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ABSTRACT

This report offers guidance for those shaping policy and designing high school mathematics courses that prepare students to be lifelong users of mathematical ideas. We have reviewed programs designed to improve high school students' understanding of mathematics by incorporating instructional technology effectively into the curriculum. Using criteria backed by research on learning and instruction, we have selected exemplary programs from among those reviewed. We emphasize the teaching of mathematics as a process of inquiry in which students learn to solve complex problems and critique mathematical arguments. We look for uses of computer and communications technology that help students become lifelong mathematics learners who are prepared to meet all sorts of challenges from computing their income taxes to interpreting data on global warming to assessing the risk of a medical procedure. We discuss: (1) Contemporary views of teaching and learning mathematics; (2) Promising uses of technology in learning and instruction; (3) Evolving uses of technology in mathematics; (4) Criteria for selecting exemplary instructional programs; (5) Reviews of programs that meet our mandatory requirements, organized by category; (6) Recommendations of exemplary programs; and (7) next steps. (Author)

Teaching and Learning High School Mathematics Through Inquiry: Program Reviews and Recommendations

Marcia C. Linn, Ph.D.
Cathy Kessel, Ph.D.
James D. Slotta, Ph.D.

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Contents

I. Introduction

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We have reviewed programs designed to improve high school students' understanding of mathematics by incorporating instructional technology effectively into the curriculum. Using criteria backed by research on learning and instruction, we have selected exemplary programs from among those reviewed.

We emphasize the teaching of mathematics as a process of inquiry in which students learn to solve complex problems and critique mathematical arguments. We look for uses of computer and communications technology that help students become lifelong mathematics learners who are prepared to meet all sorts of challenges—from computing their income taxes to interpreting data on global warming to assessing the risk of a medical procedure.

We discuss:

- Contemporary views of teaching and learning mathematics
- Promising uses of technology in learning and instruction
- Evolving uses of technology in mathematics
- Criteria for selecting exemplary instructional programs
- Reviews of programs that meet our mandatory requirements, organized by category
- Recommendations of exemplary programs
- Next steps

Contemporary Views of Teaching and Learning Mathematics

Cognitive and social research clearly indicates that educational programs should challenge students to link, connect, and integrate their ideas and to learn in authentic contexts, taking into account their perception of real world problems. (Baumgartner & Reiser, 1998; Bransford, Brown and Cocking, 1999; diSessa, 2000; Linn & Hsi, 2000; Slotta & Linn, 2000). Too often our science and mathematics curricula present ideas in isolation, fail to encourage students to link their ideas, settle for the regurgitation of facts rather than the connection of ideas, and fail to prepare students to solve the personally relevant, everyday problems they will encounter throughout their lives. Instead, we should be educating students to be lifelong learners of science and mathematics by engaging in the process of inquiry that is the very nature of these fields.

The National Council of Teachers of Mathematics (NCTM) revolutionized thinking about mathematics instruction with the publication of standards in 1989 and a subsequent elaboration published in 2000. NCTM called for significant increases in approaches to teaching and learning that foster mathematical inquiry, such as problem solving, critiquing, and project work. In addition to its recommendation that all high school students receive four full years of mathematics instruction, NCTM has identified specific instructional strategies to achieve this sort of understanding. The recent elaboration of these standards has paid greater attention to the role of instructional technology. As a result, curricular materials implementing these standards are beginning to appear, and the NCTM maintains a Web site with materials that meet the new criteria and "e-examples" that illustrate the current standards, as we describe below.

The NCTM sees clear advantages for technology, commenting, "Students can learn more mathematics more deeply with the appropriate use of technology (Dunham and Dick 1994; Sheets 1993; Boers-van Oosterum 1990; Rojano 1996; Groves 1994). Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions. In mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics" (NCTM, 2000, p. 25).

For older students, the NCTM identifies some unique opportunities for technology to enhance inquiry in the area of mathematical argumentation. For example, the NCTM notes, "One of the most important challenges in mathematics teaching has to do with the roles of evidence and justification, especially in increasingly technological environments. Using dynamic geometry software, students can quickly generate and explore a range of geometric examples. If they have not learned the appropriate uses of proof and mathematical argumentation, they might argue that a conjecture must be valid simply because it worked in all the examples they tried. Despite the possibility of students' developing such a misconception, in a classroom in which students understand the roles of experimentation, conjecture, and proof, being able to generate and explore many examples can result in deeper and more-extended mathematical investigations than might otherwise be possible" (NCTM, 2000, p. 311).

At the same time that educators and others are becoming aware that technology can improve instruction, a large body of national and international test results has revealed weaknesses in students' preparation and in their understanding of both basic and advanced mathematical ideas. Large numbers of students have difficulty solving complex problems in mathematics and fail to understand the process of inquiry. American students fall further and further behind their international counterparts as they progress through the educational system. The *Third International Mathematics and Science Study* (TIMSS) reports that the most successful American students are below average among the more than 40 countries studied, when the mathematical accomplishments at the end of high school are compared (Schmidt et al., 1999). These results, as well as other research about how students learn, have motivated teachers, schools, policy analysts, and concerned citizens to call for reform of instruction.

Opportunities for Change

Because education is a complex interconnected system, change is difficult. Many attempts at innovation succumb to the pressures of the status quo, becoming indistinguishable from the programs they seek to replace. For example, an innovative approach to problem solving may evolve into just another electronic "drill and kill" program by the time it becomes available to educators as a product. In this report, we look for exemplary programs that help to transform the teaching of mathematics based on current research studies.

Current research offers guidelines for taking advantage of what we know about how people learn. A recent National Academy of Sciences committee has produced *How People Learn*, an extremely clear and powerful summary of research on student learning (Bransford, Brown, & Cocking, 1999) that emphasizes weaknesses in the current educational system, as well as research results pointing to reforms that could improve student outcomes.

Comparing the teaching of mathematics in Japan with instruction in the United States underscores some of the shortcomings of American education. In a book synthesizing classroom observations conducted as part of the TIMSS study, Stigler and Hiebert (1999) noted that more than 95% of American in-class work requires only that students apply an approach demonstrated by the teacher. Less than half of the class time in Japan is devoted to this sort of work. In contrast, Japanese students spend far more time on complex mathematics, and generally start a topic by inventing a strategy for solving novel problems. American students study less demanding topics, and generally start by reading about or listening to the correct way to approach the problem. Essentially, the Japanese curriculum offers students more opportunities for mathematical inquiry, while the American curriculum asks American students to spend most of their time learning established approaches.

Analysis of the textbooks used in American mathematics courses supports the classroom investigations from the TIMMS research. Project 2061 recently released an analysis of textbooks for algebra and reported that the books did not emphasize the linking, connecting, and considering of mathematics concepts that underscores successful learning (AAAS, 2000 April). Textbooks did not offer sufficient opportunity for students to understand mathematics so that they could apply the ideas to novel and complex problems. These analyses coincide with complaints from the workplace that American students lack preparation for jobs requiring mathematical thinking.

Mathematics instruction in the secondary school has often resulted in fewer opportunities for women and members of various cultural groups. In addition, while women earn higher course grades in mathematics, men earn higher scores on college entrance examinations (Kessel & Linn, 1996; Linn & Kessel, in press). Two factors contribute to this problem. First, stereotyped beliefs about who can succeed in mathematics deter some from persisting in their studies of math (Eccles & Jacobs, 1986; Fennema & Leder, 1990). Second, and more importantly, traditional classroom activities and dull assignments reduce motivation and stand in the way of lifelong mathematics learning (Seymour & Hewitt, 1994). Often, mathematics problems are placed within physics or engineering contexts that disadvantage students with less interest in these topics. The likelihood of everybody succeeding is enhanced when instruction allows students to select personally meaningful projects and activities-an option that technology can support and encourage. Successful equity programs improve instruction for all students and have the greatest impact on those who are at greatest risk for learning less. When the mathematics curriculum is personally relevant to individuals and emphasizes complex projects rather than skill-based exercises, more students can become lifelong learners because their motivation is increased.

The new inquiry-based view of mathematical understanding described here departs substantially from tradition and increases the potential for frustration, as learners attempt to change their approach to mathematics while simultaneously mastering new technologies. Teachers of mathematics as inquiry have the double challenge of designing new approaches to instruction and taking advantage of new technologies.

Determining when and how to incorporate technology into the curriculum has concerned curriculum designers and policy analysts alike. Every learner will encounter new technologies and will need to learn new applications. Preparing students to be fluent users of technology in their lives must take into account the activities that students are likely to undertake. Only careful planning can ensure that students learn relevant technologies and develop the capability to adapt to the variety of technologies that will be relevant to their career and life.

Access to technology in mathematics raises an issue currently referred to as the "digital divide." Although the digital divide between older and younger Americans is narrowing, the divide between wealthier and poorer individuals and schools remains large. Wealthier schools offer their students greater access to instructional and communications technology than poorer schools. Wealthier schools also have greater resources for purchasing software and for providing professional development. It is no surprise, then, that so many reports suggest that inquiry opportunities in mathematics are far more common in the wealthier schools.

Schools bring older and younger populations together in ways that might eventually help to resolve the problem of the "digital divide." School and community programs offer one of the best ways to reach those whose economic situation keeps them from becoming fluent with technology. However, people need more than a few hours per week of access to computers to become fluent in their use, and most school programs can offer no more exposure than this. Access to a computer for a weekend could give a student a six-month advantage over someone whose only access to computers is in a class. We must address this issue in order to prepare all learners—young and old, rich and poor—to be fluent users of technology and to succeed in the workplace.

In addition, simply providing access may not resolve differences between schools serving more or less wealthy students. Several recent surveys of computer use suggest that schools differ less on access than they do on the use of technology to support inquiry—the focus of this report. Schools serving wealthier students compared to those serving poorer students use technology for problem solving, design of innovations, and research to support a point of view far more frequently. This difference often reflects a view that students must master a set of skills before they explore more complex questions. Research in schools serving poorer students, however, calls this position into question and suggests that students will learn the necessary skills when challenged to solve complex problems (Bransford, Brown, & Cocking, 1999; Lee, 1992, 1995; Krajcik, Marx, Blumenfeld, Soloway, & Fishman, 2000).

Research in classrooms, reviews of the curriculum, studies of learning, and investigations of professional development all point to the need for widespread, cumulative reform of the educational enterprise (Athens & Cohn, 1999; Schmidt, McKnight, & Raizen, 1997; Schmidt et al., 1999; Stigler & Hiebert, 1999; U.S. Department of Education & National Center for Education Statistics, 1999). Recent research looking closely at how students learn in classrooms has revealed many obstacles that prevent students from developing their own mathematical understanding. Too often, problems are presented in isolation, students are not encouraged to look for patterns, and instruction emphasizes right answers, rather than critical appraisal of the methods used to solve problems. Technologically enhanced curricular materials can address these shortcomings in the traditional curriculum while also enabling students to think mathematically. Thus, infusing the curriculum with technology can increase the effectiveness of mathematics instruction.

New programs that make effective use of technology not only offer promising instructional opportunities, but also present challenges for professional development, since they depart from traditional practice in many ways.

Promising Uses of Technology in Learning and Instruction

Technology is not a panacea guaranteed to improve test scores. Neither is it an option reserved exclusively for students in computer science courses. Rather, current technological innovations offer education two major opportunities: to increase the role of inquiry in the classroom and to prepare students for a world in which technology will play a larger and larger role in their lives. The best gift we can give this generation of students is to offer them courses that enable them to become lifelong mathematics learners and to add new technological resources regularly to their repertoire.

There is widespread agreement that students will benefit from learning with and about technology in mathematics instruction. Nevertheless, effective incorporation of information technologies into the curriculum has been controversial, difficult, and demanding. Many have worried that students might become dependent on calculators, computers, or other computational aides, and thereby not learn the fundamental facts and concepts of mathematics. Others worry that students will spend inordinate amounts of time learning new technologies-time that could be better spent learning the mathematics itself. Finding ideal uses of technology in mathematics instruction remains an active research area, and the technology itself is a "moving target," as new projects emerge on a regular basis. The recommendations in this report capture current practices and research findings and will require regular revision as new tools and new research results become available.

The discipline of mathematics has always employed a vast array of technological tools for research, everyday activity, and education. A wonderful cartoon by Gary Larson portrays a caveman counting with toes and stones. Historically, mathematics has employed the reckoning board, the abacus, the slide rule, the calculator, the computer, and manipulative materials such as blocks or three-dimensional solids. And, the technologies for mathematics have constantly become more powerful and more useful in helping mathematicians solve equations, visualize complicated surfaces, or work with dynamic systems.

The National Research Council recently called for the teaching of "fluency with information technology" by incorporating appropriate technologies at each grade and course level (NRC, 1999). Using technology to promote understanding of mathematics and to prepare students for new technologies as they become available is also a strong recommendation of the National Council of Teachers of Mathematics for educational programs from kindergarten through high school. The NCTM has recently updated its standards to articulate the role of technology more clearly and to incorporate recent advances in technology (NCTM, 2000).

Adding technology to mathematics instruction has the danger of intensifying inequitable stereotypes and reinforcing the view that mathematics is a male domain. Used effectively, technology can instead connect mathematics to problems that interest individuals traditionally underrepresented in mathematics careers and also introduce disciplines such as statistics that connect to a broader array of courses of study. Some programs have demonstrated that technological tools can be a force for equity, because they permit more personalized and project-oriented undertakings in mathematics. Technology can increase success for all students by making teachers more effective, by reducing mundane, repetitive tasks, and by freeing teachers to spend their time working with individuals and small groups.

Until recently, the enthusiasm for technology in schools has manifested itself in a focus on new wiring and new equipment. Too often we have neglected the supports needed for schools and teachers to become fluent users of technology and effective designers of mathematics programs. Many programs for schools and teachers focus on learning skills such as word processing rather than on developing the capability to be lifelong learners with and about technology. The National Research Council's report on "Fluency with Information Technology" (or FITness) stressed that skills and concepts are not sufficient and that we need to prepare everyone to use technology in their intellectual work. For teachers and schools, this intellectual work includes incorporating technology effectively, testing and customizing new curricular materials, critiquing innovations, analyzing student use of technology, and regularly updating personal understanding of technologically enhanced materials. To develop these capabilities, teachers must have the time and the opportunity to explore and reflect on the role of technology in education.

Effective use of technology in instruction requires what has been called a continuous improvement model of instruction. In this approach, teachers regularly evaluate their teaching and revise their approach both to take advantage of new technologies and to tailor instruction to their students. This process involves examining how students use technology and tailoring programs to build on students' ideas. This is a constant challenge, as each new group of students comes along with more sophisticated understanding and, at times, more experience than the school leaders in some aspects of technology, such as searching for information, playing electronic games (often with complex animation), or participating in on-line discussions. Responding to students' expertise at various levels and directing their energy to mathematically powerful uses of technology may mean modifying or augmenting an educational program that employs technology.

Programs that teachers can customize easily have a greater potential for success than those where little customization is possible. One aspect of customization is identifying problem contexts that meet the needs of students and incorporating them into instruction. In particular, students benefit when they use mathematics for tasks that are important to them in their environment, such as determining a budget for the school play or evaluating the cost-effectiveness of a computer laboratory upgrade.

Technology presses us to rethink mathematics instruction for many reasons. Some of the more sophisticated computer applications, such as MathCAD, give students the opportunity to learn more sophisticated problem-solving strategies than they could without technological support. In addition, technology can engage students more deeply in mathematical inquiry, which departs from the traditional notion of mathematics instruction and challenges teachers and schools to redesign their whole curriculum. New standards for mathematics understanding include technological tools as part of the practice of mathematics rather than as optional activities. This is one more reason for schools to rethink mathematics instruction. Schools need to consider technology in the context of all the course offerings, balance the addition of new technologies with the benefits of reusing programs in new courses, and ensure that students learn lifelong fluency with technology in mathematics.

It is not enough simply to introduce these promising curricular materials without also finding solutions to logistical problems and implementing powerful programs for professional development. Indeed, it has been difficult to evaluate some innovations because effective environments for teaching and powerful professional development programs were not in place. Rapidly changing technologies and the availability of wireless networks and portable computers for classroom not only use provide new opportunities but also point to the need for continuously revisiting logistical arrangements for using technology in instruction.

Currently, wireless networks, combined with portable computers, enable schools to place computers on carts to equip classrooms with student computers. Schools can then share these resources among teachers. This advance reduces the need for computer laboratories and enables teachers to more seamlessly incorporate technology into their everyday practice.

Professional development programs are emerging that prepare teachers to truly teach with technology rather than to use computers for personal productivity. Teachers and students both need to understand information technology and its benefits for learning. Teachers, especially, need what is called pedagogical content knowledge. By pedagogical content knowledge we refer to knowledge about how students learn from materials infused with technology. For example, to teach functions with a graphing program, teachers need to understand the potential confusions their students might develop that could interfere with success. Studies of students interpreting graphs reveal that some students see graphs as a picture rather than a relationship (Leinhardt & Zaslovsky, 1990). Understanding and anticipating these confusions prepares teachers to be effective in supporting their students as they use technology.

The continuous improvement model of professional development described above enables teachers to refine their pedagogical content knowledge as they teach a topic for the second, third, and more times. Ideally, teachers will have an opportunity to experiment with innovative programs that use technology to help them reflect on their practice and continuously improve their instruction. In situations where powerful technological programs have been combined with opportunities for continual improvement, the combination has yielded substantial benefits for learning and instruction (Linn & Hsi, 2000). At present, however, there are many obstacles to creating these opportunities, and, for many teachers, the promise of technology remains just a promise.

Evolving Uses of Technology in Mathematics

Technology offers challenges and opportunities to those designing programs for the learning and teaching of high school mathematics. New technologies are changing the nature of mathematics research, offering new approaches for learning, changing the definition of basic skills, and enhancing opportunities for instruction. Judicious incorporation of technology into high school instruction can prepare all students for lifelong learning of mathematics and enable them to use technology throughout their lives to improve their success with mathematics.

Depending on one's field of interest, today, one uses very different technologies for mathematical tasks. In mathematics, experts use a vast array of technologies, including four-function calculators, graphing calculators, spreadsheets, three-dimensional modeling, and symbolic algebra programs. Symbolic algebra programs such as Mathematica (used in *Calculus & Mathematica*, which is reviewed in this document), Maple (used with *Calculus, Concepts, Computers, and Cooperative Learning*, reviewed in this document), and MathCAD are the continuation of the calculator and the spreadsheet for advanced mathematics. Because resources such as these change rapidly and are demanding to learn, schools are well advised to incorporate them in a way that enables students to reuse the same application from course to course.

Ideally instruction would feature software useful for most learners as well as software tailored to students in specific courses. Since students will need to budget and plan their own financial activities as adults, they should encounter instruction that provides this foundation. Students planning careers in science or mathematics will use symbolic algebra and three-dimensional modeling, so, as students take more advanced courses, they would benefit from learning more complex software.

The realities of life today suggest the need to revise high school curricula to incorporate topics not heretofore considered to be part of the high school repertoire, such as statistics. Indeed, the SAT program is considering adding statistics items to the college entrance examination. New statistical and discrete mathematics courses are making successful use of technology (see Fathom software as an example), but these topics are not yet incorporated consistently into the high school curriculum. Statistics has tremendous potential for making the curriculum more relevant and accessible to students. Incorporating statistics into the pre-college curriculum and creating promising technologies to support instruction requires serious revision of current practices as well as refinement of existing technological tools.

Technologies for visualization and modeling have transformed some fields of science and mathematics and seem promising for high school instruction. Recent research demonstrates that animation of complex relationships, such as graphs of change over time, connections between components of a geometric construction, and location of a local minimum on a three-dimensional surface can help students learn these difficult topics (Confrey & Mahoney, 1996; Linn & Hsi, 2000; Schwartz & Yerushalmy, 1993). At the same time considerable research demonstrates that all animations are not successful. Often animations fail because they are too complex or too difficult to understand (e.g., Ainsworth, Bibby, & Wood, 1998).

Technologies can provide support for students by offering help or guidance for complex investigations. Carnegie Learning offers courses in algebra and geometry that provide substantial guidance for students completing the traditional curriculum. Initial studies of efforts to provide guidance so that students can carry out complex projects in open-ended modeling environments mean that more learners can use these environments (e.g., Begel, 1998, 1999; Koella & Hofer, in press). Learning environments such as the Web-based Integrated Science Environment (WISE) provide technology that helps students organize their investigations and determine next steps. This frees teachers to spend more time in one-on-one interactions with students as the technology helps support the rest of the class. Such technologies offer great promise, since teachers often avoid projects unless they have additional classroom support.

Criteria for Selecting Exemplary Instructional Programs

Some innovative and effective programs that incorporate technology go a long way toward achieving the kind of understanding of mathematical concepts that is called for by the National Council of Teachers of Mathematics (NCTM) *Standards*. However, technology is not in and of itself a panacea. Considerable research and investigation in classrooms are needed to fully realize the promise of technology for mathematics instruction.

Technology is a relatively new player in the educational enterprise, and there are both successes and failures among the experiments that have been undertaken. In this report, we identify promising directions for technology use and link those to recommendations for student understanding.

The programs we selected for review in this report met five mandatory criteria and five pedagogical criteria.

Mandatory Criteria

The mandatory criteria helped us select programs that could meet the pedagogical criteria. We define mandatory criteria below.

Engage students in mathematical inquiry. We selected programs that stimulated mathematical inquiry by requiring students to investigate mathematical problems and actively build understanding of mathematical ideas. Inquiry instruction engages students in complex, sustained, reflective reasoning about a problem that involves mathematics. Inquiry activities require students to come up with solutions, test their ideas, revise their solutions, select among methods for solving problems, and criticize solutions generated by others. Often inquiry activities involve refining the definition of a problem or require students to generate problems on their own. Commonly inquiry activities take place over days and weeks and involve critical review by peers followed by revision of solutions.

Focus on mathematics content. We selected programs that taught mathematics content appropriate to the grade level of the students as reflected in the NCTM *Standards*. We defined mathematics content broadly to include concepts such as risk assessment and estimation as well as probability and statistics. We sought programs where mathematics was used in solving problems relevant to the lives of students to ensure that students would experience the complexity of applying mathematical ideas in practice.

Be appropriate for high school students. We sought programs appropriate for high school students. In general, we placed programs primarily concerned with middle school mathematics topics in our report on K-8 science and mathematics. Some programs offer distinct inquiry activities for middle school and high school and are included in both reports.

Use information technology for inquiry teaching and learning. We sought programs that incorporate technology into instruction rather than "bolt" it on as an adjunct. Programs that mentioned optional use of calculators or computers were not reviewed unless they included specific, novel uses for the technology.

Be supported by research. We sought programs with published research on their effectiveness or programs that were designed based on research findings. We included both research reports and testimonials from users of the materials. Although we found the testimonials challenging to evaluate, they helped us understand some of the obstacles and opportunities teachers encounter in using the programs. We recognize that curriculum is only a part of the success of mathematics instruction; teachers have a substantial impact on program success, and their reflections help us understand especially the impact of school context on use of innovative technology as well as on innovative curricula (Stigler & Hiebert, 1999).

Pedagogical Criteria

Criteria that implement the pedagogical research described above were used to select the most promising programs. Taken together, these criteria describe an educational program that encourages the linked, connected, and generative understanding called for by the NCTM standards and described in *How People Learn*. A large body of research shows the benefit of these criteria (e.g., Linn & Hsi, 2000; Ma, 1999; Stigler & Hiebert, 1999).

We sought exemplary implementations of these criteria and report on how the programs selected implement one or more of these ideas. Ideally a full course curriculum would use all of these approaches to reach every student. An individual activity might use one or more of these approaches to achieve overall pedagogical success.

Be personally relevant to students (academically, practically and intellectually). We selected programs that connect to the lives of students and increase the likelihood of lifelong learning. Research shows that when educational programs connect to problems that interest students they are more motivated and also revisit the ideas in subsequent personal or educational problem-solving situations (Bransford, Brown, & Cocking, 1999; Linn & Hsi, 2000; NRC, 1999). The NCTM Illuminations series includes examples from a broad range of disciplines and from situations students are likely to encounter.

Encourage connections among the many disciplines of mathematics and science. We selected programs that make explicit connections to other disciplines, thereby increasing the relevance of mathematics for students. The NCTM (2000) emphasizes the need for connections to other disciplines, since most often students will use the mathematics they learn in other disciplines rather than in the study of mathematics. Statistics, represented in the Fathom software, is an example of a program that makes natural connections to a broad range of disciplines. The Interactive Mathematics Program is another example of a program that connects across disciplines.

Use technology to enhance understanding. We sought uses of technology that add value to instruction. In some cases (Geometer's Sketchpad, for example), technology makes mathematical concepts or ideas more visible. In other cases, technology increases opportunities for teachers or students to explain concepts more effectively (e.g., Mathematics, Modeling our World).

Meet needs of diverse populations and enable all students to achieve. We sought programs that could be customized and personalized by teachers in order to meet the needs of all the students in their classes. Research shows that students benefit when teachers can tailor instruction to the prior and subsequent lessons in the curriculum and can include varied activities to connect to the best learning strategies for all students (Bransford, Brown, & Cocking, 1999). Programs address this criterion in various ways. Some make sure that all students succeed by carefully monitoring performance, as is the case for Carnegie Learning, which also offers multiple representations to help each student participate effectively. Other projects, like StarLOGO, offer projects that students can tailor to their interests and resources.

Support peer learning. We looked for programs that engage students in collaborative problem solving or in critical review of the work of others. Collaborative opportunities give students the chance to engage in mathematical discourse and to hear the ideas of mathematics in the vocabulary of their peers. Working with peers can also motivate students to persist in activities that they might otherwise abandon, because they feel commitment to the group. Collaboration can also reinforce stereotypes and therefore needs to be monitored carefully (Linn & Hsi, 2000). Programs such as Calculus, Concepts, Computers, and Cooperative Learning use a collaborative approach, as do the Interactive Mathematics Program, Core-Plus, and Mathematics, Modeling Our World.

II. Caveats

We relied on the effectiveness information that was available in published materials. We encouraged testimonials from teachers who were asked to send us messages by the program developers. We also contacted teachers in local districts and asked for recommendations. We would have preferred to have more extensive research about nearly every program, but, with the rapid change of technology, this research is generally not available.

We sought to review all the major projects available for high school mathematics instruction. In many cases materials mentioned at Web sites or compiled by school districts were difficult to locate. We did not review programs if the software did not work, if materials were not supplied when ordered, or if materials were unavailable to us.

Several categories of materials were considered for review but not included. We did not include programs designed primarily for the home market, since they are typically more geared to entertaining than to encouraging inquiry. We also left games off the list, although some do require mathematical reasoning to succeed. Neither of these types of programs connects easily to the curriculum, and both are easily converted into activities that do not use inquiry.

We find sites that offer problems of the week promising but not sufficiently infused with technology at the current time. These sites offer challenging problems and some problems that could be enhanced with the use of technology. However, incorporating technology is left exclusively to the teacher or student.

We cannot guarantee the accuracy of the mathematics in the software or materials that we reviewed, although we did not encounter any obvious errors. In general, software designers do not make the algorithms they use for computation explicit, so accuracy cannot be assessed. In some cases there are disputes in the field-such as concerning the algorithms that should be used in symbolic algebra programs.

We relied on published accounts of professional development programs and mechanisms for teacher support. We cannot guarantee the effectiveness or quality of these programs, since we have not participated in them.

Many Internet-based programs include advertising on sites that students use. We encourage teachers and schools to develop policies about advertising and to review materials carefully to determine the extent and nature of advertising. Finally, information about publishers may change due to business mergers and acquisitions within the publishing industry. This may also affect the professional development resources offered in conjunction with particular projects.

III. Program Reviews

Program reviews are based on reports in the published literature. To the extent that information was available, we include the following information about each project:

Contact information: includes Web sites, mailing addresses, and project representatives, if available.

Description: describes the age group (e.g., grade) of students, the topics in the curriculum, the basic methodology of the project, the duration of the instruction, and the nature of the units. This section also summarizes the project's theoretical framework for learning and instruction, if any, and reviews the project's historical underpinnings, if relevant, the assessment strategies, if available, and classroom management issues, if described.

Effectiveness: reviews research that demonstrates the program's benefits, summarizing published results, including teacher reactions and student reactions, focusing on rigorous studies of learning. Effectiveness may be measured in various ways, from in-depth interviews to large-scale studies of student achievement.

Professional Development: reviews the approaches associated with the program. Does the project offer new teacher-users training to help them use the program effectively. Do teacher-training programs accompany the project? Is the training at extra cost? Are there ongoing supports for teachers who are adopting the project?

Implementation: suggests the conditions under which the program should be implemented if it is to be effective, including technological requirements in and out of school, level of technical support required, and necessary professional development.

Cost and Technical Requirements: estimates probable program costs and describes the technological requirements for schools (e.g., Internet connections, modern computers, CBLs, VCRs), students (e.g., good typing or computer skills), and teachers (e.g., Web-savvy).

Program Categories

In conducting our reviews, we first identified promising programs through an extensive survey of teachers and technology specialists, review of existing research and research synthesis, examination of recommendations from professional organizations including the American Association for the Advancement of Science, review of winners of various software awards, reviews by government expert panels, and extensive Web searches. These initial surveys of the field provided us with a large set of candidate programs, from which we selected the projects according to the criteria outlined above. As we conducted our reviews, we identified categories consistent with the enGauge framework and represented in available programs.

To organize our recommendations of exemplary programs, below, we defined the following categories: (1) Comprehensive Courses, (2) Project-based Curricula and Collaborative Projects, (3) Activities and Skill Acquisition, (4) Computational and Representational Tools, and (5) Resource Web Sites. These categories correspond to specific locations in the enGauge framework as indicated with the descriptions below. For each of these categories, we provide a brief introduction, make connections to the framework, list the projects we reviewed in table form, and present our reviews of any exemplary projects in the category.

1. Comprehensive Courses

A few groups have created complete courses for high school mathematics that take advantage of technology to promote inquiry-based understanding. Many more courses mention the option of using calculators or spreadsheets or other technology tools but fail to integrate these tools in a meaningful way. We selected comprehensive curricula that made effective and consistent use of technology.

This review category includes programs that provide all the written materials, technology, and supplies needed to support student activities and to involve students actively in a complete, full-length mathematics course.

Traditional mathematics curricula often neglect authentic inquiry. Textbooks, in particular, have neither fostered questioning from students nor encouraged sustained problem solving (Schmidt, McKnight, & Raizen, 1997). Most texts offer concise explanations, examples drawn from straightforward, often artificial contexts, and opportunities to solve set problems.

Creators of comprehensive courses that incorporate technology need to redesign the curriculum to reflect the strengths and limitations of mathematical software. Students have less need to learn the mechanics of solving problems and therefore can spend more time on the complexities of designing solutions.

Designers of comprehensive courses delivered using technology also have the opportunity to offer students feedback about their solutions. Many aspects of mathematical problem solving, such as setting up equations for a word problem, lend themselves to computer-generated feedback.

The programs in the table below offer course content for algebra, calculus, and geometry, with varying levels of support for teachers and diverse approaches to curriculum. In addition, all make use of technology to some extent. Programs, like Virtual High School, extend the traditional curriculum, providing opportunities for teachers and students to participate in courses not offered at their own school.

Comprehensive courses fit in the complex category of the enGauge framework. In addition, activities range from medium to high, depending on the choice of problems, for authenticity and constructivism.

We identified twelve courses that met our mandatory criteria and two that we deemed exemplary. The projects in this category appear in the chart below. Links to exemplary project reviews are available below.

Comprehensive Courses Reviewed	
Project/Review Link	URL
Calculus (APEX)	http://www.apex.netu.com/
Calculus and Mathematica	http://socrates.math.ohio-state.edu/
Calculus, Concepts, Computers, and Cooperative Learning	http://www.math.purdue.edu/~ccc/
Calculus Consortium	http://jws-edcv.wiley.com/college/cch
Carnegie Learning: Algebra and Geometry Tutors * Exemplary *	http://www.carnegielearning.com
College Preparatory Mathematics	http://www.cpm.org
Connected Geometry Project	http://www.edc.org/LTT/ConnGeo/index.html
Core-Plus Project	http://www.wmich.edu/cpmp
Interactive Mathematics Program * Exemplary *	http://www.mathimp.org
Mathematics: Modeling Our World	http://www.comap.com/highschool/projects/mmow/introduction.htm
University of Chicago School Mathematics Project	http://www.phschool.com/product_information/math/secondary/ucsmp.html
Virtual High School	http://vhs.concord.org/home.htm

2. Project-Based Curricula and Collaborative Projects

Project-based materials implement the NRC goal of fluency with information technology as well as the mandatory and exemplary criteria of this report in the most complete way. Projects, by their nature, focus on a compelling, relevant question and require several iterations to be completed. In addition, developers have often incorporated powerful technologies first used by experts into these materials. Expert tools, when tailored to students, have great potential to encourage the same sort of inquiry we value in scientists. Projects also generally require groups of students to collaborate and provide multiple points of entry for diverse students. Among the projects meeting our mandatory criteria, we found variation in the uses of technology and in the authenticity of the problems.

Projects fit in the complex and constructivist category of the enGauge framework. In addition, activities range from medium to high, depending on the choice of problems, for authenticity.

We identified three project-based curricula that met our mandatory criteria and one that we deemed exemplary. The projects in this category appear in the chart below. A link to the exemplary project review is available below.

Project-Based Curricula and Collaborative Projects Reviewed	
Project/Review Link	URL
IMMEX: Interactive Multimedia Exercises * Exemplary *	http://www.immex.ucla.edu/
Quantitative Literacy Series	http://www.pearsonlearning.com/dsp-publications/full_datadriven.html
University of Toronto - Mathematics Network	http://www.math.toronto.edu/awilk/MathNet/

3. Activities and Skill Acquisition

Many Internet and CD-ROM activities have been designed for high school mathematics topics. Such activities offer stand-alone opportunities for students to explore a topic such as random numbers and benefit from dynamic models, unique representations, or compelling questions. Several organizations including NCTM have reviewed the broad array of materials and selected activities with promise. To ensure that students using these activities engage in inquiry, teachers typically need to take an active role in focusing the activity on investigation and problem solving.

Typically, these activities come with all the materials needed and require no supplemental software such as a word processor or spreadsheet. Some are suited to self-paced and self-guided use (e.g., from home or after school). There is a great diversity in this category, even within the titles selected by organizations such as NCTM. Some projects feature collaboration, manipulative materials, and construction of artifacts. Others are quite simple, and may provide students only with a skills practice framework.

The success of these activities depends on teachers taking the initiative to emphasize inquiry. Unfortunately, even the projects we review here rarely meet our criteria for professional development. Generally, these projects consist of a high quality set of materials, very good intentions relating to student inquiry, and varying levels of support for students and teachers.

Activities and skill acquisition projects fit in the didactic category of the enGauge framework. In addition, activities range from low to medium, depending on the use in the classroom, for authenticity and complexity.

We identified three compendia of activities that met our mandatory criteria and one set of reviewed activities from the NCTM that we deemed exemplary. The projects in this category appear in the chart below. A link to the review of the exemplary project is provided.

Activities and Skill Acquisition Projects Reviewed	
Project	
MathXpert	http://www.mathpert.com
NCTM e-Examples	http://standards.nctm.org/document/eexamples/
NCTM Illuminations i-Math * Exemplary *	http://illuminations.nctm.org/imath/

4. Computational and Representational Tools

Computational and representational tools make it easy for students to perform "what if" experiments and to concentrate on the meaning of, for example, statistics. Learners typically manipulate objects in their simulations to test performance under different conditions. For example, with Geometer's Sketchpad, students can make a construction and test it by varying an angle or changing the size of an object. Technology projects in this category can help manipulate algebraic expressions, compute statistics, or explore representations of mathematical objects. For example, students may explore graphs of $y = ax^2$ for various values of a, or triangles with a given angle. Environments may also link different representations (e.g., functions and graphs, plane figures and their areas, or scatter plots and lines of best fit). These environments allow students to quickly examine a collection of cases and to generate conjectures, which may then be justified with a mathematical argument, disproved, or investigated further.

Students can manipulate models with a computer to make conjectures, test their ideas, and explore the rules underlying a construction. In addition to manipulating existing models, student can modify models or create their own models.

The environments described here were not developed with a particular curriculum in mind. Some are designed for use by professionals in mathematics or mathematics-based fields. Others are designed to aid student learning in particular ways. In both cases, it is important to consider the mathematical understanding that the technology may foster with regard to existing curricula and mathematics standards. Many of the tools included here are also incorporated into comprehensive curricula or projects. Many center around a particular simulation or representational environment and address very specific content learning goals. In some cases, the developers of the technology believe strongly in providing such tools to teachers, who then integrate the use of the technology into their curriculum, as is the case for Function Probe and Fathom. In other cases, the developers provide a curriculum that puts the tool in context and supports the tool. Computational and representational tools fit in the complex category of the enGauge framework. In addition, activities designed by developers or teachers can range from medium to high, depending on the choice of problems, for authenticity and constructivism.

We identified seven tools that met our mandatory criteria and three that we deemed exemplary. The projects in this category appear in the chart below. Links to the reviews of the exemplary projects are provided below.

Computational and Representational Tools Reviewed	
Project	URLs
Fathom * Exemplary *	http://www.keypress.com/catalog/products/software/Prod_Fathom.html
Function Probe	http://questmsm.home.texas.net/Products/FProbe/FPWelcome.html
Geometer's Sketchpad * Exemplary *	http://www.keypress.com/catalog/products/software/Prod_GSP.html
KaleidoMania!	http://www.keypress.com/catalog/products/software/Prod_KaleidoMania.html
Maple	http://www.maplesoft.com/products/Student/student.html
Mathematica	http://www.wolfram.com/products/mathematica/
SimCalc * Exemplary *	http://www.simcalc.com

5. Resource Web Sites (links to reviewed sites and online resources)

As innovative uses of technology to support mathematics learning proliferate, groups are beginning to review, select, and promote promising activities, projects, software, and units, much as groups have rated filmstrips in the past. While these projects do not necessarily make strong use of technology in mathematics, they nevertheless make a significant contribution to education in mathematics. In this category, we review Web sites that are carefully monitored, provide access to accurate information, and offer useful resources, such as problems of the week.

Resource Web sites do not fit into a single place in the enGauge framework. Rather, they cut across all dimensions of the framework and apply to the entire range of use of students. The resources provided by these Web Sites can thus vary dramatically in their effectiveness depending on teacher usage. The Web resources most likely to meet our criteria will be above average on the dimensions of complexity, authenticity, and constructivism.

We identified three resource Web sites that met our mandatory criteria and two that we deemed exemplary. The projects in this category appear in the chart below. Links to reviews of the exemplary projects are available.

Resource Web Sites Reviewed	
Project	URLs
Math Forum * Exemplary *	http://forum.swarthmore.edu/
NCTM Web Resources * Exemplary *	http://illuminations.nctm.org/webresources/index.html
PBS Teacher Source	http://www.pbs.org/teachersource/recommended/rec_links_math.shtm

IV. Next steps

The programs we have reviewed represent first steps toward incorporating technology effectively into the High School mathematics curriculum. The research supporting them is similarly preliminary. A vigorous, intellectually rich, and mutually supportive set of research programs is essential for continuing to improve the mathematics curriculum. We need research studies that investigate how programs evolve in classroom settings and that incorporate lessons learned from these investigations. We need to ensure that research findings are communicated widely and that the results are reported in a manner that can benefit other investigators as well as users of the programs. Too often reports are so abstract that readers cannot imagine the characteristics of the learning environment.

To support a process of continuous improvement, we need professional development opportunities that allow groups to test and revise materials jointly ¾customizing the materials to local conditions. Some projects are very conscious of professional development: they suggest implementation plans that involve all the stakeholders in a district and include plans for having experienced teachers help others. Good examples are provided by Core-Plus and the Interactive Mathematics Program.

We also need support for curriculum materials development projects that allow time for testing, review, and revision. Several of the NSF-funded projects selected as exemplary have benefited from this kind of support. This point is made clearly by Schoenfeld in *Looking Toward the 21st Century: Challenges of Educational Theory and Practice* (October 1999, p. 11):

> *We must recognize that the kinds of combined research and development efforts suggested here will require significant amounts of time and money—and that to date we have not been willing to make such investments as a nation. Early attempts, such as the science and mathematics curriculum projects initiated following the Soviet Union's successful launching of Sputnik, tended not to involve the educational research community. More recent attempts have done so, but they have not been of adequate scale. For example, I applaud the National Science Foundation for taking the initiative, earlier in this decade, to fund some major curriculum projects. Some of those projects had solid connections with the research community. But—and this is not a criticism of NSF, but of the constraints that shaped the approach they took—it doesn't take a very close look to realize that the conditions under which the curriculum developers were compelled to work were next-to-impossible. One project, for example, had a five-year grant to develop five years' worth of curriculum. Six months into the project, the project leader said she felt that she was already very far behind schedule! That kind of compressed time scale allows time for little more than the following four-step development cycle:*

- *Write instructional materials on the basis of what you know*
- *Field test the materials you've written*
- *Revise once on the basis of what you've seen*
- *Pray. That just won't do.*

We have to make a long-term investment in building instructional materials, and learning from what we do as we do it.

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