Three Spanish-speaking mothers of middle-school children, a relative of one of the mothers, a school librarian, and the researcher met regularly to establish a collaborative, two-way dialogue which would encourage social and educational equality and that would take contributions into consideration according to the validity of reasoning rather than the positions of power held by the contributors. The group had met in the past to discuss literature, and with the switch to mathematics, the researcher was concerned that the participants would not relate to the material as well as they had to literature and that the roles of teacher and student would be more clearly demarcated leading to less collaboration. The workshop met every two to three weeks and one of the main questions that arose was whether the researcher should treat the participants primarily as adult learners or as mothers. Through dialogues on algebra and geometry that arose through the mothers teaching the researcher the paper cutting technique called "papel picado," a trust was established in which the researcher was able to share her expertise while the participants were able to teach her some of their own skills that involved mathematics. The new context of learning through dialogue led to greater confidence among the participants, which improved their math learning skills. (Contains 17 references.) (MO)
Parents as Learners and Teachers of Mathematics: Toward a Two-Way Dialogue

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In a regular classroom we would not have learned it like this. Here we learn thanks to the comradship and "confianza" [trust].

In this paper I pursue the work I presented at ALM-5 and ALM-6 (Civil, 1998, 1999) on parents as adult learners of mathematics. My emphasis has been on establishing a two-way dialogue through which, while I engage the mothers in explorations about mathematics, we also collaboratively reflect on uses of mathematics in our everyday life and possible ways to establish connections between different forms of mathematics. I want to stress this collaboration aspect: we meet as a group to discuss mathematics. Granted, I come to the group as the "mathematics educator" but I am not there to "teach" but more to converse with them about mathematics. We are all learners and teachers in the group. Just recently, I have been reading about dialogic learning, in particular through the work of Flecha (2000). This work is grounded on the experiences of a group of adults in a working class neighborhood who participate in a literary circle that bears many similarities to "El Club de Literatura" in which we carry out our work in mathematics. I was struck by the similarity in orientation and will highlight some of the key ideas in Flecha’s work as part of the theoretical framework.

In this paper I focus on the process of establishing a dialogue about mathematics by looking at our experiences learning geometry. The geometry theme started through typical reform-based school mathematics tasks that allowed us to explore area and perimeter and their relationship (e.g., fixed perimeter, varying area). We then moved to a geometric exploration around the craft of “papel picado” (punched paper). The mothers taught me how to do “papel picado” and as we conversed through the practice, we brought up different mathematical questions to explore. The activity of “papel picado” provided a setting in which we all had something to learn and something to teach.

Context
Our work involves a small group of women who have been participating regularly in what we call “talleres matemácticos” (Mathematical workshops). The group consists of three mothers who have children in the middle school where we meet, one young woman who is a relative of one of the mothers, the school librarian, my colleague in the project, and myself. Other mothers have also joined us at different times, but I will focus on the group of “regulars.” These women are part of an ongoing project—“El Club de Literatura”—that started in 1995 and that centers around the reading and discussing of literature (some examples of authors read include Marcela Serrano, Angeles Mastretta, Garcia Lorca, etc.).

When the opportunity came up to expand the group activities to incorporate mathematics, these women were eager to do that. A question that came to my mind right away was: will we be able to engage in mathematical discussions that parallel the kinds of discussions and conversations that characterize the literature circle? This is a key question in our work since many of the activities we work on could probably be characterized as “mathematics for the sake of mathematics,” or “mathematicians’ mathematics” (such as the area/perimeter exploration I will discuss later in this paper), and thus, they may be less “personal” and harder to relate to (in comparison to the activities in the literature circle).

Theoretical Framework
Our work relies on three main bodies of literature: a) research on parental involvement that critically examines issues of power and perceptions of parents (especially minority and working class parents) (Henry, 1996; Vincent, 1996); b) research on adult education, especially that grounded on critical pedagogy (Benn, 1997; Frankenstein & Powell, 1994; Harris, 1991; Knijnik, 1996); c) research based on a socio-cultural approach to education (Forman, 1996; Moll, 1992; Rogoff, 1994; van Oers, 1996). (See Civil, 1999; Civil, Andrade, &
Anhalt, 2000 for some discussion of this research basis.) More recently, the work of Flecha (2000) on dialogic learning has become very helpful in my thinking about our work.

(dialogic learning) leads to the transformation of education centers into learning communities where all the people and groups involved enter into relationships with each other. In this way, the environment is transformed, creating new cognitive development and greater social and educational equality. (p. 24)

In particular, here is what he writes about one of the seven principles of dialogic learning—egalitarian dialogue:

A dialogue is egalitarian when it takes different contributions into consideration according to the validity of their reasoning, instead of according to the positions of power held by those who make the contributions. (p. 2)

Flecha points out three kinds of barriers to dialogue: a) cultural ("most members of the population are dismissed as incapable of communicating with each other using dominant knowledge" [p. 9]); b) social ("many groups are excluded from the production and evaluation of valuable knowledge ... Experiences that do not fit the mold are excluded" [p. 9]); c) personal ("many people's personal histories [and how they report them] lead to self-exclusion from many formative practices" [p. 9]).

These barriers are particularly relevant for us as the group of women we work with belongs to a minority group that has encountered many of these cultural, social, and personal barriers as women, Mexican immigrants, and working-class individuals. Furthermore, their personal histories as they relate to their learning of mathematics share many commonalities with so many other people who have been excluded from the "in-group" as mathematical learners through years of schooling that led them to believe that mathematics was not for them.

Of particular interest to our work in mathematics education is Flecha's view that dialogical learning encompasses but goes beyond constructivism:

[in dialogical learning] the meaning formation process does not depend solely on the intervention of education professionals, but also on all the people and contexts related to the student’s learning. (p. 23)

As we hear the women in the group sharing their learning with each other and with their family members, we see the role that other people and contexts play in their learning of mathematics.

The Mathematics Workshops
During the academic year 1999-2000 we met every two to three weeks for about two hours per workshop (the group kept on meeting once to twice a week with my colleague as part of the literature circle). In September we started by working on a combinatorics problem that the teachers in the research project BRIDGE were also going to try out in their classrooms (the work with the mothers is one of the components of project BRIDGE). The problem led to an investigation of patterns and of Pascal's triangle. From there, we continued with a few more sessions on counting problems and pattern problems. The group expressed an interest in learning algebra. Below is an excerpt from my journal as I reflected on this request:

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1 Project BRIDGE (Linking home and school: A bridge to the many faces of mathematics) is supported under the Educational Research and Development Centers Program, PR/Award Number R306A60001, as administered by the OERI (U.S. Department of Education). The views expressed here are those of the author and do not necessarily reflect the views of OERI.
The mothers had been asking to learn about algebra. Their children bring algebra homework and they don’t know how to help them. Algebra... what can I do that would be both meaningful and truthful to our approach, yet would allow them to help their children? They want to learn so much and I feel we have so little time in a sense. As I introduced the notation with y’s and x’s (or n’s), some of them brought up what their children are learning, they wanted to understand how to manipulate equations.

Here is my dilemma, do I start going over the often meaningless symbol manipulation that characterizes some of the algebra teaching that takes place in our schools? Or do I keep them working on the problems the way we do, discussing approaches, constructing meanings together? Eventually, we’ll reach the symbol manipulation, but hopefully in a meaningful way, but what about their immediate needs of wanting to help their children?

This excerpt captures one of my dilemmas as I work with the mothers. Do I put the emphasis on them as adult learners, or on them as mothers (Civil, 1999)? When we mention our work to others, quite often their reaction to having mathematics workshops for parents is “oh, so that they can help their children.” While this is certainly one goal, and may be what brings parents in initially, it is not our only goal. We are constantly reflecting not only on what we discuss in these workshops (issue of content) but also on why we are discussing it. We are aware that these women’s knowledge of mathematics is likely not to be recognized, considered marginal and of less value (Fasheh, 1991; Frankenstein & Powell, 1994; Harris, 1997). As Benn (1997) points out, we look for ways to connect learners’ everyday discourses with the unfamiliar discourse of academic mathematics. The mothers have made it very clear to us that they want to learn school mathematics, that is, the mathematics that their children are learning in school. As one mother writes:

I am so happy with all these mathematics workshops because I realize how to help my children understand mathematics in a different way, from a fun approach, all together as a family. ... And also for us, because one never knows when we may need/use it, and this way we move forward, and no one is going to mandate that is has to be the way they say, because we also think and solve problems.

Our Work in Geometry

Our work in geometry started with a series of activities on area and perimeter adapted from a middle school mathematics curriculum (Shroyer & Fitzgerald, 1986). Through the exploration of rectangles with constant perimeter and fixed area, as well as rectangles with constant area and varying perimeter, we continued our work in algebra through graphing, function writing, and some equation solving. The group became fascinated by the connections between the different topics and by how we went from hands-on explorations with tiles to getting a glimpse of parabolas and hyperbolas. Thus far, this was what I would describe as mathematics for the sake of mathematics. I personally enjoy these explorations and I enjoy even more seeing how others make sense out of the concepts. For example, I am interested in how they start conceptualizing what shape will have the largest or smallest area for a given perimeter and why. I am also interested in how they make sense of the graphical representation in particular in cases that I thought were rather “abstract” (for example, in looking at rectangles of area 12 tiles, we graphed one side as a function of the other; that is, since L*W = 12, we have L = 12/W; that graph gives us a branch of a hyperbola).

Throughout their work on this topic, I was clearly leading them. Because by then “confianza” [trust] had been established, these women were comfortable asking questions, challenging my and each other’s explanations. “Confianza” is an important concept in this work and these women wrote about it in the newsletter they put together in the Spring 2000:

When I integrated into the group de las Señoras, for me the most important foundation was the confianza that each one offered me.... I can say that all that I now know and have learned has been accomplished by means of the confianza (Newsletter, Spring 2000, pp. 2-3).
At last, I also have someone that more than a teacher is a friend and most importantly inspires me: Confianza [confidence, trust] the confianza that I in particular never had with any other teacher of mathematics. ... Thanks to the confianza that exists in the group we can work without problems and pose any sort of question without fear (Newsletter, Spring 2000, pp. 2-3).

Thus, even though I was leading many of the activities, I feel comfortable that had this been not of interest to them, they would have let me know. The “curriculum” is negotiated constantly.

The next topic in our exploration of geometry had them leading the activities as they taught me how to do “papel picado.” This is a traditional Mexican craft that is quite typical in our local context. Using colorful paper one creates intricate designs by cutting out shapes in a careful and pre-planned fashion. This practice involves lots of opportunities for the exploration of symmetry, and can be pushed in other directions such as finding areas and perimeters of the shapes being cut out. The task of learning to do “papel picado” was challenging for me (planning and visualizing what may come out is hard for me). As we sat around the table folding and cutting, the conversation would take different directions, but a main thread was to try to identify the mathematics in this practice. As a non-expert in “papel picado” and as someone who considers herself as rather limited visually, I was having a hard time connecting my years of academic mathematics to the practice of papel picado.

Since we had just been working on area and perimeter, one of the women in the group suggested cutting shapes for which it would be easy to find their area and then we could figure out how much of the sheet of paper had been cut out. What captured my interest in this activity is the fact that their much better understanding (than mine) of the whole process of “papel picado” made them the leaders of the activity. As I watch the videotape of that session, they sound very confident, they were in charge of the exploration and of teaching me how to make “papel picado.” They right away realized that to make 2 by 2 squares, they would have to make a 2 by 1 cut (because of the fold; when opening it, the result would be 2 by 2 squares). They also came up with a strategy to find the area that had been cut, by using the fact that many of the shapes were congruent and thus one only needed to find a few areas: “we’ll compute the area and multiply it by the number of shapes.” They suggested using grid paper to measure (we did not have enough rulers that day!). Each square on the grid was 1/4 in on its side. Each square in the “papel picado” took 4 squares on the grid (2 by 2); there were 40 squares total, so in terms of grid paper, 160 squares on the grid. How many square inches is this? This question poses a challenge, as the tendency is to divide by 4 (because of the 4 squares on the inch). This was in fact their initial reaction. We analyzed the situation and they realized that it took 16 squares on the grid to cover 1 square inch.

Our “papel picado” had circles and squares. This led to an exploration on how to find the area of a circle. The younger woman in the group kept referring to “Pi” and for example for the area of the circle (they were of diameter 2 squares on the grid), she said “6.28 (pause), ah no, that’s perimeter.” The other women in the group wanted to understand, they didn’t want to hear about all this “three point and whatever....” They did not want to use formulas that they did not know where they came from. In our next session we worked on “discovering Pi.” Then, this whole notion of Pi took a life of its own for this group of women. When I arrived at the next workshop, they had started an exploration on “applications of Pi to our daily lives.” They had collected data among their family members on the measurement around the head and their height and were working on finding a relationship among these measurements. They graphed the data, discussed it, looked for how it may relate to Pi. I contributed to the exploration (with my measurements; by bringing in graphing calculators and introducing the notion of regression), but they were leading the exploration. They explained to me (as they had been working on it outside the meetings of the “taller”) that they wanted to know if the height was equal to “Pi times the circumference of the head.” As I was trying to visualize what they were saying, the following dialogue took place:

E: So, Marta, what I do, to make it real, is our height is the circumference.
M: Ah, so what I’m calling circumference becomes
E and others: the diameter
M: the height, our height
Several voices: becomes a circle
E: the circumference of the head is the diameter [of that circle that represents the height]

What the dialogue does not capture is the ease and the confidence with which they explained to me how they were visualizing the situation. Developing this confidence in themselves as learners of mathematics may help address my dilemma of working with them as adult learners and as mothers, as the following excerpt from one of the mother's 15 year-old son shows:

Now that she [his mother] is attending these talleres she is learning in a different way, understanding the why of the formulas and where they come from and how they can be applied in her life; she shares it with the entire family and we all get involved in a mathematical reunion that is fun. We are all teachers and students at the same time, there is no difference and that there be much respect and confianza is most important. (Spring 2000 newsletter)

References

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