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Teaching Fractions: New Methods, New Resources. ERIC Digest.

The teaching of fractions continues to hold the attention of mathematics teachers and education researchers worldwide. In what order should various representations be introduced? Should multiple representations be introduced early, or one representation pursued in depth once? Does it matter if fractions are introduced as counting or as measurement? What is the relative importance of procedural, factual, and conceptual knowledge in success with fractions? These and other questions remain debated in the literature.

Following an overview of recent research on teaching and learning fractions, suggestions are offered for practice, for locating resources having direct application in the classroom, and for further reading in the research literature.

STUDENT CONCEPTIONS

The domain of skill and knowledge referred to as "fractions" or "rational numbers" has been parsed in various ways by researchers in recent years. Tzur (1999) sees children's initial reorganization of fraction conceptions as falling into three strands: (a) equidivision of wholes into parts, (b) recursive partitioning of parts (splitting), and (c) reconstruction of the unit (i. e. the whole). Recognizing this division, he suggests that teachers consider one of these strands at a time in teaching rational numbers.
Taking a psychological approach Moss and Case (1999) suggest that for whole numbers children have two natural schema, one for verbal counting and the other for global quantity comparison. In the realm of rational numbers they also see children as having two natural schema: one global structure for proportional evaluation and one numerical structure for splitting/doubling. They propose, then, as a plan for learning that teachers need to refine and extend naturally occurring processes.

Hunting's (1999) study of five-year-old children focused on early conceptions of fractional quantities. He suggested that there is considerable evidence to support the idea of "one half" as being well established in children's mathematical schema at an early age. He argues that this and other knowledge about subdivisions of quantities forming what he calls "prefraction knowledge" (p.80) can be drawn upon to help students develop more formal notions of fractions from a very early age. Similarly, based on her successful experience of teaching addition and subtraction of fractions and looking for a way to teach multiplication of fractions, Mack (1998) stresses the importance of drawing on students' informal knowledge. She used equal sharing situations in which parts of a part can be used to develop a basis for understanding multiplication of fractions; e.g. sharing half a pizza equally among three children results in each child getting one half of one third. Mack noted that students did not think of taking a part of part in terms of multiplication but that their strong experience with the concept could be developed later.

Taking an information-processing approach (Hecht, 1998) divides knowledge about rational numbers into three strands: procedural knowledge, factual knowledge, and

conceptual knowledge. Hecht's study isolated the contribution of these types of knowledge to children's competencies in working with fractions. He made two major conclusions: (a) conceptual knowledge and procedural knowledge uniquely explained variability in fraction computation solving and fraction word problem set up accuracy, and (b) conceptual knowledge uniquely explained individual differences in fraction estimation skills. The latter conclusion supports the general consensus in current research that a holistic approach to teaching of fractions is necessary with recommendations for a move away from attainment of individual tasks and towards a development of global cognitive skills.

MISTAKES TEACHERS MAKE

Based on previous research Moss and Case (1999) identified four major problems with current teaching methods in the area of fractions. The first is a syntactic rather than a semantic emphasis, which is to say that researchers have identified that teachers often emphasize technical procedures in doing fraction arithmetic at the expense of developing a strong sense in children of the meaning of rational numbers. The second problem identified is that teachers often take an adult-centered rather than a child-centered approach, emphasizing fully formed adult conceptions of rational numbers. As a result teachers often do not take advantage of students "prefractional knowledge" and their informal knowledge about fractions thus denying children a spontaneous "in" to their formal study of fractions. A third issue is the problem of teachers using representations in which rational and whole numbers are easily confused e.g. students count the number of shaded parts of a figure and the total number of parts so that each part is regarded as an independent entity or amount (Kieran cited in Moss & Case (1999). Finally, researchers have identified considerable problems in use of notation that can act as a hindrance to student development. These problems center around teachers' perceptions that the notation used for rational numbers is transparent while this has been shown not to be the case, especially with regard to decimal fractions (Hiebert, cited in Moss & Case (1999)). Tirosh (2000) conducted a study on teacher knowledge in teaching of fractions and concluded that teachers needed to pay considerably more to analysis of student errors.

NEW TEACHING APPROACHES

Moss and Case identified three different proposals on approaches to teaching of fractions that address the above mentioned problems in various ways and then propose a new curricular approach which they tested themselves in a study involving fifth and sixth grade students. The first of the older studies conducted by Hiebert and Warne (as cited in Moss & Case (1999)) was judged to have addressed primarily the syntactic and notational problems mentioned above and placed a great deal of emphasis on the use of base 10 blocks. In the second study Kieran (as cited in Moss & Case (1999)) was seen to address the syntactic and representational issues and, among other innovations, used paper folding to represent fractions in preference to pie charts. The third of the studies, conducted by Streefland (as cited in Moss & Case (1999))

attempted to address all four concerns and was based on using real-life situations to develop children's understanding of rational numbers.
Moss and Case's (1999) own approach was designed to address all four of the identified problems and was characterized by several qualities distinguishing it from previous approaches. They started with beakers filled with various levels of water and asked students to label beakers from 1 to 100 based on their fullness or emptiness. They emphasized two main strategies: halving (100 -> 50 -> 25) and composition (50 + 25 =75) in determining appropriate levels. Refining this approach they developed the notion of two place decimals with five full beakers and one three-quarter full beaker making 5.75 beakers. Four place decimals were then introduced with 5.2525 (initially, spontaneously denoted as 5.25.25 by the students) characterized as lying one quarter of the way between 5.25 and 5.26. Students eventually went on to work on exercises where fractions, decimals and percentages were used interchangeably. Moss and Case found that this approach produced deeper, more proportionally based, understanding of rational numbers. They see their approach as having four distinctive advantages over traditional approaches: (a) a greater emphasis on meaning (semantics) over procedures, (b) a greater emphasis on the proportional nature of fractions highlighting differences between the integers and the rational numbers, (c) a greater emphasis on children's natural ways of solving problems, and (d) use of alternative forms of visual representation as a mediator between proportional quantities and numerical representations (i. e. an alternative to the use of pie charts).

WORLD WIDE WEB RESOURCES

* "Visual Fractions"

This World Wide Web (WWW) site is designed to help users visualize fractions and the operations that can be performed on them. There are instructions and problems to work through for the operations of addition, subtraction, multiplication, and division, first using fractions and then working with mixed numbers. Number lines are used to picture the addition and subtraction problems while an area grid model is used to illustrate multiplication and division problems.

http://www.visualfractions.com/

"The Sounds of Fractions: Math in Music"

"Overview - You've probably heard that math and music are related, but you may not

have ever heard how or why. Objective: - Compare math and music to see how mathematical concepts of ratio, proportion, common denominator, frequency, and amplitude connect with musical elements such as time signature, pitch, tone, and rhythm"

http://www.highwired.com/Classroom/Project/0,2069,23713-68258,00.h tml

"No Matter What Shape Your Fractions Are In"

"Description: These activities are designed to cause students to think; they are not algorithmic. They do not say, To add fractions, do step one, step two, step three. Students will explore geometric models of fractions and discover relations among them. Appropriate Grades: 3rd - 6th, maybe. But precocious kindergarteners could do some of it, and middle schoolers needing another look at fractions could appreciate it as well. 'Drawing Fun Fractions' would be good for most middle school students."

http://math.rice.edu/~lanius/Patterns/

"Flashcards"

This web site was developed to help students improve their math skills interactively. Students can test their mathematics skills with Flashcards which give students practice problems to try and then gives them feedback on their answers. Students can also create and print your own set of flashcards online.

http://www.aplusmath.com/Flashcards/fractions-mult.html

FRACTIONS IN THE ERIC DATABASE

There are over 1,000 records in the ERIC database pertaining to fractions. The best way to locate those records is to search the database using one or both of the following ERIC Descriptors: "fractions" or "decimal fractions". You can narrow your search by combining these two Descriptors with others, such as teaching "methods", "educational strategies", "instructional materials", "research", "literature reviews", "mathematics

instruction", "mathematics materials", "mathematics curriculum", or "mathematics skills". You can further narrow your search by using education level Descriptors, such as "elementary education", "middle schools", "intermediate grades", or "junior high schools", or individual grade levels. You can search the database on the Web at http://ericir.syr.edu/Eric/adv_search.shtml.

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