A study was conducted to examine how principal components analysis (PCA) and Profile Analysis via Multidimensional Scaling (PAMS) can be used to diagnose individuals’ observed score profiles in terms of core profile patterns identified by each method. The standardization sample from the Wechsler Intelligence Scale for Children, Third Edition (WISC-III; 1991) was used. This sample of 2,200 cases included 200 children in each of 11 age groups from 6 to 16 years. Core profiles were estimated from PCA and PAMS analyses of the WISC-III standardization sample. Three principal components were obtained from PCA and two dimensions were identified by PAMS. When root mean squared deviations were computed from differences between observed score profiles and replicated score profiles by each method, the results indicate that the PAMS approach replicated the raw data better than the PCA approach did. The uses of both methods are discussed. (Contains 6 figures and 14 references.) (SLD)
Diagnose Test-Taker’s Profile in terms of Core Profile Patterns:
Principal Component (PC) vs. Profile Analysis via MDS (PAMS) Approaches

Se-Kang Kim†
The Psychological Corporation

Mark L. Davison
University of Minnesota-Twin Cities

†Now at Harcourt Educational Measurement. This manuscript was prepared for the annual meeting of the American Educational Research Association, April 2003, Chicago, IL.
Correspondence concerning the manuscript should be addressed to Se-Kang Kim, Harcourt Educational Measurement, 19500 Bulverde Road, San Antonio, TX 78259. Do not quote nor reproduce without the consent of authors. Electronic mail may be sent via Internet to se-kang_kim@harcourt.com.
Diagnose Test-Taker's Profile in terms of Core Profile Patterns:

Principal Component (PC) vs. Profile Analysis via MDS (PAMS) Approaches

This study is designed to examine how Principal Component Analysis (PCA) and Profile Analysis via Multidimensional Scaling (PAMS) can be used to diagnose individuals' observed score profiles in terms of core profile patterns identified by each method. In education and psychology, the most frequently used commercial standardized test batteries typically provide users with a variety of subtest scores in addition to global index scores. "Profile analysis" is a generic term used to describe the practice of distinguishing between groups of test-takers based on their unique configuration, or pattern, of subtest scores (Stanton & Reynolds, 2000). In the arena of cognitive testing, there exists a rich history of debate over the clinical utility of results from profile analyses in making differential diagnoses and designing appropriate interventions on the basis of an individual test taker's profile. According to Carroll (2000), profile analysis is useful insofar as "recommendations can be made that have known probabilities of producing selected outcomes" (p. 450).

According to this criterion, however, the literature does not reflect impressive support for the utility of profile analysis for individual test takers (see Watkins, 2000). Ipsative scores is one form of profile analysis, where a test-taker's difference scores (that result when the average scaled score across subtests is subtracted from each component subtest score) yield a "profile" of his/her subtest strengths and weaknesses. McDermott and his colleagues have shown that ipsative scores on cognitive ability tests are degraded psychometrically, are largely ineffective in discriminating between clinical groups, and are
not effective in predicting academic success (McDermott, Fantuzzo, & Glutting, 1990; McDermott, Fantuzzo, Glutting, Watkins, & Baggaley, 1992; McDermott & Glutting, 1997).

In addition to these problems, an individual's unique profile does not provide information about how profiles are related to the profiles of other test-takers. Hence, there is no way to ascertain the extent to which an individual's profile is common, or unusual, relative to a larger and more representative group. For this reason, researchers have turned their attention to the identification of "core" profiles (McDermott, et. al., 1990), which represent a smaller number of normative profile types that reflect the most commonly occurring profiles in a data set.

Methods for the identification of core profiles begin with the selection of a measure that reflects the degree of similarity/dissimilarity between individual's obtained profiles. In selecting similarity measures, profile analysis researchers must determine the extent to which a given measure is or is not sensitive to differences in a person's profile shape (i.e., the pattern of peaks and valleys across subtest or factor scores), scatter (i.e., the degree of dispersion of scores around their average), and/or elevation (i.e., the average value for all subtest or factor scores for an individual). Some similarity measures (such as the Pearson correlation coefficient) are sensitive only to differences in profile shape, while other measures (i.e., distance measures; see Aldenderfer & Blashfield, 1984) are more sensitive to elevation and scatter differences. Two most popular methods for profile analysis are introduced: Cluster and modal profile analyses.

Clustering procedures (Aldenderfer & Blashfield, 1984) provide an alternative for identifying "core" profiles in a data set. The essence of a cluster analysis is to classify
objects into meaningful sets, where the objects within each set are more similar to each other than the objects of other sets, and each set is relatively unique. The cluster analysis yields groups of people. The mean of each subtest score for all people within the cluster is interpreted to be a description of the profile that characterizes the cluster. “Modal profile analysis” (MPA), a hybrid of cluster and Q-factor analysis (Cattell, 1967), yields clusters that vary in terms of profile shape. MPA identifies the most frequently occurring and replicable profile shapes in a dataset, which thereafter are referred to as “modal profiles” (Skinner & Lei, 1980; Pritchard, Livingston, Reynolds, & Moses, 2000). MPA is useful to those who wish to focus on individual differences in profile pattern, but is of less use to those interested in individual differences in level, as this eliminates the influence of profile elevation differences in identifying meaningful profiles (Pritchard et. al, 2000).

According to Davison and Kuang (2000), MPA relies on ipsatized scores that have been standardized to have a variance of 1.00. This procedure permits an individual’s profile to reflect the pattern of the original scores but not the level (average of all subtest scores) of the profile. For researchers who value information provided by a profile’s level parameter, the MPA profile analysis method may not be the wisest choice. In addition, Davison and Kuang (2000) note that MPA can be difficult to apply when sample sizes are quite large, because the matrix to be factored is a respondents by respondents matrix.

Similarly, cluster analysis can be difficult to use with large samples. The cluster analysis method begins with computation of a proximity matrix (usually a correlation or distance matrix) defined over all possible pairs of people (not tests). As the sample size grows the size of this proximity matrix grows almost exponentially, and at some point becomes too large for analysis. Due to the proximity measures employed, the test profiles
describing the various clusters have differed primarily in terms of profile level, not shape or scatter (e.g., Glutting, McGrath, Kamphaus, & McDermott, 1992; Konold, Glutting, & McDermott, 1977). Thus, the clusters have mainly described individual differences in overall profile level or general intelligence, rather than individual differences in profile pattern. Researchers who are primarily interested in profile pattern may not wish to employ the cluster methods. Because of shortcomings of cluster and MPA approaches in profile analysis, we introduce two methods, Principal Component Analysis (PCA) and Profile Analysis via Multidimensional Scaling (PAMS), which include both level and pattern information about profiles of interest.

**PCA Model vs. PAMS Model**

In the PCA model, measures can be explained by linear combinations of mean item scores, component scores, component loadings, and error terms (e.g., measure = item mean + component score × loading + error). The component scores in the model are associated with people and index the subject sensitivity to variation among items along components. The parameters associated with the items are component loadings.

Similarly, in the PAMS model, measures can be explained by linear combinations of level (which is actually mean score of a person), weights, MDS coordinates (scale-values), and error terms (e.g., measure = level + weight × scale-value + error). Here, weights are associated with people, which reflect the subject sensitivity to variation among items along dimensions and the scale-values are item parameters.

Davison (1985) showed that after excluding the first component of PCA the rest of the components corresponded to the MDS solution, when the same correlation matrix was analyzed. In other words, K - 1 components in the K principal components solution,
where the first component was excluded, can be replicated by a \( K - 1 \) dimensional solution. The first component is considered a "g" factor (or a general intelligence) in cognitive ability test settings. It is straightforward to establish a connection between the first component score and the level parameter in PAMS since the first component score represents overall ability for an individual and the level parameter is an average score of a person who took the test battery (e.g., WISC-III).

Three connections are made in the present study between; (1) the level parameter in PAMS and the first component score in PCA; (2) dimension scale-values and component loadings; and (3) person weights and component scores. To do this, \( K \) components and \( K - 1 \) dimensions are extracted using the same data set and \( K - 1 \) component loadings (excluding the first one) and \( K - 1 \) dimension scale-values are used for coordinates of core profiles. Then both component scores and person weights in PAMS are estimated by regressing observed scores of individuals onto test parameters (component loadings or dimension scale-values) and used as correspondence indices between test-takers’ score profiles and core profiles for each method. Component core profiles are superimposed on PAMS core profiles in a figure to aid in visual inspection. In addition, randomly selected children’s score profiles are diagnosed/interpreted in terms of core profile patterns for each method (PCA or PAMS).

Method

The standardization sample from the Wechsler Intelligence Scale for Children - Third Edition (WISC-III) (Wechsler, 1991) was used. The sample of 2200 cases included 200 children in each of 11 age groups ranging from 6 to 16 years. The sample included
100 male and 100 female participants in each age group. For each age group in the standardization sample, the proportions of Whites, Blacks, Hispanics, and other ethnic groups were based on the group proportions of children aged 6-16 in the U. S. population according to the 1988 Census survey.

WISC-III includes 13 cognitive subtests; Information (IN), Similarities (SI), Arithmetic (AR), Vocabulary (VO), Comprehension (CO), Digit Span (DS), Picture Completion (PC), Coding (CD), Picture Arrangement (PA), Block Design (BD), Object Assembly (OA), Symbol Search (SS), and Mazes (MZ). The first six subtests were assigned to Verbal Factor and the other seven subtests were assigned to Performance Factor (Wechsler, 1991). For group factors, IN, SI, VC, & CO were assigned to Verbal Comprehension Factor; PC, PA, BD, OA, & MZ were assigned to Perceptual Organization Factor; AR & DS were assigned to Freedom from Distractibility Factor; and CD & SS were assigned to Processing Speed Factor (Wechsler, 1991). All these 13 subtests are also used for the current study.

First, core profiles are estimated from PCA and PAMS analyses of the WISC-III standardization sample. In fact, principal components excluding the first principal component from PCA and dimensions from PAMS are interpreted as core profiles. A 3-component solution is used for PCA and a 2-dimensional solution is used for PAMS. The first component corresponds to the level parameter and the second and third components correspond to the first and the second dimensions in the PAMS solution, respectively. The first component scores for individuals are used to determine elevations of observed profiles for PCA. The second and third component scores are used to quantify the correspondence between PCA core profile patterns and observed profiles, as do person
weights in PAMS.

Three sets of correlations are computed to examine consistency between PCA and PAMS approaches in profile analysis; (1) correlations between PCA and PAMS core profiles; (2) correlations between the first component score and the level parameter; and (3) correlations between the second & third component scores and two person weights (on two dimensions). Finally, observed score profiles for children are diagnosed in terms of core profile patterns. For example, a person with a substantial weight on Verbal Ability, but a trivial weight on Performance Ability is predicted to do better on Verbal Ability tests than on Performance Ability tests. If the value of the level parameter for this person is equal to 1.0, this person's overall ability is above average (since the level parameter is standardized to zero mean). In other words, this person's overall score is above average. In order to examine which method replicates observed profiles for individuals better, root mean squared deviations (RMSD) are computed from differences between observed score profiles and replicated score profiles by each method.

Results

Using 2200 participants of the WISC-III standardization sample, three principal components were obtained from PCA and two dimensions were identified by PAMS. The 3-component solution explained 61% of the total variance of the WISC-III standardization sample. The first component that represents the overall cognitive ability, so called g, solely occupied 70% (=43/61) of the explained variance (which is 61% of the total variance) and the other two components that identify core profile patterns occupied 30% (=18/61) of the explained variance.
For the 2-dimensional solution of PAMS, Stress (using Kruskal’s stress formula 1) was .12 and R-squared was .95. The R-squared value is the proportion of variance of disparities that are accounted for by their corresponding distances. The disparities are Euclidean distance values among monotonically transformed original dissimilarity data and the distances are Euclidean distances among coordinates of dimensions. The stress value indicates the degree of discrepancy between disparities and distances.

Since the second and the third components were assumed to correspond the first and the second dimensions in PAMS, respectively, they were assumed core profiles and were labeled. Note that the first principal component was assumed to correspond the level parameter in PAMS. The second component and the first dimension were label Performance vs. Verbal Profile, where Performance Factor (since WISC-III Tech. Manual identified Performance as a single factor) located on the positive side and Verbal Factor (WISC-III Tech. Manual labeled Verbal as a single factor) located on the negative side (see Figure 1).

Consistent with the results shown in the WISC-III technical manual, Performance Factor in profile analysis (both PCA and PAMS) included CD, PA, BD, OA, SS, & MZ except PC, which were all on the positive side and Verbal Factor included IN, SI, AR, VO, & CO except CO, which were on the negative side. The subtest PC was supposed to be on the positive side in the profile since this subtest was considered to measure performance skills, but different from our expectation, it was close to 0 in PCA and was on the negative side in PAMS. To inspect visual similarity/correspondence of profile patterns between PCA and PAMS solutions, the second component was superimposed upon the first dimension and the third dimension was upon the second dimension as shown
The third component and the second dimension were labeled **Attention/Concentration vs. Perceptual Organization Profile** since DS, CD, and SS distinctively appeared on the positive side and PC, PA, BD, OA, & MZ appeared on the negative side. The subtest DS that was assigned to Freedom from Distractibility Factor and CD & SS were assigned to Processing Speed Factor, whereas PC, PA, BD, OA, & MZ were assigned to Perceptual Organization Factor (Weschler, 1991). Other subtests were ignored since they were all flat and close to zero (see Figure 1).

To quantify measures of the correspondence between PCA and PAMS results, correlations were computed. First, correlations between PCA and PAMS core profiles were computed; $corr(DIM1, PC2) = .85$ and $corr(DIM2, PC3) = .92$. Second, a correlation between the first component score and the level parameter was computed; $corr(sc_{1(1)}, c_p) = 1.00$. Finally, correlations between the second & third component scores and two person weights (on two dimensions) were computed; $corr(sc_{p(2)}, \omega_{p(1)}) = .87$ and $corr(sc_{p(3)}, \omega_{p(2)}) = .94$. And then, observed score profiles for children were diagnosed in terms of core profile patterns identified by either PCA or PAMS. For example, a person with a substantial weight on Verbal Ability, but a trivial weight on Performance Ability would be predicted to do better on Verbal Ability tests than on Performance Ability tests. Or vice versa is true. For illustration, four individuals
were selected from the original data (N=2,200).

**Diagnosis of Observed Profiles Based on Core Profile Patterns**

For Participant #25, first, the height of the observed profile was examined:

\[(s_{c_{25(1)}}, c_{25}) = (-.52, .31)\]

where \(s_c\) refers to the first principal score, \(c\) represents the score for the level parameter, the subscript (e.g., 25) represents the participant's number, and the value of () in the subscript of \(s_c\) is for the order of principal component. These two scores determine the height of the Participant #25’s observed (score) profile. Since these two scores are standardized (e.g., Mean=0.00 and SD=1.00) or \(z\)-scores, the negative values \((s_{c_{25(1)}} = -.52\) and \(c_{25} = .31\) for Participant #25) means that the participant scored below average overall. To identify that Participant #25 belongs to which core profile, the second and third component scores and person weights on dimensions (simply dimension weights) were examined:

\[(s_{c_{25(2)}}, \omega_{25(1)}) = (2.83, 3.33)\] and \[(s_{c_{25(3)}}, \omega_{25(2)}) = (-.46, -.04)\]. This person is expected to perform better on Performance subtests than on Verbal subtests (see Figure 2).

However, Participant #152’s height of the observed profile was, \((s_{c_{152(1)}}, c_{152}) = (1.01, .86)\), which indicates that the participant scored above average. Weights for patterns were: \((s_{c_{152(2)}}, \omega_{152(1)}) = (-1.81, -1.86)\) and \((s_{c_{152(3)}}, \omega_{152(2)}) = (.17, .38)\). Since Participant #152 had substantial weights on the first core profile (Performance vs. Verbal Profile) but negative, while weights on the second profile were trivial. This person’s raw
score (or observed) profile is expected to be similar to the mirror image of the first core profile (Verbal Factor is on the positive and Performance Factor is on the negative side) and to perform better on Verbal subtests rather than on Performance subtests (see Figure 2).

Insert Figure 2 about here.

For Participant #8, \( (s_{c8(1)}, c_8) \) = (-1.09, -1.18), which indicates that the participant scored much below average overall. Weights for patterns were \( (s_{c8(2)}, \omega_{8(1)}) \) = (-.30, -.13) and \( (s_{c8(3)}, \omega_{8(2)}) \) = (1.11, 1.41). Since Participant #8 had substantial weights on the second core profile (Attention/Concentration vs. Perceptual Organization Profile), but trivial ones on the first core profile. This person’s observed profile is expected to be similar to the second core profile and to perform better on Attention/Concentration subtests than on Perceptual Organization subtests (see Figure 3).

The last example is Participant #113. The person’s profile height was \( (s_{c113(1)}, c_{113}) \) = (1.37, 1.45), which indicates that this person performed much better than average overall. Pattern weights were \( (s_{c113(2)}, \omega_{113(1)}) \) = (1.36, .76) and \( (s_{c113(3)}, \omega_{113(2)}) \) = (1.51, 1.26). This person’s weights were substantial on both core profiles, and it is expected that this person could do well on positive side-domains of two core profiles. In other words, Participant #113 would do well on subtests requiring Performance skills and
In order to examine which method replicates observed profiles for individuals better, root mean squared deviations (RMSD) were computed from differences between observed score profiles and replicated score profiles by each method. RMSD from PCA was 1.23 and RMSD from PAMS was 1.04. This result indicates that the PAMS approach replicated the raw data better than the PCA approach did.

Discussion and Educational Importance

Two of the most popular profile approaches, cluster analysis and MPA, have shortcomings. The clustering approach differentiates core profiles in terms of profile level, not pattern, whereas the MPA approach differentiates core profiles only in terms of profile pattern, not level. Therefore, researchers who are interested in both profile pattern and level may find these two approaches inappropriate. The profile level can be used to separate whether or not a specific person belongs to a certain group. For example, in the screening procedure for employment, the level parameter is a mean score of the test, for an individual, and thus determines whether examinees pass a certain criterion. Once some individuals are qualified with mean scores (or levels of their observed score profiles) above the criterion, they can be scrutinized by diagnosing their profile patterns. For example, if a
company wants to hire people with a specific profile pattern identified by the company (say, it is Performance Ability Profile: PAP), then the company will hire those examinees who have passed the initial screening and possess profile patterns identified in the PAP. When it is necessary to classify people in a given pool, the profile pattern has an important use.

Similarly, this illustration can easily be applied to clinical settings. Employing profile level and pattern information, clinicians or school psychologists use results from profile analyses to make differential diagnoses. The profile analysis results can then be used to design appropriate interventions for individuals. Therefore, in this study, we introduce PCA and PAMS methods that include both profile level and pattern information and evaluate these two methods.
References


Figure Captions

Figure 1. PCA Core Profiles Superimposed on PAMS Core Profiles.

Figure 2. Observed Profile Patterns Superimposed on Core Profile 1 and Mirror Image of Core Profile 1.

Figure 3. Observed Profiles Superimposed on Core Profile 2 and on Linearly Combined Core Profiles.
Figure 1. PCA Core Profiles Superimposed on PAMS Core Profiles

Core Profile 1: PC_2 vs. DM_1
(r = .85)
Core Profile 2: PC_3 vs. DM_2

(r = .92)
Figure 2. Observed Profile Patterns Superimposed on Core Profile 1 and Mirror Image of Core Profile 1
Participant #152 vs. DIM_1
(c = .86, w1 = -1.86, w2 = .38)
Figure 3. Observed Profiles Superimposed on Core Profile 2 and on Linearly Combined Core Profiles

Participant #8 vs. DIM_2
\(c = -1.18, w1 = -0.13, w2 = 1.41\)
Participant #113 vs. (DIM_1 + DIM_2)
(c = 1.45, w1 = .76, w2 = 1.26)
Diagnose Test-Taker's Profile in terms of Core Profile Patterns:

Principal Component (PC) vs. Profile Analysis via MDS (PAMS) Approaches

Author(s): Se-Kang Kim & Mark L. Davison

Corporate Source: Harcourt Educational Measurement

Publication Date: 4/24/03

In order to disseminate as widely as possible timely and significant materials of interest to the educational community, documents announced in the monthly abstract journal of the ERIC system, Resources in Education (RIE), are usually made available to users in microfiche, reproduced paper copy, and electronic media, and sold through the ERIC Document Reproduction Service (EDRS). Credit is given to the source of each document, and, if reproduction release is granted, one of the following notices is affixed to the document.

If permission is granted to reproduce and disseminate the identified document, please CHECK ONE of the following three options and sign at the bottom of the page.

I hereby grant to the Educational Resources Information Center (ERIC) nonexclusive permission to reproduce and disseminate this document as indicated above. Reproduction from the ERIC microfiche or electronic media by persons other than ERIC employees and its system contractors requires permission from the copyright holder. Exception is made for non-profit reproduction by libraries and other service agencies to satisfy information needs of educators in response to discrete inquiries.

Signature: Se-Kang Kim

Printed Name/Position/Title: Psychometrician

Organization/Address: Harcourt Educational Measurement

Telephone: 210-339-5542

E-Mail Address: Se-Kang.Kim@harcour.com

Date: 5/30/03