There are millions of web pages available on the World Wide Web. Using the word "math" or "maths" in one of the many search engines such as Alta Vista brings forth details of many thousands of mathematics web pages. All around the world, mathematicians are more than willing to share their knowledge with others. However, this can present more problems than it solves. For example, what is available? Who checks the veracity of material published on the web? Who decides on its worth? Is the material copyrighted? Is one likely to be swamped by the sheer volume of what is available? Can the web be used to deliver courses in place of the standard chalk-and-talk delivery? Will this lead to a standardization in content and delivery? The emphasis of this paper is on the likely impact of such material on teaching, learning, and assessment and the issues resulting from such impact.

(Author/MVL)
Use of the internet in teaching mathematics  
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Introduction

There are millions of web pages available on the World Wide Web (WWW). Using the word 'math', or 'maths', in one of the many 'search engines' (such as Alta Vista), brings forth details of many thousands of mathematics web pages. All around the world, mathematicians are more than willing to share their knowledge with others. However, this can present more problems than it solves. For example, what is available? Who checks the veracity of material published on the web? Who decides on its worth? Is the material copyright? Is one likely to be swamped by the sheer volume of what is available? Can the web be used to deliver courses in place of the standard chalk-and-talk delivery? Will this lead to a standardisation in content and delivery?

It is not the intention to provide a review of available teaching and learning material on the web in this group report. Rather, the emphasis is on the likely impact of such material on teaching, learning and assessment and the issues resulting from such impact. For a more in-depth look at mathematics contents of web sites see the other report on mathematics and the Internet elsewhere within this Proceedings. However, the reader is encouraged to visit sites that describe issues relating to the use of web technology in course delivery such as “w4t” [1], COSE (Creation of Study Environments) [2] and WOLF (Wolverhampton Online learning Framework) [3]. The reader can also see details of MLE’s (Managed Learning Environments) and VLE’s (Virtual learning Environments) at such sites as [4], [5] and [6].

What is Available?

Generally, the use of the Internet can be categorised in a number of ways. For example:

- A repository of a lecturer’s printed handouts, tutorial questions, etc, quite often still in .txt, .doc, .rtf or .pdf format.
- Pages of tutorial material presented in full web HTML format - sometimes dynamic, interactive web pages, but quite often just an HTML form of notes and handouts.
- Full dynamic web pages quite often with the inclusion of Java applets (small programs that run over the WWW).
• Gateways to other mathematical web sites (e.g. Mathgate [7])
• Publications, Research papers, etc. (e.g. CTI Maths & Stats Newsletter on-line [8])
• Bulletin Boards, Chat Rooms, Telephony, Video Conferencing, etc. for students and tutors to exchange information on their course/module material, administration, etc.
• Major mathematical organisations (e.g. IMA, LMS, RSS, AMS)
• Web sites offering paid tutorial help (or even the completion of assignments!)
• Commercial web sites, advertising mathematical software or perhaps offering software downloads. (E.g. computer-based assessment via Question Mark Computing [9].)

Advantages and Disadvantages of using the WWW

The above list may suggest that there is nothing on the web that can't be done using traditional notes, tutorial software and learning resources centres at a university campus. So what are the advantages of using the WWW? Assuming easy access to the WWW at all times, an answer has to be accessibility, immediacy and quantity.

• The student no longer has to depend on being present on the right day at the right time 'to pick up the notes'.
• Mislaid notes or solutions to exercises no longer constitute a crisis when they're on the web.
• A workshop tutorial using a particular piece of software needn't be attended if the program is available over the web. (The negative ramifications of this will be discussed later in this paper.)
• Computer animations, illustrative or interactive software can become platform-independent and hence available outside your specific computer laboratory.
• Formative assessments, particularly in "drill" areas can be taken at any time, any place.
• Other material relating to the subject course can be referenced easily, whether it originates in the U.K. or elsewhere. (As an aside, this accessibility also means that it is possible to share your work with others around the world via the Internet, as will be discussed later).
• The Internet is 'immediate'. Using search engines, a student can find and download mathematical material from anywhere around the world at any time of day or night - and the quantity available is immense, as suggested above.
• Delivery of software over the Internet has the important advantage of 'robustness' - students cannot generally interfere with the set-up of software on a web site server and hence the provision of the material is potentially more secure. However, security issues generally remain (see later).
• Up to the time of writing, a search for 'math applets' using Alta Vista has shown a burgeoning number of sites with mathematical applets. Since these are necessarily dynamic, they are more likely to engage the student and, further, give the students sense of 'owning their own learning'. (see [10]).

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Since web pages are a dynamic medium, it is possible to ensure that the student can always receive the latest version of any text file or piece of software. Once material is produced in electronic format, it is relatively easy to upgrade. Additional benefits may be that it can then be converted to specific forms for students with special needs, e.g. voice for deaf students; Braille for blind students or large text for visually impaired students. Access to the material 24 hours per day, 7 days per week from other locations than a single dedicated computer lab can also facilitate access for other disabled students. However, there are currently a number of obvious disadvantages in the use of the WWW to consider. For example:

- Despite there being many mathematics web pages, at present there is a problem with producing such pages because of the lack of mathematical symbols available, although there is hope on the horizon in the form of MathML (see for example [11]) - a standard for mathematical ‘word processing’ on web pages. We would hope that this becomes available soon. In the meantime, one can see web pages with mathematical symbols generated from ASCII characters. Even producing mathematics notes as a text document can cause problems. One of the authors of this report has saved some tutorial worksheets that were converted from Word 97 to ‘portable document format’ (.pdf), but when downloaded from the web on some Unix systems, none of Word’s Equation Editor’s equations were displayed!

- Dynamic mathematics tutorial software is readily available on CD-ROM (see Mathwise [12], for example). Up until recently, producing such material over the WWW has proved impossible (except as downloadable files). With the advent of the Java programming language, it has become possible to write programs (applets) that are loaded via the web page to be executed by the web user, [10].

- Since the use of the Internet is rapidly evolving, it is not always possible to keep abreast of all the web sites that can be useful to mathematics lecturers and their students. Consequently it is important to ‘bookmark’ (save in your list of favourite web sites) important sites that offer guidance on using the Internet in teaching, learning and assessment. In a manner similar to the role of Abstract indices for reviewing mathematical literature, ‘gateway’ sites are needed. These sites will not only recommend useful mathematical web sites but also offer information on relevant conferences, books, etc. We will discuss further the need for such gateways later in this report.

- Designing and maintaining web sites can be a major issue for staff. In many cases, design and maintenance of web sites falls on the shoulders of the individual lecturer. It would save time and effort if universities or perhaps mathematics communities had some agreed ‘web-site template’ that allowed easy ‘cut-and-paste’ development for web page development rather than all lecturers ‘doing their own thing’. In some cases, enthusiasm for the benefits of the web can be diminished by the tedium of proper maintenance.
• Although greater access for disabled students has been listed as an advantage, there may be cases when certain types of students are disadvantaged. For example, gender and cultural bias using this technology may need to be considered and addressed.

The advantage of ‘distance learning’ has been discussed above. However, in distance learning, students talking to students (and lecturers) is still considered important and a potential disadvantage of the WWW is that this communication disappears. There are ways to overcome this using technology. For example, Open University students are given instruction in the use of 'First Class', a package which makes possible the use of threaded e-mail to enable such dialogue. Several distance learning courses make use of the chat-room facility provided by the package WebCT [13]. BLACKBOARD [14] offers similar discussion facilities as well as being an assessment tool. Even within universities, many lecturers are now using e-mail to ensure that all students are made aware of course requirements and commitments and to post course/module notes. This can happen between institutions where the Internet is also being used for telephony (a call to America at local telephone rates, for example) and video conferencing.

Implications of the use of WWW

In many respects the implications of WWW use for teaching mathematics are similar to those faced by lecturers and students using any other distance learning material. The potential advantages of accessibility, immediacy and quantity discussed in section 3, together with the popularity (especially by Government) and mass appeal of the web, have brought these issues into sharper focus.

(a) Teaching

As indicated above, the introduction of web-based resources opens up a wide spectrum of potential teaching arrangements. On the one hand, the lecturer has full control over the presentation of the subject material and the materials produced. Web delivery complements traditional lectures with the main advantage being that the materials provided are more accessible as discussed above. Staff remain in control of the whole process on a week by week basis. At the other end of the spectrum stands the complete automated delivery of notes, back-up materials, links and assessment (both summative and formative) over the web, needing no human interaction to run it except for the compiling of essential statistics (read marks) at the end by a well trained operator. In this case, after the module has been set up, the whole process is controlled (in terms of pace of learning) by the student. In many respects, this is how many existing forms of computer-based mathematics teaching software such as CALMAT [15] are used now.

Somewhere in between these two extremes lies a whole gamut of possibilities. A lecturer may have less control over the learning activities when they become more student-centred. There can be different levels of the use of technology, from putting text documents on the web, to generating interactive software, to deepen understanding, and to using web-delivered assessment to manage the students' learning process.
Increasingly, issues concerning the amount and nature of independent learning and other 'graduate skills' are becoming central issues in programme design and quality audits such as Subject Review (see [16]). Hence the use of the web as an 'independent learning device' (together with an evaluation of its use?) is likely to be an expectation rather than a choice in future programme construction. Web-based activities should not be seen as the only form of learning – student presentations, student group work, etc. should all feature.

It is clear that the extra effort in increasing the access to materials, or indeed introducing access to other materials (elsewhere), is done in the hope that it may facilitate the students' engagement with the course or module. This will very likely happen to some who might find they enjoy it more than before; others certainly will fail to take the opportunity to engage in this way. But, the possibility to make more and different materials available to students may help them to find something that suits them individually in coming to terms with the course or module requirements. What is needed is more examples of successful use of web material that can be shared amongst the mathematics community.

It may now be opportune to consider changes to the role of the lecturer. There may well exist very good notes on a topic, or videos of charismatic teachers, all considered to be successful learning materials, so that a traditional oral lecture becomes an unnecessary duplication. In such situations, the local lecturer's role may change to that of managing students' learning either individually or in small groups. This could include monitoring their progress through set assessment tasks, or their attendance. The lecturer's time can be spent diagnosing an individual student's problems or small group problems rather than concentrating on the delivery of subject matter, although there should be no suggestion that staff-student contact time is necessarily reduced. The role of the lecturer is discussed further in the management section that follows.

Staff-student interaction can be on-line, using chat rooms or bulletin boards, to avoid repetition of explanations. This could also encourage student-student interaction, but in an environment which can be moderated by the lecturer. Such schemes often benefit from an element of seeding of the discussions that take place. Using e-mail complements traditional ways of contacting staff and may be more desirable in certain circumstances. But these should not be seen as substitutes for personal contacts between lecturer and students. Technology, like WebCT [13], can help the lecturer by logging assessment scores and averages, how much login-time (and when) for each student, which can be useful at the final assessment.

(b) Student Learning

In an environment where teaching involves asking students to interact with web-based materials, it is likely that students will, in general, have to take more responsibility for the management of their own learning. Experience with distance learning material
suggests that students should be given regular milestones to achieve (tests, reports, etc.) to help structure their learning. These milestones would allow them to attempt/complete their study at a pace and a time of their own choosing. Students may have to study the same material a number of times before some degree of accomplishment is obtained, thus allowing them to pass on to the next milestone. It could be that different students studying the same material could reach a given milestone at different times. Students should be able to identify their own particular problems and resolve them with a tutor if necessary. In short, the possibility of producing real 'independent learners' exists with this more easily accessible technology. In early years, student access to the system and/or student attempts at formative assessments may need to be monitored to help the lecturer identify problems early on.

As indicated, 'virtual laboratories' offer the possibility of student 'learning by doing' or 'active learning' rather than the passive learning that can happen in a purely lecture-based course. Indeed, ultimately the student may have the freedom to make informed decisions that control the amount of formal attendance (if any at all) at an H.E. institution he/she requires to complete a course (including the assessment process). Formal meetings with personal tutors may be the only formal attendance requirement.

It is considered unlikely that this extreme, of almost complete remoteness of students from university and lecturing staff, will prevail. Nor indeed is it considered desirable as much anecdotal evidence suggests that student’s learning benefits from social interaction with each other as well as interaction with teachers in tutorial sessions. Any move towards increased independent/distance learning should be monitored carefully and is considered more likely to be seen in the later years of an undergraduate course (in a manner similar to the demands of final year project work). The potential step-change in student learning habits from school/college to first year at university could be harmful to students. Mathematics education research could be useful in identifying student-learning issues in such a learning environment.

(c) Assessment

In theory, the remote access possibilities of the web could lead to some or all assessment taking place at anytime or in any place, if software based assessment is used. Formal written examinations at a set time of the year could be replaced wholly or in part by a computerised assessment, which is made accessible to a given student at a given time with the agreement of assessor and student. Different students on the same course could be similarly assessed at different times, each student having a slightly different computer-generated assessment. Web-based software such as Question Mark PERCEPTION [9] allow this in some form and one of the authors of this report has had experience of so using PERCEPTION to assess a class of 150+ engineering mathematics students rather than using a formal written examination paper. In the same way, referrals could be attempted a number of times (although this idea of considering assessment like a ‘driving test’ may not be considered desirable). Disabled students could benefit from easier access to assessment material.
Video conferencing via the web would allow the possibility of remote vivas taking place and the recording of such vivas if required for future confirmation of correct examination protocols.

Computer-based assessment has the potential advantage of automatic and immediate marking and feedback to an individual student. Also, it may be possible (desirable?) to share questions and/or examination papers in common modules between Universities so as to reduce preparation time and maybe help to ensure comparable standards between universities. It is not considered likely that the above scenario, even if desired, is attainable in the near future. Reasons for such a statement include:

- The input of mathematical symbols and expressions on the web is not yet easy or standard.
- Setting computer-based questions, even in multiple-choice form, is not straightforward (if done properly) and does not cover all aspects of assessment. Hence there is still a strong need for written examinations involving such things as proofs, for example.
- Plagiarism, an issue unresolved now, is potentially more of a problem with remote access assessment, even if video conferencing is employed.
- The technology needs to be robust and secure to ensure not only that students can access the actual assessment but that a database containing student marks and other assessment records is maintained with access available only to appropriate people. Assessors can become a hostage to technology (see management section below).
- A heavier reliance on computer-based assessment could disadvantage some students who lack I.T. skills and who may not be able to finance their own individual web access.
- The whole issue of degree classification, number of referral opportunities, number of final year examinations sat at a time, etc. would need to be reconsidered at both local and national level.

New initiatives like AIM [17] do try to extend the functionality of computer based assessment for mathematics by using the power of computer algebra packages. A number of issues regarding attempts at more localised computer-based assessment (not necessarily web-based), have arisen and been reviewed by Race et. al. [18].

(d) Management

As described above, there is a continuum of possible models of the use of the web, each with its own "management" issues, from the relatively simple one in which only the notes for the course are on the web and are used to augment the lectures, through to the 'all-singing all-dancing' module in which all the material is on the web, making full use of the audio and video capabilities, and with links to other web-sites for individual parts of
some (or all) of the topics. Each model has its own validity, related to the purpose of the module and the intentions of the lecturer, and may thus represent "good practice".

There can be a problem if the student becomes too involved in the whole "web-process". As suggested above, it is advisable to set frequent milestones so that students have a framework in which to schedule their work. The need to plan the format of the course will increase because of the greater variety of possibilities and pathways. Obviously, the course/module guide to students at the start of the course/module should outline the format in detail.

The role of the lecturer may change from the "conventional" stand-in-front-of-the student-person to that of a "learning manager", i.e. someone who suggests pathways for learning to the students and who can act as a safety-net for those who experience problems. The lecturer may choose the pace with which the students are exposed to different ideas from other web sites and how these relate to the "home"-course. Providing a selection of vetted web links may help students daunted by the seemingly endless supply of materials and guard to some extent against exposure to inferior quality materials.

The increased variety of presentation offered by the use of the Internet may attract students with increasing diverse enrolment patterns; there may be more students wishing to enrol for part-time degrees or even for only individual modules. Lecturers will have to be able to organise the management of the module to cope with this diversity and the central administration of the university will also have to have in place systems for keeping track of the progress of individual students.

This control will vary (should vary?) for students in (say) first year courses/modules where recommendations to access the web may be very proscribed (e.g. visit site A and comment on the approach given there) and for students in later years where, one would hope, they have developed a level of independence and control over their own learning and its pace. It would be advisable, especially at the first year level, for the course handbook to be very explicit about what the students must do, what they should probably not do and what milestones have been put in place. The Open University course handbook is an example of this kind of material.

Accessibility using the Internet means not only that materials can easily be shared but also that the development costs of those materials can be shared. (Those wishing to keep their material within their own institutions can place the material on an institution-wide Intranet.) Whilst it is clear that there may be an overall increase in the workload for lecturers if they work in isolation, there is a potential advantage in a model in which lecturers from different institutions co-operate over the preparation and delivery of a web-based course. This co-operation could include the preparation and delivery of web-based assessment and of the resulting feedback to the students. The organisation of such courses is, itself, made simpler by the web where lecturers at different institutions can view and then comment upon draft web-pages prepared by their colleagues; the exchange of information has been made much easier by the advent of the Internet. In fact (and this
may not be considered to be an advantage!) the process of quality control (e.g. through the QAA) may become easier if the External Examiners can view the courses through the web.

With the development of video conferencing, the idea of sharing courses across universities becomes viable. In particular, this could be a way to broaden the provision in a subject area independent from the actual staff expertise in a given location. Although progress is being made in providing the infrastructure to enable this, the current financing arrangements and the competitive nature of HE Institutions may impede progress on the implementations of these ideas.

Individual lecturers will want to think very carefully before embarking on a project to set up a web-based course. They will, of course, wish to consider the points raised above and in other sections of this report and come to a judgement of the viability of the project in terms of the proposed outcome from the project compared with the effort that would be needed for its creation. They will also need to bear in mind the institutional support that they will have available to them, whether or not the appropriate resources (software, hardware and technical support) are readily available.

Consideration should also be given to any issues of copyright and intellectual property rights and to whom they belong. Once you have published the material on the Internet it now becomes available to a much wider audience. This, of course, can cause additional problems such as the author being now legally responsible for the material. This may need the lecturers to seek advice (possibly legal) and, conceivably, for a contract for the preparation of the module to be entered into with the university. This problem is more likely to arise when it is thought that the material might be attractive to an audience outside the institution concerned.

(e) Tools

The main problem associated with the development of material on the Internet has been the necessity to learn and produce HTML. Over the past few years there has been a variety of software tools which have made it readily possible for users to transfer their static lecture notes very easily onto the web via Word, Latex etc. The software currently available appears to be in its infancy but developments are occurring at a significant pace. One of the major factors influencing the development of Web based material is time and most current Web authoring packages require a considerable amount of investment before one can become proficient in the development of Web based material. There is significant scope for the development of a standard Web authoring tool which is both easy to use and has the capability to generate mathematical equations and allows the user to experiment with different standard Web page designs through the use of a development kit. As noted earlier, there are still problems in the production of mathematical equations and the need for platform-independent software which will allow the user to view the Web pages.
While most producers of academic web content will be familiar with Word and Latex for document presentation on the web and some will be familiar with HTML, few will be familiar with programming dynamic mathematical software for web. Here the would-be programmer is confronted with having to learn a programming language such as Java. Those already familiar with C or C++ will not find this too onerous whilst novices will want to hire the services of a professional programmer – so incurring possible cost. It is possible to buy software translators that transcribe programs written in Visual Basic, for example, for web delivery. However, Java is the most widely adopted approach since the latest versions of web browsers such as Internet Explorer and Netscape are capable of running Java software through the browsers themselves.

At present there is little monitoring or even a consistent strategy for producing or, for that matter, locating mathematical topics on the Internet. It would be a considerable improvement if a repository of Internet sites was produced and information on them consistently stored via key words and Meta tags which were recognised standards in the Mathematical community. This would then allow the development of new bespoke mathematical search engines, such as MathGate [7], to locate specific mathematical web sites. The database would have hyperlinks to the Web sites and would contain a standard front page indicating the content and use of the material contained within the site. A peer review process would also form an essential part of feedback on the material and the basis for further development.

(f) Resources

A move to Internet delivery of course material would have a significant financial impact on an academic institution as the majority of staff would have to become proficient in the use of this new technology, even if tools were developed to make web page design easier. ‘e-learning tools’ such as Lotus Learning Space [19] and the e-learning products promised soon by Microsoft may help to produce coordinated, Institutional-wide, web-based learning environments at a reasonable time cost to the individual academic staff member.

There is also a need for ‘standard’ software (computer algebra software such as Maple, Derive, Mathematica etc., IT software such as Word, Excel, etc. ...) to link up seamlessly with Internet browsers so that a student remote from a university campus can use all the software tools required with a minimum of fuss and delay.. Financial strategies for web based materials need to be implemented at institutional/governmental level.

The Future

The implementation of the National Grid for Learning over the next few years will raise the level of expectations on web-based materials with which students will arrive at university. So Higher Education must embrace the opportunities the Internet provides.
As described, the implications for change in teaching and learning are enormous. However, care must be exercised for at least two reasons:

- technology changes so rapidly that any heavy investment in a current technology or methodology may soon be superseded, e.g. the impending introduction of MathML
- there is as yet limited evidence of the benefits for the students of web-based learning and assessment.

We look to organisations such as LTSN (MSOR), together with other mathematical societies both national and international, to initiate research into the effectiveness of web-based learning, to disseminate the result of that research, and, through the LTSN staff development program, to train staff in the emerging technologies. We also look to them to assist in the evaluation of available materials, to encourage peer review and the establishment of common standards.

We take it as a given that the control over the teaching and learning processes using web-based materials remains with academic staff

Summary of Issues

Using the Internet could, for example, make it possible

- to increase the interaction with students on tutorials
- to increase the amount of one-to-one teaching
- to guarantee experts are lecturing on advanced topics
- to allow a more careful management of student activities where appropriate

On the other hand, the introduction of the Internet

- may tempt managers to discard human resources, i.e. academic staff
- will most likely lead to a change in the type of activities that need to be done, and, consequently a redeployment of staff and other resources
- requires very high set-up costs for the initial provision of courses

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