This paper presents a case-study of a mathematics teacher who has substantively developed her mediational strategies and her conceptions of mathematical knowledge as a result of her participation in an in-service program. Her new teaching practices raise serious dilemmas for her in relation to her teaching context, in particular, the limited knowledge of her pupils. Using Vygotsky's notion of mediation in the zone of proximal development, it is argued that her successes, dilemmas and constraints raise important questions for mathematics teacher development programs in such contexts. In particular the position of contextually grounded pedagogical content knowledge is considered. (Author)
Mathematics teacher development and learner failure: challenges for teacher education

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This paper presents a case-study of a mathematics teacher who has substantively developed her mediational strategies and her conceptions of mathematical knowledge as a result of her participation in an in-service programme. Her new teaching practices raise serious dilemmas for her in relation to her teaching context, in particular, the limited knowledge of her pupils. Using Vygotsky's notion of mediation in the zone of proximal development, it is argued that her successes, dilemmas and constraints raise important questions for mathematics teacher development programmes in such contexts. In particular the position of contextually grounded pedagogical content knowledge is considered.

Introduction

"I can't just go to the next lesson, I can't...I can't just go. You have seen maybe two, three, four students they understand what they are doing but I cannot just go to the next lesson because two, three students understand ... if I don't finish I'll be sorry but I think it will be better if the things that I have done they understand, unlike to finish the syllabus only to find that they...they know nothing of the things I've done. ... if I don't finish the syllabus of course I will be worried but it will be better if they know one thing that I have treated with them.”

The above quote expresses the dilemma of a teacher who wants to work with pupils' meanings but also needs to ensure that pupils learn the knowledge of the curriculum. Jaworski (1994) and Edwards and Mercer (1987) have documented how teachers in England manage this tension. The teacher who said the above words is a secondary mathematics teacher in a rural, over-crowded, under-resourced South African school. For her the dilemma is acute, because so many of her pupils fail mathematics. Her context introduces a second dilemma, implicit in the above quote. Does she focus on building the mathematical knowledge of the majority of pupils or does she focus on the few who might pass? Focussing on the majority would require working at a much lower level, she would not cover much of the Grade 10 syllabus, and would therefore disadvantage the few who are at Grade 10 level and might be able to pass. If she focuses on the few relatively more successful pupils, she will disadvantage the majority. Similar dilemmas

1 I am noting this failure as a reality which needs to be addressed. I do not wish to set up deficit explanations, and I particularly do not think it is attributable to either the teacher or the learners.

2 These pupils are successful relative to the pupils who fail badly in this school. In reality their mathematical knowledge is also severely limited and in schools with better results, they would be considered weak.
are experienced by all teachers. However, the pervasiveness of failure in this teacher's context foregrounds the dilemmas and makes them more acute. In this paper I will argue that such teacher dilemmas present particular challenges for research and teacher education in such contexts.

To do this, I will present some of the story of the teacher whose dilemmas I have outlined above. She was a student on an in-service programme and a participant in a research project which followed a sample of teachers who enrolled in the programme in 1996. She was a successful student on the course, both in our terms and in hers. She feels that through the course she has changed from being an angry, frustrated teacher, to someone who is able to understand pupils' difficulties, both in and outside of the classroom. The research observed that she was able to appropriate and use in her classroom much of what she had learned on the course, in particular with respect to pedagogy. She elicits and hears her pupils' meanings and attempts to work with them in order to develop new knowledge. In Vygotsky's terms she attempts to mediate in the zone of proximal development (Vygotsky, 1978). We (Brodie, Davis and Lelliott, 1999) have characterised her as a teacher who has substantively taken up learner-centred mediational strategies from the programme. However, the interactions in her classroom and her subsequent dilemmas raise new questions in this context.

What happens when such a teacher adopts learner-centred practices? This paper will describe classroom incidents, her reflections on them, and the dilemmas that arise for her. On the basis of these dilemmas, it will address the question as to what the notion of mediation means when pupils' knowledge base does not enable access to new concepts. Thus the paper will raise important questions about learner-centred mathematics teaching in South Africa, and contribute to arguments that the notion of learner-centred pedagogy and its theoretical underpinnings may need to be tempered with local realities, particularly, but not only in developing countries (Tatto, 1999; Clark, 1999).

The teacher and her context

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3 The programme provides a Further Diploma in Education, and is known as the FDE.
4 The aims, methods and first two years of analysis from this project have been written up in Adler et al, 1997, 1998. This paper is based on the data of one teacher over the three years of the project. This teacher was observed, and her data analysed over the three years by Kgethi Setati, Jill Adler and myself.
Ms Nhlapo* teaches mathematics and science in a rural secondary school in the Northern Province. She originally trained as a primary school teacher, but moved to help found her current school when the community decided that a secondary school was necessary in the area. She has been teaching Grade 8-10 mathematics since 1995, and feels that the FDE has given her the confidence to move into Grades 11 and 12 (in 1999 she is teaching grade 11). Ms Nhlapo's knowledge of mathematics is strong in relation to the rest of our sample, although there are certainly limitations. She did well in her mathematics courses in the FDE and in her previous studies. Although she is primary-trained, she is clearly competent and confident in the topics that she teaches up to grade 10 level and her conceptual difficulties in class originate in very subtle mathematical issues.

Ms Nhlapo's school is very poor. There are eleven classrooms for 705 pupils and 18 staff members. Seven out of the eleven classrooms are not complete, with windows, doors, floors and roofs missing. There are not enough desks and chairs, and many that are there are broken and hardly functional. The classrooms have small portable chalkboards which are not always visible from all points in the classroom. The school receives limited numbers of textbooks. The number of pupils in a class ranges from 40 to 120, and the classrooms are built for 40. There are high rates of absenteeism among pupils, with 10 or more pupils absent from most of Ms Nhlapo's lessons. There is also much teacher absenteeism.

Many pupils in the school are overage and some are refugees from Mozambique. In order to get a sense of what pupils know and can do in mathematics we examined their books and tested the pupils*. A detailed examination of the books of Ms Nhlapo's Grade 10 class suggests that much of the work in the syllabus is not covered, and of those sections that are covered, only the easier concepts and examples are dealt with. As the quote at the beginning of the paper shows, Ms Nhlapo is acutely aware of how much of the syllabus she has not covered, and she is extremely concerned about it.

The pupils' test results in the table below tell a depressing story. The April and June tests were set by Ms Nhlapo based on what she had taught. The research tests were developed by the research team to try to access pupils' conceptual and procedural knowledge. Ms Nhlapo's Grade 10 pupils were given the Grade 9 test and were able to complete only the very basic questions. Questions requiring slightly more complex mathematics, or a combination of skills were not done successfully.

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5 This is a pseudonym
6 The purpose of the tests was to see what testing could tell us about pupils' knowledge. In fact they told us very little beyond the fact that pupils know very little. In order to get at the complexities of pupils' knowledge we would need more sophisticated instruments.
No. in class | No. wrote | pass | fail | below 20%
--- | --- | --- | --- | ---
April | 44 | 42 | 9 | 33 | 19
June | 44 | 36 | 8 | 28 | 15
Research (Gr 9) | 44 | 34 | 1 | 33 | 28

Poor test results were not unique to Ms Nhlapo's class. Similar results were pervasive in our research, they get worse in the higher grades, and culminate in an overwhelming matric failure rate in South Africa. Pervasive failure as a consequence of an under-resourced context limits possibilities for effective mediation in the zone of proximal development. Given her lack of material resources, Ms Nhlapo views herself and her pupils as her primary resources and tries to work extensively with pupils' knowledge in relation to mathematical knowledge. She tries to probe pupils' meanings and work with their ideas through a questioning strategy which breaks with conventional classroom discourse. However, her new style of questioning brings her dilemmas to the fore, makes them more acute and raises difficulties with the kinds of mathematical knowledge she is trying to teach.

**The “Teacher's Dilemma”**

Ms Nhlapo describes the aim of her questioning strategy as follows:

"where I use question and answer ... I like them to tell me why they say this ... I don't just take the answers I do follow up to ask why they say that, whether it's correct or wrong I like to follow up because they might be guessing or something like that or to be able to help other students.” (my emphasis)

Consistent with her aim of working with pupils' meanings and misconceptions, Ms Nhlapo often probes for a range of answers and then asks pupils to explain why they have answered in a particular way, before settling on the right answer. Thus she does not evaluate immediately, but rather waits for a number of ideas to be in the public domain before doing so. In doing this, she breaks substantially from the conventional I-R-F exchange structure with a strong evaluative component, where the teacher goes for the correct answer and ignores wrong answers (Edwards and Mercer, 1987). This practice was evident to a limited extent in Ms Nhlapo's teaching in 1996 and she used it more frequently in 1997 and 1998. For example:

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7 It is crucial to note here that the above results cannot serve as an indication of whether Ms Nhlapo is a successful teacher or not. We did not investigate whether there was any improvement in her pupils' mathematical knowledge and thinking as a result of her improved teaching. The above table serves to underline one of the major constraints that Ms Nhlapo works with, her pupils' weak subject knowledge.
T: How many terms \[\text{writes 4x3x2}\]
P1: 3 terms
P2: 0 terms
P3: 1 term
P4: 0 terms
T: Anyone with a different answer?
P5: 2 terms
T: Can you count your 2 terms
P5: Its 1 term
T: (to pupil 5) Why now 1 term?
(silence)
T: Can you tell us why 3 terms?
P1: \(4 \times 3 \times 2\)
T: Meaning you have counted the numbers 4, 3 and 2
Why are you saying 1 term? Its correct to understand 1 term but why do you say it?
P: (inaudible)
T: You multiply it and get 1 number so its one term.
Now who said 0 terms, Confidence?\

Here, Ms Nhlapo allows all possible answers to be tabled before asking for reasons and evaluating them. Her tone of voice in her questions: “why 1 term?” and “why 3 terms?” is the same and does not suggest which one she favours. Even when a correct explanation has been achieved she continues to ask for explanations for wrong answers. This is a considerable break with conventional participation and evaluation patterns in the classroom. The difficulty in developing and sustaining such an approach cannot be underestimated, particularly when there are pressures of time and limited pupil knowledge. That Ms Nhlapo continues to use it after three years suggests that she finds it a useful teaching strategy. Her rationale for this teaching strategy, expressed in the quote above, is firstly that she does not have to do all the explaining, pupils who understand can do it for her, which may help the others, and secondly it enables her to identify and work with pupils' misconceptions and mistakes.

There are however a number of limitations to the way this mode of interaction is played out in Ms Nhlapo's classrooms. Firstly, although Ms Nhlapo is careful not to favour particular answers and explanations, in the end she must emphasise the correct answer, because this is what pupils need to know. Thus Ms Nhlapo finds herself firmly in the teacher's dilemma (Jaworski, 1994; Edwards and Mercer, 1987). The teacher wants to hear, acknowledge and affirm a range of pupil ideas. However, ultimately she must approve the right answer.

The teacher's dilemma is one in which all teachers find themselves and can be understood from a Vygotskian analysis of teaching. Teaching is constituted as mediation by a knowledgeable other in the zone of proximal development. Teachers mediate between what pupils know and can do, and what they must learn, the knowledge and actions (or

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8 This extract has been reconstructed from field notes, not transcribed from a video tape so it may not reflect the exact words of the pupils in some cases.
scientific concepts) of society (Vygotsky, 1978; 1986; Wertsch, 1984). The “gap” of the zpd is necessary for fruitful interaction and learning to take place. Good teaching involves negotiating this gap and the tensions that arise from working in it. However, what happens when the zone is too large?

A second example from Ms Nhlapo's classroom illustrates just how large the zpd can be. Ms Nhlapo had written the expression $3i \times 3b \times 3a + 6z$ on the board, and asked the question: “how many terms?”. One pupil answered that there are four terms in the expression. Ms Nhlapo asked why, and the pupil did not answer. Another pupil clicked his hand to answer and Ms Nhlapo reprimanded him saying: “give him time to think”. When the (first) pupil still did not answer, Ms Nhlapo tried to scaffold his thinking, asking “what are terms separated by?”. The pupil answered: “addition and subtraction”. She then asked: “what are the terms in the expression?”. He did not answer and she called him up to the board asking him to write an addition sign. He wrote a subtraction sign and she asked the class if he's correct. The class laughed and she reprimanded them. She then wrote an addition sign and explained in XiTsonga to the pupil.

In reflecting on this incident in her interview, Ms Nhlapo said:

“I called him because I could see that Mm, Mm...there is more problem in him than just a minor problem that I was thinking, because ...I asked him whether he seen a negative or addition sign there, which makes him say four terms, while it was multiplication, and he said addition sign. Then I said maybe it's my handwriting because multiplication and addition are...the same sort of, if you don't put them nicely and my handwriting is not good. So I thought maybe he might think my multiplication sign is addition. I thought that way when I called him to come and write the addition sign. I was amazed when he wrote subtraction...So I said oh my God then it means it's a serious problem, meaning I should just sit with him down and try to help him more. I will try to find some ways, I don't know how but I could see that he's having a serious problem not just a minor problem. It's very serious and you can see he's older. ..... I was shocked, I didn't even know what to say, really I didn't know.... I was worried, I was worried. I didn't know what to think quickly or what to say even now I'm still worried...”

I shared Ms Nhlapo's shock when the incident occurred. It is untenable that a pupil in Grade 8 draws a subtraction sign when asked to draw an addition sign, even if he may be feeling panic that he has not so far managed to answer the teacher's questions and on the spot in front of the class. The above incident is significant in that Ms Nhlapo tried to follow through the thinking of an individual pupil in order to deal with his difficulty. However, she was unable to do so in a manner which shielded him from ridicule, which helped his learning of mathematics, and which helped the class to progress with their work. The incident shows the level of pupil difficulty that Ms Nhlapo must deal with among her many pupils, and the difficulty of doing this through probing misconceptions and difficulties in class. Thus although she views her pupils' knowledge as a resource, it can be a severely constraining factor in her teaching. Further, in developing a strategy
which is helpful for some pupils in some cases, i.e.: probing pupils' knowledge, Ms Nhlapo creates new challenges for herself, which she is yet to develop ways of dealing with.

**New challenges for the teacher**

The first of Ms Nhlapo's new challenges was mentioned in the introduction. Her new questioning strategies reveal her pupils' knowledge to her and so she is more aware of its limitations. Her learner-centred approach does not allow her to ignore these, she must work with them. However, her pupils' limited knowledge base means that her initial attempts might create zpd's that are too large. Learner-centred pedagogy would then require that she creates appropriately smaller zones which support the learners' progress towards the ultimate goal. This is what she tries to do, but given where many of her pupils are, she does not often achieve her teaching goals, and this disadvantages the stronger pupils. Ms Nhlapo makes a choice in this regard, she chooses to work with the knowledge of the majority. However, given the enormity of this task, the implications of her choice are that most of her pupils' achievements in mathematics are not improving\(^9\) and she is not managing to cover enough of the syllabus. The zone is too wide for her to negotiate in ways in which she is trying to at the moment. We could ask whether she manages to create a zpd for most pupils.

A second, related challenge concerns the mathematics that Ms Nhlapo is trying to teach. She characterises her mathematical goals as:

> "I would like my pupils to learn the...the content itself, the language in mathematics using correct language... My aim is to teach my students to...to use these concepts, mathematical concepts correctly and the calculations itself to calculate properly"

Ms Nhlapo emphasises developing the language and concepts of mathematics with her pupils. This is something that she has learned about from the FDE programme. She integrates aspects of the programme, by using pedagogical strategies that she learned about in one part of the programme, with teaching mathematical language that she learned about in another.

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\(^9\) We did not set out to investigate in a textured way what pupils were learning as their teacher changed her practice, and thus we cannot identify even slight, improvements in their mathematical thinking which may result from her increased attention to their existing knowledge. These may be there, and may make the substantial effort she is putting into changing her practice worthwhile. However, what is clear is that Ms Nhlapo's pupils' achievement and knowledge remain extremely poor. There are no obvious indications of improved learning.
Unfortunately, it may be the case that the strategies she chooses are not the most appropriate to teach what she intends to. In the above extracts, Ms Nhlapo is working on the definition of the notion “term” in an algebraic expression. Probing pupils' responses in these cases may help her in identifying misconceptions. However, in both extracts above, the range of possible answers that pupils might give is limited, and ultimately there is only one correct answer, because of the definitional nature of the task. Almost all of the pupils' meanings that Ms Nhlapo elicits in her lessons are definitional or procedural (when she works with calculations, another key aspect of her teaching).

It may be the case that many pupils cannot read the ground rules (Edwards and Mercer, 1987) of her new form of classroom discourse. They may not understand why she keeps asking “why”, or that her questioning of ideas does not necessarily mean they are incorrect. Moreover, because the tasks ultimately only allow for one correct answer, the pupils might become confused when a range of possibilities are put up and not strongly evaluated, particularly given their difficulties with mathematics. They may not be able to identify which are the correct answers. Ms Nhlapo could address this by trying to make her discourse rules explicit. Ms Nhlapo might also try to refine her use of the question “why”. When she asks “why” she is asking for different kinds of responses at different times, including clarification, justification and explanation.

The question arises as to whether the strategies Ms Nhlapo has worked so hard to achieve might be more appropriate if she set more exploratory tasks, or worked more conceptually. This aspect of her practice is one that Ms Nhlapo has not yet developed and so we do not have the empirical means to answer the question in this case. However, remembering the size of the zone between where pupils are and where they need to be, we might predict that Ms Nhlapo would face similar problems and we might be challenged to think about how to help her face them.

**New challenges for teacher development**

What can we learn from Ms Nhlapo's teaching which can help us to think more appropriately about learner-centred teaching in contexts characterised by pervasive and acute failure and limited pupil knowledge? I have argued that Ms Nhlapo struggles to mediate between existing and new knowledge because of the large distance between these. She makes choices to close the gap and work at lower levels, and still she does not succeed. I have also argued that there are mismatches between her pedagogical strategies and the mathematics that she is trying to teach.
An analysis of the FDE programme shows that Ms Nhlapo's achievements in her classroom come from thoughtful study and participation in the programme. The mathematics teaching course in the programme emphasises pedagogical strategies and is complemented in this respect by education courses. The mathematics content courses have helped her to develop a broader view of what she might aim to do in the classroom and have increased her confidence and competence in teaching mathematics. However, these two aspects have not yet been brought together to help her to think about pedagogical content knowledge. By this I mean, how might this teacher bring together her knowledge of pedagogical strategies and her knowledge of her pupils' mathematical thinking to help her think about the most appropriate strategies to create and mediate within appropriately sized zones of proximal development.

In order to achieve this, two key research and development projects are necessary in South Africa. First, we need to develop more accurate pictures of what South African mathematics learners know and can do at various levels in the education system, ie we need to find out about learners' actual levels of development. Researchers and teacher educators need to focus on learners if we are to enable teachers to do so. This is a substantial research project, and requires theoretical frames and research instruments which can capture what pupils really do know, in context. Second, we then need to develop, together with teachers, more realistic possibilities for teachers to work with pupils' knowledge. In “zpd” language, we need to consider how to create successive zpd's which move from where learners are to where they need to be. One example would be to refine possible teaching strategies. For example, Ms Nhlapo could benefit from reflection on her different purposes in asking “why”, and what mathematical and pedagogical function her questions serve in the zone of proximal development.

What I am suggesting here is that contextually grounded pedagogical content knowledge should be a key aspect of teacher development programmes. This is the case in all contexts. In contexts characterised by pervasive failure, particular pedagogical content knowledge is required which enables teachers to mediate large or non-existent zones of proximal development. Dilemmas experienced by teachers such as Ms Nhlapo should be foregrounded in our thinking about mediation and made the object of discussion with teachers. Hopefully in this way resolutions might be found in relation to particular mathematical knowledge in particular circumstances.

References


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