In a series of mathematics education workshops in which teachers from adult basic education and vocational education worked together to design teaching situations on particular contents in mathematics in order to make explicit and bring into reflection the teaching strategies used by each group. The workshops constituted a common space of interaction for jointly designing teaching situations and creating common discourses that narrowed the distance between normative curriculum and actual teaching or real curriculum. In order for adult educators to teach effectively they must fulfill the following conditions: (1) they must provide a context for the teaching; (2) the knowledge taught must constitute a solution adapted to the problem; (3) there must be didactic variables that lead to the development of logic in a fruitful way; and (4) they must take into account students' previous experience. (Two teaching situations from the reference situations of a hairdressing class and a cooking class are described and a discussion of proportionality, including how proportional situations are recognized and how they can be modeled effectively is included. The document includes an appendix containing problems from each of the teaching situations, 5 figures, and 12 references.) (MO)
Math In-Service Training for Adult Educators

Juan Carlos Llorente, Marta Porras, and Rosa Martinez
Universidad Nacional del Comahue, Argentina

I.- Introduction
The paper reports an experience developed in the south of Argentina with adult educators. We will concentrate on a series of workshops on math education. The workshops were part of a larger State project aiming at constructing a unified curriculum for adult basic education and vocational education. The workshops were seen as a tool to make explicit the actual teaching strategies in math education and, at the same time, as a way of creating conditions for teachers from adult basic education and vocational education to work together.

The aim was to create conditions to make explicit and bring into reflection the teaching strategies used by teachers of adult basic education and vocational education. Teachers (with formal training) and teachers of vocational courses (without formal training) participated in a series of workshops where they were asked to design a teaching situation on a particular content in the area of mathematics.

We will present some examples of the situations designed during the workshops. Then, we will discuss the situations considering the limits and possibilities of the teaching strategies regarding the particular content involved. For the analysis we will follow the theory of Brousseau.

II.- Didactic Theoretical Background
Education is a social event in which individuals “share meaning” in such a way that they understand the reality that surrounds them, consequently fostering their own development in everyday life. If we admit that school must prepare citizens for a satisfactory integration into the life of the society to which they belong, this conception claims for an adequate understanding of those realities of the culture in which they live and the capacity to make a profitable use of the necessary techniques of a certain work, “Every social project for teaching and learning is dialectically constituted with the identification and designation of savoir’s contents as the contents to be taught” (Chevallard, 1997).

The relationships students establish throughout the teaching-learning process depend, among other factors, on the interactions they have with such notions. At school, that relationship is generally segmented and algorithmic. It is essential that students are given concepts they consider of interest. At the same time, these contents must have sens. As Brousseau states (1993), the student builds the sens of notions, since he knows when to use them, whenever he utilizes them as problem solving tools, that is to say, taking the relationships he establishes with the mathematical notion and the teaching situation as a starting point.

Brousseau declares that all knowledge involved can be distinguished by one or more situations in which the sens of that notion is preserved. Adequate situations must be situations in which the mathematical notion to be taught is the best solution. If the sens of notions is not contemplated, the teaching-learning process is reduced to a mere circulation of official savoir.

Depending on the way the problem is posed to the students, the situations given in the classroom may have different “status”: to take decisions, to solve or communicate something new, to justify knowledge. Brousseau (1993), accordingly, identifies situations of action, formulation, validation, and institutionalization.

The study of a situation, of the conditions that will allow some notion to work in the classroom, demands the organization of notions in such a way that they can explain their origin, questions, and problems that have been posed. It does not have to do with the reproduction of its historical development, but it is a matter related to the organization of an environment where this savoir can “live.”
Within the framework of the Theory of Didactic Situations the work of teachers is, to an extent, inverse to that of the mathematician’s: he must look for situations that provide sens to the notions he has to impart and he must create the conditions (Llorente, 1998) that make those notions form part of the knowledge of a student at a given time. In order to transform the students’ answers and knowledge into a cultural and communicable savoir, students, with the help of teachers, will have to go back on their steps and take their productions out of context and make them impersonal in such a way that they can identify them with the savoir that is being developed within the cultural and scientific community of their time.

Taking into account all these general aspects the teacher will have to fulfil certain conditions:

- the context that will provide sens to teaching. In adult education, loom, cooking, or hairdressing classes can offer a network of referential situations that teachers can use to organize teaching situations.
- the knowledge to be taught constitutes a solution adapted to the problem.
- didactic variables that lead to the development of knowledge in a fruitful way.
- the student’s previous experiences so that he/she can try a resolution method, although he/she cannot solve it completely. This condition is especially relevant in adult education since they reach formal education with an amount of knowledge that has to be taken into account. If this previous knowledge is not taken into account, there is a serious risk of changing the sens of knowledge through the posing of childish situations.

III. The Analysis of Two Teaching Situations Outlined During the Workshops
We will base our analysis on two teaching situations outlined as from the reference situations of a hairdressing class and of a cooking class, respectively. These situations, which were outlined for teaching purposes, capture important aspects of the notion of proportionality, even though they can be approached by intuitive actions. We will try to make explicit, as best we can, the scope of these situations, considering the different aspects of knowledge that are involved in the solution process being practiced. We will also point out some of the conditions that are related to the study of the modeling process of the notion by means of functions, based on the sens of the latter.

It is evident that proportionality occupies an important position in culture outside the school. This is the reason why we favor productions based on this subject of savoir in the workshops, in order to try and demonstrate how this instrument may be a valid one to recover teaching strategies.

We can say that the reference situations proceeding from cooking, knitting, sewing, and hairdressing classes may provide a range of teaching situations that could be modeled through linear functions, proportional functions.

In order to select the situations that will be put forth in the classroom, one should take into account issues related to:

- how proportional situations are recognized;
- and, once they have been deemed to be proportional, how they will be modeled effectively.

The teaching situations designed by the teachers would fit into the distinction made by Brousseau on action situations. These situations may contribute to a first approach to a teaching content.
Modeling Through Functions

A proportionality relation is a bivariate function \( f: A \to B \), where \( A \) and \( B \) are subgroups of the real numbers so

\[
f(x) = k \cdot x \text{ with } k \text{ as a real constant other than zero.}
\]

This function verifies the following properties:

**Property 1:** \( f(\alpha \cdot x) = \alpha \cdot f(x) \quad \alpha \in \mathbb{R}, x \in A, (\alpha \cdot x) \in A \)

**Property 2:** \( f(x_1 + x_2) = f(x_1) + f(x_2) \quad \forall x_1, x_2 \in A \text{ and } (x_1 + x_2) \in A \)

III. 1.- Analysis of the First Situation

The situation taken as reference on topics related to hairdressing practices would offer a possible context for dealing with dependency and variability, two aspects that characterize functions.

In the situation presented by the teachers, the question is to find out the necessary volume in \( \text{cm}^3 \) of hydrogen peroxide needed for a tube of dye, in order to dye an average sized head. The figures given in the situation do not provide clarity regarding the variation of hydrogen peroxide content in relation to the tubes of dye used; that is, the concept of "one magnitude being directly proportional to another" is vague.

In solving this situation, operations with rational numbers are involved. It is probable that the students' experiences with rational number calculus allow them to apply scalar procedures that account for property 1.

If this experience does not show what is expected, it is possible to make use of the well-known "reduction to the unit" procedure or the "rule of three" technique. In the reduction to the unit procedure, the search for the image of 1 is distinctly expressed, but the relation between the two spaces (\( \text{cm}^3 \) of hydrogen peroxide-dye tubes) is not included, but it remains as a scalar procedure. The reduction to the unit procedure is often mistaken for the functional one (that expresses a relation between two magnitude spaces) because the means of calculating are the same. The difference between these is that, in the reduction to the unit procedure, what is sought is the image of 1, while in the functional procedure it is a \( k \) (constant). This \( k \) allows you to go from one magnitude space to another one. When applying the "rule of three" some "rites" are followed that in most cases are based on learned techniques rather than on the resolution of the question. How can a correspondence rule be recognized as proportional?

Number selection can be considered as a didactic variable as it enables the development of different procedures. In this case, changing the numbers to favor the use of procedures of the functional type would turn out to be "unnatural." To avoid this, a ratio of price and volume (in \( \text{cm}^3 \)) could be used.

For example, it would be convenient to count on data that ease the choice between buying a 40 \( \text{cm}^3 \) dye tube at \$4 or a 47 \( \text{cm}^3 \) tube at \$4.50, an ordinary situation in which it is easier to find the relation between 40 and 4 than between others.

<table>
<thead>
<tr>
<th>Dye volume ( (\text{cm}^3) )</th>
<th>Price $</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>47</td>
</tr>
<tr>
<td>47</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dye volume ( (\text{cm}^3) )</th>
<th>Price $</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>4.70</td>
</tr>
</tbody>
</table>

\[ + 10 \]

1 In compulsory schooling, the proportionality situations are generally restricted to \( N \) or \( Q^* \).
The selection of these numbers contributes to the construction of sens in the law of correspondence. In this case it is the correspondence between two magnitude spaces: volume of the dye and price.

III. 2.- Analysis of the Second Situation

The situation designed in the cooking class can be dealt with by means of intuitive actions, that is, doubling, tripling, and quadrupling the quantity of each ingredient. We can infer that these actions correspond to property 1.

The numbers proposed in the second instruction can also be solved intuitively. In this case properties 1 and 2 are combined and there are different proportional relations between each of the ingredients and the number of cookies. Although it is possible that properties 1 and 2 may be involved in each relation, it is senseless to carry out a mathematical analysis to explain them since an intuitive solution would suffice.

The situation would have to be conditioned so that the work in the classroom is not only intuitive but so that there is also room for the study of the properties as well as the laws of correspondence, that is, how and when some things change in relation to others.

The different relations between ingredient content and number of cookies could be presented in order to recognize proportional situations. The questions posed in each situation should not only consider the relation between the amounts used of each ingredient and the number of cookies that can be obtained, but they should also drive at establishing a relation among all the ingredients used. This conforms to the need for a deeper analysis of how and when proportional relations are present or not.

For instance, taking the basic recipe we can pose the following question:

A: For 1 kg of flour, how much butter is needed?

In the solution, at least two procedures could arise that express different relations:

The amount of butter used may be calculated on the basis of the variation of the amount of flour used.

<table>
<thead>
<tr>
<th>Amount of flour</th>
<th>amount of butter</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>200</td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
</tbody>
</table>

Or the amount of butter can be calculated using the ratio between the amount of flour and the number of cookies, and then using the result obtained to find out the amount of butter needed based on the number of cookies previously obtained. It is highly probable for this procedure to turn up if questions were previously formulated regarding the amounts of different ingredients and the number of cookies that can be obtained, or vice versa.

<table>
<thead>
<tr>
<th>Amount of flour</th>
<th>number of cookies</th>
<th>number of cookies</th>
<th>amount of butter</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>50</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>1000</td>
<td>125</td>
<td>125</td>
<td></td>
</tr>
</tbody>
</table>
B: How many eggs are needed for 150 g of butter?

The question poses the relation between number of eggs and amount of butter.

For 200 grs. butter, 2 eggs;
For 100 grs. butter, 1 egg;
For 125 grs. butter, 1 egg;

We can see that sometimes the relation established is proportional (as in A). Instead, the number of eggs is not proportional to the change in the amount of butter for making cookies, what will give elements to discern a proportionality situation from others.

If in the resolution we wanted to establish a relationship between the amounts of the two measures in question, giving rise to the law that governs the dependency between these two measures, we would need to pose a different situation.

A situation should be put forth, whereby its solution promotes the inclusion of the correspondence law in order to establish predictions and to take decisions.

For example:
The facade of a house is to be repaired. There are two possibilities:

A: The cost per m$^2$ of repair work is $5.
B: The cost per m$^2$ of repair work is $3.50 plus $20 invested in a tool needed for the job.

In which cases should we choose A and in which B?

In example B there is a cost which is proportional to the extension in m$^2$ that need to be repaired, and $20 of initial cost should be added to each example. In example A total expenditure is proportional to the m$^2$ that have been repaired.
Procedures involving the rule of three are not appropriate due to operating cost. The optimal procedure consists of finding the answer in the chart where the relation of the two options is shown. In order to choose the most convenient option, it is necessary to carry out an analysis, the scope of which should not be limited to obtaining results by calculating the particular figure variations. The use of a chart is interesting because it allows us to make predictions and take decisions comparing figures.

This situation tries to show the weakness of the application of the rule of three. In order to make a decision we need a series of organized data. To achieve this we may draw a chart or a graph expressing these relations. It is necessary to develop a comprehensive study of the functions that model the relations expressing the options.

Remarks
The selection of situations for dealing with proportionality should foresee that through retrieving intuitive procedures we allow for the development of knowledge. An analysis of the comprehensive characteristics of the function is preferred, rather than a segmented study based on techniques of resolution such as the "rule of three."

It remains the teacher's responsibility to continue with the teaching of each content, and therefore making the students understand the social significance of the knowledge skills involved (definitions, properties, graphic and colloquial representations, etc.).

As Brousseau (1995, p. 27) states:

The resolution of the problem may give the student the idea that there was nothing new to learn. Yet, being aware of having replaced an old and culturally identified strategy for another "invented" by himself, he will find it difficult to state that this innovation is a new learning. Why must it be identified as a method if it seems to come so easily when needed? How could an individual on his own identify from all the decisions he has taken those that derive from the situation and that could be useful in other situations from those which are purely local and occasional?

The social conditions of learning by adaptation, rejecting the principle of knowledge intervention of a third person to obtain an answer, tends to suppress the identification of this answer as something new, and therefore as something that derives from the acquisition of knowledge.

IV.- Conclusions
Finally we would like to point out the potentialities of the workshops as a tool for narrowing the distance between prescription and practice, that is, between normative curriculum and actual teaching or real curriculum.

We understand that any attempt to build a unified curriculum demands that two communities of practitioners—adult basic educators and vocational educators—be brought together. Therefore the workshops constitute common spaces of interaction for jointly designing teaching situations and create common discourses between the two communities of practitioners.

Another point we would like to make is that the intervention of a specialist in mathematics during the workshops proved to be relevant in controlling the logical structure of the content but avoiding didactic suggestions that could darken the educators' teaching strategies.
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Appendix

First Situation

1)

The dying of an average head demands the use of 75 cm³ of hydrogen peroxide per 1.50 tubes of dye. How many cm of hydrogen peroxide are needed for a base dye of one tube?

Possible resolution:

<table>
<thead>
<tr>
<th>Hydrogen Peroxide (cm³)</th>
<th>Dye Tubes</th>
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<tbody>
<tr>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>75</td>
<td>1.5</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>150</td>
<td>3</td>
</tr>
<tr>
<td>200</td>
<td>4</td>
</tr>
</tbody>
</table>

1.5 tubes ——— 75 cm³
1 tube ——— 75 + 1.5 = 50 cm³

2)

With one liter of hydrogen peroxide (1000 cm³), how many base dyes can we prepare?

Second Situation

1) 200 grs. butter
    200 grs. sugar
    2 eggs
    400 grs. flour

To make 50 cookies of 6 cm diameter

If I wanted to make 100 cookies, how much would I need of each ingredient?

- For 150 cookies?
- For 200 cookies?

2) According to the ingredients given in this example:

500 grs butter
500 grs sugar
5 eggs
1 kg flour

How many 6 cm diameter cookies will be obtained?
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Printed Name/Position/Title: Juan Carlos Llorente - Director - Doctor in Education

Telephone: 54-2941-423466

Fax: 54-2941-423503

E-Mail Address: jllorente@PAIDEIA

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