme of study. However, experience in some institutions shows reluctance on the students’ part to undertake the programme, doubtless drowned out by pressures on the course elsewhere; therefore it is essential that the specific programme be integrated into the general programme in a way that maximises effectiveness at minimum cost. To do this we propose to carry out the following analysis at the focus group universities of Birmingham, Bristol & Keele:

(a) Validate the results of selected fresher student groups produced by two distinct diagnostic tests by cross evaluation between the two.
(b) Evaluate various student support materials in order to recommend those, which may provide the better learning and support resource.
(c) Provide a case study on how diagnostic test and support material may be evaluated.

This part of the study will begin during the first week in the autumn semester 2001 when the scheduled diagnostic tests take place. The members of the group will investigate the different student support environments in the context of diagnostic testing, student-directed support study together with computer based formative assessment to follow through.
Diagnostic Testing

A number of diagnostic testing procedures were considered for the purpose, none of which is considered perfect. The following test/system were selected because they are easy to operate, computerised, already in full use, and the group is familiar with them. They are:

1. The Nottingham/Keele Test (NK) developed by Stephen Hibberd at the University of Nottingham
2. The Test and Learn (TAL) System developed at the University of Bristol by John Sims Williams and Mike Barry.

The NK test is showing its age but gives a good presentation of the skills of students and has been evaluated in the past. The TAL system is a 1000+ question bank of multiple choice questions in mathematics at interface and foundation level. It is WWW based, but the output presented to students is maybe a little terse; they are given a full review of their test, which includes advice of how to get right any question wrongly answered. Both tests overlap in the skills and knowledge they assess, so some benchmarking between them is possible. The main mission is to follow established practice in the focus universities using designated groups of students to draw comparisons and thus proceeding as follows:

(i) Keele University will continue with the NK diagnostic test that has been used for some years but will use a sample of students to take the TAL diagnostic test. This will permit benchmarking of the tests.
(ii) Bristol University Engineering Mathematics Department will use the TAL diagnostic test for all their students and select a dedicated sample to take the NK test, i.e. mirroring the procedure at Keele.
(iii) Birmingham University will ask a dedicated sample of students to use both the NK and TAL tests, conducting other relevant diagnostic testing as necessary.

1.2 Student Support

After the diagnostic test students will be assigned to other support materials including:

1. Mathwise Modules (Techniques of Differentiation; Applications, Min and Max)
2. AIM material currently being used at the University of Birmingham. ‘Alice Interactive Mathematics (AIM)’ is a computer system that assists in the teaching of some parts of university mathematics. It was developed at the Ghent University in Belgium (see http://mat-mt2.bham.ac.uk:8080/).
3. TAL material (tests generated from the database)
4. Selected paper based material.

1.3 Post Trial Testing

After a suitable period each group of students in the trial will be asked to take one of two TAL tests appropriate to the work they have studied. A statistical comparison will be carried out to analyse the effectiveness of the various support materials. The students will then be asked to complete a questionnaire to investigate their general response to remedial work, diagnostic testing and more subjective issues such as:

- deficiencies in knowledge due to inadequate preparation or syllabus coverage
- deficiencies in mathematical ability
- attitude problems towards mathematics.
Part 2. A review of current methods so as to advise on best practice

The report ‘Measuring the Mathematics Problem’ [3] estimated that at least 60 departments in the UK give mathematical diagnostic tests to their new undergraduates. Naturally these tests take a variety of forms and are linked to remedial measures in a variety of ways. The decline in mathematical ability is well documented, but a systematic survey of diagnostic tests, especially in conjunction with remedial measures, is still lacking, though some progress in that direction has been made (LTSN Internal Report).

We propose to compare and evaluate the practice of diagnostic testing and associated student support in the UK as comprehensively as possible by surveying the practice in about 12 universities, in relatively equal proportions: firstly, where diagnostic testing is carried out and remedial action taken; secondly, where diagnostic testing is carried out but without remedial action; and thirdly, where there is neither diagnostic testing nor remedial action. To conduct such a survey properly one really needs to forewarn the universities in detail and then sit down with the recipients of the survey, both staff and students alike, so that questions are properly understood and responded to in a consistent manner. The team expects to have covered the first part of their UMTC remit (see Part 1) in their own universities by the end of the autumn Term 2001, and will be able to refine their structured questions for others in the light of their findings. Part 2, or the survey phase, would then proceed in January-April 2002. Already there is some knowledge upon which to build ideas. The preliminary survey of reports on diagnostic testing (e.g. [1]) has highlighted the following questions that we will be considered in the proposed comparison: -

- Should the test be computer based or paper based?
- Should the test be conducted prior to arrival at university? Can part of the remedial work be done prior to students' arrival? How effective is sending out reading lists?
- How should tests be marked?
- In what way should feedback to students be given?
- How can tests be kept short, yet be comprehensive?
- Should the tests be a take-home test or should it be taken in class? How does it need to be supervised?
- How can testing/student support be conducted resource-effectively?
- Under what circumstances and how easily can a test/student support scheme be transferred between different institutions?
- How can students be motivated to undertake remedial work?
- How do you identify good students?
- How do you test for 'ability', rather than 'knowledge'?
- How do you make students take responsibility for their own learning?

By considering existing tests/student support schemes under these, as well as other criteria, we intend to produce a survey of current practice in diagnostic testing and student support. We intend to pay particular attention to developing a framework for the evaluation of diagnostic testing and student support, as well as to recommendations for 'best practice'. The outcome of this study should facilitate the choice, introduction (and upgrading) of diagnostic testing and student support in departments that so far have little diagnostic testing and little or no student support.
We further propose to test this framework and recommendations in an inter-university comparison using a diagnostic package to produce a comparative assessment of students' mathematical ability at entrance into university in a variety of subjects relying on mathematics. For this the experience gained in Part 1 will be very useful.

References

2. Quinney D.A. 'Diagnostic Testing in Mathematics' http://www.keele.ac.uk/depts/ma/diagnostic/
Working Group Brief: Study Skills for Mathematics

Introduction
The aim of this group will be to update and redraft the booklet Study Skills in Mathematics in preparation for its publication in both electronic and printed form. Members will review other currently available books, examine what constitutes "study skills", and produce a detailed outline for publication within three months.

The current edition of Study Skills in Mathematics, published by Sheffield Hallam University Press, was originally compiled and edited from the reports of three working groups with the above title at UMTC 1979. It was subsequently revised and updated at UMTC 1990 and in 1998 by Pam Bishop and Laurence Nicholas.

The book is used widely in departments (sales exceeded 2800 in both 1999 and 2000) and the view has been expressed that it should be revised again. Using data from the publishers the LTSN Maths, Stats & OR Network has surveyed 35 departments, 9 of which replied. Of these 8 gave the guide free to every student in the first year, that is approximately 925 students. The Network is now preparing a survey of student response to the 1998 edition, which will be available by the time of the conference.

The Remit
1. To briefly review other currently available published books aimed at helping students to develop mathematical study skills. This includes books of a “how to solve mathematical problems” and “how to write proof” nature. Some suggested titles are in the Appendix below.

2. To examine with an open mind what constitute “study skills” and the way students develop them. To reassess in the light of student feedback how the current draft of the book helps students.

3. To produce an outline of the updated booklet at a level of detail sufficient to allow its completion by members of the group who will be designated at the working party. The finished booklet should be ready to send out with the proceedings in both printed and electronic form approximately three months from the conference.

Resources
The conference organisers will endeavour to provide a copy of each of the following books for reference purposes. However, to aid discussion, members of this working group are invited to read one or more in advance of the meeting and to bring further copies with them.

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<thead>
<tr>
<th>Title</th>
<th>Author</th>
<th>Publisher etc</th>
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<tbody>
<tr>
<td>How to Prove it</td>
<td>Dan Vellmann</td>
<td>CUP, 1994</td>
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<tr>
<td>How to Read and Do Proofs</td>
<td>Daniel Solow</td>
<td>John Wiley, 1990</td>
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<tr>
<td></td>
<td></td>
<td>ISBN 0-471-51004-1</td>
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<tr>
<td>How to Solve Mathematical Problems</td>
<td>Wayne Wickelgreen</td>
<td>Dover, 1995</td>
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<td></td>
<td>ISBN 0-486-28433-6</td>
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<tr>
<td>How to Ace Calculus - The Streetwise Guide</td>
<td>Colin Adams, Joel Hass,</td>
<td>MAA Catalog Code: HAC/W</td>
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<tr>
<td></td>
<td>and Abigail Thompson</td>
<td>240 pp., Paperbound, 1998</td>
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<tr>
<td></td>
<td></td>
<td>ISBN 0-7167-3160-6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Price: $14.95</td>
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<tr>
<td>An Introduction to Mathematical Reasoning</td>
<td>P J Eccles</td>
<td>CUP, 1997</td>
</tr>
<tr>
<td>100% Mathematical Proof</td>
<td>Talyor J and Garnier R</td>
<td>Wiley, 1996</td>
</tr>
<tr>
<td>An Accompaniment to Higher Mathematics</td>
<td>Exner, G.R., Bucknell University, USA</td>
<td>Springer, 1991</td>
</tr>
<tr>
<td>Learning and Doing Mathematics</td>
<td>J Mason</td>
<td>Macmillan Education, 1988</td>
</tr>
<tr>
<td>Thinking Mathematically</td>
<td>J Mason</td>
<td>Addison Wesley, 1985</td>
</tr>
<tr>
<td>Proof</td>
<td>Plumpton C, Shipton E and Perry R L</td>
<td>Macmillan Education 1984</td>
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Study Skills for Mathematics

Members of the Working Group

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Activities

The activities of this Working Group were directed solely at reviewing and updating the booklet "Study Skills in Mathematics". A ‘Report’ as such was agreed to be unnecessary and distracting. Instead the end product of their discussion and work at UMTC 2001 is the publication of the revised edition of the booklet.
Introduction

Web Delivered Assessment (as compared with Computer Aided Assessment) offers new possibilities but also new challenges. But of course, many of the issues associated with Computer Aided Assessment will also be present if the assessment is delivered via the web. This working group will consider the various issues associated with Web Delivered Assessment, including both pedagogical and practical ones; it may consider how we as teachers relate to these new tools. It may evaluate and compare currently available tools and discuss features not yet provided. It may chart some examples of "good practice" or how the use of Web Delivered Assessment relates to the model for the learning cycle, which underpins the set-up of a course.

The role of assessment in a course often deserves more consideration. These days, assessment is expected to refer to the stated learning outcomes of a course, but is often still very much restricted to traditional written papers. Advances in technology over recent decades now make it possible to replace parts of or the whole of this assessment by computer aided assessment (CAA). There are now also possibilities to have this assessment delivered via the WWW, where students may e-mail or submit material through a web-based form. This creates new possibilities:

- CAA is no longer confined to one particular computer lab, but can simultaneously be executed over several computer labs, even if they contain different hardware or run different operating systems. This opens the possibility to use CAA as parts of the summative assessment of larger classes.
- CAA becomes more portable even within a course: it can be used again even if problems (timetable, technical faults, etc) force you to use a different computer lab.
- CAA can be administered remotely, allowing students to sit tests from home, their hall of residence or different labs on campus, at any time (24 hours a day, seven days a week sometimes).
- CAA can be used to tests students in different countries, allowing distance learning or closer liaison between certain Universities and their feeder schools overseas.

The tools for CAA and Web Delivered Assessment are still being developed, with problems still persisting in the support for Mathematics on the web. Some packages are developed for general use (QuestionMark, tests in WebCT, etc) but others try to use the power of Computer Algebra Packages to break out of the restrictions that these packages may contain (e.g. AIM). There are several issues associated with the use of Web Delivered Assessment. Some of these also occur in the use of other forms of CAA:

issues of a pedagogical nature:
- what do we assess;
- how do we assess it;
- why do we assess it;

the practical ones:
- how do we know the work is the student's own, i.e. "Security";
- are we limited to some form of multiple choice, or do we allow traditional style;
- how do (and should) we control access to other web sites during a test;
- what prior IT training is needed;
- how does this affect our role as a teacher.

The Remit

Consider the various issues associated with using Web Based Assessment for Mathematics or how we as teachers relate to these tools. Identify some of the key issues. You could suggest what technology could be utilised now to use Web Based Delivery, and what is required for the future, to promote its use. You could suggest some examples of the use of Web Delivered Assessment in a course, and how their use relate to the model for the learning cycle which underpins the set-up of the course.

Indicative Resources

1. Diagnosys - http://www.staff.ncl.ac.uk/john.appleby/diagpage/diagindx.htm
3. Issues of computer based assessment: http://www.calm.hw.ac.uk/calmpubs.htm
7. LTSN Mathematics, Statistics and Operational Research http://ltsn.mathstore.ac.uk
Issues for Web-Delivered Assessment

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1. Introduction

Recent years have seen the development of sophisticated computer aided assessment tools used on university networks. Examples include TASMAT, the assessment arm of CALMAT, DIAGNOSYS, a knowledge based system for testing basic mathematical skills, and the assessment engine developed for the TLTP Mathwise project (see also appendix (i)). These are examples of Computer Aided Assessment (CAA). Increasingly however, such assessment tools are being developed for internet delivery, primarily using the Web.

In this report we look specifically at some of the issues relating to Web-based assessment delivery (as compared to CAA). First we outline what we identify as the advantages and disadvantages of Web-based assessment. This is followed by a brief account of some recent developments. Then some of the teaching and learning implications of Web-based assessment are considered under two headings, purposes of assessment, and types and modes of assessment. The report continues with a discussion of the practical issues involved in adopting Web-based assessment such as implementation, the implication for resources, and problems related to security. After this there is a brief discussion of possible future developments and, finally, the Group’s conclusions. There is also a list of useful references and an appendix highlighting some details from the references.

2. Advantages and Disadvantages

In evaluating the advantages of Web-delivered assessment, the Group was careful to distinguish between Web-based assessment and Computer Aided Assessment in a non Web-based environment. The latter has many advantages, most of which also apply to Web-delivered assessment. However, the group wished to identify advantages that are exclusive to Web-delivered assessment.

The only advantage of Web-delivered assessment unanimously acknowledged by the Group is that such assessment need not be restricted to specific locations or times. Apart from this considerable factor, the Web-delivered approach appears to offer few advantages over conventional Computer Aided Assessment. However, use of the Web could lead to:

- formative feedback that is more readily available to students and which can help to promote more efficient learning.
• collaboration between geographically separated institutions in devising databanks of suitable assessment materials that could then be made widely available via the Web. This would reduce the duplication of effort that is currently invested across institutions in producing conventional assessment materials. The Group was of the belief that the quantity and quality of materials so produced could surpass that of materials produced by individuals working in isolation.

• benefits to the lecturer by allowing large groups of dispersed students to be managed by exception.

• easier access to assessment for disabled students (see for example the work of the Accessible Educational Media (AEM) Group at the Open University) and students who find it difficult to attend assessment at specific University locations.

Of course, there are disadvantages such as:

• the set-up costs for commercially produced software, such as Perception, can initially appear relatively expensive both in terms of site licence cost and staff time needed to produce assessments.

• Computer Aided Assessment may force the assessor into a particular type of questioning such as multi-choice assessment (although it is evident that this is becoming less of a problem as the software develops). In particular, the typesetting of mathematics on the web is still an issue.

• the possibility of student plagiarism is greater when assessment is taken remotely without an invigilator.

These issues are discussed below in Sections 4 and 5.

3. Recent Developments

The use of a search engine and a key phrase such as ‘Web-based assessment’ will produce a large number of hits, many of which report how traditional CAA has been implemented on the Web. While these are useful in themselves, they do not necessarily focus on issues relating to the advantages and disadvantages of the Web as a delivery platform. The authors have found that a useful starting point is the paper “Web-based Assessment: Two UK Initiatives” [1] which describes instances of claimed practice in the use of CAA on the Web. It also includes a discussion of the advantages and limitations of Web-based CAA although it is clear that many of these apply equally to traditional CAA. Security and authentication issues, discussed later in this report, are also considered. The paper [1] concludes with examples of other CAA developments and relevant references. In a similar way the paper “Web-based Assessment Software: Fit for purpose or squeezed to fit?” [2] reviews the software package Perception and its use in Web-based assessment delivery. Other useful Web-sites are included in the references at the end of this report. Readers may also find the Journal of Computer-Aided Learning, Volume 17, No 3 (September 2001) interesting, since it is dedicated to the subject of computers and assessment.
4. Educational Issues

4.1 Purposes of assessment

Teaching in Higher Education was once exclusively of the face-to-face variety. However, developments in Computer Aided Learning, CAA and the Web have enabled an increasing emphasis to be placed on distance-learning franchises. At the same time, the number of people entering Higher Education is increasing much more rapidly than resources. The Group believes that it is therefore imperative that methods of assessment which do not rely upon conventional means are investigated and evaluated.

As indicated above in Section 2, the Web offers an efficient method of delivering modes of self-assessment for independent learners. It also offers a means of providing both formative and summative assessment that is not overly burdensome or onerous for staff. The frequency of testing may either be self-paced to meet the needs of the individual, or may be determined by the tutor. In either case, students may receive rapid or even instantaneous feedback on their efforts (see for example [3]). This can be an important factor in providing and maintaining motivation, particularly for students who have limited access to tutorial assistance. Even where students have ample access to tutorials, Web-delivered assessment may still provide an exciting alternative dimension to the learning experience. We believe that this helps to meet the expectations of today's computer literate generation. From the staff point of view an additional benefit is the ability to generate an up-to-date profile of students, and this will enable precise and efficient help to be put in place.

4.2 Types and Modes of Assessment

Conventional methods of assessment employ a spectrum of approaches. These range from highly structured closed questions with multiple-choice answers to open ended questions, such as existence proofs, with solutions that are not well suited to machine evaluation. Whilst it is evident that the former is most easily implemented via the Web, nevertheless Web-delivered assessment need not be restricted to such approaches. For example, the MathWise assessment engine enables longer, more traditional examination questions to be set by the process of embedding and ‘hiding’ stages of the solution, which a student can access (at a cost) if they are stuck.

As well as multiple-choice answers, modern software will cater for answers which require the input of mathematical symbols and expressions. Techniques for parsing of text and the location of graphics also allow machine comprehension of a wider range of responses from the students. However, even though subsequent improvements in technology are likely to increase the available range of types of assessment, the Web itself is unlikely to provide anything that will not also be available in a more conventional setting.

Tutors may also have greater flexibility to customise the assessment and to provide appropriate restrictions for the time allowed for student to complete assessments, the deadline date for submissions, the number of attempts, and so on.
5. Practical Issues

5.1 Individual versus Institutional Web-sites

A number of Web-sites have been developed by individuals to satisfy the needs of their students and themselves; these sites tend to be limited in their appeal to a wider audience. Whilst the energy and resourcefulness of these individuals is to be applauded we nevertheless believe that such developments are best co-ordinated at the institutional level, and even possibly across institutions, e.g. in consortia.

5.2 Implementation

There is a wide range of implementation styles ranging from the individual Web-site through to the corporate image presented by a University adopting a Virtual Learning Environment, such as WebCT (http://latis.ex.ac.uk/webct.htm) or Blackboard (http://www.ncl.ac.uk/ltsu/Bbhome.htm). An example of the development of a corporate image may be found at the University of Huddersfield where the VLE 'Blackboard' is administered and maintained centrally by a Learning and Teaching Unit. This has the advantage that lecturers with no special Web knowledge are able to place their course notes and other module information on a readily available system. It also means that students accessing a number of different courses need to learn the navigation of only one system. However, assessment capabilities can be somewhat limited in such VLEs and it is sometimes evident that VLE developers have not always given the same attention to assessment as they have to learning and teaching issues.

5.3 Resources

There are two distinct phases in the development of a Web-based assessment system; the creation phase followed by the delivery phase. The resource requirement of these two phases is different though interdependent. Institutions typically tend to underestimate the time and effort required in constructing and maintaining these systems. We suggest that a time allowance be available for individuals to undergo appropriate staff development. The process is not simply a matter of dumping one's lecture notes and other items on the Web. The type of material and the way in which it is assessed will, in all probability, need to be completely rethought. For example, if Web-delivered assessment takes place at locations that are separated by geography or time, then similar topics will need to be assessed in a similar fashion if a uniform standard is to be maintained though there might be some random element in the allocation of questions.

5.3.1 Hardware

We would expect a student to have appropriate access to a PC. This could be either via the student's own machine connected to the Web or via a PC in the Institution's own laboratory. If a course is delivered within an institution, then the provision of sufficient computer laboratory space for the regular tutorials is essential.

5.3.2 Software

Questions are required before assessments can be made. These may be found in question banks such as TASMAT, or may be individually constructed with appropriate software. A very widely used commercial package for question construction is Perception, a Web-
based version of Question Mark. The latter does require a familiarity with its features and offerings and a period of staff development would be recommended before it is used in anger. In addition to a major package such as Perception a number of other units, typically MathType, TeX and LaTeX, can be very useful in the typesetting of questions that include mathematical script.

5.4 Security

Issues of security and authentication have existed as long as students have been assessed but they have become particularly acute with the advent of Web-based assessment. If remote assessment without supervision is allowed, how can an assessor be confident that the work is solely that of a given student? We do not believe that there is a complete solution to this problem at present but we do have a number of suggestions to make. Specific anti-plagiarism measures include the following.

Use of accredited assessment centres
This is already common practice with the Open University. All the usual security measures can then be implemented.

Use of passwords and IDs
This is normal practice and provides a minimal measure of security and authentication.

Invigilated summative tests
It is recommended that where possible summative tests should take place in a controlled environment with an invigilator present.

Final written examinations
Whilst computer assessment may be used to motivate students it is recommended that a written examination, taken under normal conditions, is used to supplement the computer based assessment.

Regular and frequent testing of students
Whilst collusion is a real possibility for a single test it is less likely to occur for a series of tests of short duration and high frequency.

Student records
Maintaining a computerised record of a student’s progress enables efficient comparison of formative and summative results and hence makes it easier to detect wild fluctuations in performance.

Bio-metric authentication
There is commercially available software that is capable of identifying students from the rhythm of their keystrokes. More speculatively, Web cameras and voice recognition software could be incorporated into the system so that a visual and aural watch could be maintained on the candidate. See (http://www.biopassword.com).
6. **Future Developments**

The delivery of assessment to off-campus students, sometimes at overseas distance-learning centres, may well be one of the prime advantages of using Web-based assessment, but it is probable that other potential benefits will emerge over time.

Currently many institutions are collaborating in the development of CAA (see the CAA Centre Web-site [www.CAACentre.ac.uk](http://www.CAACentre.ac.uk) for details), and this collaboration is likely to develop both nationally and internationally. This introduces the possibility of producing national/international databases of assessment questions together with appropriate learning material. In principle, students from one institution could be graded using assessments provided by another, thus encouraging the development of internationally recognised qualifications. This may have a profound effect on institutional control of assessment procedures and the roles of external examiners.

A co-ordinated national/international collection of subject-specific assessment questions may be of benefit both to tutors and students. On a national level, such a collection could be realised if an organisation such as the LTSN were given the remit of identifying existing databanks of assessment material that are available to public access via the Web. A further programme for the construction of a national databank might then also be implemented. This, however, would require careful coordination as well as considerable extra funding.

The benefits to tutors of such a databank would include a great saving of preparation time and the possibility of new applications. Students would gain opportunities for more assessment practice and an increased appreciation of the many possible uses of the material they study. For such moves to be successful however, the reluctance of academics to use material prepared by others may need to be overcome.

Given that encryption procedures appear to be sufficiently mature to support secure systems, the verification of student responses to questions is not a significant issue. Nevertheless, even allowing for advances in the field of biometrics (see for example [http://www.biopassword.com](http://www.biopassword.com)) the authentication of a student taking an assessment at a remote location is likely to remain problematic. The authentication in this system is through keystroke recognition and has a claimed success rate very close to 100%. One can easily imagine that authentication systems using palm prints and retina scans, amongst others, might also provide useful security for Web-based assessment procedures, provided that costs are acceptable.

7. **Conclusions**

We believe that Web-based assessment has a great deal to offer the educational world as a vehicle for flexible delivery of material and is an excellent facility for the support of distance-learning. It is possible to conceive of a system in which students in many areas, both in the UK and the rest of the world, can gain the advantages of a good education without the need to attend many, if any, formal classes. The Open University has, of course, been successfully operating on these lines for more than 20 years, but their mode of delivery and means of assessment have been predominantly along conventional lines.
With the advent of the Web however, a different system of distance-learning, in which a major part of the material and much of the assessment is Web-based, is a distinct possibility. For such a system to match the success of the Open University a number of issues must first be addressed. In this report we have identified some of these, particularly the issue of resources and the problem of security. The former is a question of political will whilst the latter requires technical advance, but neither is insurmountable. Some systems are already available for authentication of individuals at distant or insecurely monitored locations.

Even given these undoubted attributes of the Web, and the possibilities it holds, we favour a model of delivery in which there is a substantial amount of lectures, with associated material being made freely available on the Web. In addition to lectures, we also feel that regular tutorial meetings are of great benefit. We are of the opinion that, all else being equal, face-to-face teaching, with its personal ingredient and immediacy, will always be the ideal form of education.

8. References


See also these web-sites:
A. http://www.caacentre.ac.uk/
B. http://www.scaan.ac.uk
C. http://www.lboro.ac.uk/service/flu/licaa/
D. http://www.biopassword.com
E. http://cvu.strath.ac.uk/ae/
F. http://www.imsproject.org/
Appendix

Exemplars of current practice in Web-based assessment also include:

(i) Learning Teaching Support Network list (The LTSN mission states that it exists to ‘promote high standards in the learning and teaching of Maths, Stats & OR by encouraging knowledge exchange, innovation and enterprise, leading to an enhancement of the learning experience for students’.)
http://ltsn.mathstore.ac.uk/articles/maths-caa-series/index.htm
http://ltsn.mathstore.ac.uk/articles/maths-caa-series/au2001/index.htm
http://ltsn.mathstore.ac.uk/articles/maths-caa-series/ju2001/index.htm

(ii) Two UK initiatives (Abstract: This paper describes two UK initiatives aimed at promoting and disseminating best practice in the use of computer assisted assessment. Both initiatives are tackling the technological, pedagogical and organisational aspects of using the Web to support student assessment. Some of the more interesting problems being tackled include the interoperability of assessment systems and the use of XML to represent questions and tests. Both groups have also looked at the security requirements of online student assessment and are beginning to develop strategies that are technically feasible, pedagogically sound and work within existing organisations).
http://evu.strath.ac.uk/dave/ausweb2k/presentation/

(iii) University of Wolverhampton (The following is an extract from their Web site: ‘The Computer Based Assessment project at the University of Wolverhampton has produced a range of programs to replace traditional written tests with computerised versions. The primary objective was to save the lecturer time by leaving the program to write, mark and give feedback on the test! Our research (see the references below) has shown that these tests can also have a significant impact on student learning outcomes, particularly increasing subject confidence and perception of understanding.’)
http://cba.scit.wlv.ac.uk
http://www.csv.warwick.ac.uk/alt-E/alt-C96/demos.html#eleven

(iv) SCAAN (The Universities of Glasgow, Heriot-Watt and Strathclyde established the Scottish Computer Assisted Assessment Network (SCAAN) in mid-1999. It is funded through the SHEFC WebTools initiative which is designed to disseminate information and evaluate Web-based learning tools. The particular focus of this project is Web-based assessment.)
http://www.scaan.ac.uk/

(v) Charter Oak State College (The 'school without walls' holds no classes but instead grant credits based upon faculty evaluation of onsite and distance-learning courses transferred from regionally accredited colleges and universities, non-collegiate sponsored instruction, standardized test, special assessment, contract learning and portfolio assessment. Degrees offered at both Associate and Baccalaureate levels.)
http://www.cosc.edu/
Appendix
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